INTERPRETATION OF A HYPERONS SPIN POLARIZATION MEASUREMENTS

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based on the paper: 2102.02890 [hep-ph]

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ANGULAR MOMENTUM IN HEAVY-ION COLLISIONS

Non-central heavy-ion collisions create systems with large global orbital angular momenta

$$oldsymbol{L}_{
m init} \, \sim 10^5 \hbar$$

F. Becattini, F. Piccinini, J. Rizzo, PRC 77 (2008) 024906

Part of the angular momentum can be transferred from the orbital to the spin sector

$$m{J}_{
m init} = m{L}_{
m init} = m{L}_{
m final} + m{S}_{
m final}$$

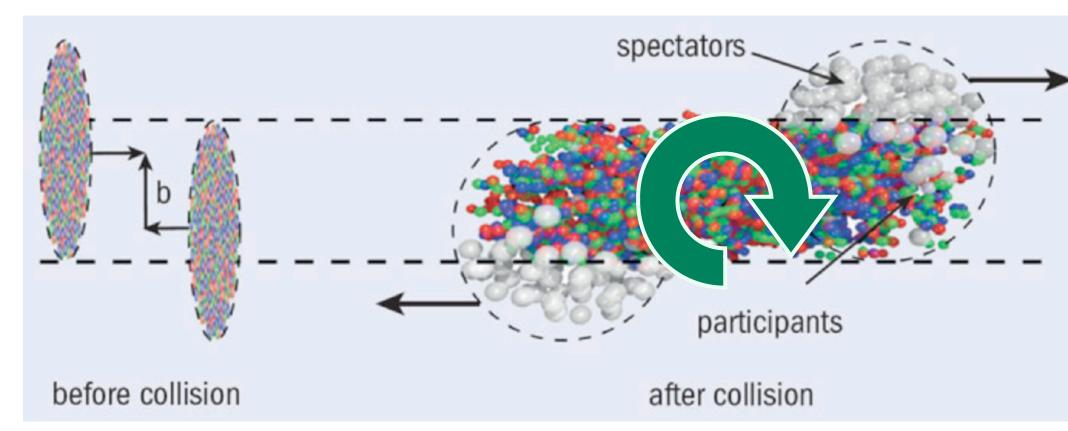
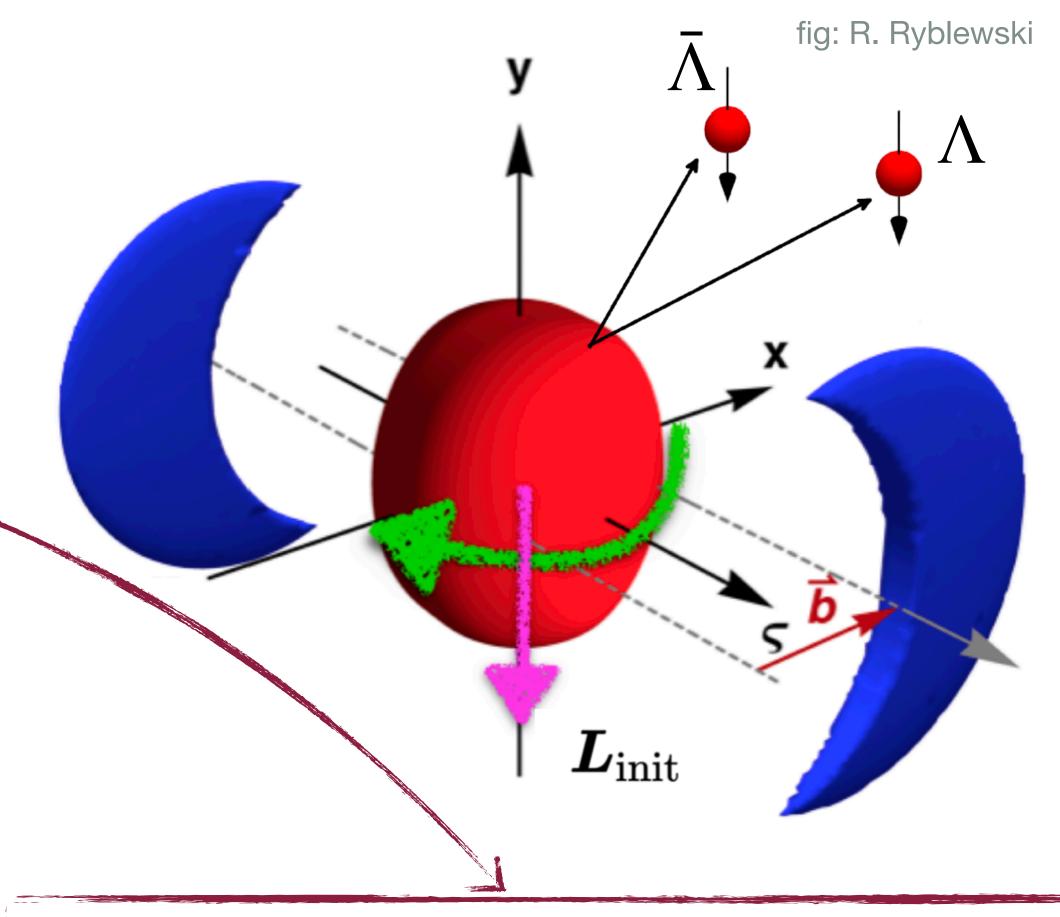


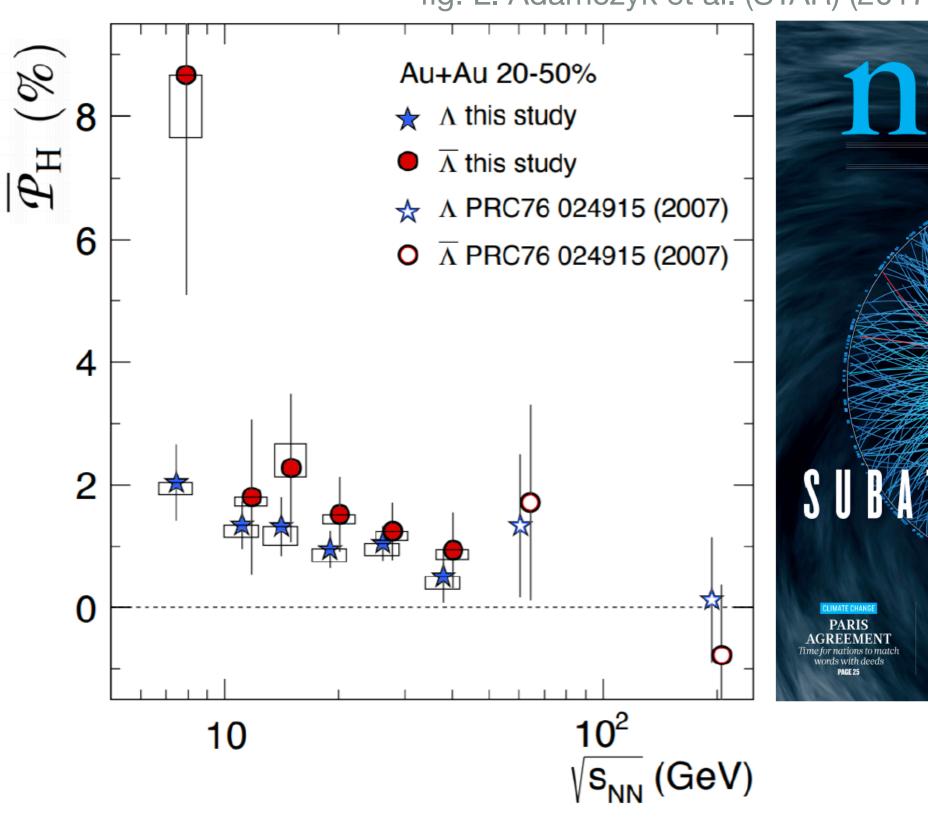
fig: M. Lisa, SQM 2016

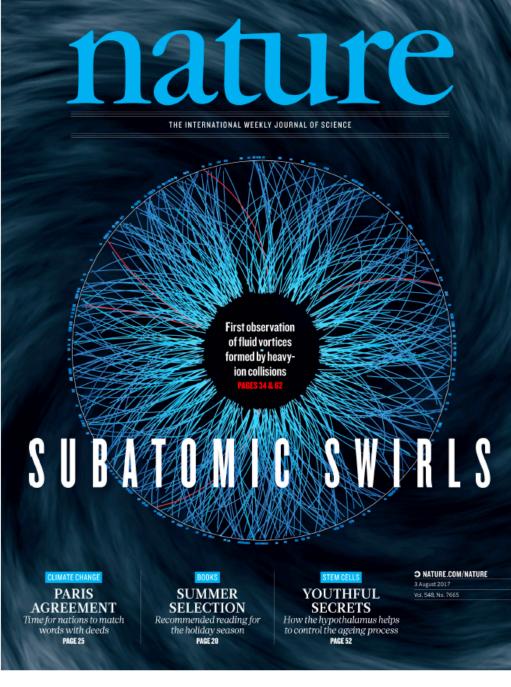


Emitted particles are expected to be polarized along the system's orbital angular momentum

