Development of transverse flow for small and large systems in conformal kinetic theory

Clemens Werthmann

in Collaboration with Sören Schlichting and Victor Ambrus

based on PRD 105 (2022) 014031 and WiP

Bielefeld University

CRC-TR 211
Strong-interaction matter under extreme conditions

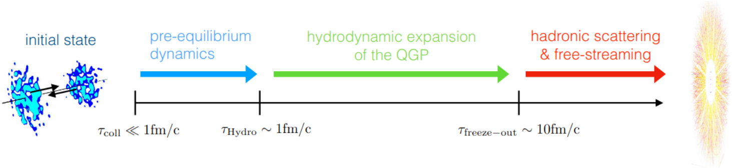
DFG

UNIVERSITÄT BIELEFELD

HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

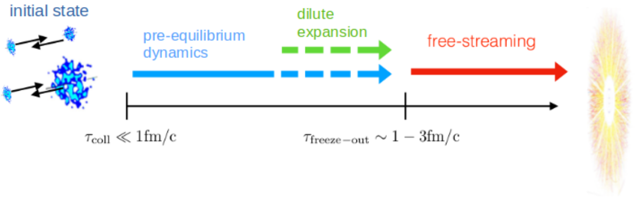
Standard model of heavy ion collisions

Spacetime evolution dominated by hydrodynamic phase



- early stage requires non-equilibrium description, but system quickly equilibrates
- strongly interacting QGP leaves imprints of thermalization and collectivity in final state observables
- transport description after hadronization

Very dilute, hydrodynamics not necessarily applicable



- still collective behaviour is observed!
- collectivity can also be explained in kinetic theory, a microscopic description which does not rely on equilibration
- limit of large interaction rate is hydrodynamics!

Aim

Case study in simplified kinetic theory description on full range from small to large system size with comparison to hydrodynamics based on transverse flow

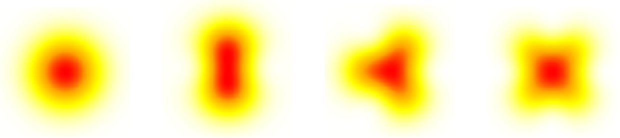
Model and Setup

- describing time evolution of boost-invariant phase space distribution of massless bosons using Boltzmann equation in conformal RTA

$$p^\mu \partial_\mu f = -\frac{p^\mu u_\mu}{\tau_R}(f - f_{\mathrm{eq}})\ , \quad \tau_R = 5\frac{\eta}{s}T^{-1}$$

  - initialized with vanishing longitudinal pressure and no momentum anisotropies
  - time evolution of f depends only on opacity $\hat{\gamma} = \left(5\frac{\eta}{s}\right)^{-1}\left(\frac{30}{\nu_{\mathrm{eff}}\pi^2}\frac{1}{\pi}\frac{\mathrm{d}E_\perp^{(0)}}{\mathrm{d}\eta}R\right)^{1/4}$

    Kurkela, Wiedemann, Wu EPJC 79 (2019) 965
  - energy weighted d.o.f.: dependence on IS only in energy density
- first study: simple initial energy density introducing only one eccentricity at a time



(no ecc.) $(n = 2)$ $(n = 3)$ $(n = 4)$

Analytical and numerical treatment

CRC-TR 211

- typical values of $\hat{\gamma}$
  - min. bias pp: $\hat{\gamma} \approx 0.88 \left(\frac{\eta/s}{0.16}\right)^{-1} \left(\frac{R}{0.4\,\mathrm{fm}}\right)^{1/4} \left(\frac{\mathrm{d}E_\perp^{(0)}/\mathrm{d}\eta}{5\,\mathrm{GeV}}\right)^{1/4} \left(\frac{\nu_{\mathrm{eff}}}{40}\right)^{-1/4}$
  - central PbPb: $\hat{\gamma} \approx 9.2 \left(\frac{\eta/s}{0.16}\right)^{-1} \left(\frac{R}{6\,\mathrm{fm}}\right)^{1/4} \left(\frac{\mathrm{d}E_\perp^{(0)}/\mathrm{d}\eta}{4000\,\mathrm{GeV}}\right)^{1/4} \left(\frac{\nu_{\mathrm{eff}}}{40}\right)^{-1/4}$

