Spin-thermal shear coupling in relativistic nuclear collisions

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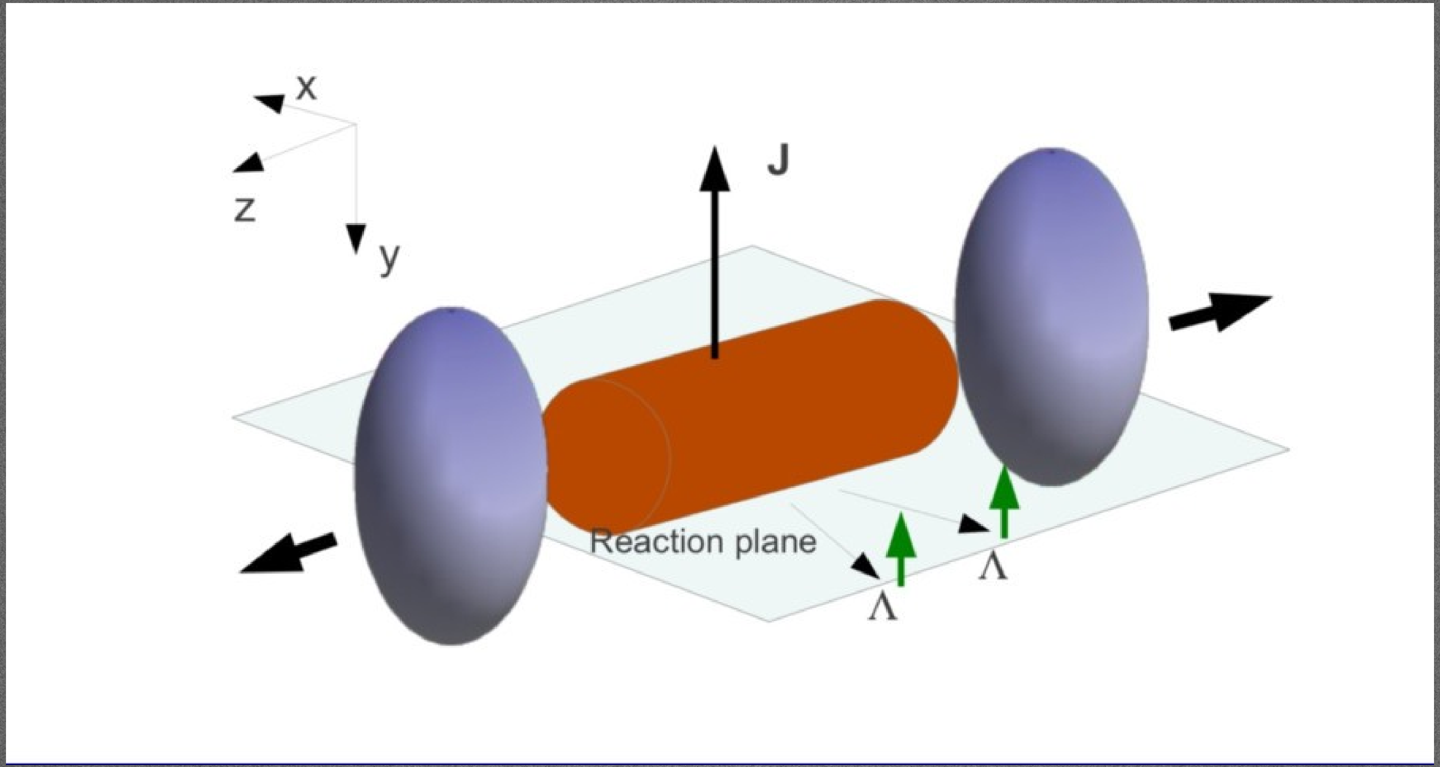
Peripheral collisions: large angular momentum

Peripheral collisions ➡ Angular momentum ➡ Global polarization w.r.t. reaction plane



- Polarization estimated at quark level by spin-orbit coupling

  Z. T. Liang, X. N. Wang, Phys. Rev. Lett. 94 (2005) 102301

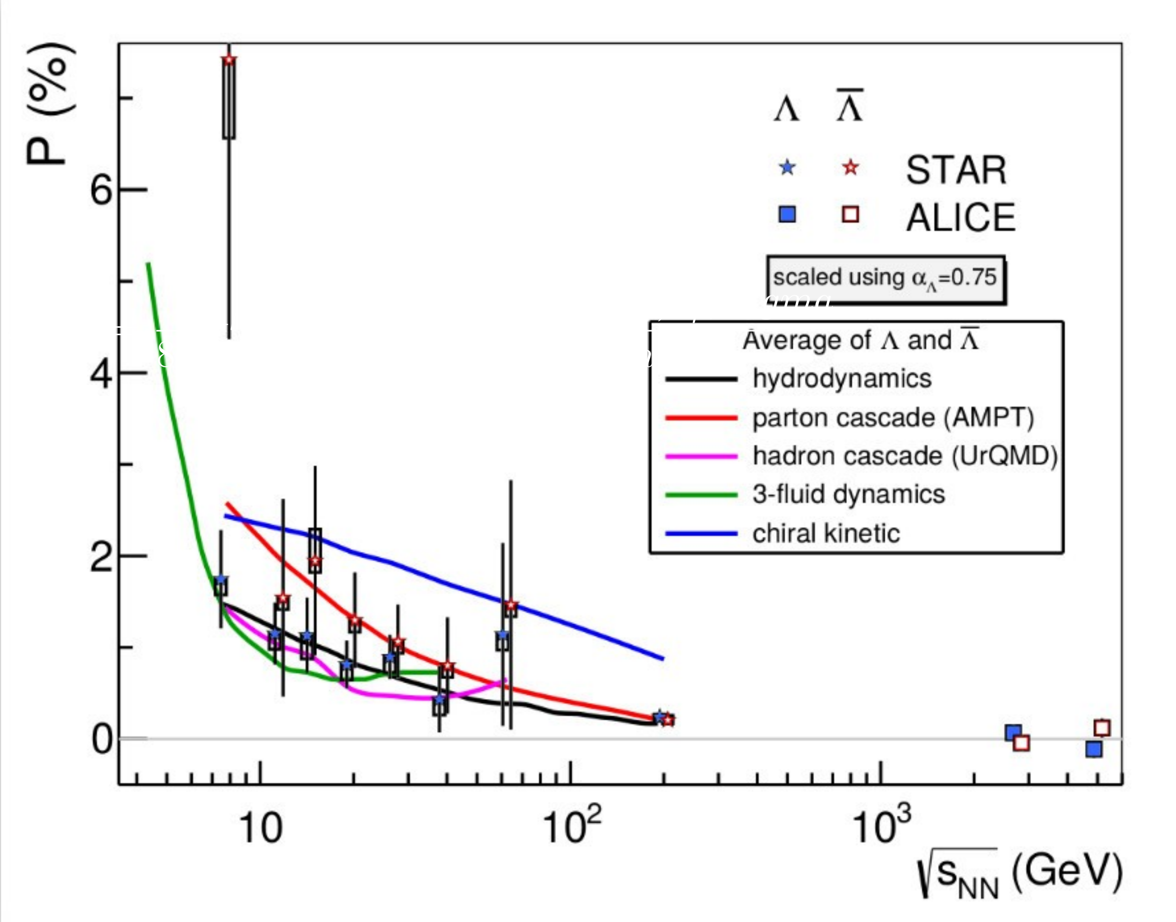
- By local thermodynamic equilibrium of the spin degrees of freedom

  F. Becattini, F. Piccinini, Ann. Phys. 323 (2008) 2452; F. Becattini, F. Piccinini, J. Rizzo, Phys. Rev. C 77 (2008) 024906

Spin ∝(thermal) vorticity

Agreement between hydrodynamic **predictions** and the data

Global polarization



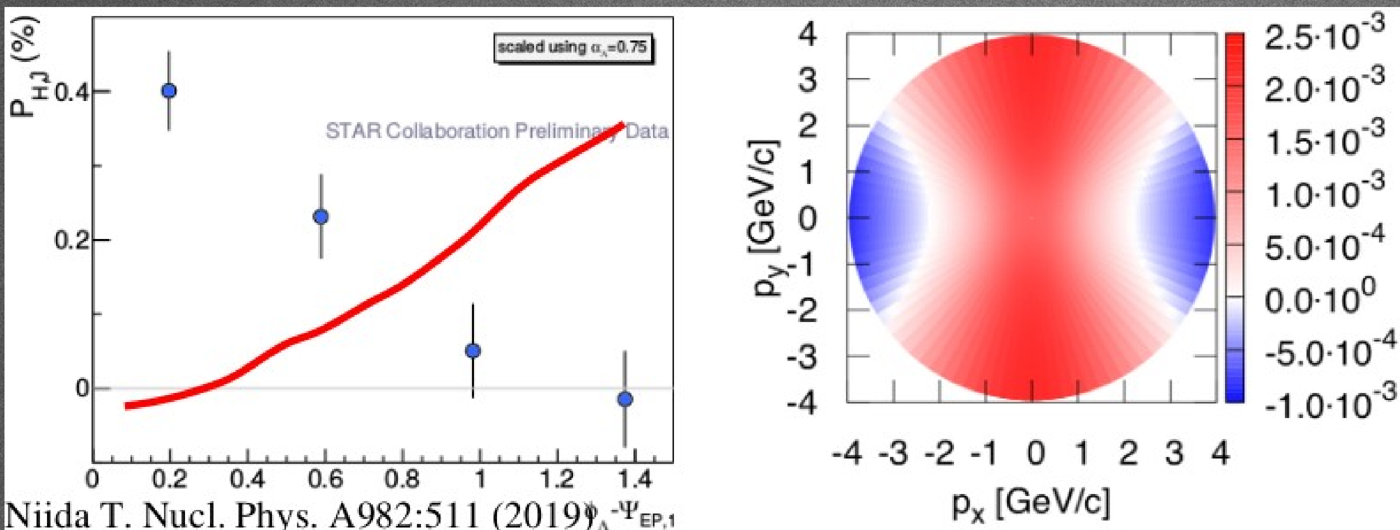
F. Becattini, V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338:32 (2013)