Measurement of Λ and $\bar{\Lambda}$ spin polarization

fig: L. Adamczyk et al. (STAR) (2017), Nature 548 (2017) 62-65





... the hottest, least viscous – and now, most vortical – fluid produced in the laboratory . . .

$$\omega = (P_\Lambda + P_{ar{\Lambda}}) k_B T/\hbar \sim 0.6 - 2.7 imes 10^{22} \ {
m s}^{-1}$$

Self-analysing parity-violating hyperon weak decay allows to measure Λ polarization

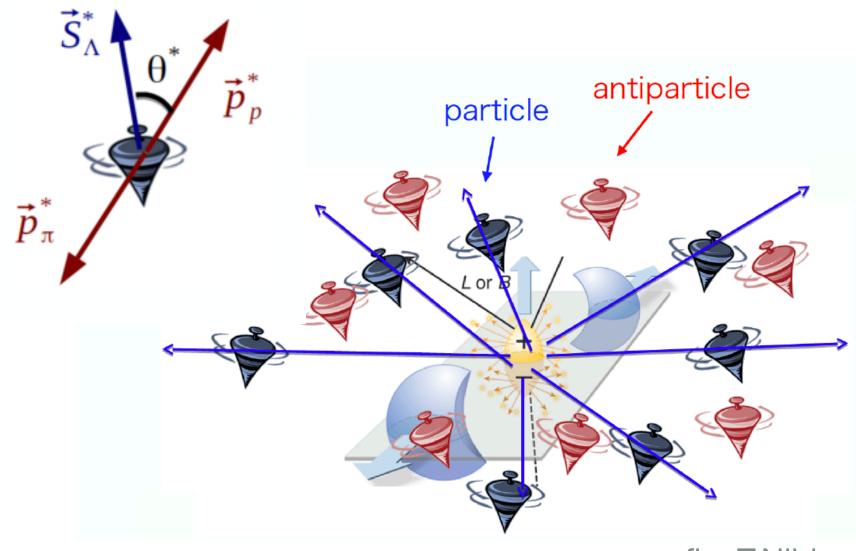
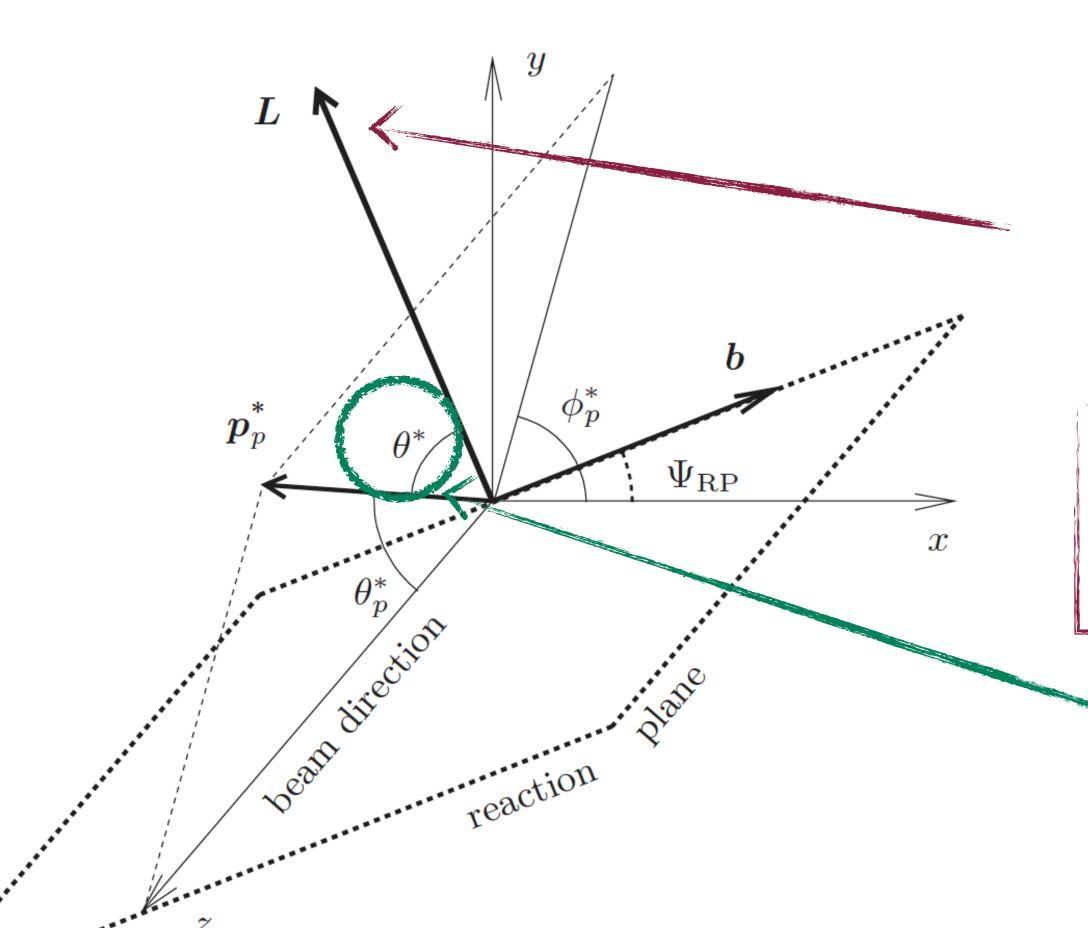


fig: T.Niida

$$rac{dN}{d\Omega^*} = rac{1}{4\pi}ig(1+lpha_{
m H}{f P}_{
m H}\cdot{f p}_{
m p}^*ig)$$

Measurement of Λ and $\bar{\Lambda}$ spin polarization



As the outcome of the spin polarization experiments, one cites the magnitude of the polarization along orbital angular momentum of the system defined in system's center-of-mass (COM) frame

However, to determine the magnitude of the polarization one studies momentum distributions of protons emitted in the weak decay of Lambdas which are measured in the Λ rest frame (Λ RF)

