Shear-Induced Polarization & Spin Hall Effects in heavy-ion collisions

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Spin Hall Effects: arXiv: 2201.12970
Global polarization

- **Spin-orbital coupling** in non-central heavy ion collisions

- **Signals observed at STAR BES energy:**
  

- **Data described by the statistic calculation**

\[
S^\mu(p) \leftarrow \omega_{\nu p}(x)
\]

**Hydrodynamics:**

BF, K. Xu, X-G, Huang, H. Song, Phys.Rev.C 103 (2021) 2, 024903

**Transport model:**

local polarization: ‘Sign puzzle’

- Different trend/sign in $P_y(\phi)$ and $P_z(\phi)$ results
- Long exist in hydrodynamic and transport calculations

See also:

Karpenko, Becattini, EPJC 77 (2017) 4, 213
D. Wei, et al., PRC 99 (2019) 014905
X. Xia, et al., PRC 98 (2018) 024905
Becattini, Karpenko, PRL 120 (2018) 012302
I. Shear Induced Polarization (SIP)

---- toward solving the local polarization puzzle

BF, S. Liu, L.-G. Pang, H. Song and Y. Yin
Shear Induced Polarization (SIP) by BF, S. Liu, L.-G. Pang, H. Song, Y. Yin, Phys.Rev.Lett. 127 14, 142301(2021)

Axial Wigner function from CKT (Chen, Son, Stephanov, PRL 115 (2015) 2, 021601)

\[ \mathcal{A}^\mu = \sum_\lambda \left( \lambda p^\mu f_\lambda + \frac{1}{2} \frac{\epsilon^{\mu\nu\alpha\rho} p_\nu u_\alpha \partial_\rho f_\lambda}{p \cdot u} \right) \]

Expand \( \mathcal{A}^\mu \) to 1\(^{st} \) order gradient of the fields:

\[ \mathcal{A}^\mu = \frac{1}{2} \beta n_0 (1 - n_0) \left\{ \epsilon^{\mu\nu\alpha\lambda} p_\nu \partial_\alpha u_\lambda + 2 \epsilon^{\mu\nu\alpha\lambda} u_\nu p_\alpha [\beta^{-1}(\partial_\lambda \beta)] - 2 \frac{p^2_\perp}{\varepsilon_0} \epsilon^{\mu\nu\alpha\rho} u_\nu Q_\alpha^\lambda \sigma_\rho^\lambda \right\} \]

- \( \sigma^{\mu\nu} \): shear stress tensor (symmetric)
- No free parameter
- Identical form by linear response theory with arbitrary mass (S. Liu and Y. Yin, JHEP 07 (2021) 188)

\[ Q^{\mu\nu} = - p_\perp^\mu p_\perp^\nu / p_\perp^2 + \Delta^{\mu\nu} / 3 \]

\[ \sigma^{\mu\nu} = \frac{1}{2} (\partial_\perp^\mu u_\nu + \partial_\perp^\nu u_\mu) - \frac{1}{3} \Delta^{\mu\nu} \partial_\perp \cdot u \]
Shear Induced Polarization (SIP)

Axial Wigner function from CKT (Chen, Son, Stephanov, PRL 115 (2015) 2, 021601)

\[ A^\mu = \sum_\lambda \left( \lambda p^\mu f_\lambda + \frac{1}{2} \epsilon^{\mu\nu\alpha\rho} p_\nu u_\alpha \partial_\rho f_\lambda \right) \]

Expand \( A^\mu \) to 1st order gradient of the fields:

\[ A^\mu = \frac{1}{2} \beta n_0 (1 - n_0) \left\{ \epsilon^{\mu\nu\alpha\lambda} p_\nu \partial_\alpha u_\lambda + 2 \epsilon^{\mu\nu\alpha\lambda} u_\nu p_\alpha [\beta^{-1} (\partial_\lambda \beta)] - \frac{2 p_\perp^2}{\varepsilon_0} \epsilon^{\mu\nu\alpha\rho} u_\nu Q_\alpha \lambda \sigma_{\rho\lambda} \right\} \]

Vorticity

T gradient (spin Nernst effect)

Shear-Induced Polarization

Total \( P^\mu = [\text{Vorticity}] + [\text{T gradient}] + [\text{Shear}] \)
Shear Induced Polarization (SIP)