$$S^\mu(p) = \frac{1}{8m}\epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F(1-n_F)\partial_\rho \beta_\sigma}{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F}$$

$$n_F = \left(\mathrm{e}^{\beta\cdot p - \zeta} + 1\right)^{-1}$$

Different models of the collision, same formula for polarization

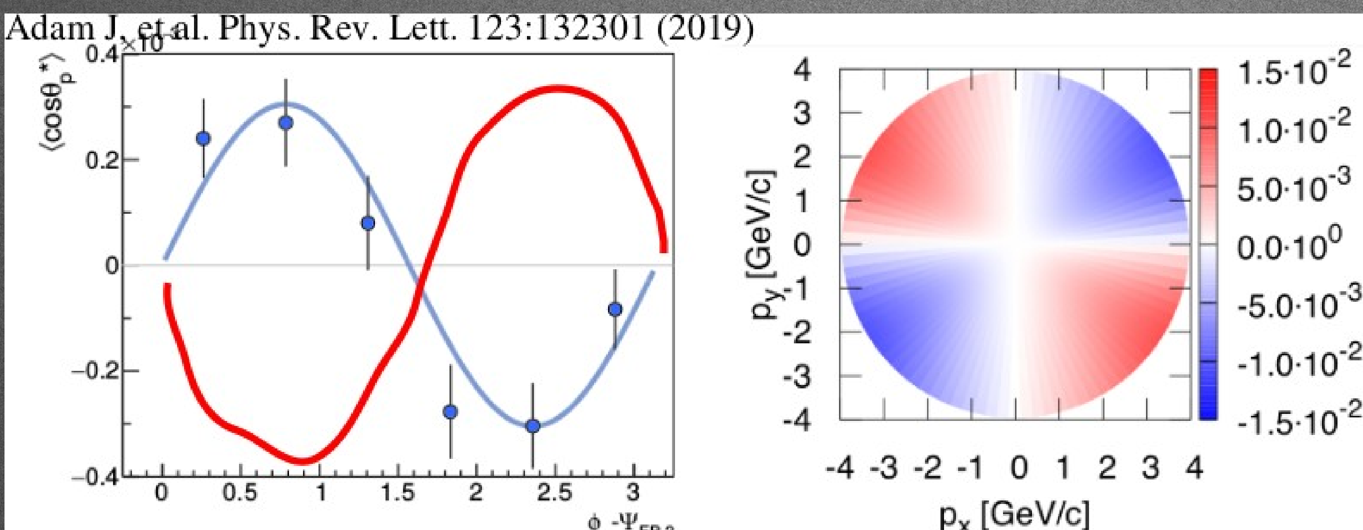
Puzzles: momentum dependence of polarization



Au + Au, 20 - 60 %

$\sqrt{s_{NN}} = 200 \text{ GeV}$

Theory prediction



Not the effect of decays:

X. L. Xia, H. Li, X.G. Huang and H. Z. Huang, Phys. Rev. C 100 (2019), 014913

F. Becattini, G. Cao and E. Speranza, Eur. Phys. J. C 79 (2019) 741

Polarization from Wigner function

F. Becattini, Lect. Notes Phys. 987 (2021) 15-52.

The covariant Wigner function of the free Dirac field:

$$W(x,k)_{AB} = \frac{1}{(2\pi)^4}\int \mathrm{d}^4y\, \mathrm{e}^{-\mathrm{i}k\cdot y}\langle : \bar{\Psi}_B(x+y/2)\Psi_A(x-y/2) : \rangle$$

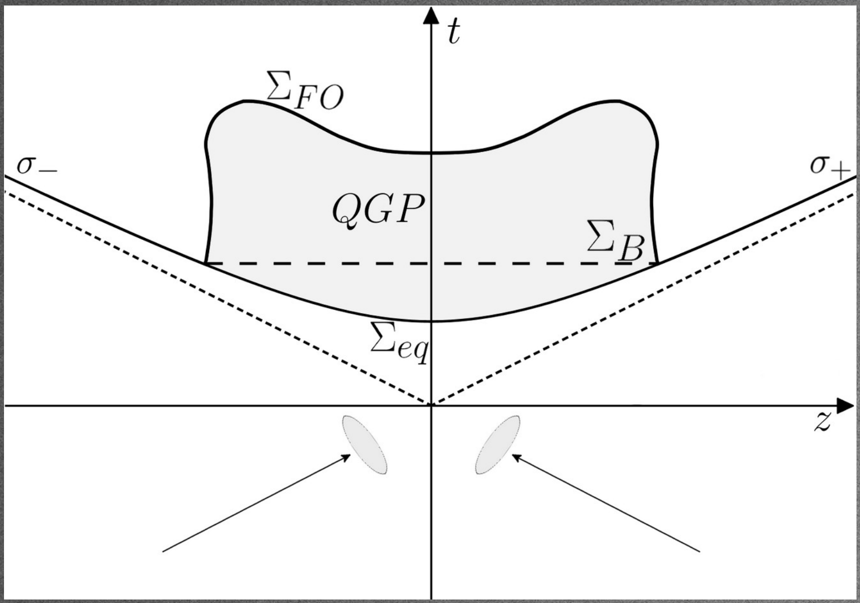
where:

$$\langle \widehat{X} \rangle = \mathrm{tr}\left(\widehat{\rho}\,\widehat{X}\right)$$

It allows to calculate the mean spin vector:

$$S^\mu(p) = \frac{1}{2}\frac{\int \mathrm{d}\Sigma\cdot p\;\mathrm{tr}_4\left(\gamma^\mu\gamma^5 W_+(x,p)\right)}{\int \mathrm{d}\Sigma\cdot p\;\mathrm{tr}_4 W_+(x,p)}$$

Local thermodynamic Equilibrium (Zubarev theory)



D.N. Zubarev, et al, Theor. Math. Phys. 1979, 40, 821
C.G. van-Weert, Ann. Phys. 1982, 140, 133
F. Becattini, MB, E. Grossi, Particles 2 (2019) 2, 197-207;
MB, Lect. Notes Phys. 987 (2021) 53-93.

$$\beta^\nu = \frac{u^\nu}{T}$$

$$\widehat{\rho} = \frac{1}{Z}\exp\left[-\int_{\Sigma_{eq}} \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta\,\widehat{j}^\mu\right)\right]$$

With the Gauss theorem:

$$\widehat{\rho} = \frac{1}{Z}\exp\left[-\int_{\Sigma_{FO}} \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta\,\widehat{j}^\mu\right) + \underbrace{\int_\Theta \mathrm{d}\Theta \left(\widehat{T}^{\mu\nu}\nabla_\mu\beta_\nu - \widehat{j}^\mu\nabla_\mu\zeta\right)}_{\text{Dissipative}}\right]$$

$$\simeq \widehat{\rho}_{\mathrm{LE}} = \frac{1}{Z}\exp\left[-\int_{\Sigma_{FO}} \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta\,\widehat{j}^\mu\right)\right]$$

Hydrodynamic Limit

$$W(x,k) \simeq W(x,k)_{\mathrm{LE}} = \mathrm{tr}\left(\widehat{\rho}_{\mathrm{LE}}\widehat{W}(x,k)\right)$$
$$= \frac{1}{Z}\mathrm{tr}\left(\exp\left[-\int_{\Sigma_{FO}} \mathrm{d}\Sigma_\mu(y)\left(\widehat{T}^{\mu\nu}(y)\beta_\nu(y) - \zeta(y)\,\widehat{j}^\mu(y)\right)\right]\widehat{W}(x,k)\right)$$