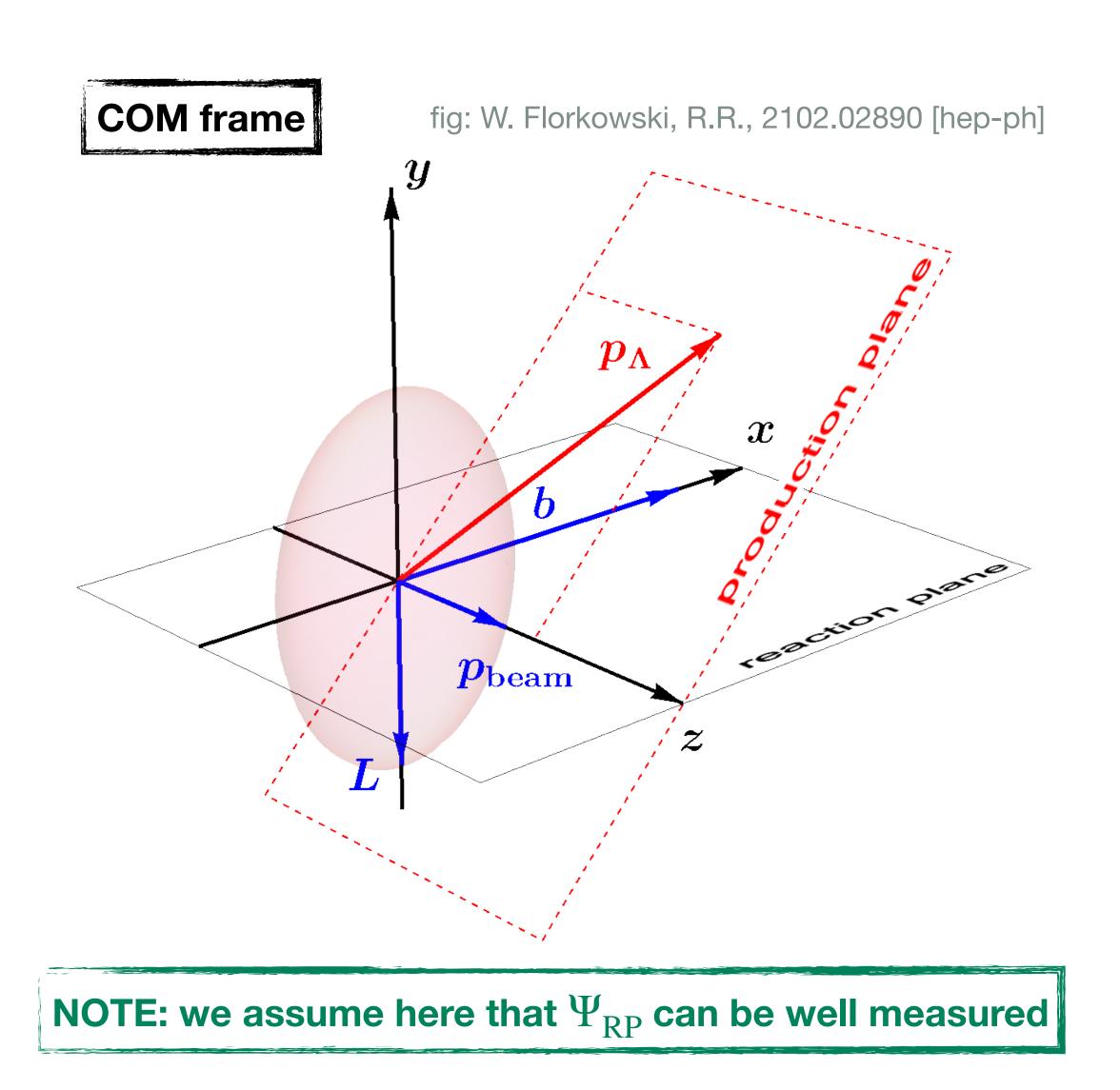
$$\frac{dN}{d\cos\theta^*} \sim 1 + \alpha_H P_H \cos\theta^*$$

These two frames are linked by a non-trivial Lorentz transformation.

Interpretation of the results will depend on it!

fig: B. I. Abelev et al. (STAR) PRC 76, 024915 (2007)

CENTER-OF-MASS FRAME IN HEAVY-ION COLLISION



Transformation from COM to ΛRF

canonical boost

$$\mathcal{L}^{\mu}_{\nu}(-\boldsymbol{v}_{\Lambda}) = \begin{bmatrix} \frac{E_{\Lambda}}{m_{\Lambda}} & -\frac{p_{\Lambda}^{1}}{m_{\Lambda}} & -\frac{p_{\Lambda}^{2}}{m_{\Lambda}} & -\frac{p_{\Lambda}^{3}}{m_{\Lambda}} \\ -\frac{p_{\Lambda}^{1}}{m_{\Lambda}} & 1 + \alpha p_{\Lambda}^{1} p_{\Lambda}^{1} & \alpha p_{\Lambda}^{1} p_{\Lambda}^{2} & \alpha p_{\Lambda}^{1} p_{\Lambda}^{3} \\ -\frac{p_{\Lambda}^{2}}{m_{\Lambda}} & \alpha p_{\Lambda}^{2} p_{\Lambda}^{1} & 1 + \alpha p_{\Lambda}^{2} p_{\Lambda}^{2} & \alpha p_{\Lambda}^{2} p_{\Lambda}^{3} \\ -\frac{p_{\Lambda}^{3}}{m_{\Lambda}} & \alpha p_{\Lambda}^{3} p_{\Lambda}^{1} & \alpha p_{\Lambda}^{3} p_{\Lambda}^{2} & 1 + \alpha p_{\Lambda}^{3} p_{\Lambda}^{3} \end{bmatrix}$$

$$\alpha \equiv 1/\left(m_{\Lambda} \left(E_{\Lambda} + m_{\Lambda}\right)\right)$$

 $p'^{\mu} = \widehat{\mathcal{L}}^{\mu}_{\;\;
u} \left(-oldsymbol{v}_{\Lambda}
ight) p^{
u}.$

COM frame

$$p_{\Lambda}^{\mu}=\left(E_{\Lambda},p_{\Lambda}^{1},p_{\Lambda}^{2},p_{\Lambda}^{3}
ight)$$



 $S'({m p}_{\Lambda})$ Λ rest frame

$$p'^\mu_\Lambda=(m_\Lambda,0,0,0)$$

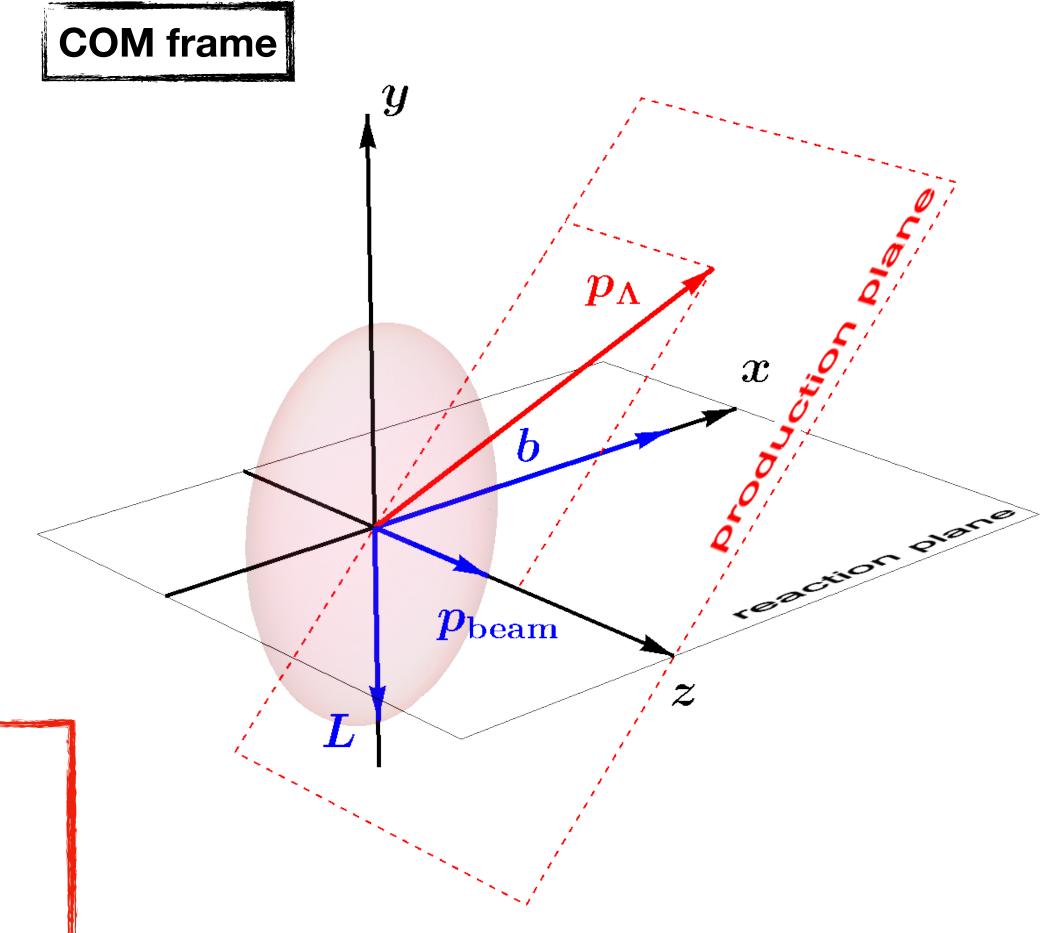


fig: W. Florkowski, R.R., 2102.02890 [hep-ph]

TRANSFORMATION OF ORBITAL ANGULAR MOMENTUM

In non-central heavy-ion collisions, a substantial orbital part is generated at the initial stage

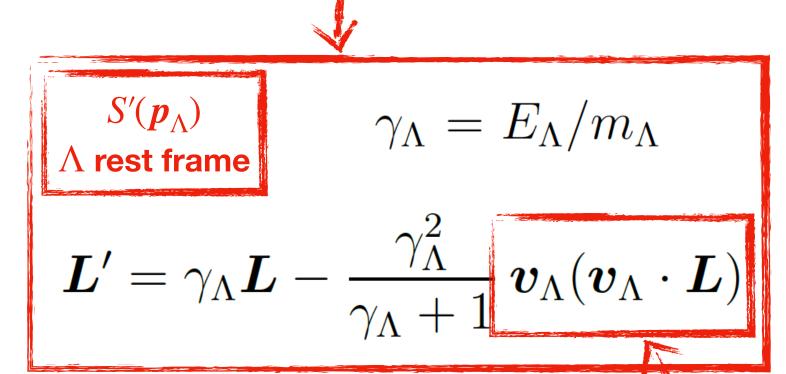
transforms like the spatial components of an antisymmetric tensor

$$J^{\mu\nu} = L^{\mu\nu} + S^{\mu\nu}$$

$$L^{k} = -\frac{1}{2} \epsilon^{kij} L^{ij}$$

$$K^{i} = -L^{0i} = 0$$

 Λ in its rest frame "sees" different direction of angular momentum !



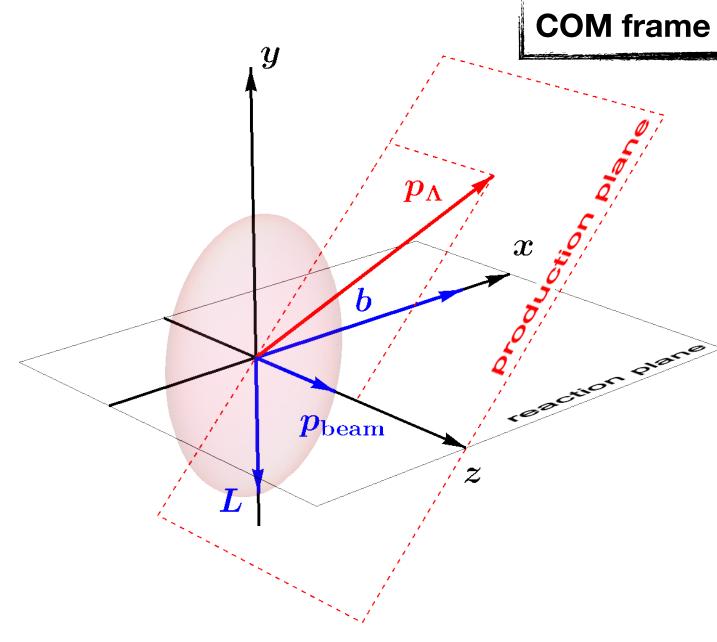


fig: W. Florkowski, R.R., 2102.02890 [hep-ph]

relativistic correction

COM frame

 \boldsymbol{L}

 $\mathbf{K} = 0$



TRANSFORMATION OF ORBITAL ANGULAR MOMENTUM

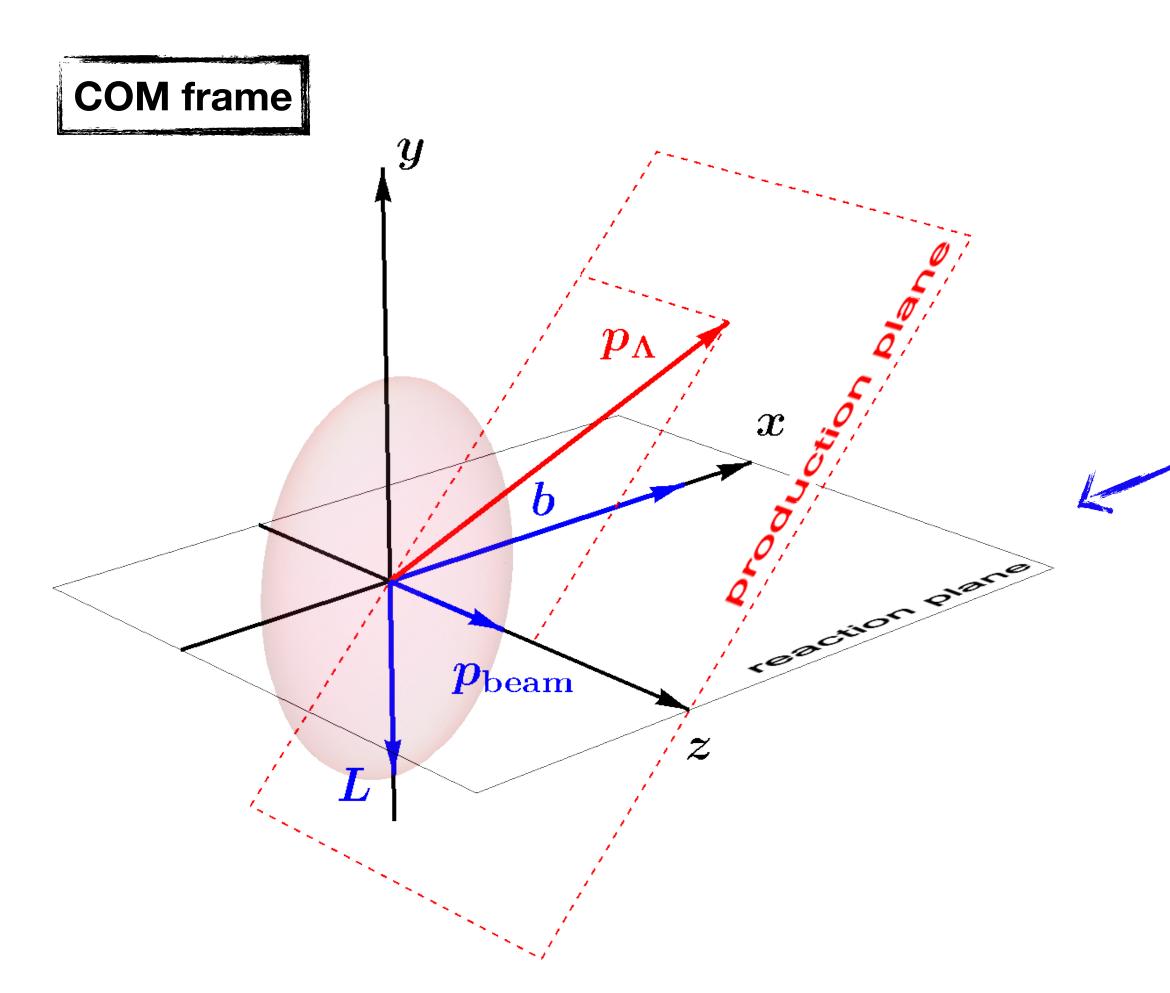
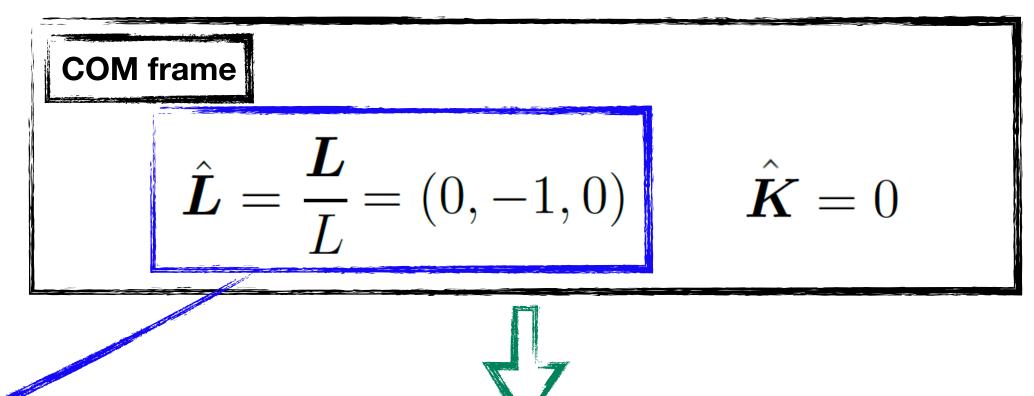


fig: W. Florkowski, R.R., 2102.02890 [hep-ph]



$$S'({m p}_{\Lambda})$$
 Λ rest frame

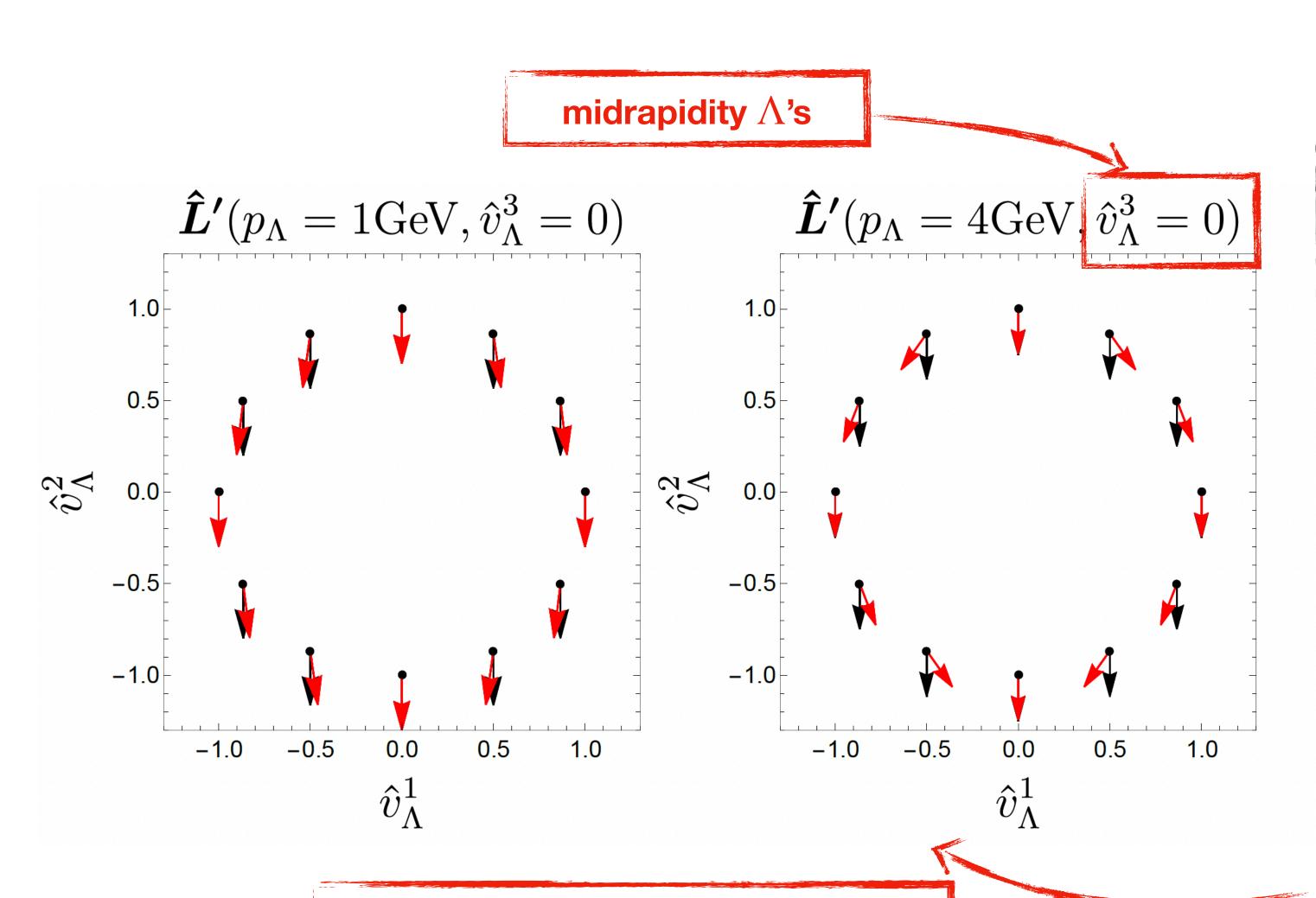
$$\hat{L}'^{1} = \left(1 - (v_{\Lambda}^{2})^{2}\right)^{-1/2} \frac{\gamma_{\Lambda}}{\gamma_{\Lambda} + 1} v_{\Lambda}^{1} v_{\Lambda}^{2},$$

$$\hat{L}'^{2} = \left(1 - (v_{\Lambda}^{2})^{2}\right)^{-1/2} \left(\frac{\gamma_{\Lambda}}{\gamma_{\Lambda} + 1} v_{\Lambda}^{2} v_{\Lambda}^{2} - 1\right)$$

$$\hat{L}'^{3} = \left(1 - (v_{\Lambda}^{2})^{2}\right)^{-1/2} \frac{\gamma_{\Lambda}}{\gamma_{\Lambda} + 1} v_{\Lambda}^{3} v_{\Lambda}^{2}.$$

$$\hat{K}' = \frac{(v_{\Lambda}^{3}, 0, -v_{\Lambda}^{1})}{\sqrt{(v_{\Lambda}^{1})^{2} + (v_{\Lambda}^{3})^{2}}}$$

TRANSFORMATION OF ORBITAL ANGULAR MOMENTUM



change of the system's angular momentum direction due to relativistic effects

COM frame

$$\hat{L} = \frac{L}{L} = (0, -1, 0)$$
 $\hat{K} = 0$



 $S'({m p}_{\Lambda})$ rest frame

$$\hat{L}'^{1} = \left(1 - (v_{\Lambda}^{2})^{2}\right)^{-1/2} \frac{\gamma_{\Lambda}}{\gamma_{\Lambda} + 1} v_{\Lambda}^{1} v_{\Lambda}^{2},$$

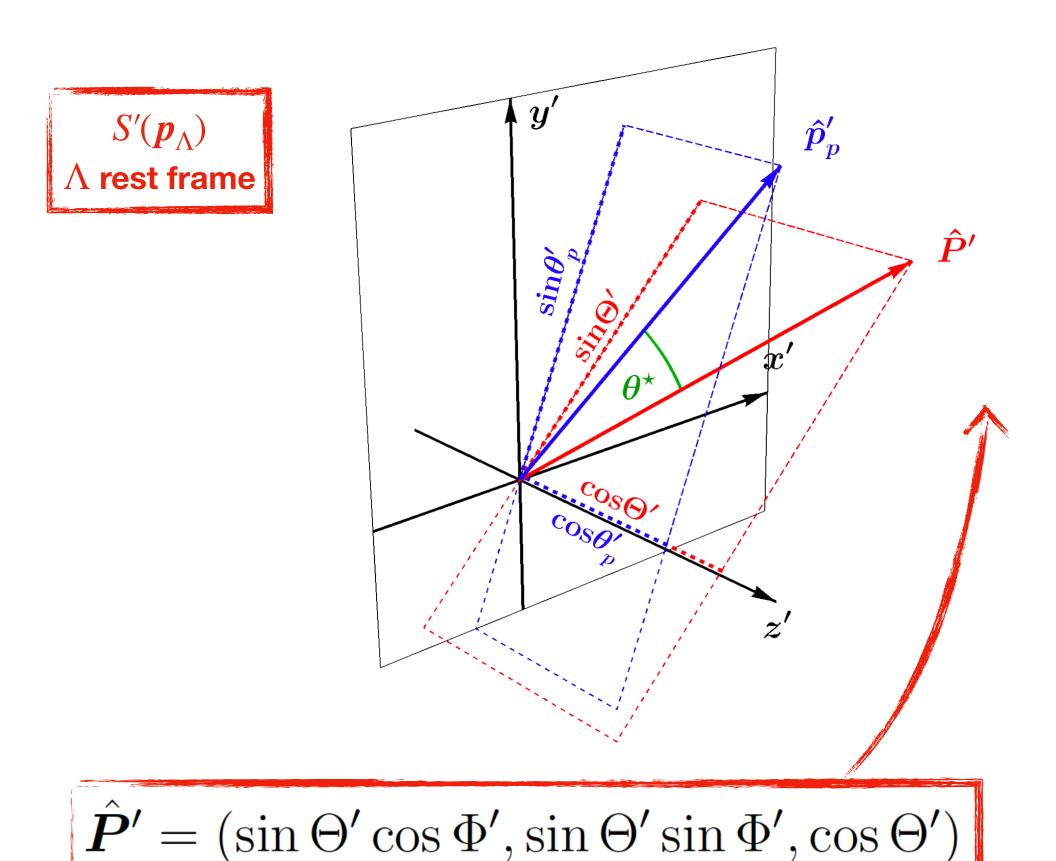
$$\hat{L}'^{2} = \left(1 - (v_{\Lambda}^{2})^{2}\right)^{-1/2} \left(\frac{\gamma_{\Lambda}}{\gamma_{\Lambda} + 1} v_{\Lambda}^{2} v_{\Lambda}^{2} - 1\right)$$

$$\hat{L}'^{3} = \left(1 - (v_{\Lambda}^{2})^{2}\right)^{-1/2} \frac{\gamma_{\Lambda}}{\gamma_{\Lambda} + 1} v_{\Lambda}^{3} v_{\Lambda}^{2}.$$

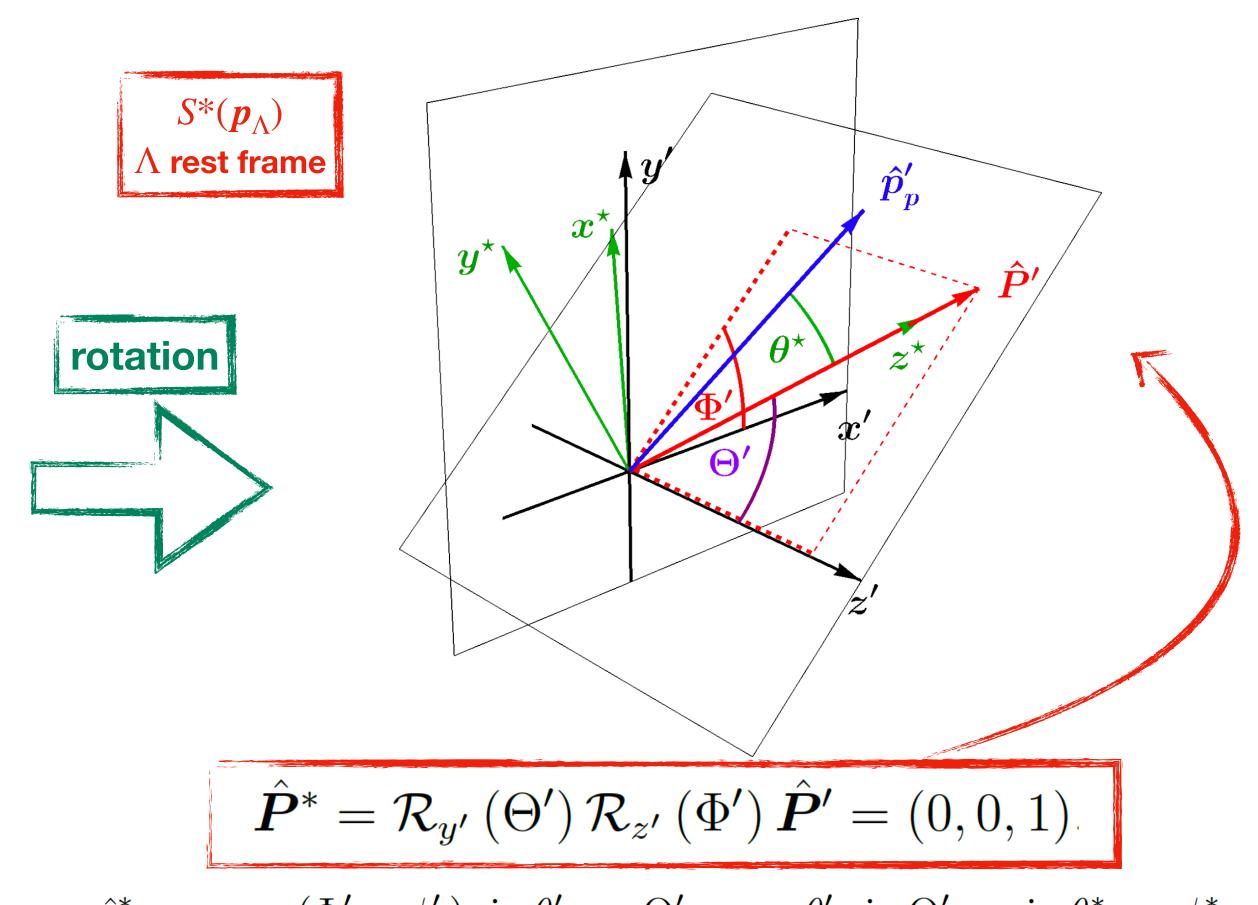
$$\hat{K}' - \frac{(v_{\Lambda}^{3}, 0, -v_{\Lambda}^{1})}{(v_{\Lambda}^{3}, 0, -v_{\Lambda}^{1})}$$

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$S'(p_{\Lambda})$ vs $S^*(p_{\Lambda})$ Λ rest frames and the weak decay



$$\hat{\boldsymbol{p}}_p' = \left(\sin \theta_p' \cos \phi_p', \sin \theta_p' \sin \phi_p', \cos \theta_p'\right)$$



$$\hat{p}_{p,x}^* = \cos(\Phi' - \phi_p')\sin\theta_p'\cos\Theta' - \cos\theta_p'\sin\Theta' \equiv \sin\theta^*\cos\phi^*,$$

$$\hat{p}_{p,y}^* = -\sin(\Phi' - \phi_p')\sin\theta_p' \equiv \sin\theta^*\sin\phi^*,$$

$$\hat{p}_{p,z}^* = \cos(\Phi' - \phi_p')\sin\theta_p'\sin\Theta' + \cos\theta_p'\cos\Theta' \equiv \cos\theta^*.$$

A WEAK DECAY LAW

$$S^*(oldsymbol{p}_{\Lambda})$$
 rest frame

$$\frac{dN_p^{\text{pol}}}{d\Omega^*} = \frac{1}{4\pi} \left(1 + \alpha_{\Lambda} \mathbf{P}^* \cdot \hat{\mathbf{p}}_p^* \right)$$

$$\hat{m{P}}'\cdot\hat{m{p}}'_p=\hat{m{P}}^*\cdot\hat{m{p}}_p^*=\cos heta^*$$

$$S'({m p}_{\Lambda})$$
 Λ rest frame

$$\frac{S'(p_{\Lambda})}{\Lambda \text{ rest frame}} \qquad \frac{dN_p^{\rm pol}}{d\Omega'} = \frac{1}{4\pi} \left[1 + \alpha_{\Lambda} P \left(\cos(\Phi' - \phi_p') \sin \theta_p' \sin \Theta' + \cos \theta_p' \cos \Theta' \right) \right]$$

$$\langle \hat{p}'_{p,x} \rangle = \int \left(\frac{dN_p^{\text{pol}}}{d\Omega'} \right)$$

$$\langle \hat{p}'_{p,y} \rangle = \int \left(\frac{dN_p^{\text{pol}}}{d\Omega'} \right)$$

 $\langle \hat{p}'_{p,x} \rangle = \int \left(\frac{dN_p^{\text{pol}}}{d\Omega'} \right) (\sin \theta'_p)^2 \cos \phi'_p \, d\theta'_p \, d\phi'_p = \frac{1}{3} P' \alpha_{\Lambda} \sin \Theta' \cos \Phi',$ $\langle \hat{p}'_{p,y} \rangle = \int \left(\frac{dN_p^{\text{pol}}}{d\Omega'} \right) (\sin \theta'_p)^2 \sin \phi'_p \, d\theta'_p \, d\phi'_p = \frac{1}{3} P' \alpha_{\Lambda} \sin \Theta' \sin \Phi',$

$$\langle \hat{p}'_{p,z} \rangle = \int \left(\frac{dN_p^{\text{pol}}}{d\Omega'} \right) s$$

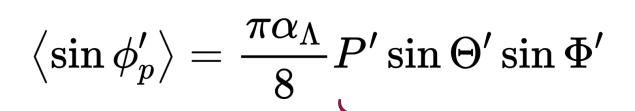
 $\langle \hat{p}'_{p,z} \rangle = \int \left(\frac{dN_p^{\text{pol}}}{d\Omega'} \right) \sin \theta'_p \cos \theta'_p d\theta'_p d\phi'_p = \frac{1}{3} P' \alpha_{\Lambda} \cos \Theta'.$

$$\langle\ldots
angle = \int\!\left(rac{dN_p^{
m pol}}{d\Omega'}
ight)\!(\ldots)\sin heta_p'd heta_p'd\phi_p'$$

the polarization vector can be obtained from the averaged values of the proton three-momentum components measured in $S'(p_{\wedge})$

$$\mathbf{P}' = P' \left(\sin \Theta' \cos \Phi', \sin \Theta' \sin \Phi', \cos \Theta' \right) = \frac{3}{\alpha_{\Lambda}} \left(\langle \hat{p}'_{p,x} \rangle, \langle \hat{p}'_{p,y} \rangle, \langle \hat{p}'_{p,z} \rangle \right)$$

Interpretation of the Λ polarization measurement



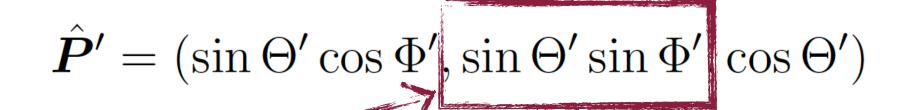
$$P_H = P' \sin \Theta' \sin \Phi'$$

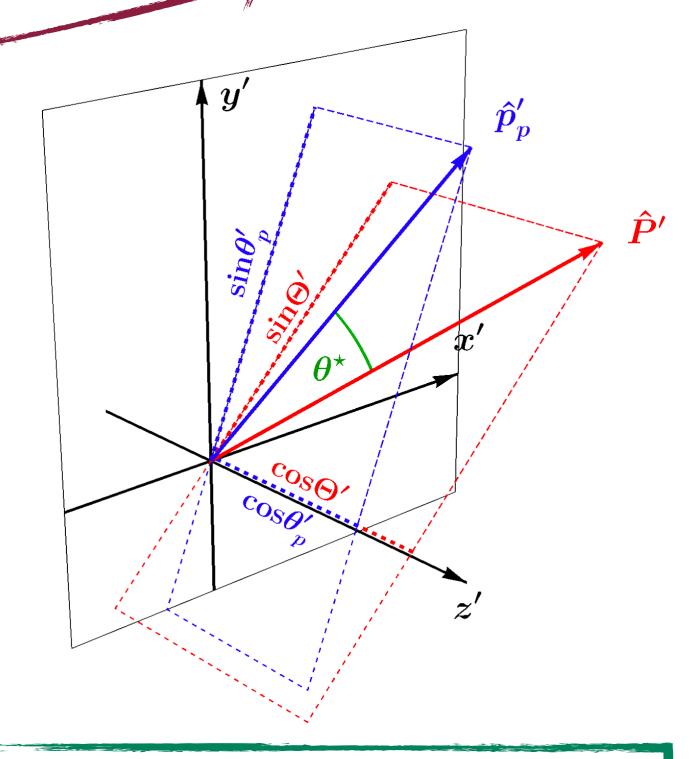
$$P_H = \frac{8}{\pi \alpha_{\Lambda}} \langle \sin \phi_p' \rangle$$

"y" component of the polarization three-vector measured in the ΛRF

not the component of the polarization along the orbital angular momentum, as the "y" directions is COM and $\Lambda \rm RF$ differ

$$\left\langle\cos\phi_p'
ight
angle=rac{\pilpha_\Lambda}{8}P'\sin\Theta'\cos\Phi'$$





it is tempting to measure also mean $\langle\cos\phi_p'\rangle$ — ratio of the two would give us information about the angle Φ'

CORRELATION WITH GLOBAL ANGULAR MOMENTUM

direction of the orbital angular momentum that is "seen" by the spin in the Λ rest frame

$$\hat{\boldsymbol{L}}' = \left(1 - (\boldsymbol{v}_{\Lambda} \cdot \hat{\boldsymbol{L}})^{2}\right)^{-1/2} \left(\hat{\boldsymbol{L}} - \frac{\gamma_{\Lambda}}{\gamma_{\Lambda} + 1} \boldsymbol{v}_{\Lambda} (\boldsymbol{v}_{\Lambda} \cdot \hat{\boldsymbol{L}})\right)$$

components of the polarization vector

$$m{P}' = rac{3}{lpha_{\Lambda}}ig(ig\langle \hat{p}_{p,x}' ig
angle, ig\langle \hat{p}_{p,y}' ig
angle, ig\langle \hat{p}_{p,z}' ig
angleig)$$

projection of the polarization vector along the direction of the global angular momentum

$$\hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' = \left(1 - (\boldsymbol{v}_{\Lambda} \cdot \hat{\boldsymbol{L}})^{2}\right)^{-1/2} \left(\hat{\boldsymbol{L}} \cdot \boldsymbol{P}' - \frac{\gamma_{\Lambda}}{\gamma_{\Lambda} + 1} \boldsymbol{v}_{\Lambda} \cdot \boldsymbol{P}' \ \boldsymbol{v}_{\Lambda} \cdot \hat{\boldsymbol{L}}\right)$$

the advantage of $\hat{L}'\cdot P'$ compared to $\hat{L}\cdot P'$ is that the spin polarization of each Λ irrespectively of its three-momentum in COM, is projected on the same physical axis corresponding to L in COM

ESTIMATES OF RELATIVISTIC EFFECTS

let us assume that

$$P' = P'\hat{L}$$



$$\hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' = P' \left(1 - v_2^2 \right)^{-1/2} \left(1 - \frac{v_2^2}{1 + \sqrt{1 - v^2}} \right) \equiv P' F_P(\boldsymbol{v}).$$

$$v = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

average value for Λ in momentum range (m,n) GeV

$$\langle \hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' \rangle_{m-n} = P' \frac{\int_{v_{(m)}}^{v_{(n)}} dv \int d\Omega F_P(\boldsymbol{v}) F_T(v)}{\int_{v_{(m)}}^{v_{(n)}} dv \int d\Omega F_T(v)}$$

$$\langle \hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' \rangle_{2-3} = 0.97 P'$$

$$\langle \hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' \rangle_{3-4} = 0.94 P'$$

$$\langle \hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' \rangle_{4-5} = 0.92 P'$$

$$\langle \hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' \rangle_{5-6} = 0.90 P'$$

$$F_T(v) = N \left[\exp\left(\frac{m_{\Lambda}}{T_{\text{eff}}\sqrt{1 - v^2}}\right) + 1 \right]^{-1}$$

$$v_{(n)} = \tanh \left[\sinh^{-1} \left(\frac{n \, \text{GeV}}{m_{\Lambda}} \right) \right]$$

$$T_{\rm eff} = 150 \, \mathrm{MeV}$$

relativistic effects may reach 10% for the most energetic Λ studied at STAR

INCLUDING REACTION PLANE DEPENDENCE

$$\Psi_{\mathrm{RP}} \neq 0$$



$$\hat{\boldsymbol{L}} = -\left(\cos\left(\Psi_{\mathrm{RP}} + \pi/2\right), \sin\left(\Psi_{\mathrm{RP}} + \pi/2\right), 0\right)$$

$$\hat{\boldsymbol{L}} \cdot \boldsymbol{P}' = -\frac{8}{\pi \alpha_{\Lambda}} \langle \sin(\phi_p' - \Psi_{\text{RP}}) \rangle = -\frac{8}{\pi \alpha_{\Lambda}} \int \left(\frac{dN_p^{\text{pol}}}{d\Omega'} \right) \sin\left(\phi_p' - \Psi_{\text{RP}}\right) d\Omega'$$

explicit dependence of the results on the reaction plane angle which is not directly measured

$$egin{aligned} \left\langle \sin\!\left(\phi_p' - \Psi_{ ext{EP}}^{(1)}
ight)
ight
angle_{ ext{ev.}} &= \left\langle \sin\!\left(\phi_p' - \Psi_{ ext{RP}} - \left(\Psi_{ ext{EP}}^{(1)} - \Psi_{ ext{RP}}
ight)
ight)
ight
angle_{ ext{ev}} &\equiv \left\langle \sin\!\left(\phi_p' - \Psi_{ ext{RP}}
ight)
ight
angle_{ ext{ev}} R_{ ext{EP}}^{(1)} \ & \left\langle \cos\Delta\Psi
ight
angle_{ ext{ev.}} &\equiv R_{ ext{EP}}^{(1)} & \Delta\Psi &\equiv \Psi_{ ext{EP}}^{(1)} - \Psi_{ ext{RP}}
ight
angle & \left\langle \sin\Delta\Psi
ight
angle_{ ext{ev.}} &= 0 \end{aligned}$$

$$\langle \hat{\boldsymbol{L}} \cdot \boldsymbol{P}' \rangle_{\text{ev.}} = -\frac{8}{\pi \alpha_{\Lambda}} \langle \sin(\phi_p' - \Psi_{\text{RP}}) \rangle_{\text{ev.}} = -\frac{8}{\pi \alpha_{\Lambda}} \frac{\langle \sin(\phi_p' - \Psi_{\text{EP}}^{(1)}) \rangle_{\text{ev.}}}{R_{\text{EP}}^{(1)}}$$

INCLUDING REACTION PLANE DEPENDENCE

$$\hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' = \left(1 - (\boldsymbol{v}_{\Lambda} \cdot \hat{\boldsymbol{L}})^{2}\right)^{-1/2} \left(\hat{\boldsymbol{L}} \cdot \boldsymbol{P}' - \frac{\gamma_{\Lambda}}{\gamma_{\Lambda} + 1} \boldsymbol{v}_{\Lambda} \cdot \boldsymbol{P}' \ \boldsymbol{v}_{\Lambda} \cdot \hat{\boldsymbol{L}}\right)$$

$$\hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' = \hat{\boldsymbol{L}} \cdot \boldsymbol{P}' - \frac{\gamma_{\Lambda}}{\gamma_{\Lambda} + 1} \boldsymbol{v}_{\Lambda} \cdot \boldsymbol{P}' \ \boldsymbol{v}_{\Lambda} \cdot \hat{\boldsymbol{L}} + \frac{1}{2} (\boldsymbol{v}_{\Lambda} \cdot \hat{\boldsymbol{L}})^{2} \hat{\boldsymbol{L}} \cdot \boldsymbol{P}'$$

$$\langle \boldsymbol{v}_{\Lambda} \cdot \boldsymbol{P}' \ \boldsymbol{v}_{\Lambda} \cdot \hat{\boldsymbol{L}} \rangle_{\text{ev.}} = \frac{8}{\pi \alpha_{\Lambda}} \langle \cos(\phi_{\Lambda} - \phi'_{p}) \sin(\phi_{\Lambda} - \Psi_{\text{RP}}) \rangle_{\text{ev.}}$$
$$\frac{\langle \cos(\phi_{\Lambda} - \phi'_{p}) \sin(\phi_{\Lambda} - \Psi_{\text{EP}}^{(1)}) \rangle_{\text{ev.}}}{\langle \cos(\phi_{\Lambda} - \phi'_{p}) \cos \Delta \Psi \rangle_{\text{ev.}}}$$

INCLUDING REACTION PLANE DEPENDENCE

$$\hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' = \left(1 - (\boldsymbol{v}_{\Lambda} \cdot \hat{\boldsymbol{L}})^{2}\right)^{-1/2} \left(\hat{\boldsymbol{L}} \cdot \boldsymbol{P}' - \frac{\gamma_{\Lambda}}{\gamma_{\Lambda} + 1} \boldsymbol{v}_{\Lambda} \cdot \boldsymbol{P}' \ \boldsymbol{v}_{\Lambda} \cdot \hat{\boldsymbol{L}}\right)$$

$$\hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' = \hat{\boldsymbol{L}} \cdot \boldsymbol{P}' - \frac{\gamma_{\Lambda}}{\gamma_{\Lambda} + 1} \boldsymbol{v}_{\Lambda} \cdot \boldsymbol{P}' \quad \boldsymbol{v}_{\Lambda} \cdot \hat{\boldsymbol{L}} + \frac{1}{2} (\boldsymbol{v}_{\Lambda} \cdot \hat{\boldsymbol{L}})^{2} \hat{\boldsymbol{L}} \cdot \boldsymbol{P}'$$

$$\langle (\boldsymbol{v}_{\Lambda} \cdot \hat{\boldsymbol{L}})^2 \hat{\boldsymbol{L}} \cdot \boldsymbol{P}' \rangle_{\text{ev.}} = -\frac{8}{\pi \alpha_{\Lambda}} \langle \sin^2(\phi_{\Lambda} - \Psi_{\text{RP}}) \sin(\phi_p' - \Psi_{\text{RP}}) \rangle_{\text{ev.}}$$

$$\langle \sin^2(\phi_{\Lambda} - \Psi_{\rm EP}^{(1)}) \sin(\phi_p' - \Psi_{\rm EP}^{(1)}) \rangle_{\rm ev.} = M_1 \langle \cos \Delta \Psi \cos(2\Delta \Psi) \rangle_{\rm ev.}$$

 $+ (M_2 + M_3) \langle \cos \Delta \Psi \sin^2 \Delta \Psi \rangle_{\rm ev.}$

$$\langle \sin(2(\phi_{\Lambda} - \Psi_{\rm EP}^{(1)})) \cos(\phi_p' - \Psi_{\rm EP}^{(1)}) \rangle_{\rm ev.} = (M_2 + 2(M_3 - 2M_1)) \langle \cos \Delta \Psi \cos(2\Delta \Psi) \rangle_{\rm ev.}$$
$$-2(M_3 - 2M_1) \langle \cos^3 \Delta \Psi \rangle_{\rm ev.},$$

$$M_1 = \langle \sin^2(\phi_{\Lambda} - \Psi_{\rm RP}) \sin(\phi_p' - \Psi_{\rm RP}) \rangle_{\rm ev.}$$

 $M_2 = \langle \sin(2(\phi_{\Lambda} - \Psi_{\rm RP})) \cos(\phi_p' - \Psi_{\rm RP}) \rangle_{\rm ev.},$

$$M_3 = \langle \sin(\phi_p' - \Psi_{\rm RP}) \rangle_{\rm ev.}$$

SUMMARY

We have discussed the interpretation of the spin polarization measurements in relativistic heavy-ion collisions

We have shown that the appropriate interpretation of the relation between the spin direction (measured in the Λ RF) and the orbital angular momentum of the system (measured in the COM frame) requires that the direction of the angular momentum is boosted to the Λ RF

We have given the necessary formula that may be used to average the measured polarization of Λ with different momenta in the COM frame

THANK YOU FOR YOUR ATTENTION!