Expand $A^\mu$ to first order gradient of the fields:

$$A^\mu = \frac{1}{2} \beta n_0 (1 - n_0) \left\{ e^{\mu\nu\alpha\lambda} p_{\nu} \partial_\alpha (\beta u)_{\lambda} + 2 e^{\mu\nu\alpha\lambda} u_{\nu} p_{\lambda} \left[ \beta^{-1} (\partial_\lambda \beta) \right] - \frac{p_{\perp}^2}{\varepsilon_0} e^{\mu\nu\alpha\lambda} u_{\nu} Q_{\alpha} \sigma_{\lambda} \right\}$$

- **Vorticity**
- **T gradient** (spin Nernst effect)
- **Shear-Induced Polarization**

To one-loop order (in charge neutral fluid)

Thermal vorticity

$$\omega_{\mu \nu} = \frac{1}{2} (\partial_\nu (\beta u_\mu) - \partial_\mu (\beta u_\nu))$$

$$\rho = \frac{p_{\perp}^2}{\varepsilon_0} e^{\mu\nu\alpha\lambda} u_{\nu} Q_{\alpha} \sigma_{\lambda}$$

Total $P^\mu = [\text{Vorticity}] + [\text{T gradient}] + [\text{Shear}]$
Shear Induced Polarization (SIP)

Axial Wigner function

To one-loop order (in charge neutral fluid)

\[ \mathcal{A}^\mu = \frac{1}{2} \beta n_0 (1 - n_0) \left\{ \epsilon^{\mu \nu \alpha \lambda} p_\nu \partial_\alpha (\beta u)_\lambda \right. \\
\left. + 2 \epsilon^{\mu \nu \alpha \lambda} u_\nu p_\alpha [\beta^{-1} (\partial \beta)] - \frac{2 \nu^2}{\varepsilon_0} \epsilon^{\mu \nu \alpha \rho} u_\nu q_\alpha \lambda \right\} \]

Vorticity

T gradient (spin Nernst effect)

Shear-Induced Polarization

Thermal vorticity

\[ \omega_{\mu \nu} = \frac{1}{2} (\partial_\nu (\beta u_\mu) - \partial_\mu (\beta u_\nu)) \]

Total \( P^\mu = [\text{Vorticity}] + [\text{T gradient}] + [\text{Shear}] \)

Total \( P^\mu = [\text{Thermal vorticity}] + [\text{Shear}] \)

See also: Kumar@Tues. & Buzzegoli@Tues.

The only new effect
‘Λ equilibrium’ vs. ‘S-quark memory’

\[ P^\mu(p) = \frac{\int d\Sigma^\alpha p_\alpha A^\mu(x, p; m)}{2m \int d\Sigma^\alpha p_\alpha n(\beta \varepsilon_0)} \]

Spin Cooper-Frye:

‘Λ equilibrium’
\[ \tau_{\text{spin}, \Lambda} \to 0 \]
Polarization of Λ-hyperon
\[ P^\mu_\Lambda(p) \]
F. Becattini (2013)
and later hydrodynamic (transport) calculations

‘S-quark memory’
\[ \tau_{\text{spin}, \Lambda} \to \infty \]
Polarization of S-quark
\[ P^\mu_\Lambda(p) = P^\mu_S(p) \]

Competition of $P_z$: Grad T vs. SIP

Total $P^\mu = [\text{vorticity}] + [\text{T grad}] + [\text{SIP}]$

- [SIP]: " + $\sin(2\phi)"$ structure for $P_z$ (same as exp.)
- Total polarization: a competition between [SIP] and [Grad T]

BF, S. Liu, L. -G. Pang, H. Song, Y. Yin, PRL 127 14, 142301(2021)
\[ P_z(\phi) \text{ with SIP} \]


Total \( P^\mu = [\text{vorticity}] + [T \text{ grad}] + [\text{SIP}] \)

\[ = [\text{thermal vorticity}] + [\text{SIP}] \]

- In the scenario of ‘S-quark memory’, the total \( P^\mu \) with SIP qualitatively agrees with data
\[ P_y(\phi) \text{ with SIP} \]


Total \( P^\mu \) = [vorticity] + [T grad] + [SIP]
= [thermal vorticity] + [SIP]