Expand the β and ζ fields from the point x where the Wigner operator is to be evaluated

$$\beta_\nu(y) \simeq \beta_\nu(x) + \partial_\lambda\beta_\nu(x)(y-x)^\lambda + \cdots$$
$$\int_\Sigma \mathrm{d}\Sigma_\mu \widehat{T}^{\mu\nu}(y)\beta_\nu = \beta_\nu(x)\int_\Sigma \mathrm{d}\Sigma_\mu \widehat{T}^{\mu\nu}(y) = \beta_\nu(x)\widehat{P}^\nu$$

$$\widehat{\rho}_{LE} \simeq \frac{1}{Z}\exp\left[-\beta_\nu(x)\widehat{P}^\nu - \frac{1}{2}(\partial_\mu\beta_\nu(x) - \partial_\nu\beta_\mu(x))\widehat{J}^{\mu\nu}_x - \frac{1}{2}(\partial_\mu\beta_\nu(x) + \partial_\nu\beta_\mu(x))\widehat{Q}^{\mu\nu}_x + \cdots\right]$$

$$\widehat{J}^{\mu\nu}_x = \int \mathrm{d}\Sigma_\lambda\left[(y-x)^\mu\widehat{T}^{\lambda\nu}(y) - (y-x)^\nu\widehat{T}^{\lambda\mu}(y)\right] \quad \widehat{Q}^{\mu\nu}_x = \int \mathrm{d}\Sigma_\lambda\left[(y-x)^\mu\widehat{T}^{\lambda\nu}(y) + (y-x)^\nu\widehat{T}^{\lambda\mu}(y)\right]$$

$$\widehat{\rho}_{LE} \simeq \frac{1}{Z} \exp\left[-\beta_\nu(x)\widehat{P}^\nu + \frac{1}{2}\varpi_{\mu\nu}(x)\widehat{J}^{\mu\nu}_x - \frac{1}{2}\xi_{\mu\nu}(x)\widehat{Q}^{\mu\nu}_x + \cdots \right]$$

$$\varpi_{\mu\nu} = -\frac{1}{2}\left(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu\right)$$

Thermal vorticity
Adimensional in natural units

$$\xi_{\mu\nu} = \frac{1}{2}\left(\partial_\mu \beta_\nu + \partial_\nu \beta_\mu\right)$$

Thermal shear
Adimensional in natural units

At global equilibrium the thermal shear vanishes because of the Killing equation

Thermal shear coupling gives rise to
Non-dissipative non-equilibrium phenomena

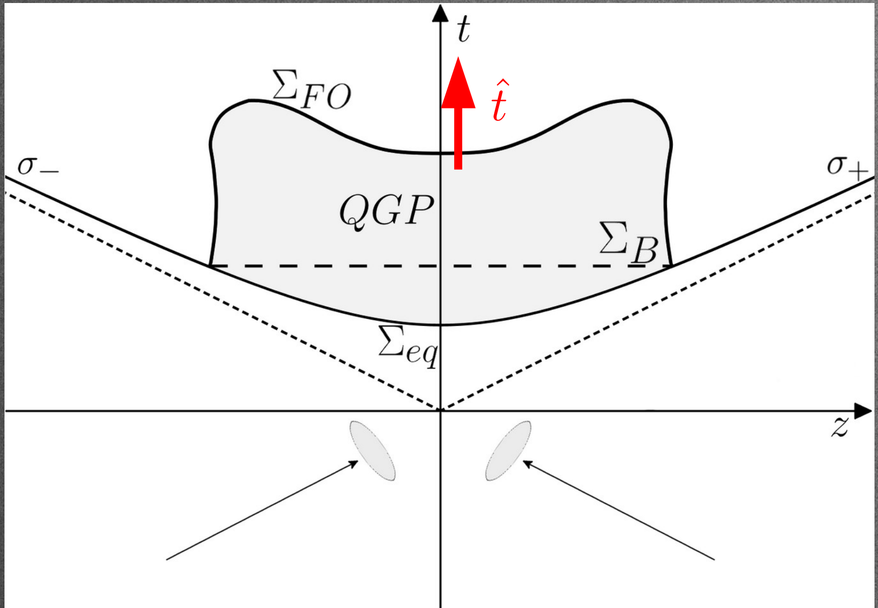
Spin polarization induced by thermal shear

Using **linear response theory** we eventually obtain:

$$S^{\mu}_{\xi}(p) = -\frac{1}{4m}\epsilon^{\mu\nu\sigma\tau}\frac{p_\tau p^\rho}{\varepsilon}\frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F(1-n_F)\hat{t}_\nu \xi_{\sigma\rho}}{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F}$$

F. Becattini, MB, A. Palermo, Phys. Lett. B 820 (2021) 136519

Same (not precisely the same) formula obtained by Liu and Yin with a different method:
S. Liu, Y. Yin, JHEP 07 (2021) 188



Dependence on a specific vector is not surprising as this term arises from the correlator

$$\langle \widehat{Q}^{\mu\nu}_x \widehat{W}(x,k) \rangle$$

But Q is not a tensor and, unlike J, it does depend on the hypersurface

Application to relativistic heavy ion collisions

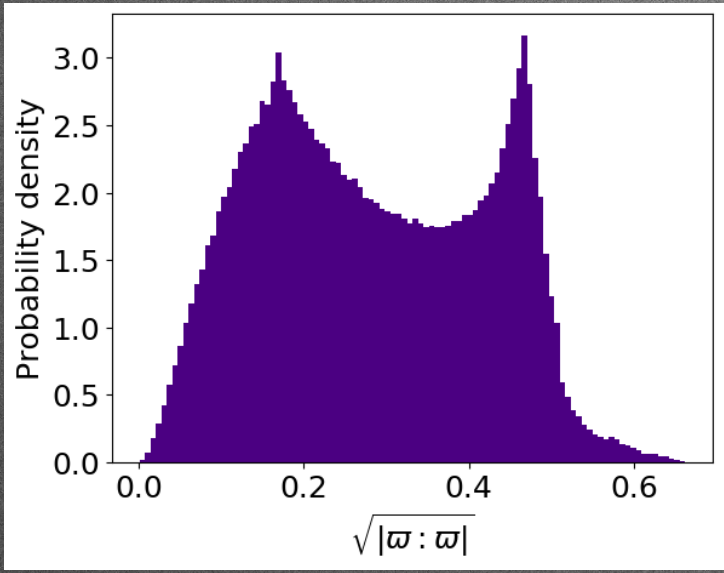
F. Becattini, MB, A. Palermo, G. Inghirami and I. Karpenko, Phys. Rev. Lett. 127 (2021) 27, 272302

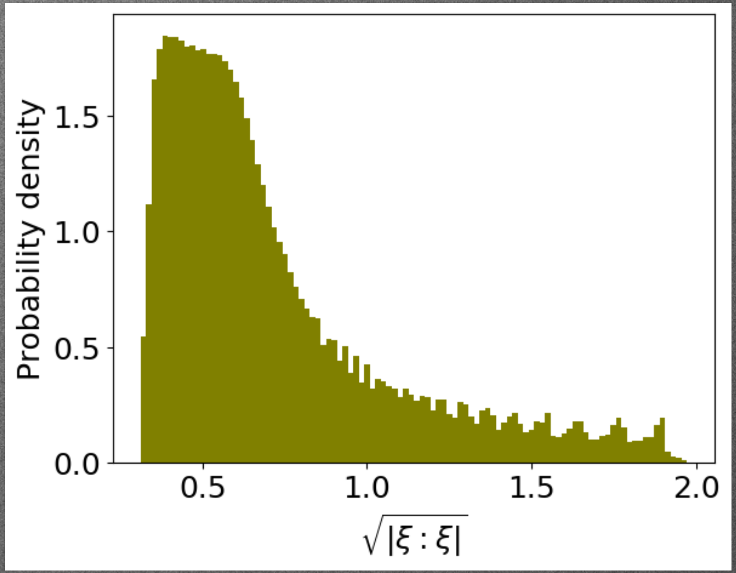
$$S^\mu = S^\mu_\varpi + S^\mu_\xi$$

$$S^\mu_\varpi(p) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F(1-n_F)\varpi_{\rho\sigma}}{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F}$$

$$S^\mu_\xi(p) = -\frac{1}{4m}\epsilon^{\mu\nu\sigma\tau}\frac{p_\tau p^\rho}{\varepsilon}\frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F(1-n_F)\hat{t}_\nu \xi_{\sigma\rho}}{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F}$$

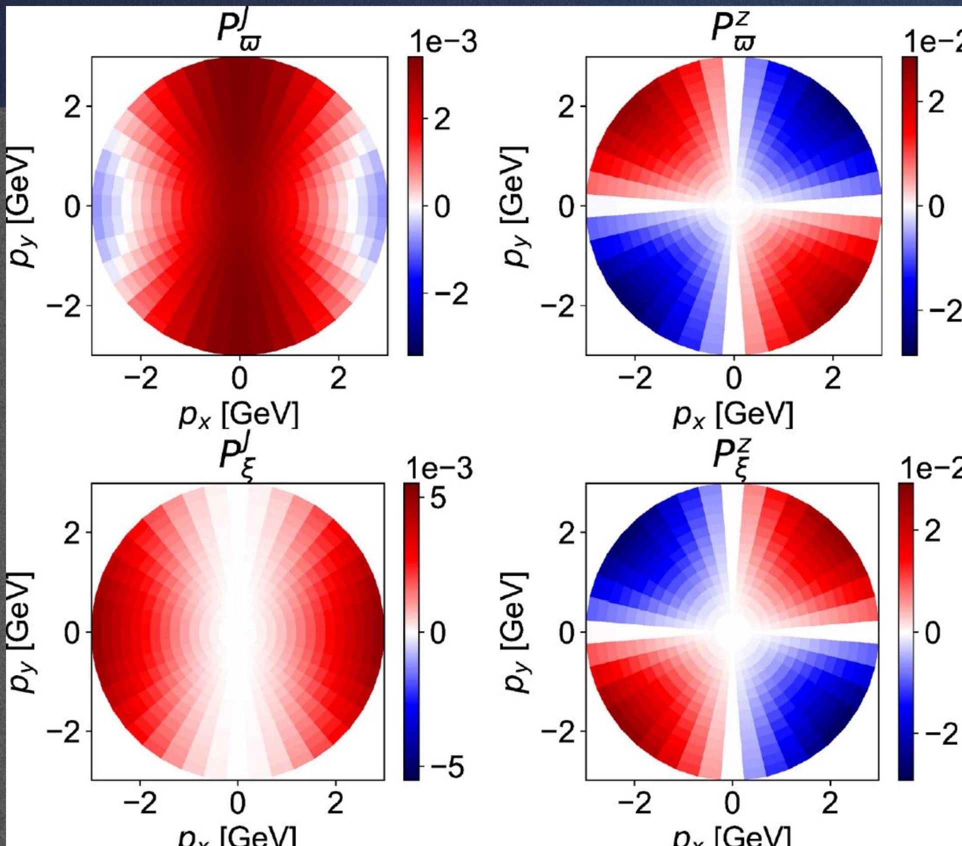
Modulus of thermal-vorticity and thermal-shear at the freeze-out hypersurface

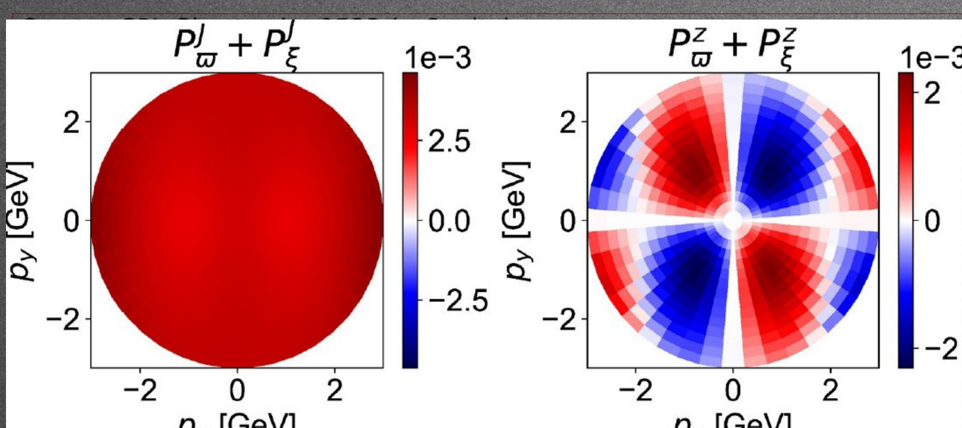




Back to the sign puzzle

F. Becattini, MB, A. Palermo, G. Inghirami and I. Karpenko, PRL 127 (2021) 27, 272302





Based on the hydrodynamic code VHLLE (author I. Karpenko) tuned to reproduce AuAu momentum spectra at RHIC top energy.
Similar output with ECHOQGP (main author G. Inghirami).

Right pattern!

Not sufficient to restore the agreement between data and model
Calculations fully consistent with:
B. Fu, S. Liu, L. Pang, H. Song and Y. Yin, Phys.Rev.Lett. 127 (2021) 14, 142301

Recent analysis of sheat induced polarization in the two formulations:
S. Ryu, V. Jupic, C. Shen, PRC 104 (2021)
S. Alzhrani, S. Ryu, C. Shen, 2203.15718

Are we using the most appropriate approximations?

The formulas we have derived are based on a Taylor expansion of the density operator

$$W(x,k)_{LE} = \frac{1}{Z}\mathrm{tr}\left(\exp\left[-\int_{\Sigma_{FO}} \mathrm{d}\Sigma_\mu(y)\left(\widehat{T}^{\mu\nu}(y)\beta_\nu(y) - \zeta(y)\,\widehat{j}^\mu(y)\right)\right]\widehat{W}(x,k)\right)$$

$$\beta_\nu(y) \simeq \beta_\nu(x) + \partial_\lambda\beta_\nu(x)(y-x)^\lambda + \cdots$$

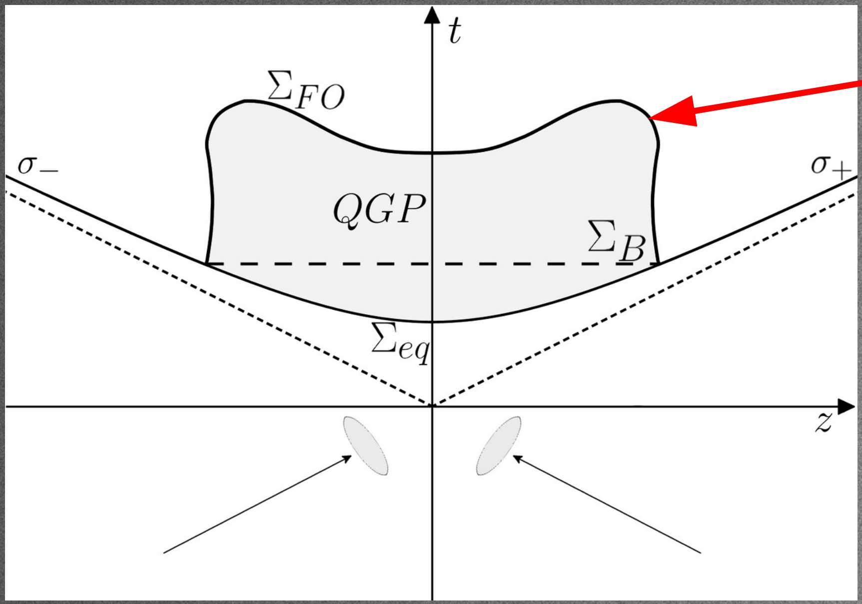
$$\widehat{\rho}_{LE} \simeq \frac{1}{Z}\exp\left[-\beta_\nu(x)\widehat{P}^\nu - \frac{1}{2}(\partial_\mu\beta_\nu(x) - \partial_\nu\beta_\mu(x))\widehat{J}^{\mu\nu}_x - \frac{1}{2}(\partial_\mu\beta_\nu(x) + \partial_\nu\beta_\mu(x))\widehat{Q}^{\mu\nu}_x + \cdots\right]$$

This is generally correct, but it is an approximation after all.

Can we find a better approximation for our special case?

Isothermal local equilibrium

The most appropriate setting for relativistic heavy ion collisions at very high energy!



At high energy, Σ_{FO} expected to be T= constant!

$$\beta^\mu = (1/T)u^\mu$$

$$\widehat{\rho}_{LE} = \frac{1}{Z}\exp\left[-\int_{\Sigma_{FO}} \mathrm{d}\Sigma_\mu \widehat{T}^{\mu\nu}\beta_\nu\right] = \frac{1}{Z}\exp\left[-\frac{1}{T}\int_{\Sigma_{FO}} \mathrm{d}\Sigma_\mu \widehat{T}^{\mu\nu}u_\nu\right]$$

Only NOW u can be expanded!

$$u_\nu(y) \simeq u_\nu(x) + \partial_\lambda u_\nu(x)(y-x)^\lambda + \cdots$$

$$\widehat{\rho}_{LE} \simeq \frac{1}{Z}\exp\left[-\beta_\nu(x)\widehat{P}^\nu - \frac{1}{2T}(\partial_\mu u_\nu(x) - \partial_\nu u_\mu(x))\widehat{J}^{\mu\nu}_x - \frac{1}{2T}(\partial_\mu u_\nu(x) + \partial_\nu u_\mu(x))\widehat{Q}^{\mu\nu}_x + \cdots\right]$$

Spin mean vector at leading order with isothermal local equilibrium (ILE)

Readily found by replacing the gradients of β with those of u

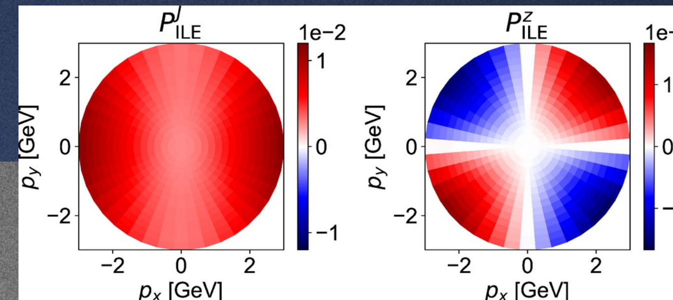
$$S^\mu_{\rm ILE}(p) = -\epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int_\Sigma {\rm d}\Sigma \cdot p \, n_F (1-n_F) \left[\omega_{\rho\sigma} + 2\, \hat{t}_\rho \frac{p^\lambda}{\varepsilon} \Xi_{\lambda\sigma}\right]}{8 m T_{\rm FO} \int_\Sigma {\rm d}\Sigma \cdot p \, n_F}$$

$$\omega_{\rho\sigma} = \frac{1}{2}\left(\partial_\sigma u_\rho - \partial_\rho u_\sigma\right)$$ Kinematic vorticity

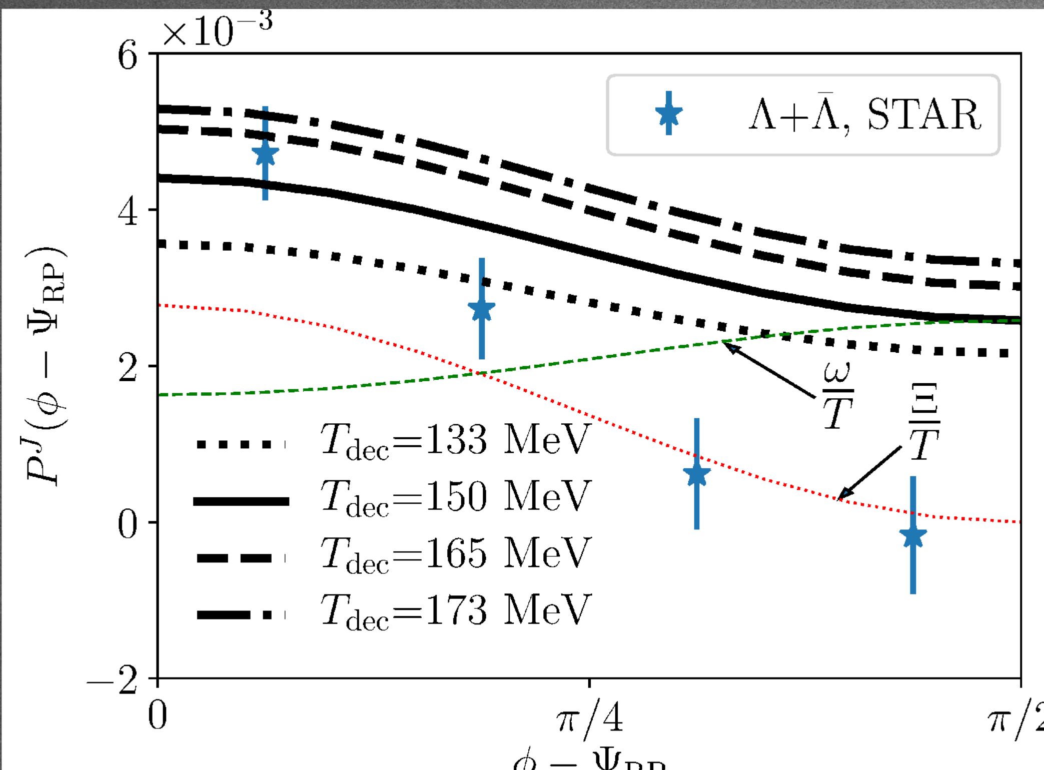
$$\Xi_{\rho\sigma} = \frac{1}{2}\left(\partial_\sigma u_\rho + \partial_\rho u_\sigma\right)$$ Kinematic shear

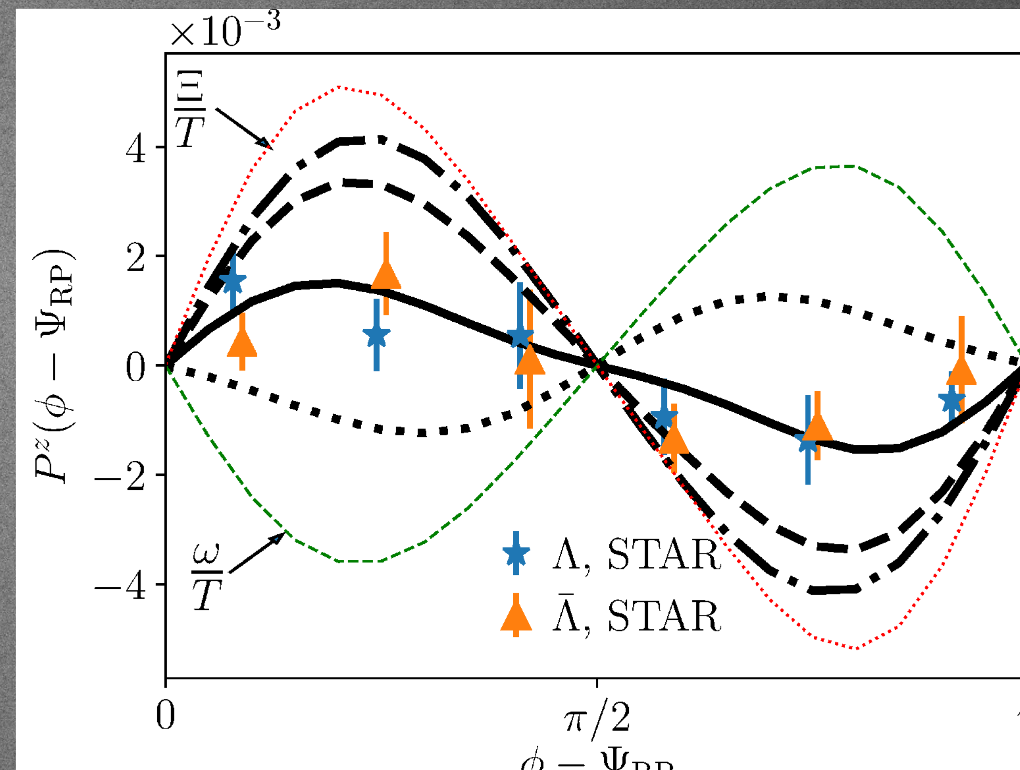
Isothermal local equilibrium: result

Apply the new formula (for primary hadrons):



$$S^\mu_{\mathrm{ILE}}(p) = -\epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F(1-n_F) \left[\omega_{\rho\sigma} + 2\,\hat{t}_\rho \frac{p^\lambda}{\varepsilon} \Xi_{\lambda\sigma}\right]}{8mT_{\mathrm{dec}} \int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F}$$





- Sensitive to the decoupling temperature T_{dec}
- Quantitative agreement with the data for realistic T_{dec}=150 MeV

Decomposition of thermal shear

$$\Xi_{\rho\sigma} = \frac{1}{2}\left(A_\rho u_\sigma + A_\sigma u_\rho\right) + \sigma_{\rho\sigma} + \frac{1}{3}\theta\Delta_{\rho\sigma} \quad \omega_{\rho\sigma} = A_\rho u_\sigma - A_\sigma u_\rho + \frac{1}{2}\epsilon_{\rho\sigma\mu\nu}\omega^\mu u^\nu$$

A is the acceleration field
ω is the rotation field

$$\sigma_{\mu\nu} = \frac{1}{2}(\nabla_\mu u_\nu + \nabla_\nu u_\mu) - \frac{1}{3}\Delta_{\mu\nu}\theta$$

$$\theta = \nabla \cdot u$$

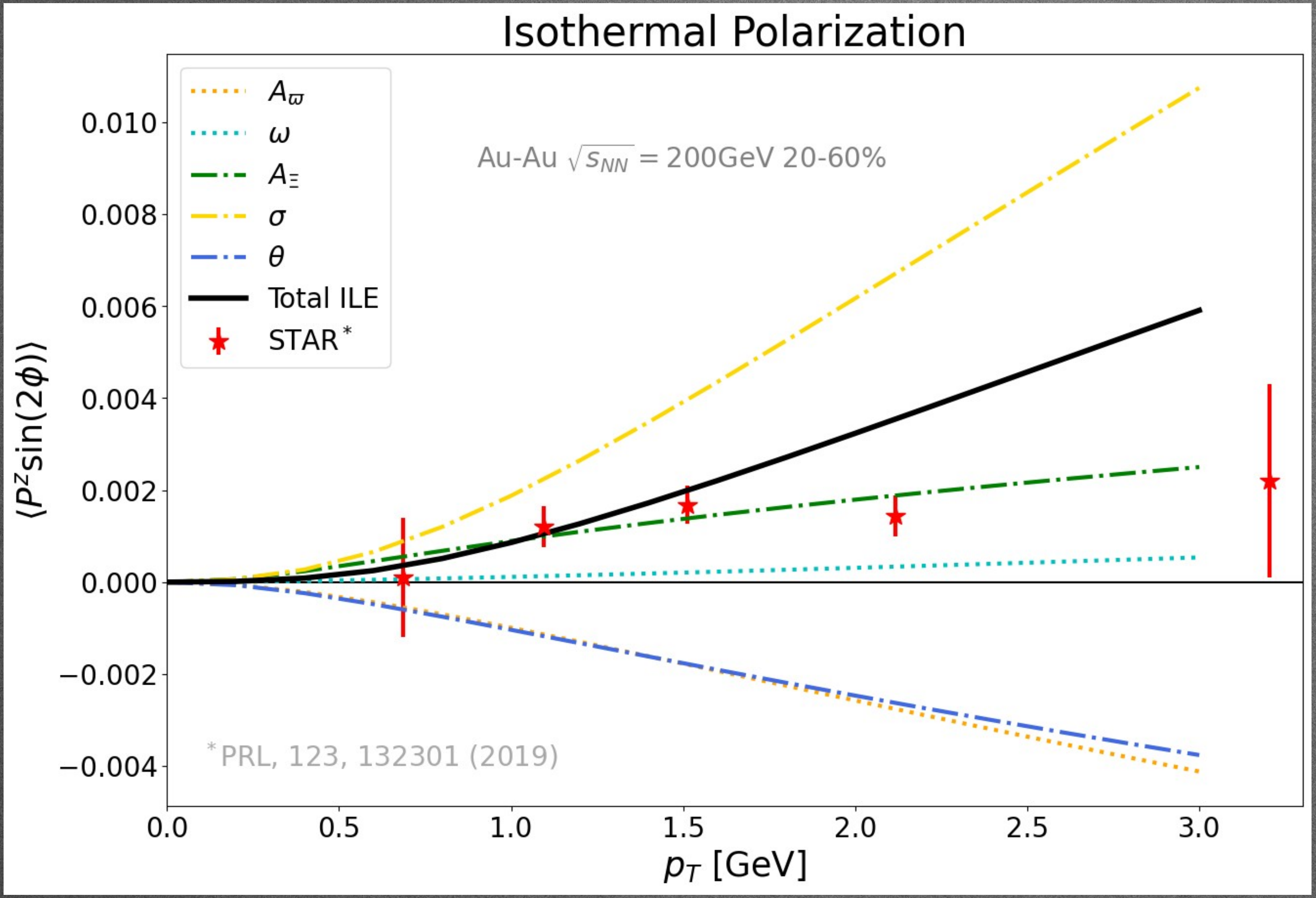


Figure by A. Palermo

Summary and Outlook

- Quantum statistical mechanics: proper tool
- Thermal shear and isothermal hadronization are able to solve the sign puzzle

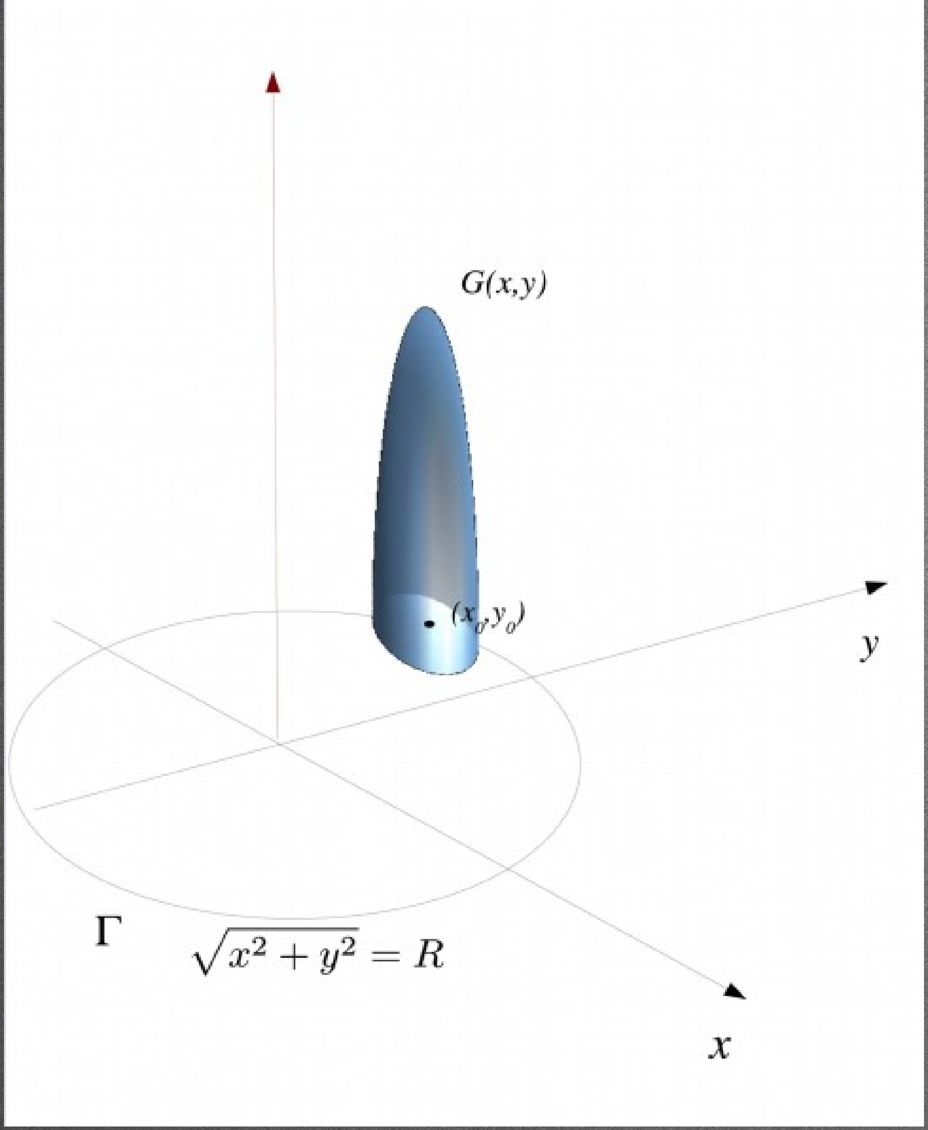
Future investigations

- Inclusion of the decays contribution to spin polarization;
- Anlaysis on the dependence on viscosities and initial conditions;
- Predictions at other energies.

Backup

Understand the point: a simple example

Task: approximate the integral:



$$W = \int_\Gamma \mathrm{e}^{\sqrt{x^2+y^2}} G(x,y)\mathrm{d}s$$

Where G(x,y) is a peaked function around the point (x_0 ,y_0) on the circle.

Since G is peaked, one can Taylor expand the exponent about (x_0 ,y_0)

$$W \simeq \mathrm{e}^{\sqrt{x_0^2+y_0^2}} \int_\Gamma \mathrm{e}^{x_0(x-x_0)/R+y_0(y-y_0)/R} G(x,y)\mathrm{d}s$$

$$= \mathrm{e}^R \int_\Gamma \mathrm{e}^{x_0(x-x_0)/R+y_0(y-y_0)/R} G(x,y)\mathrm{d}s$$

$$= \mathrm{e}^R \int_\Gamma \mathrm{e}^{\boldsymbol{\nabla} r|_{(x_0,y_0)}\cdot(\boldsymbol{x}-\boldsymbol{x}_0)} G(x,y)\mathrm{d}s$$

But it is just pointless if we integrate over the circle!

$$W = \mathrm{e}^R \int_\Gamma G(x,y)\mathrm{d}s$$

In the previous example, the Taylor expansion at first order introduces an undesired term:

$$W = \mathrm{e}^R \int_\Gamma G(x,y)\mathrm{d}s \qquad W \simeq \mathrm{e}^R \int_\Gamma \mathrm{e}^{\boldsymbol{\nabla} r|_{(x_0,y_0)}\cdot(\boldsymbol{x}-\boldsymbol{x}_0)} G(x,y)\mathrm{d}s$$

Exact — With gradient of r expansion

which is proportional to the gradient of the constant quantity on the circle, perpendicular to the integration line. This term does not vanish in the integration!

Similarly, for an isothermal hadronization, the inclusion of temperature gradients results in an additional, undesirable contribution proportional to the gradient of T, perpendicular to Σ_{FO}:

$$\frac{1}{2}\left[(\partial_\mu T)u_\nu(x) - (\partial_\nu T)u_\mu(x)\right]\widehat{J}^{\mu\nu}_x + \frac{1}{2}\left[(\partial_\mu T)u_\nu(x) + (\partial_\nu T)u_\mu(x)\right]\widehat{Q}^{\mu\nu}_x$$

Non-relativistic limit

Does it have a non-relativistic limit? Let us decompose it

$$\xi_{\sigma\rho} = \frac{1}{2}\partial_\sigma \left(\frac{1}{T}\right) u_\rho + \frac{1}{2}\partial_\rho \left(\frac{1}{T}\right) u_\sigma + \frac{1}{2T}\left(A_\rho u_\sigma + A_\sigma u_\rho\right) + \frac{1}{T}\sigma_{\rho\sigma} + \frac{1}{3T}\theta\Delta_{\rho\sigma}$$

A is the acceleration field

$$\sigma_{\mu\nu} = \frac{1}{2}(\nabla_\mu u_\nu + \nabla_\nu u_\mu) - \frac{1}{3}\Delta_{\mu\nu}\theta$$

$$\theta = \nabla \cdot u \qquad \Delta_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$$

All terms are relativistic (they vanish in the infinite c limit) EXCEPT grad T terms, which give rise to

$$\mathbf{S}_\xi = \frac{1}{8}\mathbf{v} \times \frac{\int \mathrm{d}^3\mathrm{x}\, n_F(1-n_F)\nabla\left(\frac{1}{T}\right)}{\int \mathrm{d}^3\mathrm{x}\, n_F}$$

There is an equal contribution in the NR limit from thermal vorticity

Why do we have a dependence on Σ?

$$\widehat{J}_x^{\mu\nu} = \int \mathrm{d}\Sigma_\lambda \ \left[(y-x)^\mu \widehat{T}^{\lambda\nu}(y) - (y-x)^\nu \widehat{T}^{\lambda\mu}(y)\right]$$

$$\widehat{Q}_x^{\mu\nu} = \int \mathrm{d}\Sigma_\lambda \ \left[(y-x)^\mu \widehat{T}^{\lambda\nu}(y) + (y-x)^\nu \widehat{T}^{\lambda\mu}(y)\right]$$

The divergence of the integrand of $J^{\mu\nu}$ vanishes, therefore it does not depend on the integration hypersurface (it is a constant of motion) and

$$\widehat{\Lambda}\widehat{J}_x^{\mu\nu}\widehat{\Lambda}^{-1} = \Lambda_\alpha^{-1\mu}\Lambda_\beta^{-1\nu}\widehat{J}_x^{\alpha\beta}$$

The divergence of the integrand of $Q^{\mu\nu}$ does not vanish, therefore it does depend on the integration hypersurface and

$$\widehat{\Lambda}\widehat{Q}_x^{\mu\nu}\widehat{\Lambda}^{-1} \neq \Lambda_\alpha^{-1\mu}\Lambda_\beta^{-1\nu}\widehat{Q}_x^{\alpha\beta}$$

Linear response theory

$$e^{\widehat{A}+\widehat{B}} = e^{\widehat{A}} + \int_0^1 \mathrm{d}z\, e^{z\widehat{A}}\, \widehat{B}\, e^{-z\widehat{A}}\, e^{\widehat{A}} + \cdots ,$$

$$\widehat{A} = -\beta(x)\cdot\widehat{P}, \quad \widehat{B} = \frac{\varpi_{\nu\lambda}(x)}{2}\widehat{J}_x^{\nu\lambda} - \frac{\xi_{\mu\nu}(x)}{2}\widehat{Q}_x^{\mu\nu}$$

$$\langle \widehat{W}_{ab}^{+}(x,k)\rangle_{\mathrm{LE}} \simeq \langle \widehat{W}_{ab}^{+}(x,k)\rangle_{\beta(x)} + \Delta_{\varpi} W_{ab}^{+}(x,k) + \Delta_{\xi} W_{ab}^{+}(x,k)$$

$$\Delta_{\varpi} W_{ab}^{+}(x,k) = +\frac{\varpi_{\mu\nu}(x)}{2}\int_0^1 \mathrm{d}z\, \langle \widehat{W}_{ab}^{+}(x,k)\widehat{J}^{\mu\nu}_{x+\mathrm{i}z\beta(x)}\rangle_{\beta(x)}$$

$$\Delta_{\xi} W_{ab}^{+}(x,k) = -\frac{\xi_{\mu\nu}(x)}{2}\int_0^1 \mathrm{d}z\, \langle \widehat{W}_{ab}^{+}(x,k)\widehat{Q}^{\mu\nu}_{x+\mathrm{i}z\beta(x)}\rangle_{\beta(x)}$$

$$= -\xi_{\kappa\rho}(x)\int_0^1 \mathrm{d}z \int_\Sigma \mathrm{d}\Sigma_\lambda(y)(y-x)^\kappa \langle \widehat{W}_{ab}^{+}(x,k)\widehat{T}^{\lambda\rho}(y+\mathrm{i}z\beta(x))\rangle_{\beta(x)}$$

Wigner function-thermal shear term

using the normal mode expansion of the Dirac field and standard thermal field theory techniques

$$\langle \widehat{W}^{+}(x,k)\rangle_{\beta(x)} = (m+\gamma^{\mu}k_{\mu})\,\delta(k^2-m^2)\theta(k_0)\frac{1}{(2\pi)^3}n_{\mathrm{F}}\,(k)$$

$$\Delta_{\xi}W^{+}_{ab}(x,k) = -\frac{\xi_{\kappa\rho}(x)}{(2\pi)^6}\int_0^1 \mathrm{d}z \int_{\Sigma}\mathrm{d}\Sigma_{\lambda}(y)(y-x)^{\kappa}\int\frac{\mathrm{d}^3\mathrm{p}}{2\varepsilon_p}\int\frac{\mathrm{d}^3\mathrm{p}'}{2\varepsilon_{p'}}\delta^4\left(k-\frac{p+p'}{2}\right)\times$$

$$\times\underline{\mathcal{T}^{\lambda\rho}(p,p')_{ab}}\,\mathrm{e}^{\mathrm{i}(p-p')\cdot(x-y)}\mathrm{e}^{z(p-p')\cdot\beta}n_{\mathrm{F}}(p)(1-n_{\mathrm{F}}(p'))$$

$$\mathcal{T}^{\lambda\rho}(p,p')_{ab} = \frac{1}{4}\left[(\not{p}'+m)\gamma^{\lambda}(\not{p}+m)\right]_{ab}(p^{\rho}+p'^{\rho})+\frac{1}{4}\left[(\not{p}'+m)\gamma^{\rho}(\not{p}+m)\right]_{ab}(p^{\lambda}+p'^{\lambda})$$

Wigner function- thermal shear Gauss theorem

$$\Delta_\xi W^+_{ab}(x,k) = \Delta_B W^+_{ab}(x,k) + \Delta_\Omega W^+_{ab}(x,k)$$

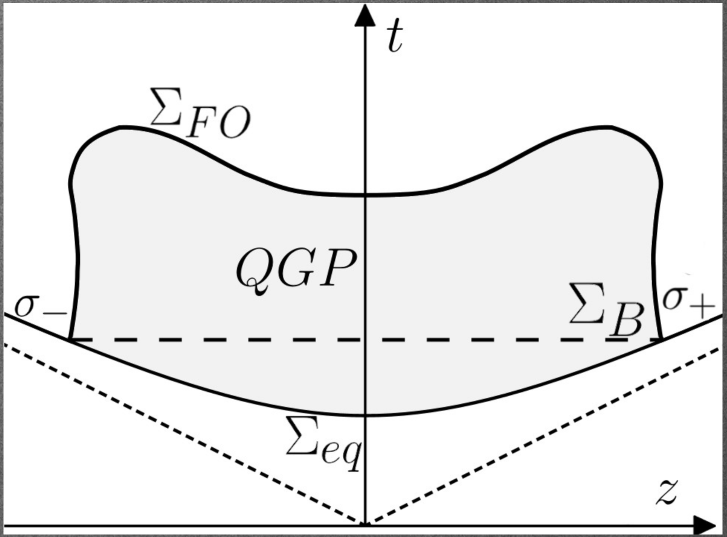
$$\Delta_B W^+_{ab}(x,k) = -\frac{\xi_{\kappa\rho}(x)}{(2\pi)^6}\int_0^1 \mathrm{d}z \int_{\Sigma_B} \mathrm{d}\Sigma_\lambda(y)(y-x)^\kappa \int \frac{\mathrm{d}^3\mathrm{p}}{2\varepsilon_p}\int \frac{\mathrm{d}^3\mathrm{p}'}{2\varepsilon_{p'}}\delta^4\left(k-\frac{p+p'}{2}\right)$$
$$\times \mathcal{T}^{\lambda\rho}(p,p')_{ab}\mathrm{e}^{\mathrm{i}(p-p')\cdot(x-y)}\mathrm{e}^{z(p-p')\cdot\beta} n_\mathrm{F}(p)(1-n_\mathrm{F}(p'))$$

$$\Delta_\Omega W^+_{ab}(x,k) = -\frac{\xi_{\kappa\rho}(x)}{(2\pi)^6}\int_0^1 \mathrm{d}z \int_{\Omega_B} \mathrm{d}^4 y \int \frac{\mathrm{d}^3\mathrm{p}}{2\varepsilon_p}\int \frac{\mathrm{d}^3\mathrm{p}'}{2\varepsilon_{p'}}\delta^4\left(k-\frac{p+p'}{2}\right)$$
$$\times \mathcal{T}^{\rho\kappa}(p,p')_{ab}\mathrm{e}^{\mathrm{i}(p-p')\cdot(x-y)}\mathrm{e}^{z(p-p')\cdot\beta} n_\mathrm{F}(p)(1-n_\mathrm{F}(p'))$$

Volume contribution:

$$\int_{\Omega_B} \mathrm{d}^4 y\, \mathrm{e}^{\mathrm{i}(p-p')\cdot(x-y)} \simeq \delta t (2\pi)^3 \delta^3(\boldsymbol{p}-\boldsymbol{p}')$$