$\Rightarrow$ treat problem both analytically (for small $\hat{\gamma}$) and numerically

linearized analytical treatment

- "opacity expansion" in number of scatterings

$$\text{0th order}: \quad p^\mu \partial_\mu f^{(0)} = 0 \;,$$

$$\text{1st order}: \quad p^\mu \partial_\mu f^{(1)} = C[f^{(0)}]$$

Heiselberg, Levy PRC 59 (1999) 2716

Borghini, Gombeaud EPJC 71 (2011) 1612

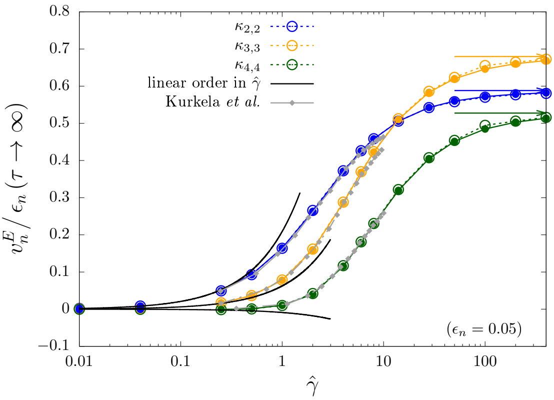
- expansion parameter $C_{\mathrm{RTA}}[f] \sim \hat{\gamma}$
- linearize also in eccentricity

numerical treatment

- nonlinear in both opacity and eccentricity
- Relativistic Lattice Boltzmann solver for energy-weighted d.o.f.

Ambrus, Blaga PRC 98 (2018) 035201

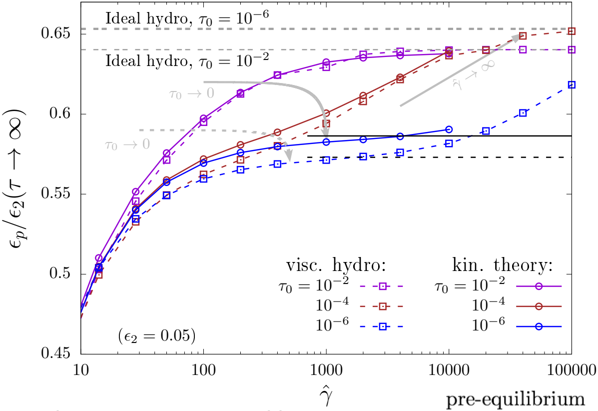
Opacity dependence of response coefficients



- linearized results tangential to curve of numerical results at small $\hat{\gamma}$
- agreement with previous results

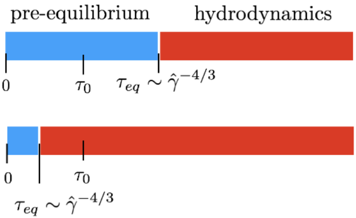
  Kurkela, Taghavi, Wiedemann, Wu PLB 811 (2020) 135901
- saturation at large $\hat{\gamma}$, expectation: hydrodynamic behaviour

Comparison to Hydrodynamics



unphysical limit

physical limit

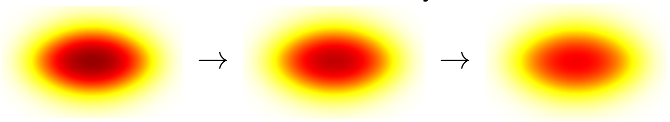


- agreement at large τ_0: no pre-equilibrium
- small τ_0: pre-equilibrium causes discrepancies
- convergence only in unphysical order of limits

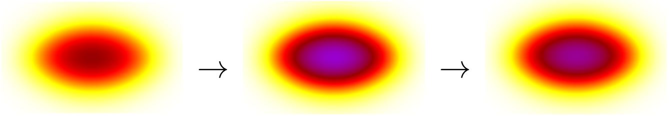
$\Rightarrow$ Why is pre-equilibrium important for observables that develop at $\tau \sim R$?

- $\tau \ll R$: only longitudinal expansion
  - local Bjorken flow cooling: follows universal attractor curve

τe in kin. theory



τe in visc. hydro

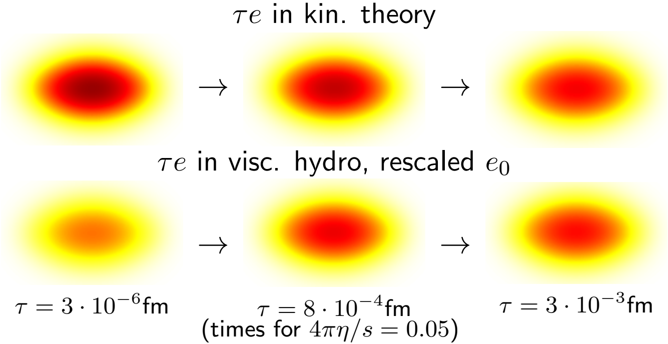


$\tau = 3 \cdot 10^{-6}$fm $\tau = 8 \cdot 10^{-4}$fm $\tau = 3 \cdot 10^{-3}$fm

(times for $4\pi\eta/s = 0.05$)

- dynamics depend on local energy density $\Rightarrow$ inhomogeneous cooling
  - decrease of eccentricity before transverse flow develops

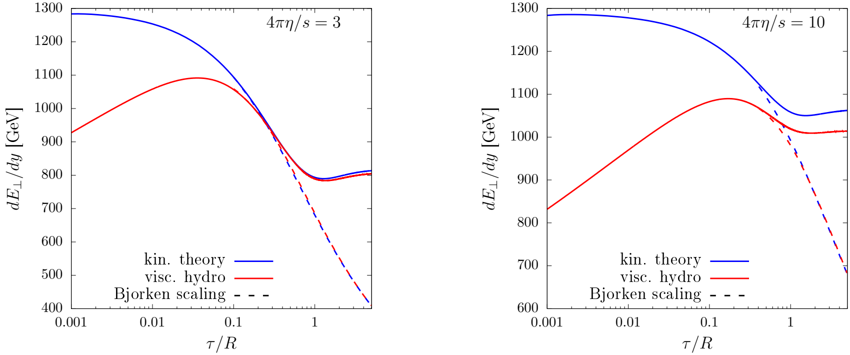
- idea: counteract difference in pre-equilibrium by different hydro initialization

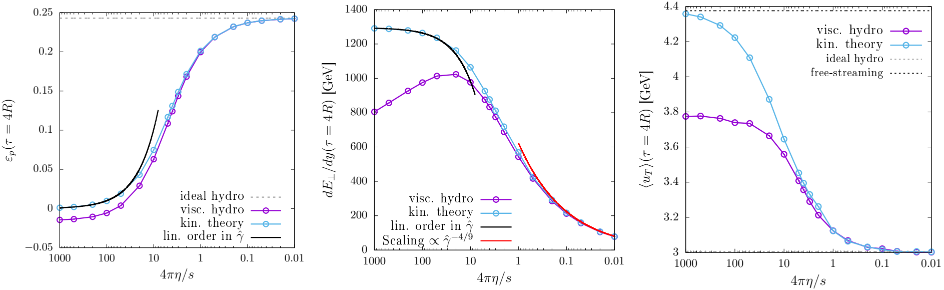


- more realistic initial condition: average profile (Pb+Pb 30-40%)
  - fixed profile: vary $\hat{\gamma}$ via η/s: $\hat{\gamma} \approx 11 \cdot (4\pi\eta/s)^{-1}$

Initializing on the attractor

- initialize hydro on its attractor to match Bjorken flow cooling at late times
- accuracy depends on timescale separation of longitudinal cooling and transverse expansion

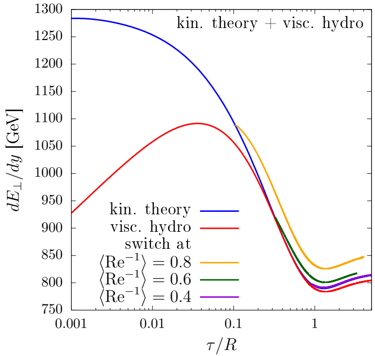


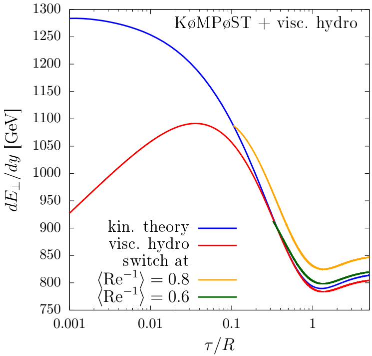


- when initialized on the attractor, observables in ideal hydro and viscous hydro agree perfectly with kinetic theory at large opacities
- hydrodynamics valid for $4\pi\eta/s \lesssim 3$ (for Pb+Pb 30-40%)

Hybrid schemes

- idea: evolve system in kinetic theory until $\langle \mathrm{Re}^{-1} \rangle$ drops to specific value, then match $T^{\mu\nu}$ to hydro code
- system immediately starts following similar evolution to a pure hydro run
  - switching too early causes errors in pre-equilibrium
  - results from late switching times more accurate than rescaled hydro
- works just as well with KøMPøST, but with limited range of applicability





Summary

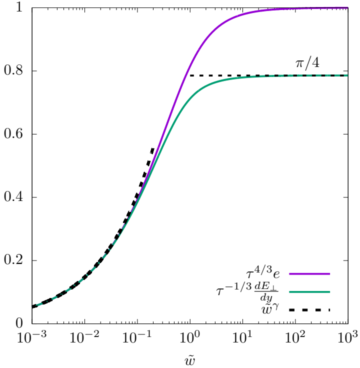
- kinetic theory description covers full range in opacity from small to large systems
- naive comparison to hydrodynamics: disagreement even at large opacities!
  - difference during pre-equilibrium
  - eccentricity decreases before onset of transverse expansion
- different setup of hydrodynamic simulations can bring agreement at large opacities
  - initializing hydrodynamics on its early-time attractor
  - hybrid models with kinetic theory for pre-equilibrium

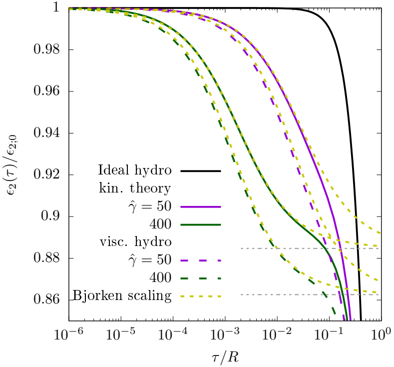
Backup

Early time Bjorken scaling

CRC-TR 211

- $\tau \ll R$: no transverse expansion, system locally behaves like 0+1D Bjorken flow
  - universal attractor curve scaling in the variable $\tilde{w}(\tau, \mathbf{x}_\perp) = \frac{T(\tau,\mathbf{x}_\perp)\tau}{4\pi\eta/s}$
    Giacalone, Mazeliauskas, Schlichting, PRL 123 (2019) 262301
  - $\tilde{w} \gg 1$: $\tau^{4/3} e = \text{const.}$, $\tau^{1/3} \frac{dE_\perp}{dy} = \text{const.}$
  - $\tilde{w} \ll 1$: model dependent power law $\tau^{4/3} e \sim \tilde{w}^\gamma$





- inhomogeneous cooling changes energy density profile