- In the scenario of ‘S-quark memory’, the total \( P^\mu \) with SIP qualitatively agrees with data
From RHIC to LHC

Same hydrodynamic model: AMPT + MUSIC
(LHC parameter from EPJC 77 (2017) 9, 645)

- “Strange Memory” scenario qualitatively describes the centrality & $p_T$ dependence
- More precise model needed to quantitative description
The 3rd order Fourier coefficient of $P_z$ is comparable to $f_2$ in both Au+Au and Ru+Ru systems. Spin polarization also probes the initial state fluctuations.

$$f_n = \langle P_z \sin[n(\phi - \Psi_n)] \rangle = \frac{\int p_T dp_T d\phi dy \int p \cdot d\sigma A(x,p) \sin[n(\phi - \Psi_n)]}{\int p_T dp_T d\phi dy 2m\int p \cdot d\sigma f(x,p)}$$

- Non-zero $f_3$ is comparable to $f_2$ in both Au+Au and Ru+Ru systems.
- Spin polarization also probes the initial state fluctuations.
II. Spin Hall Effects (SHE) at RHIC-BES

How about with finite $\mu_B$?

$$A^\mu(x, p) = \beta f_0(x, p)(1 - f_0(x, p))\varepsilon^{\mu\nu\sigma\rho} \times \left( \frac{1}{2} p_\nu \partial_\alpha u_\rho - \frac{1}{T} u_\nu p_\alpha \partial_\rho T - \frac{p_\perp^2}{\varepsilon_0} u_\nu Q_\alpha \sigma_\rho \lambda - \frac{q_B}{\varepsilon_0 \beta} u_\nu p_\alpha \partial_\rho (\beta \mu_B) \right),$$

- vorticity
- T-gradient
- SIP
- SHE
Spin Hall Effects (SHE)

**In condensed-matter**

- Transverse spin current induced by spin-orbital coupling under external electric field
  \[ \vec{s} \propto \vec{p} \times \vec{E} \]
- Probes transport properties in quantum materials with theory under QED
- Has been observed in semiconductors, metal and insulators at room temperature or below

**In hot QCD matter**

- With similar form, replacing electric field \( \vec{E} \) to baryon chemical potential gradient \( \vec{\nabla} \mu_B \)
  \[ \vec{p}_\pm \propto \pm \vec{p} \times \vec{\nabla} \mu_B \]
- Thermal vorticity
- Shear-Induced Polarization
  S. Liu and Y. Yin, JHEP 2021, BF, et al., PRL 2021
  F. Becattini, et al., PLB 2021, PRL 2021
- Spin Hall Effects (SHE)
- Another mechanism for spin generation under QCD
- Probes the properties of QCD matter at extremely high temperature (\(~10^{12}\)K)
Spin Hall Effects (SHE)  


Axial Wigner function $\mathcal{A}^\mu$ expansion with finite chemical potential:

$$\mathcal{A}^\mu(x, p) = \beta f_0(x, p)(1 - f_0(x, p))\varepsilon^{\mu\nu\alpha\rho} \times \left( \frac{1}{2} p_\nu \partial_{\alpha}^\perp u_\rho - \frac{1}{T} u_\nu p_\alpha \partial_\rho T - \frac{p_\perp^2}{\varepsilon_0} u_\nu Q^\lambda_{\alpha} \sigma^\rho_{\lambda} - \frac{q_B}{\varepsilon_0\beta} u_\nu p_\alpha \partial_\rho (\beta \mu_B) \right),$$

- Induced by $\mu_B$ gradient: more important at RHIC-BES
- Spin current generation: search SHE signal in differential observables like $P^\mu(\phi)$
- Opposite contribution for particles / anti-particles

Spin Cooper-Frye type formula:

- "\(\Lambda\) equilibrium" scenario
- "Strange memory" scenario

Same hydrodynamic model: AMPT + MUSIC

BF, K. Xu, X-G. Huang, H. Song, Phys.Rev.C 103 (2021) 2, 024903

See also: S.Ryu, et al., PRC 104 (2021) 5, 054908 (Global effect)  
S. Liu and Y. Yin, PRD 104 (2021) 5, 054043 (B-W model)
Individual contributions to $P_z(\phi)$ and $P_y(\phi)$