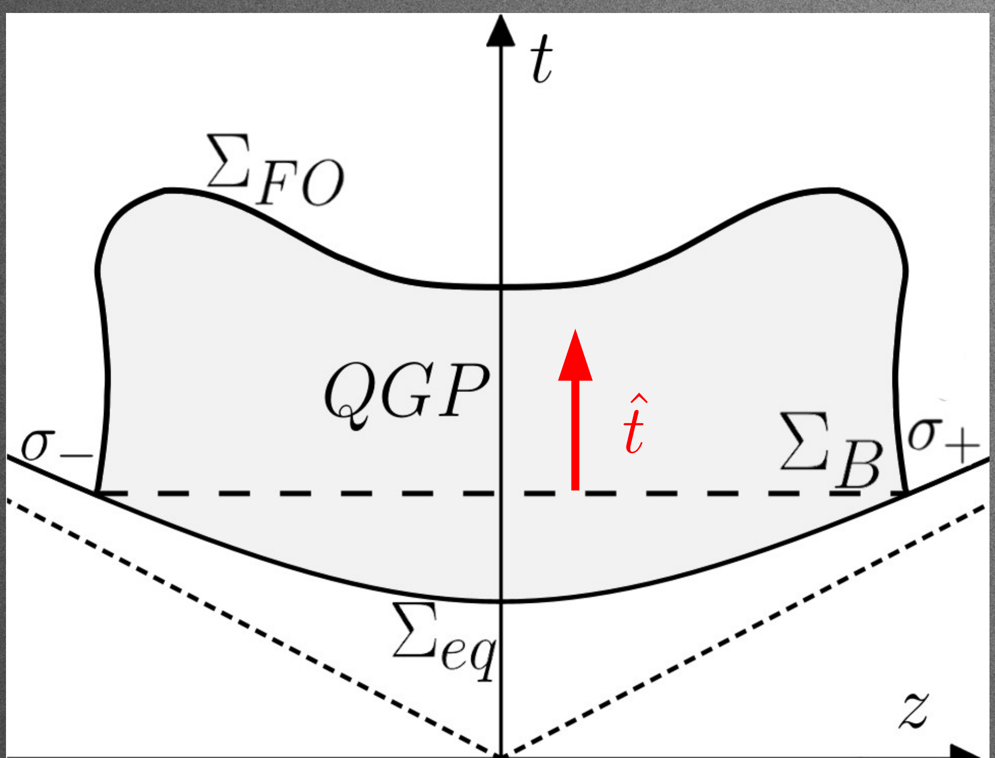
$$S^\mu_{\xi,\Omega} \propto \mathrm{tr}\left[\gamma^\mu\gamma^5\Delta_\Omega W^+(x,k)\right] = 0$$



Wigner function- thermal shear Base contribution

$$\int_{\Sigma_B} \mathrm{d}\Sigma_\lambda(y)(y-x)^\kappa \mathrm{e}^{\mathrm{i}(p-p')\cdot(x-y)} = \int_{\Sigma_B} \mathrm{d}^3 y\, \hat{t}_\lambda (y-x)^\kappa \mathrm{e}^{\mathrm{i}(p-p')\cdot(x-y)}$$

$$\simeq -\mathrm{i}\hat{t}_\lambda \Delta^\kappa_{\kappa'} (2\pi)^3 \frac{\partial}{\partial p'_{\kappa'}} \delta^3(\boldsymbol{p}-\boldsymbol{p}') + \hat{t}_\lambda \hat{t}^\kappa (2\pi)^3 \Delta t \delta^3(\boldsymbol{p}-\boldsymbol{p}')$$



$$\Delta^{\mu\nu} = \eta^{\mu\nu} - \hat{t}^\mu \hat{t}^\nu, \qquad \Delta t = (y-x)\cdot\hat{t}$$

in the center-of-mass frame: $\hat{t}_\lambda = \delta^0_\lambda$

Polarization

First order correction on thermal shear:

$$S^\mu_\xi(k) = \frac{1}{2}\frac{\int \mathrm{d}\Sigma \cdot k \; \mathrm{tr}\left(\gamma^\mu\gamma^5 \Delta_B W^+(x,k)\right)}{\int \mathrm{d}\Sigma \cdot k \; \mathrm{tr}\, \langle \widehat{W}^+(x,k)\rangle_{\beta(x)}}$$

$$\langle \widehat{W}^+(x,k)\rangle_{\beta(x)} = (m + \gamma^\mu k_\mu)\,\delta(k^2 - m^2)\theta(k_0)\frac{1}{(2\pi)^3} n_{\mathrm{F}}(k)$$

$$\Delta_B W^+_{ab}(x,k) = -\frac{\xi_{\kappa\rho}(x)}{(2\pi)^6}\int_0^1 \mathrm{d}z \int_{\Sigma_B} \mathrm{d}\Sigma_\lambda(y)(y-x)^\kappa \int \frac{\mathrm{d}^3\mathrm{p}}{2\varepsilon_p}\int \frac{\mathrm{d}^3\mathrm{p}'}{2\varepsilon_{p'}}\delta^4\left(k - \frac{p+p'}{2}\right)$$

$$\times \mathcal{T}^{\lambda\rho}(p,p')_{ab}\mathrm{e}^{\mathrm{i}(p-p')\cdot(x-y)}\mathrm{e}^{z(p-p')\cdot\beta} n_{\mathrm{F}}(p)(1-n_{\mathrm{F}}(p'))$$

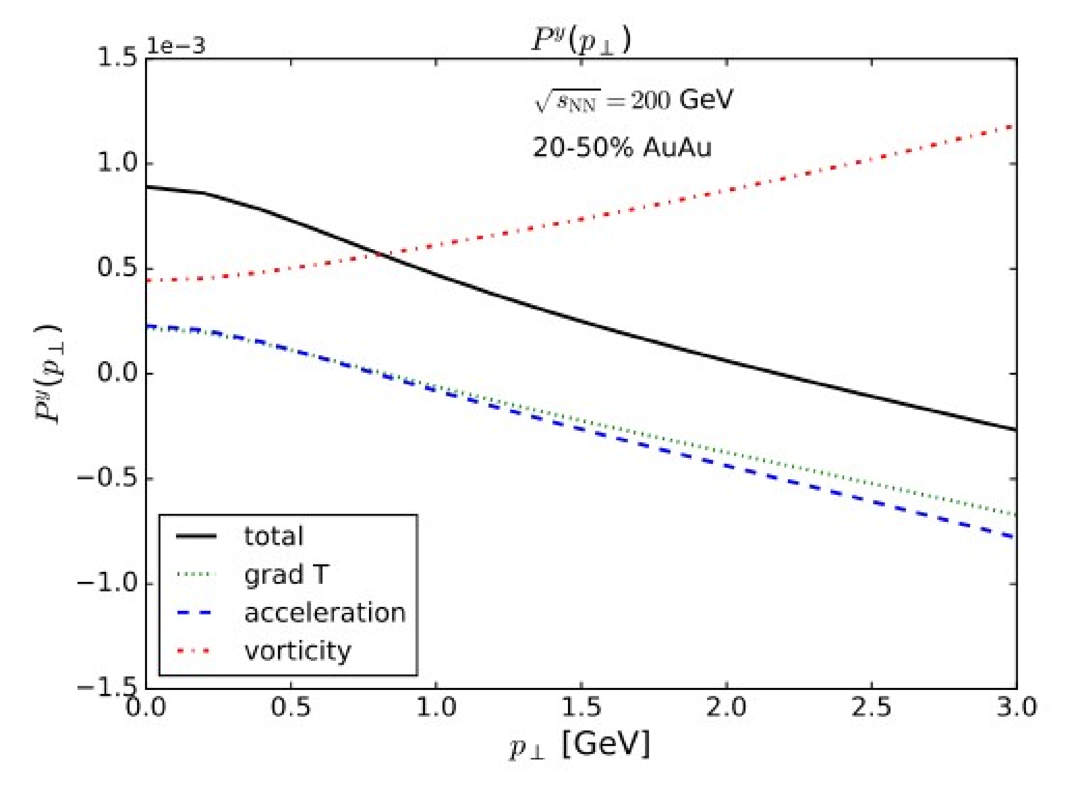
Isothermal local equilibrium

The most appropriate setting for relativistic heavy ion collisions at very high energy!

Both thermal shear and thermal vorticity include temperature gradients

$$\varpi_{\mu\nu} = -\frac{1}{2}\left(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu\right) \qquad \xi_{\mu\nu} = \frac{1}{2}\left(\partial_\mu \beta_\nu + \partial_\nu \beta_\mu\right) \qquad \beta^\mu = (1/T)u^\mu$$

Thermal gradients do contribute to the polarization



$$\mathbf{S}^* \propto \frac{\hbar}{k_B T}\mathbf{u} \times \boldsymbol{\nabla} T + \frac{\hbar}{k_B T}\left(\boldsymbol{\omega} - (\boldsymbol{\omega}\cdot\mathbf{u})\mathbf{u}/c^2\right) + \frac{\hbar}{k_B T}\mathbf{A} \times \mathbf{u}/c^2$$

I. Karpenko, Nucl. Phys. A 982 (2019), 519522, talk given at QM 2018