Total $P^\mu = [\text{vorticity}] + [T \ \text{grad}] + [\text{SIP}] + [\text{SHE}]$

$\vec{P}_{\text{SHE}} \propto \pm \vec{p} \times \vec{\nabla} \mu_B$

- **SHE**: “$\sin(2\phi)$” on $P_z$ & “$\cos(2\phi)$” on $P_y$
- The magnitude of SHE is comparable to other effects
- Opposite SHE for particles and anti-particles
Net spin polarization: $P_{\text{net}}(\phi)$

The ‘net’ spin polarization used to extract SHE signals

Net $P_z(\phi)$: increase with decreasing collision energy

Net $P_y(\phi)$: non-monotonic behavior from SHE
The 2nd order Fourier coeff. of $P_z^{\text{net}}(\phi)$ & $P_y^{\text{net}}(\phi)$

**Possible obsv. in exp.**

From the distribution function

$$f(x, p) = \left( e^{(\varepsilon_0 - q_B \mu_B)\beta} + 1 \right)^{-1}$$

**Monotonic increasing**
The 2\textsuperscript{nd} order Fourier coeff. of $P_z^\text{net}(\phi)$ & $P_y^\text{net}(\phi)$

From the distribution function

$$f(x,p) = \left(e^{(\epsilon_0 - q_B \mu_B)\beta} + 1\right)^{-1}$$

Possible obsv. in exp. Au+Au, 20-50%

Separation induced by SHE

Possible obsv. in exp. Au+Au, 20-50%

Monotonic increasing

\begin{itemize}
  \item $\langle P_z^\text{net} \sin(2\phi) \rangle$
\end{itemize}

BF, L.-G. Pang, H. Song, Y. Yin
arXiv: 2201.12970
The 2nd order Fourier coeff. of $P_z^{\text{net}}(\phi)$ & $P_y^{\text{net}}(\phi)$

- Monotonic increasing
- Non-monotonic behavior

Possible obsv. in exp.  
\[ \langle P_z^{\text{net}} \sin(2\phi) \rangle \]

Possible obsv. in exp.  
\[ \langle P_y^{\text{net}} \cos(2\phi) \rangle \]

BF, L.-G. Pang, H. Song, Y. Yin  
arXiv: 2201.12970
Summary

Spin Hall Effects: arXiv: 2201.12970

Total $P^\mu = [\text{vorticity}] + [\text{Grad T}] + [\text{SIP}] + [\text{SHE}]$

Shear-Induced Polarization

"Strange memory" + Shear-Induced Polarization

Describes $P_z(\phi)$ and $P_y(\phi)$ qualitatively at top RHIC and LHC

Spin Hall Effects

$\vec{P}_\pm \propto \pm \vec{p} \times \vec{\nabla} \mu_B$

- Particle – Anti-particle separation
- Relevant for RHIC-BES and RHIC/LHC forward rapidity
- Scenario independent
Back up
Hydrodynamic gradients

Derivatives of the velocity field:

\[ \partial_\mu u_\nu(x) \]

Anti-symmetric: vorticity

\[ \omega^\mu = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} u_\nu \partial_\alpha u_\beta \]

Symmetric: shear stress

\[ \sigma^{\mu\nu} = \frac{1}{2} (\partial_\perp u_\nu + \partial_\perp u_\mu) - \frac{1}{3} \Delta^{\mu\nu} \partial_\perp \cdot u \]

In heavy-ion, condensed matter ...

Spin polarization

[Strain induced polarization]

In crystal physics:

Crooker and Smith, PRL (2005) 94, 236601

will be discussed in this talk
Total $P_z(\phi)$ and $P_y(\phi)$ with SHE

\[ \text{Total } P^\mu = [\text{vorticity}] + [\text{Grad } T] + [\text{SIP}] + [\text{SHE}] \]

\[ \vec{P}_{\text{SHE}} \propto \pm \vec{\rho} \times \vec{\nabla} \mu_B \]

- Separation between particles and anti-particles by SHE

- Different local polarization w/o SHE:
  - Change the space-time of emitted particles
  - Pauli blocking

  O. Vitiuk, et al., PLB 2020
  R-H. Fang, et al., PRC 2016

- Scenario independent
- Do not use EoS-s95p-PCE widely used in hydro calculations!

NEoS:

S95p-v1: