

Shear-Induced Polarization & Spin Hall Effects in heavy-ion collisions

Baochi Fu

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with S. Liu, L.-G. Pang, H. Song and Y. Yin

Shear-Induced Polarization: Phys.Rev.Lett. 127 14, 142301(2021)

Spin Hall Effects: arXiv: 2201.12970

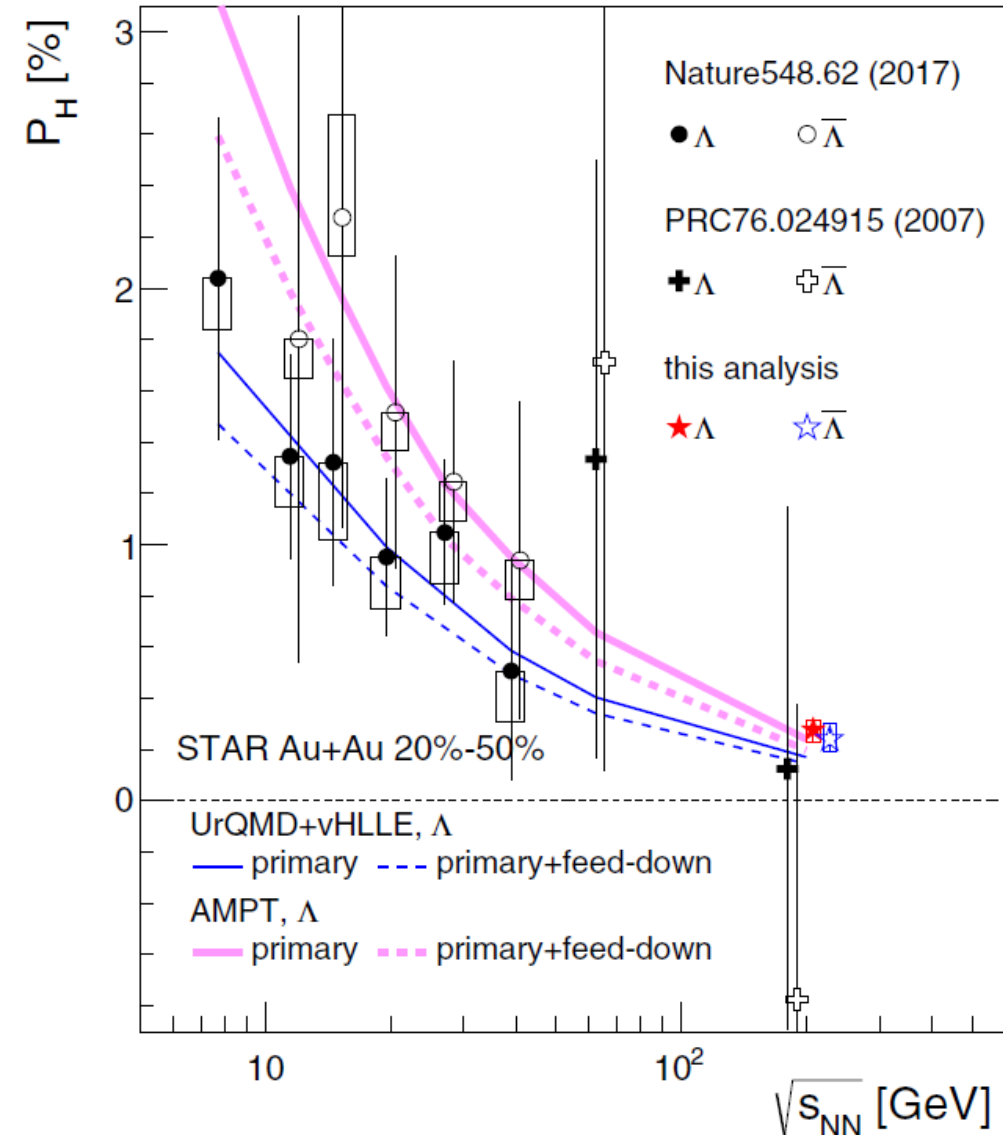


北京大學
PEKING UNIVERSITY



Global polarization

STAR, Phys.Rev.C 98 (2018) 014910



- Spin-orbital coupling in non-central heavy ion collisions
- Signals observed at STAR BES energy:
[STAR Collaboration, Nature 548, 62 \(2017\)](#)
- Data described by the statistic calculation

$$S^\mu(p) \leftarrow \varpi_{\nu\rho}(x)$$

Hydrodynamics:

I. Karpenko, F. Becattini, Eur.Phys.J.C 77 (2017) 4, 213

BF, K. Xu, X-G, Huang, H. Song, Phys.Rev.C 103 (2021) 2, 024903

Transport model:

H. Li, L. Pang, Q. Wang, X. Xia, Phys.Rev. C96 (2017) 054908

D. Wei, W. Deng, X. Huang, Phys.Rev. C99 (2019) 014905

local polarization: 'Sign puzzle'

- Different trend/sign in $P_y(\phi)$ and $P_z(\phi)$ results
- Long exist in hydrodynamic and transport calculations

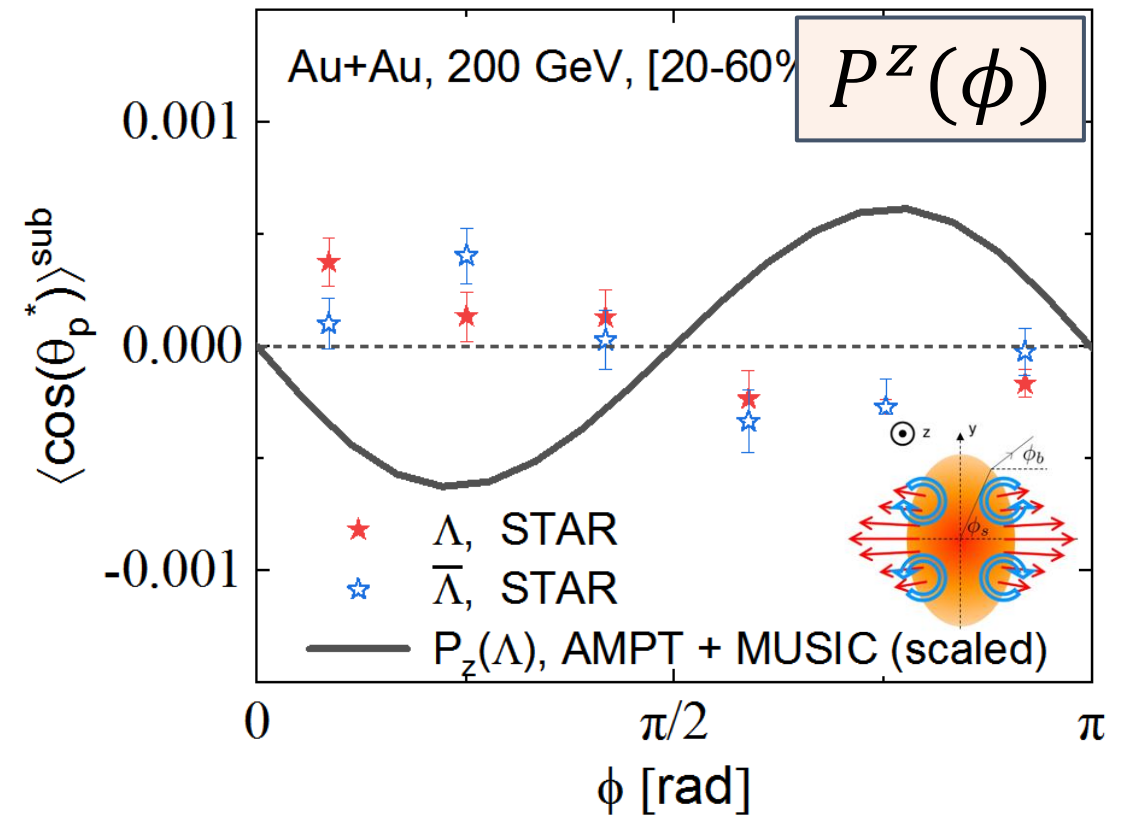
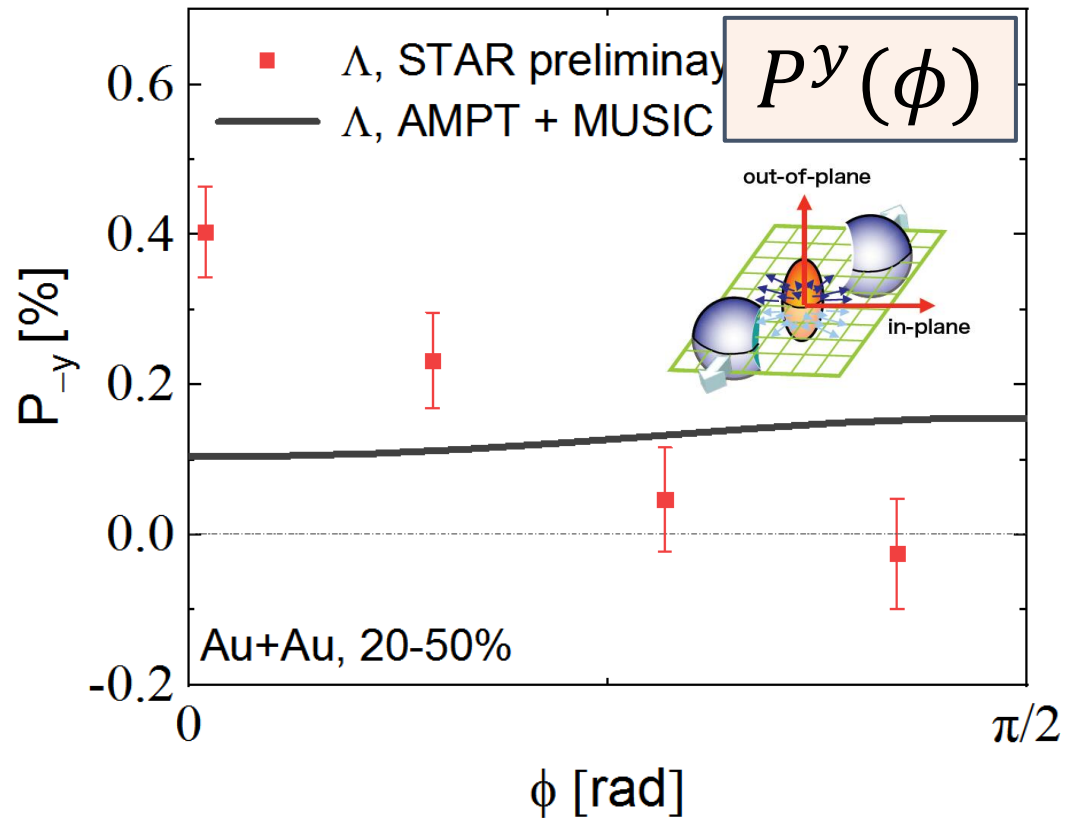
See also:

Karpenko, Becattini, EPJC 77 (2017) 4, 213

D. Wei, et al., PRC 99 (2019) 014905

X. Xia, et al., PRC 98 (2018) 024905

Becattini, Karpenko, PRL 120 (2018) 012302



I. Shear Induced Polarization (SIP)

---- toward solving the local polarization puzzle

BF, S. Liu, L.-G. Pang, H. Song and Y. Yin
Phys.Rev.Lett. 127 14, 142301(2021)

Shear Induced Polarization (SIP)

BF, S. Liu, L. -G. Pang, H. Song, Y. Yin,
Phys.Rev.Lett. 127 14, 142301(2021)

Axial Wigner function from CKT ([Chen, Son, Stephanov, PRL 115 \(2015\) 2, 021601](#))

$$\mathcal{A}^\mu = \sum_\lambda \left(\lambda p^\mu f_\lambda + \frac{1}{2} \frac{\epsilon^{\mu\nu\alpha\rho} p_\nu u_\alpha \partial_\rho f_\lambda}{p \cdot u} \right)$$

Expand \mathcal{A}^μ to 1st order gradient of the fields:

$$\mathcal{A}^\mu = \frac{1}{2} \beta n_0 (1 - n_0) \left\{ \underbrace{\epsilon^{\mu\nu\alpha\lambda} p_\nu \partial_\alpha^\perp u_\lambda}_{\text{Vorticity}} + \underbrace{2\epsilon^{\mu\nu\alpha\lambda} u_\nu p_\alpha [\beta^{-1}(\partial_\lambda \beta)]}_{\text{T gradient (spin Nernst effect)}} - \underbrace{2 \frac{p_\perp^2}{\epsilon_0} \epsilon^{\mu\nu\alpha\rho} u_\nu Q_\alpha^\lambda \sigma_{\rho\lambda}}_{\text{Shear-Induced Polarization}} \right\}$$

- $\sigma^{\mu\nu}$: shear stress tensor (symmetric)
- No free parameter
- Identical form by linear response theory

$$Q^{\mu\nu} = -p_\perp^\mu p_\perp^\nu / p_\perp^2 + \Delta^{\mu\nu} / 3$$

$$\sigma^{\mu\nu} = \frac{1}{2} (\partial_\perp^\mu u^\nu + \partial_\perp^\nu u^\mu) - \frac{1}{3} \Delta^{\mu\nu} \partial_\perp \cdot u$$

with **arbitrary mass** ([S. Liu and Y. Yin, JHEP 07 \(2021\) 188](#))

Shear Induced Polarization (SIP)

BF, S. Liu, L. -G. Pang, H. Song, Y. Yin,
Phys.Rev.Lett. 127 14, 142301(2021)

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$$\text{Total } P^\mu = [\text{Vorticity}] + [\text{T gradient}] + [\text{Shear}]$$

Shear Induced Polarization (SIP)

BF, S. Liu, L. -G. Pang, H. Song, Y. Yin,
Phys.Rev.Lett. 127 14, 142301(2021)

Axial Wigner fu

To one-loop order (in charge neutral fluid)

$$\epsilon^{\mu\nu\alpha\lambda} p_\nu \partial_\alpha (\beta u)_\lambda$$

Thermal vorticity

$$\varpi_{\mu\nu} = \frac{1}{2} (\partial_\nu (\beta u_\mu) - \partial_\mu (\beta u_\nu))$$

Expand \mathcal{A}^μ to

$$\mathcal{A}^\mu = \frac{1}{2} \beta n_0 (1 - n_0) \left\{ \epsilon^{\mu\nu\alpha\lambda} p_\nu \partial_\alpha^\perp u_\lambda + 2 \epsilon^{\mu\nu\alpha\lambda} u_\nu p_\alpha [\beta^{-1} (\partial_\lambda \beta)] - 2 \frac{p_\perp^2}{\epsilon_0} \epsilon^{\mu\nu\alpha\rho} u_\nu Q_\alpha^\lambda \sigma_{\rho\lambda} \right\}$$

Vorticity

T gradient
(spin Nernst effect)

Shear-Induced Polarization

$$\text{Total } P^\mu = [\text{Vorticity}] + [\text{T gradient}] + [\text{Shear}]$$

Shear Induced Polarization (SIP)

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Phys.Rev.Lett. 127 14, 142301(2021)

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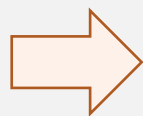
$$\mathcal{A}^\mu = \frac{1}{2} \beta n_0 (1 - n_0) \left\{ \epsilon^{\mu\nu\alpha\lambda} p_\nu \partial_\alpha^\perp u_\lambda + 2 \epsilon^{\mu\nu\alpha\lambda} u_\nu p_\alpha [\beta^{-1} (\partial_\lambda \beta)] - 2 \frac{p_\perp^2}{\epsilon_0} \epsilon^{\mu\nu\alpha\rho} u_\nu Q_\alpha^\lambda \sigma_{\rho\lambda} \right\}$$

Vorticity

T gradient
(spin Nernst effect)

Shear-Induced Polarization

$$\text{Total } P^\mu = [\text{Vorticity}] + [\text{T gradient}] + [\text{Shear}]$$



$$\text{Total } P^\mu = [\text{Thermal vorticity}] + [\text{Shear}]$$

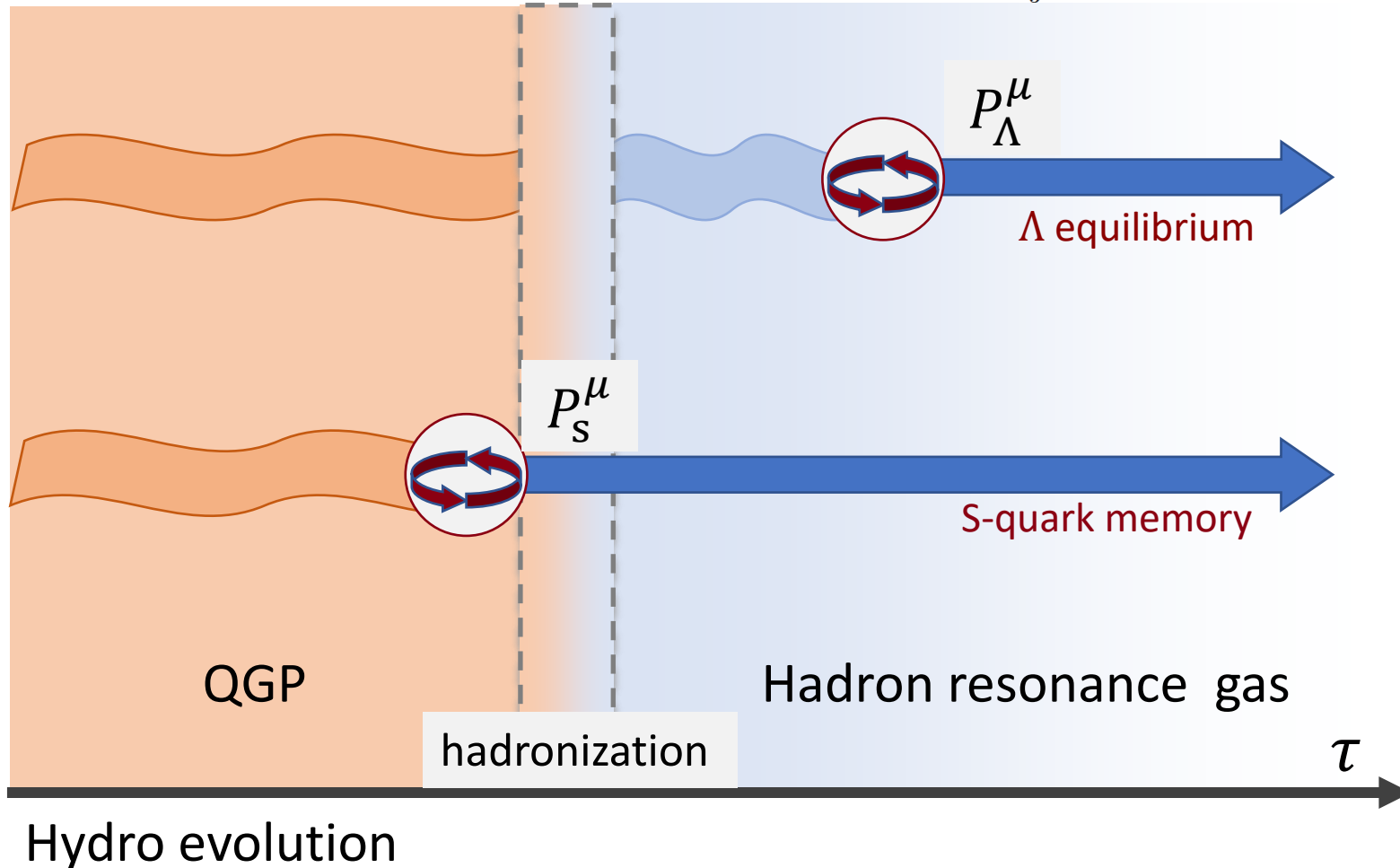
See also: Kumar@Tues. & Buzzegoli@Tues.

The only new effect

' Λ equilibrium' vs. 'S-quark memory'

BF, S. Liu, L. -G. Pang, H. Song, Y. Yin,
Phys.Rev.Lett. 127 14, 142301(2021)

Spin Cooper-Frye:
$$P^\mu(\mathbf{p}) = \frac{\int d\Sigma^\alpha p_\alpha \mathcal{A}^\mu(x, \mathbf{p}; m)}{2m \int d\Sigma^\alpha p_\alpha n(\beta\varepsilon_0)}$$



' Λ equilibrium'

$\tau_{\text{spin}, \Lambda} \rightarrow 0$

Polarization of Λ -hyperon

$P_\Lambda^\mu(p)$

F. Becattini (2013)
and later hydrodynamic(transport) calculations

'S-quark memory'

$\tau_{\text{spin}, \Lambda} \rightarrow \infty$

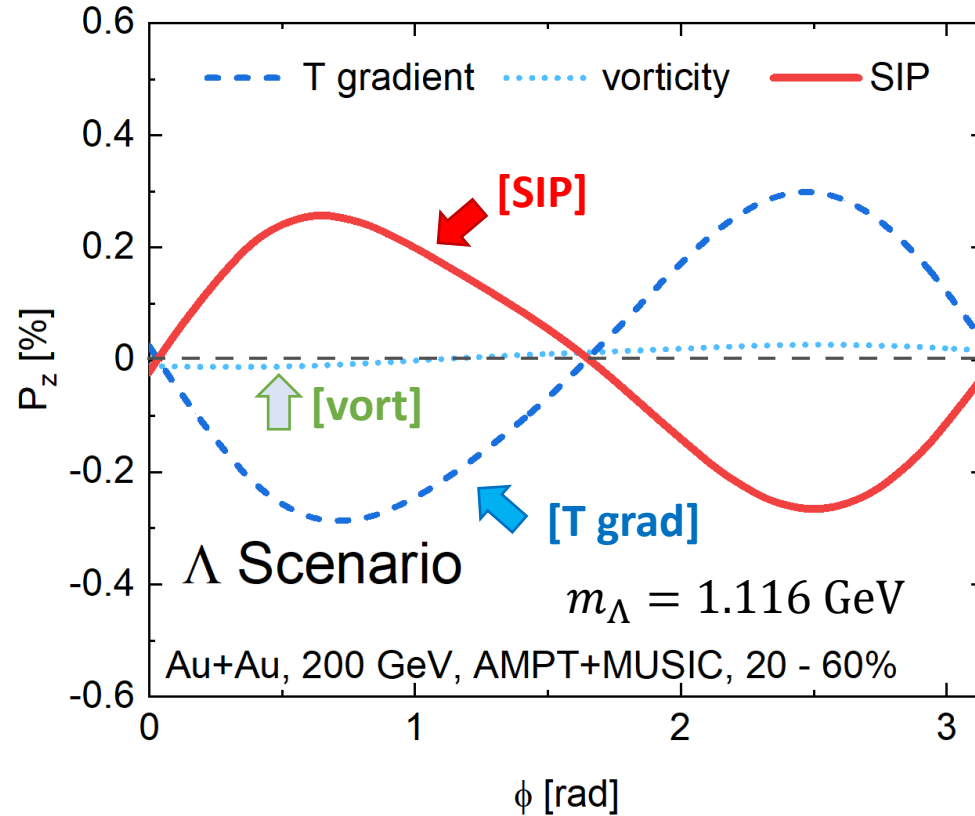
Polarization of S-quark

$P_\Lambda^\mu(p) = P_S^\mu(p)$

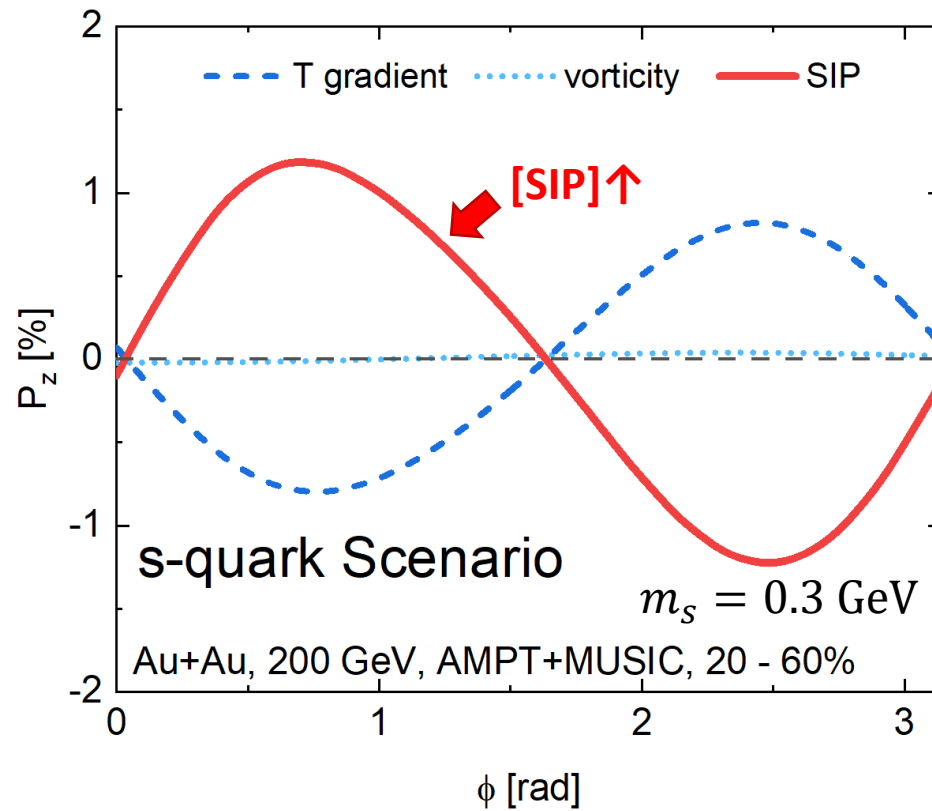
Z.-T. Liang, X.-N. Wang, PRL 94 (2005) 102301

Competition of P_z : Grad T vs. SIP

BF, S. Liu, L. -G. Pang, H. Song, Y. Yin, PRL 127 14, 142301(2021)



$$\text{Total } P^\mu = [\text{vorticity}] + [\text{T grad}] + [\text{SIP}]$$



Competition between:

$$\text{T-grad: } \epsilon^{\mu\nu\alpha\lambda} u_\nu p_\alpha [\beta^{-1} \partial_\lambda \beta]$$

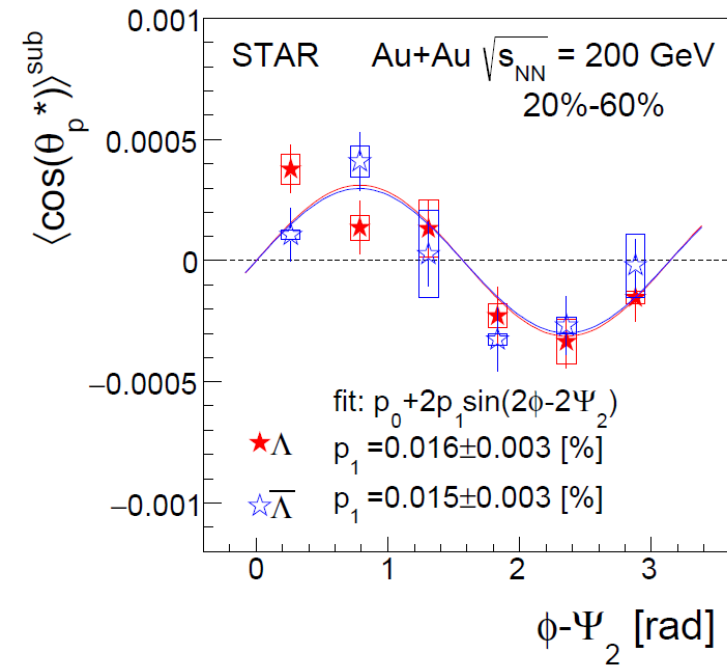
$$\text{Shear: } \epsilon^{\mu\nu\alpha\rho} u_\nu p_\rho \left(\frac{p^\lambda}{\epsilon_0} \right) \partial_{(\alpha} u_{\lambda)}$$

- [SIP]: " + sin(2φ)" structure for P_z (same as exp.)
- Total polarization: a competition between [SIP] and [Grad T]

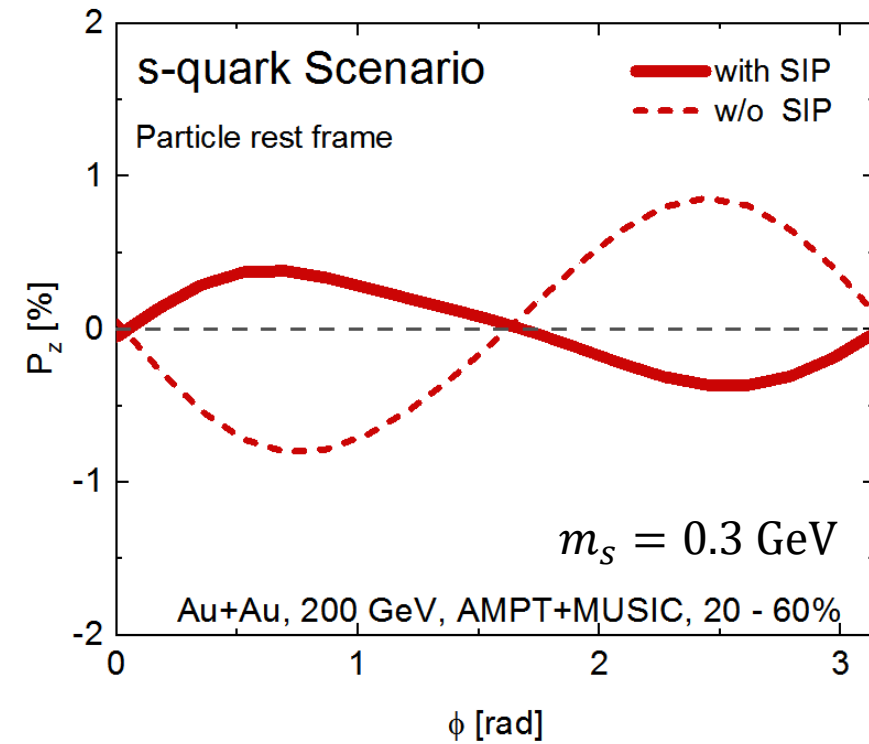
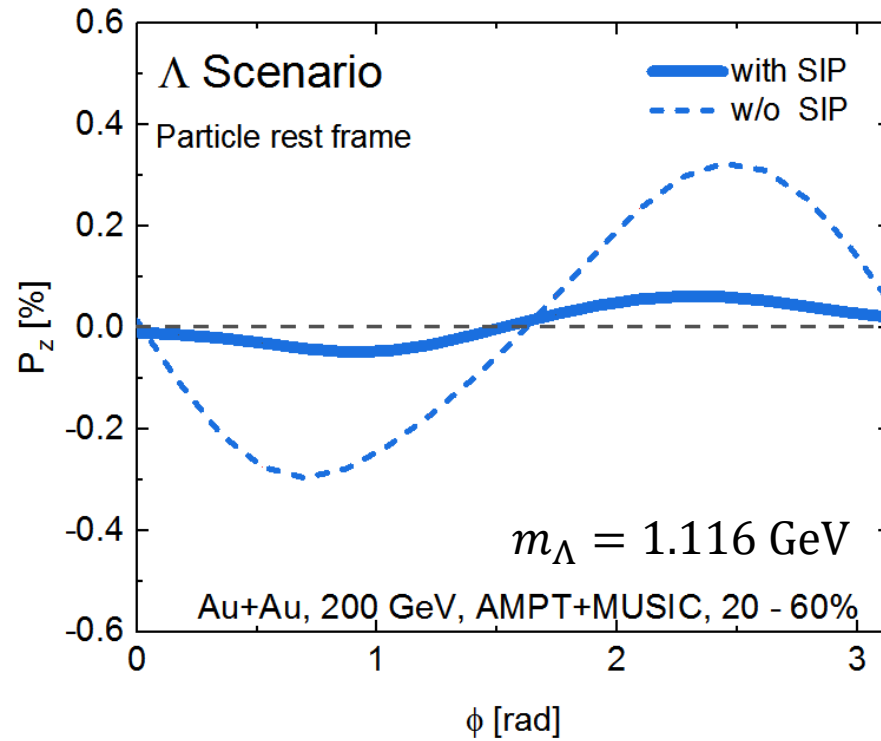
$P_z(\phi)$ with SIP

BF, S. Liu, L. -G. Pang, H. Song, Y. Yin,
Phys.Rev.Lett. 127 14, 142301(2021)

$$\begin{aligned} \text{Total } P^\mu &= [\text{vorticity}] + [\text{T grad}] + [\text{SIP}] \\ &= [\text{thermal vorticity}] + [\text{SIP}] \end{aligned}$$



STAR, PRL 123 (2019) 132301



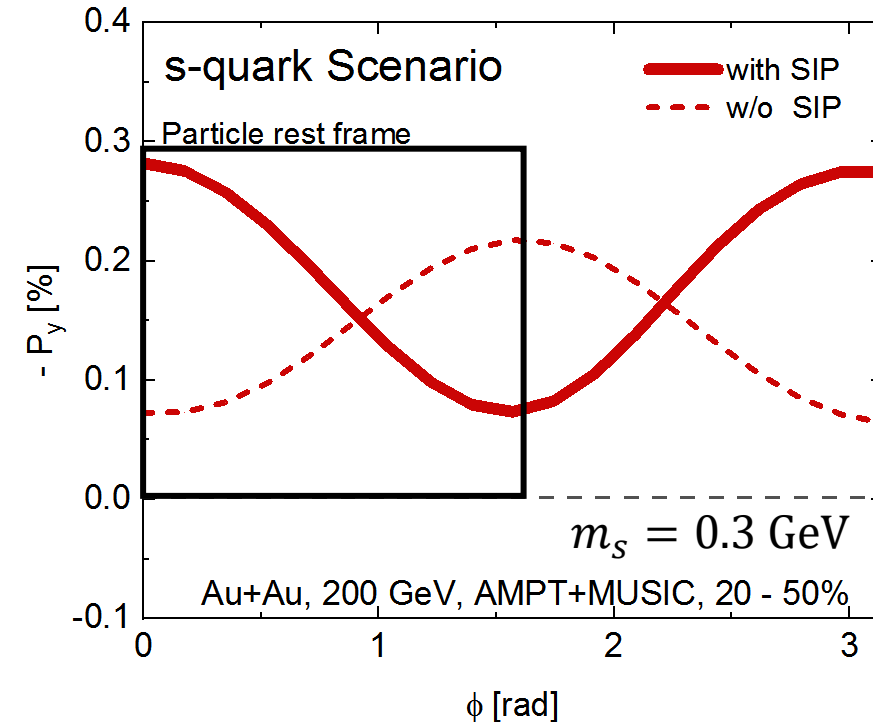
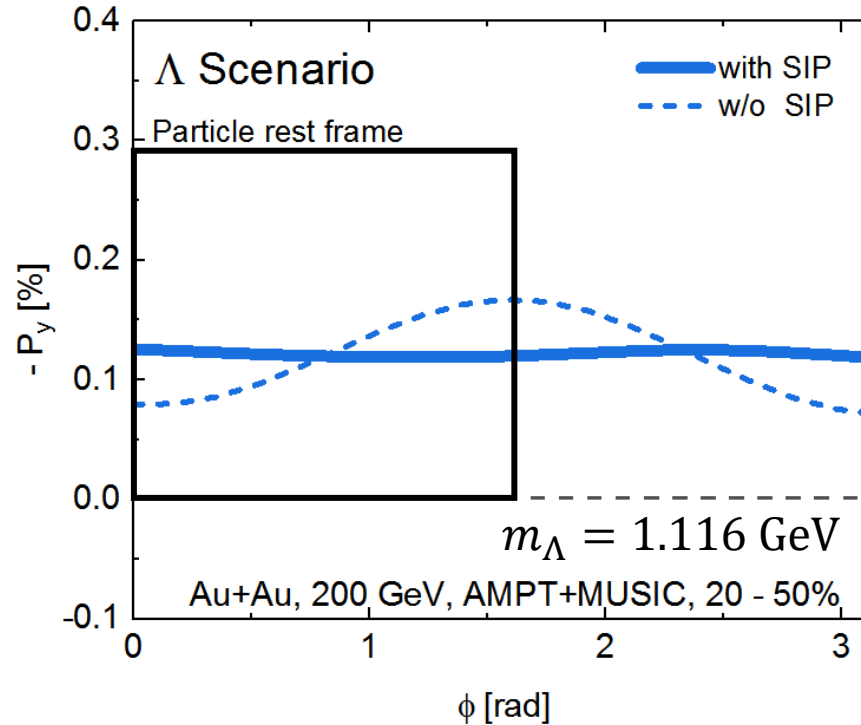
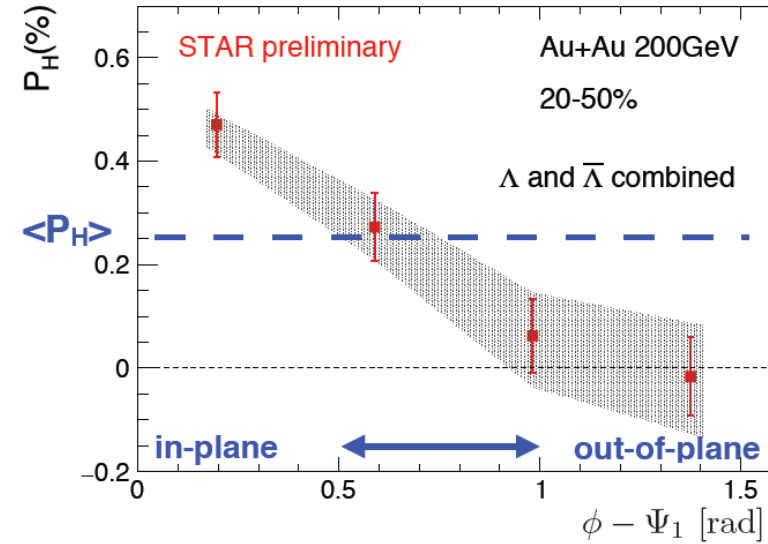
- In the scenario of ‘S-quark memory’, the total P^μ with SIP qualitatively agrees with data

$P_y(\phi)$ with SIP

BF, S. Liu, L. -G. Pang, H. Song, Y. Yin,
Phys.Rev.Lett. 127 14, 142301(2021)

$$\begin{aligned} \text{Total } P^\mu &= [\text{vorticity}] + [\text{T grad}] + [\text{SIP}] \\ &= [\text{thermal vorticity}] + [\text{SIP}] \end{aligned}$$

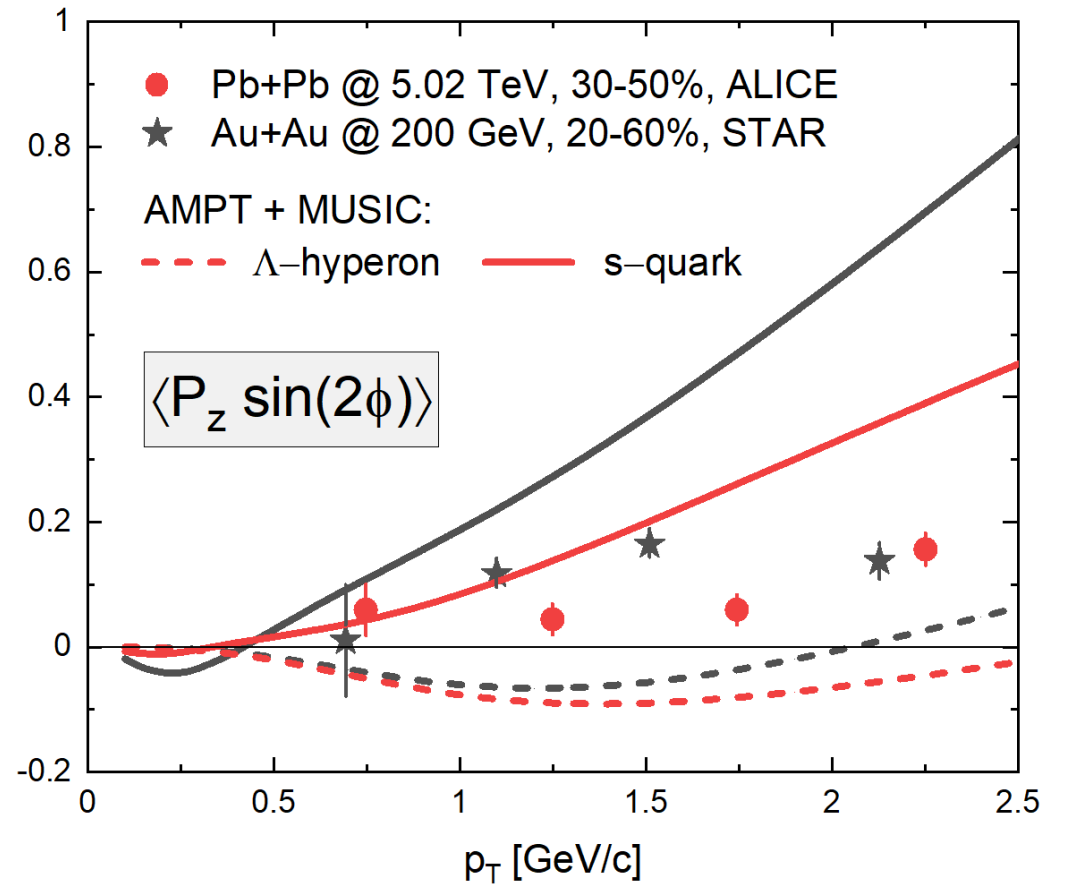
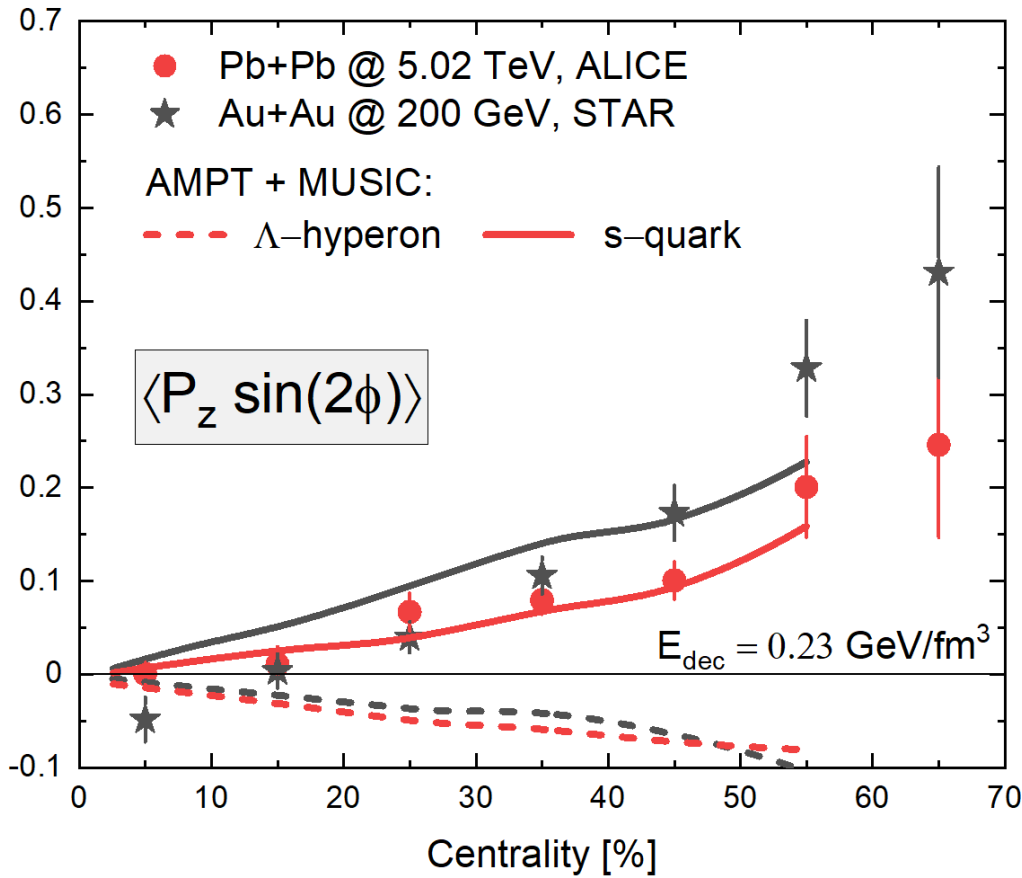
STAR, NPA, 982 (2019) 511-514



- In the scenario of ‘S-quark memory’, the total P^μ with SIP qualitatively agrees with data

From RHIC to LHC

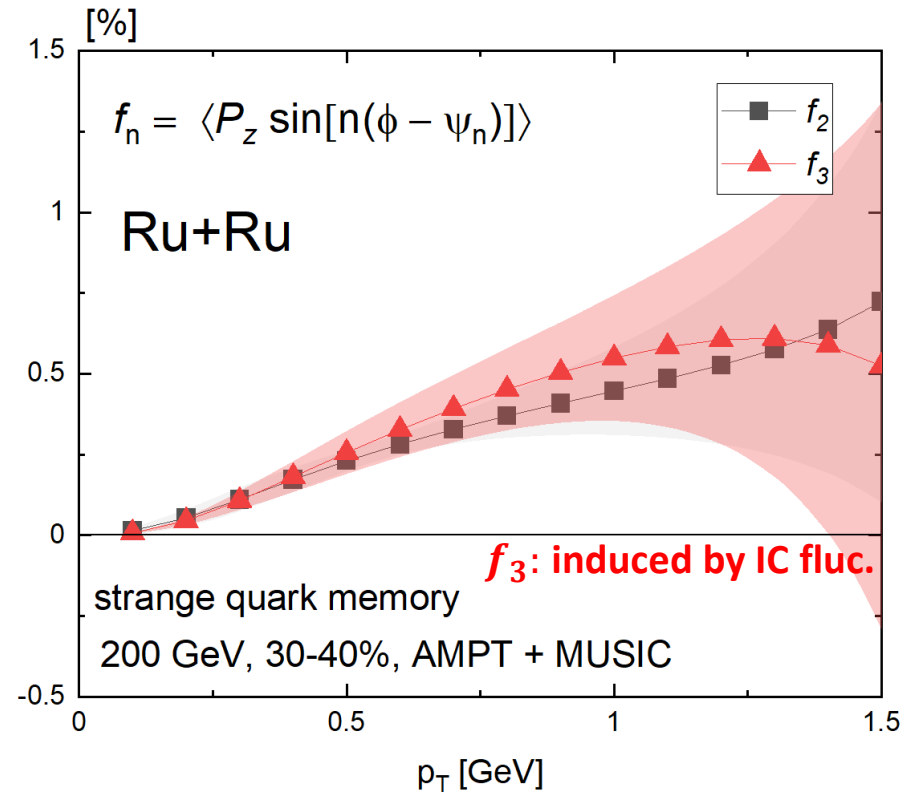
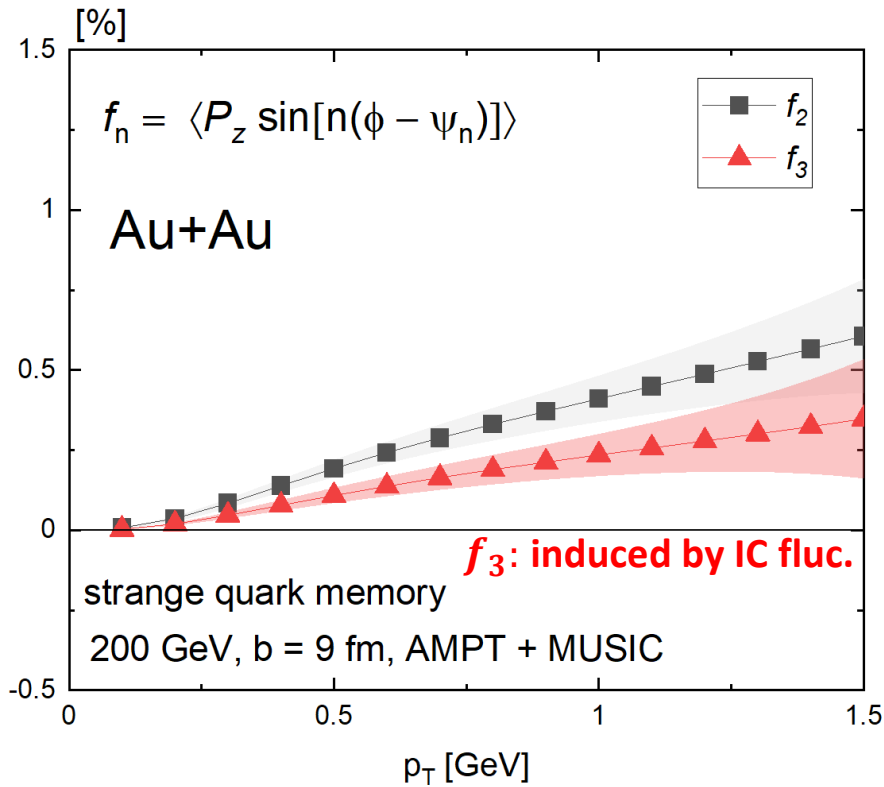
Same hydrodynamic model: AMPT + MUSIC
 (LHC parameter from EPJC 77 (2017) 9, 645)



- “Strange Memory” scenario qualitatively describes the centrality & p_T dependence
- More precise model needed to quantitative description

The 3rd order Fourier coefficient of P_z

Event-by-event AMPT + MUSIC



$$f_n = \langle P_z \sin[n(\phi - \Psi_n)] \rangle = \frac{\int p_T dp_T d\phi dy \int p \cdot d\sigma \mathcal{A}^\mu(x, p) \sin[n(\phi - \Psi_n)]}{\int p_T dp_T d\phi dy 2m \int p \cdot d\sigma f(x, p)}$$

- Non-zero f_3 is comparable to f_2 in both Au+Au and Ru+Ru systems
- Spin polarization also probes the initial state fluctuations

II. Spin Hall Effects (SHE) at RHIC-BES

BF, L.-G. Pang, H. Song and Y. Yin, arXiv: 2201.12970

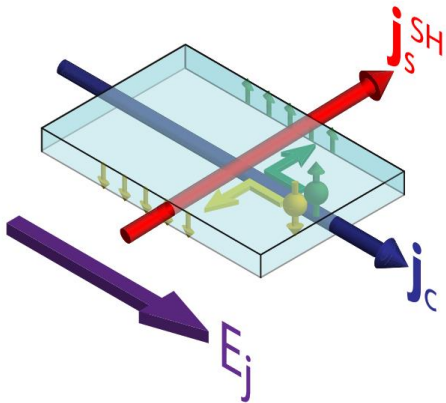
How about with finite μ_B ?

$$\mathcal{A}^\mu(x, p) = \beta f_0(x, p)(1 - f_0(x, p))\varepsilon^{\mu\nu\alpha\rho} \times \left(\underbrace{\frac{1}{2}p_\nu \partial_\alpha^\perp u_\rho}_{\text{vorticity}} - \underbrace{\frac{1}{T}u_\nu p_\alpha \partial_\rho T}_{\text{T-gradient}} - \underbrace{\frac{p_\perp^2}{\varepsilon_0}u_\nu Q_\alpha^\lambda \sigma_{\rho\lambda}}_{\text{SIP}} - \underbrace{\frac{q_B}{\varepsilon_0\beta}u_\nu p_\alpha \partial_\rho(\beta\mu_B)}_{\text{SHE}} \right),$$

Spin Hall Effects (SHE)

In condensed-matter

- Transverse spin current induced by spin-orbital coupling under external electric field



$$\vec{s} \propto \vec{p} \times \vec{E}$$

S. Meyer, et al., Nature Materials, 2017

J. Sinova, et al., Rev. Mod. Phys. 2015

- Probes transport properties in quantum materials with theory under **QED**
- Has been observed in semiconductors, metal and insulators at **room temperature** or below

In hot QCD matter

- With similar form, replacing electric field \vec{E} to baryon chemical potential gradient $\vec{\nabla}\mu_B$

$$\vec{P}_{\pm} \propto \pm \vec{p} \times \vec{\nabla}\mu_B$$

Spin Polarization

Thermal vorticity

F. Becattini, et al., Annal Phys. 2013

Shear-Induced Polarization

S. Liu and Y. Yin, JHEP 2021, BF, et al., PRL 2021

F. Becattini, et al., PLB 2021, PRL 2021

Spin Hall Effects (SHE)

In this talk

- Another mechanism for spin generation under **QCD**
- Probes the properties of QCD matter at **extremely high temperature** ($\sim 10^{12}$ K)

Spin Hall Effects (SHE)

BF, L.-G. Pang, H. Song and Y. Yin, arXiv: 2201.12970

Axial Wigner function \mathcal{A}^μ expansion with finite chemical potential:

$$\mathcal{A}^\mu(x, p) = \beta f_0(x, p)(1 - f_0(x, p))\varepsilon^{\mu\nu\alpha\rho} \times \left(\underbrace{\frac{1}{2}p_\nu \partial_\alpha^\perp u_\rho}_{\text{vorticity}} - \underbrace{\frac{1}{T}u_\nu p_\alpha \partial_\rho T}_{\text{T-gradient}} - \underbrace{\frac{p_\perp^2}{\varepsilon_0}u_\nu Q_\alpha^\lambda \sigma_{\rho\lambda}}_{\text{SIP}} - \underbrace{\frac{q_B}{\varepsilon_0\beta}u_\nu p_\alpha \partial_\rho(\beta\mu_B)}_{\text{SHE}} \right),$$

- Induced by μ_B gradient: more important at RHIC-BES
- Spin current generation: search SHE signal in differential observables like $P^\mu(\phi)$
- Opposite contribution for particles / anti-particles

$$\vec{P}_\pm \propto \pm \vec{p} \times \vec{\nabla}\mu_B$$

Spin Cooper-Frye type formula:

- “ Λ equilibrium” scenario
- “Strange memory” scenario

Same hydrodynamic model: AMPT + MUSIC

BF, K. Xu, X-G, Huang, H. Song, Phys.Rev.C 103 (2021) 2, 024903

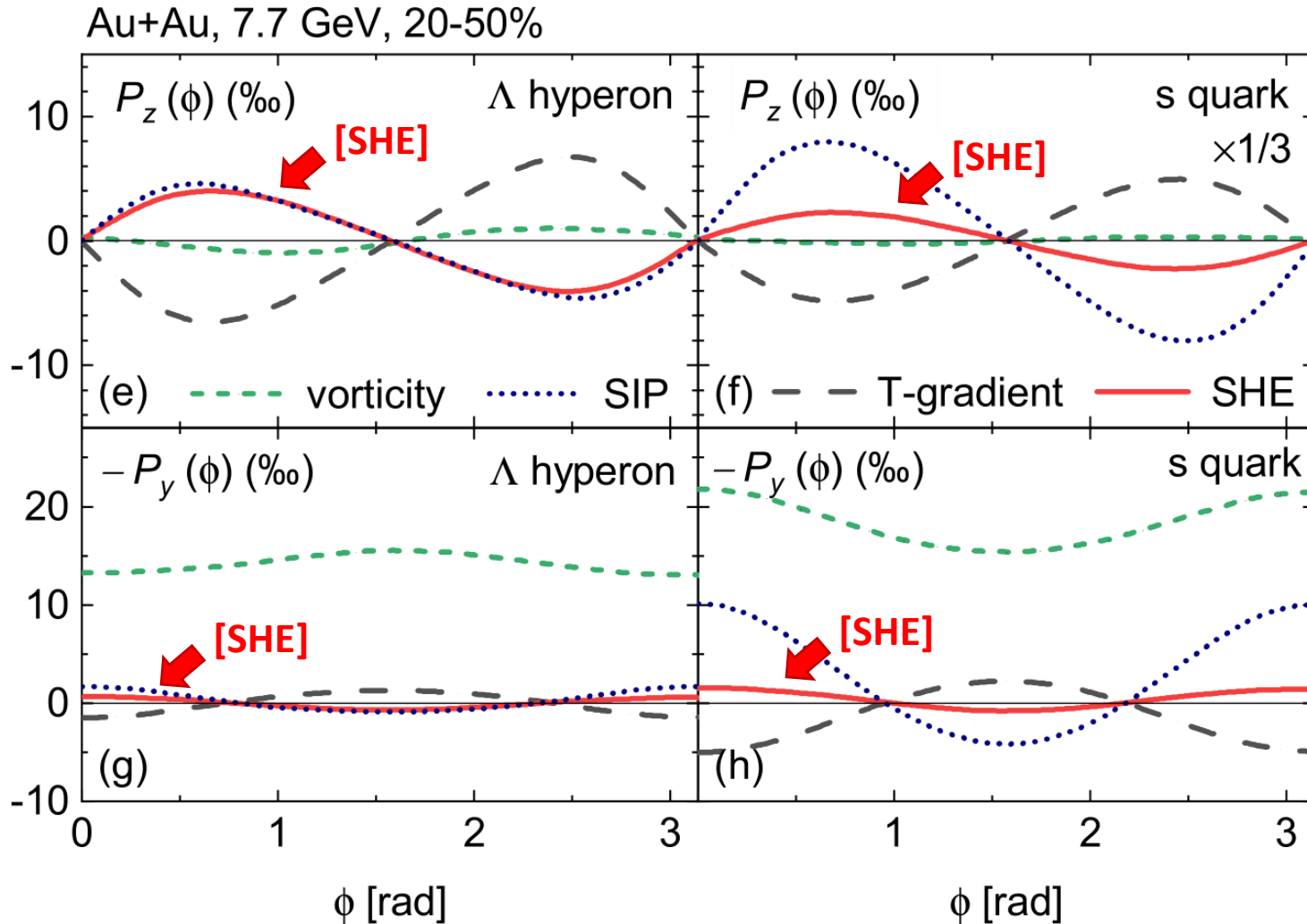
See also: S.Ryu, et al., PRC 104 (2021) 5, 054908 (Global effect)
S. Liu and Y. Yin, PRD 104 (2021) 5, 054043 (B-W model)

Individual contributions to $P_z(\phi)$ and $P_y(\phi)$

BF, L.-G. Pang, H. Song and Y. Yin,
arXiv: 2201.12970

$$\text{Total } P^\mu = [\text{vorticity}] + [\text{T grad}] + [\text{SIP}] + [\text{SHE}]$$

$$\vec{P}_{\text{SHE}} \propto \pm \vec{p} \times \vec{\nabla} \mu_B$$



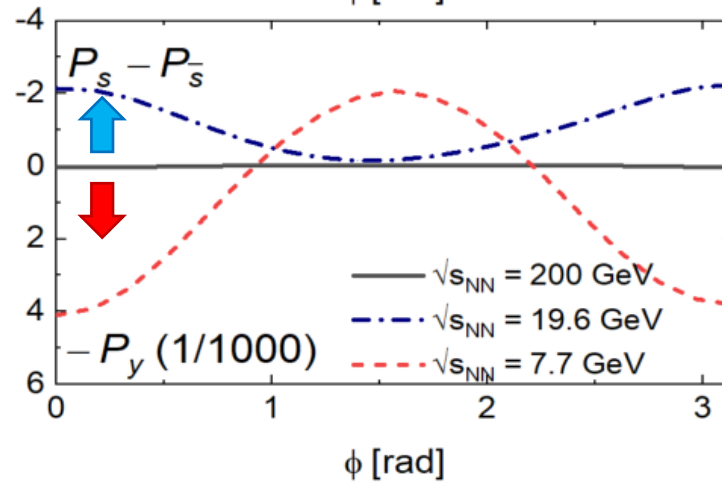
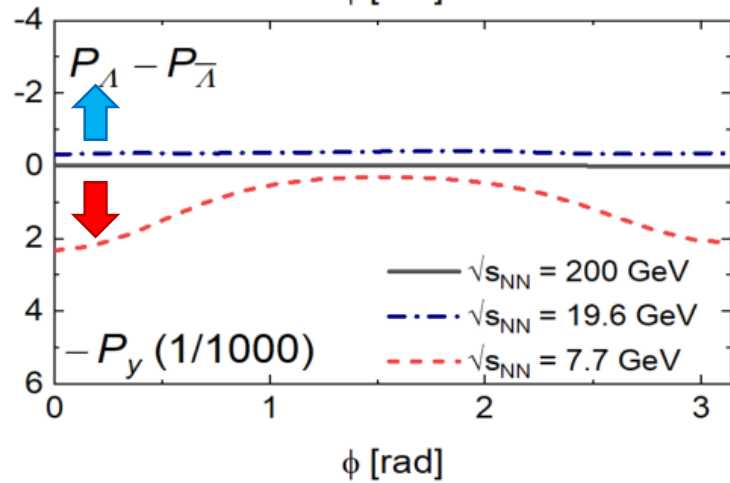
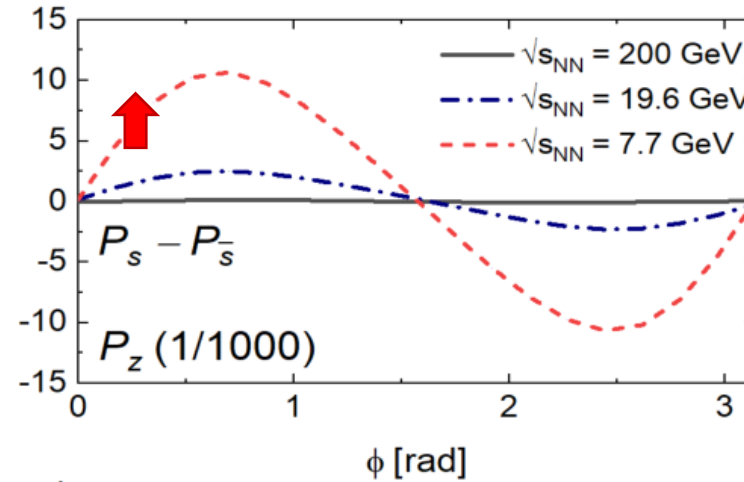
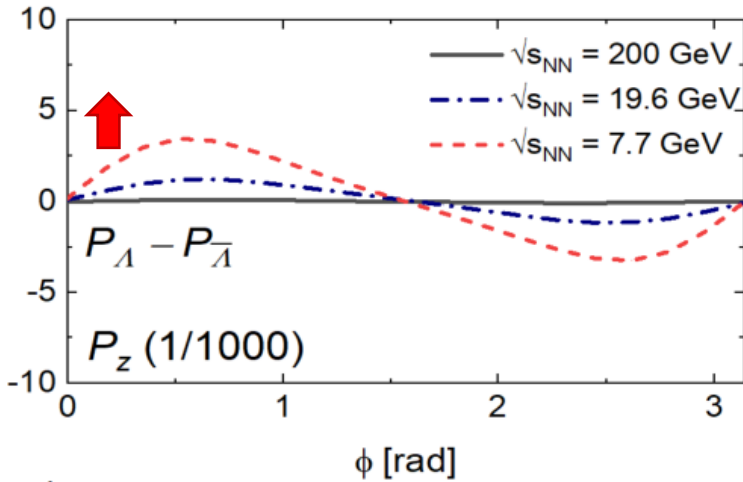
- SHE: “ $\sin(2\phi)$ ” on P_z & “ $\cos(2\phi)$ ” on P_y
- The magnitude of SHE is comparable to other effects
- Opposite SHE for particles and anti-particles

Net spin polarization: $P^{\text{net}}(\phi)$

BF, L.-G. Pang, H. Song and Y. Yin, arXiv: 2201.12970

$$P_{\Lambda}^{\text{net}} \equiv P_{\Lambda}(\phi) - P_{\bar{\Lambda}}(\phi)$$

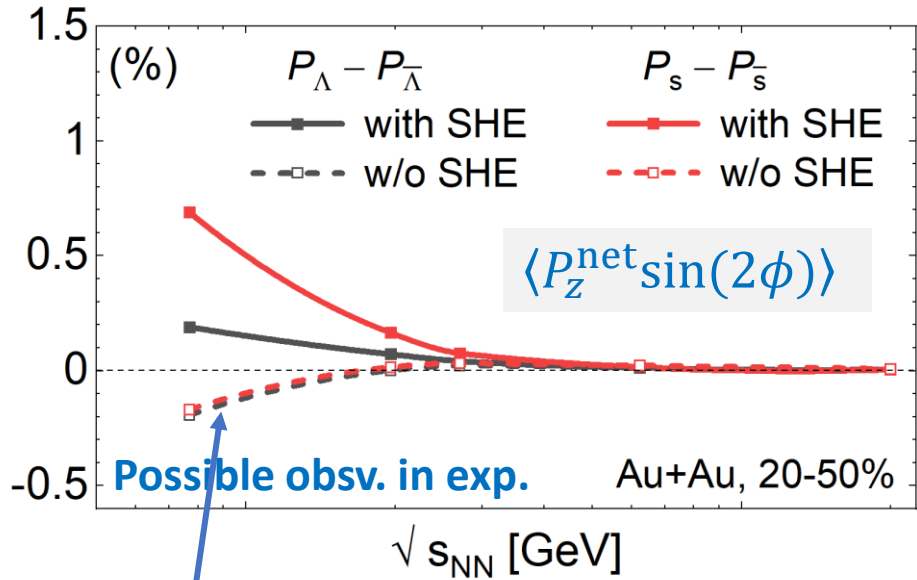
$$P_s^{\text{net}} \equiv P_s(\phi) - P_{\bar{s}}(\phi)$$



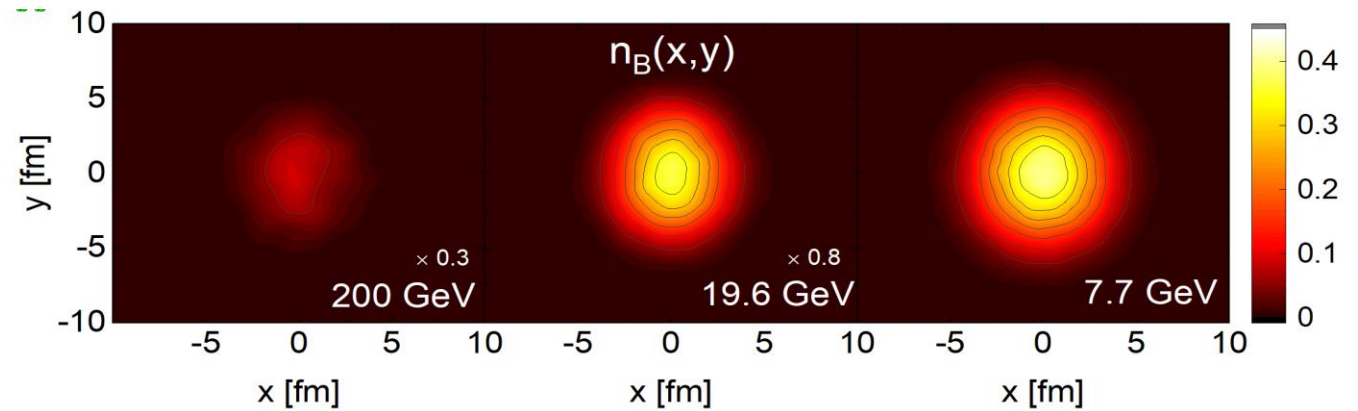
- The 'net' spin polarization used to extract SHE signals
- Net $P_z(\phi)$: increase with decreasing collision energy
- Net $P_y(\phi)$: non-monotonic behavior from SHE

The 2nd order Fourier coeff. of $P_z^{\text{net}}(\phi)$ & $P_y^{\text{net}}(\phi)$

BF, L.-G. Pang, H. Song, Y. Yin
arXiv: 2201.12970



- Monotonic increasing

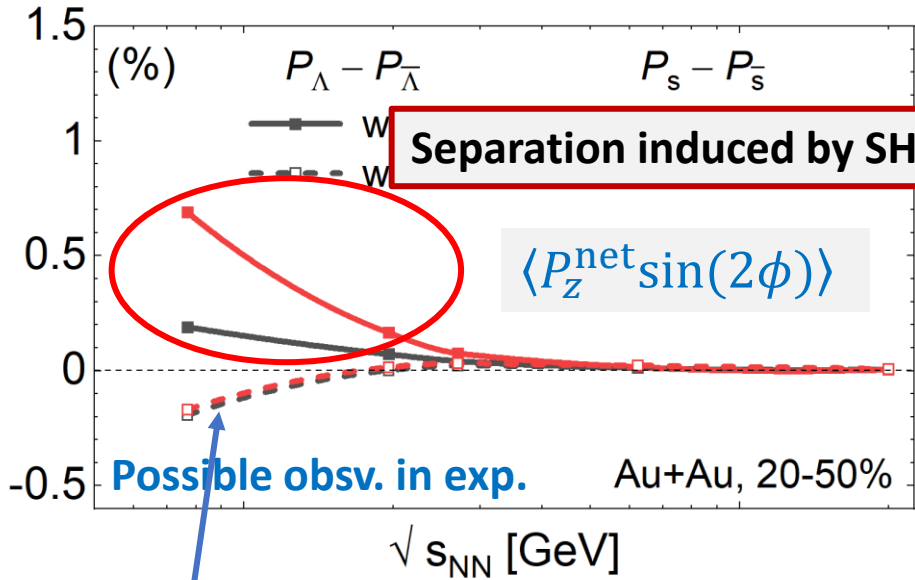


From the distribution function

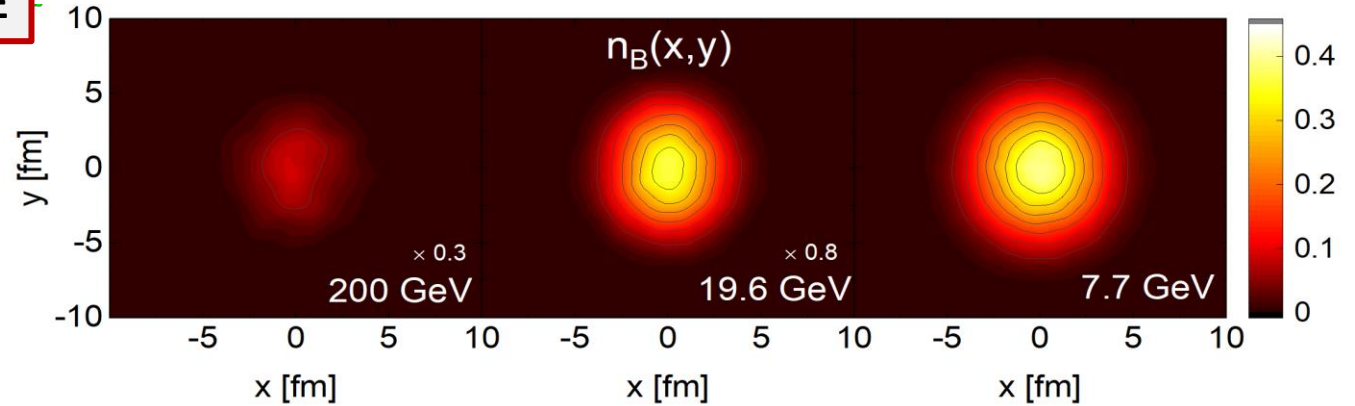
$$f(x, p) = \left(e^{(\epsilon_0 - q_B \mu_B) \beta} + 1 \right)^{-1}$$

The 2nd order Fourier coeff. of $P_z^{\text{net}}(\phi)$ & $P_y^{\text{net}}(\phi)$

BF, L.-G. Pang, H. Song, Y. Yin
arXiv: 2201.12970



- Monotonic increasing

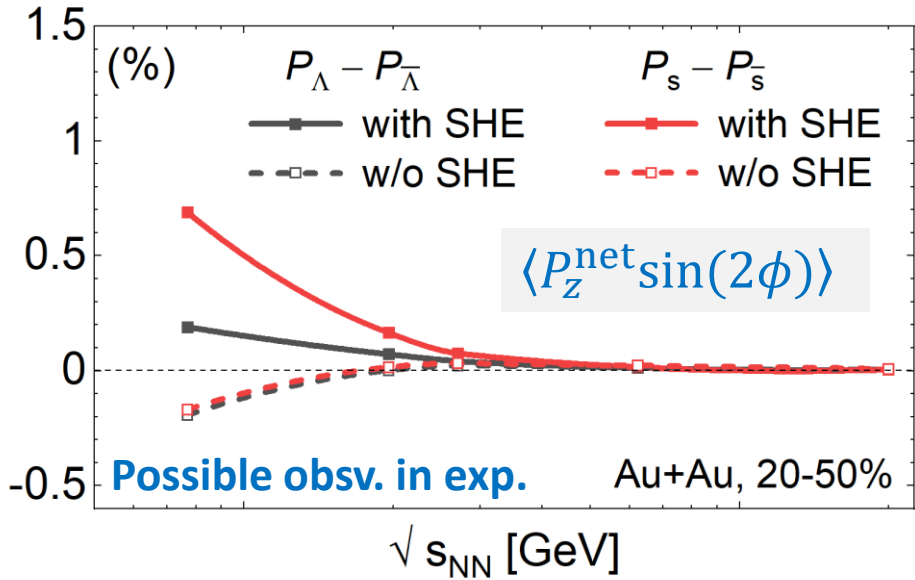


From the distribution function

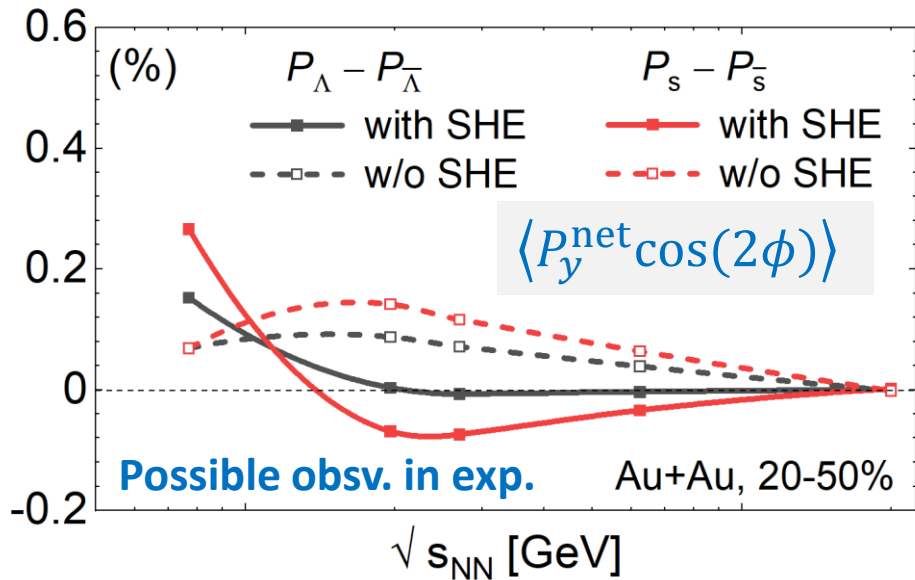
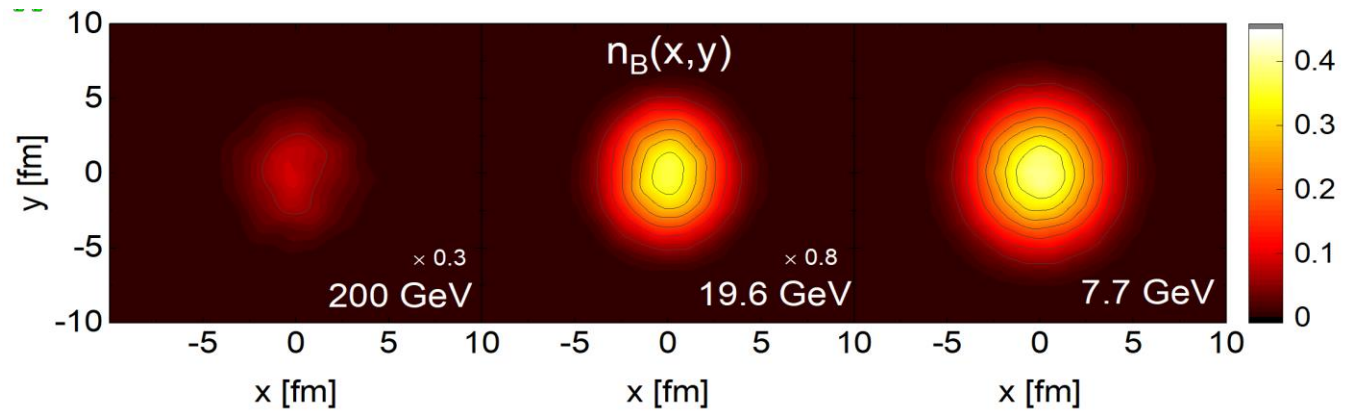
$$f(x, p) = \left(e^{(\epsilon_0 - q_B \mu_B) \beta} + 1 \right)^{-1}$$

The 2nd order Fourier coeff. of $P_Z^{\text{net}}(\phi)$ & $P_y^{\text{net}}(\phi)$

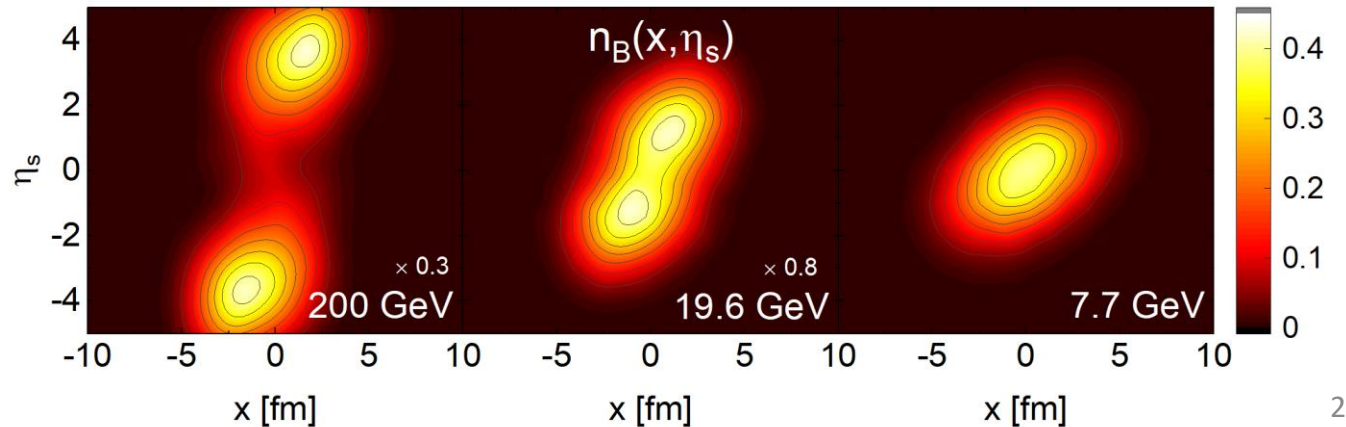
BF, L.-G. Pang, H. Song, Y. Yin
arXiv: 2201.12970



- Monotonic increasing



- Non-monotonic behavior



Summary

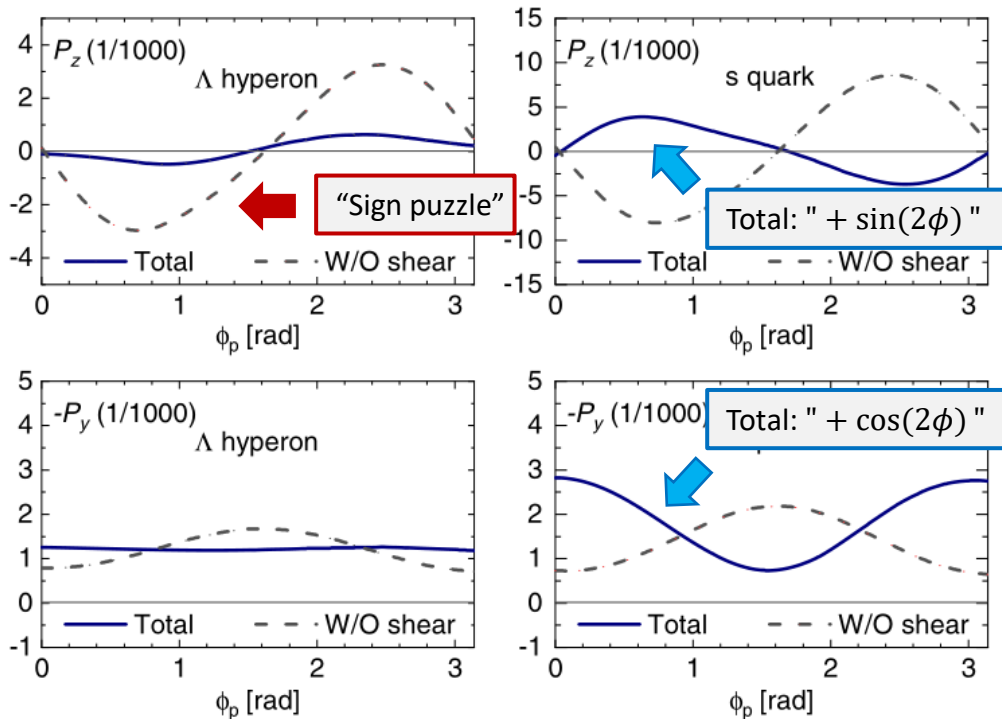
Shear-Induced Polarization: Phys.Rev.Lett. 127 14, 142301(2021)
Spin Hall Effects: arXiv: 2201.12970

$$\text{Total } P^\mu = [\text{vorticity}] + [\text{Grad T}] + [\text{SIP}] + [\text{SHE}]$$

Shear-Induced Polarization

“Strange memory” + Shear-Induced Polarization

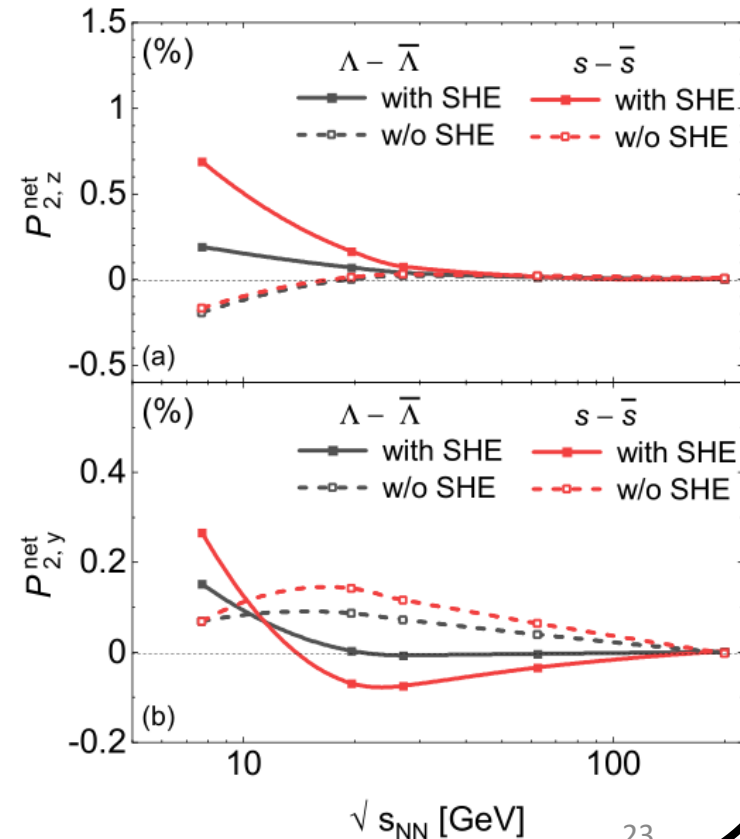
Describes $P_z(\phi)$ and $P_y(\phi)$ qualitatively at **top RHIC and LHC**



Spin Hall Effects

$$\vec{P}_\pm \propto \pm \vec{p} \times \vec{\nabla} \mu_B$$

- Particle – Anti-particle separation
- Relevant for RHIC-BES and RHIC/LHC forward rapidity
- Scenario independent



Back up

Hydrodynamic gradients

Derivatives of the velocity field:

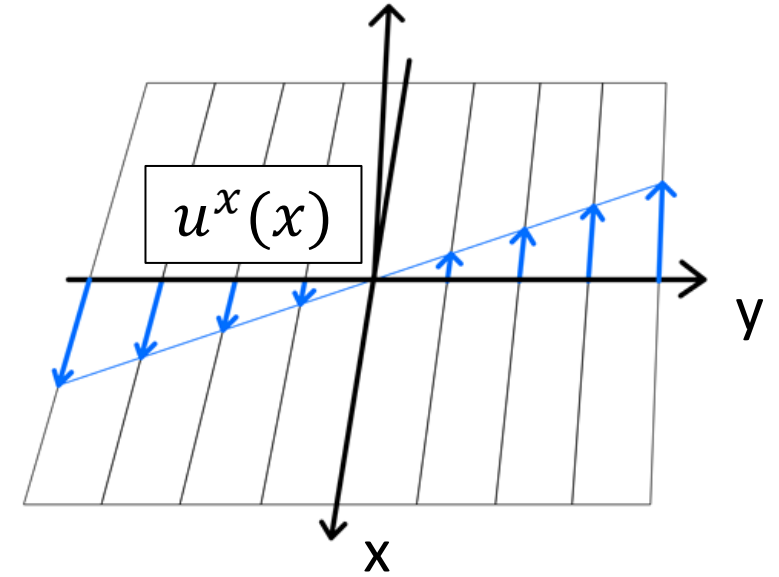
$$\partial_\mu u_\nu(x)$$

Anti-symmetric: vorticity

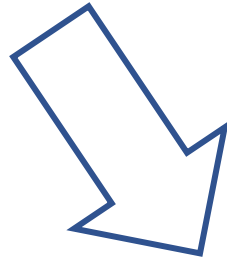
$$\omega^\mu = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} u_\nu \partial_\alpha^\perp u_\beta$$

Symmetric: shear stress

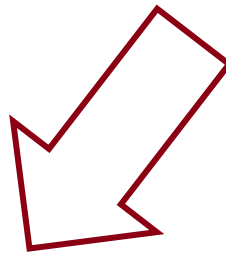
$$\sigma^{\mu\nu} = \frac{1}{2} (\partial_\perp^\mu u^\nu + \partial_\perp^\nu u^\mu) - \frac{1}{3} \Delta^{\mu\nu} \partial_\perp \cdot u$$



In heavy-ion,
condensed matter ...



Spin polarization



?

will be discussed in this talk

[Strain induced polarization]
In crystal physics:

Crooker and Smith, PRL (2005) 94, 236601

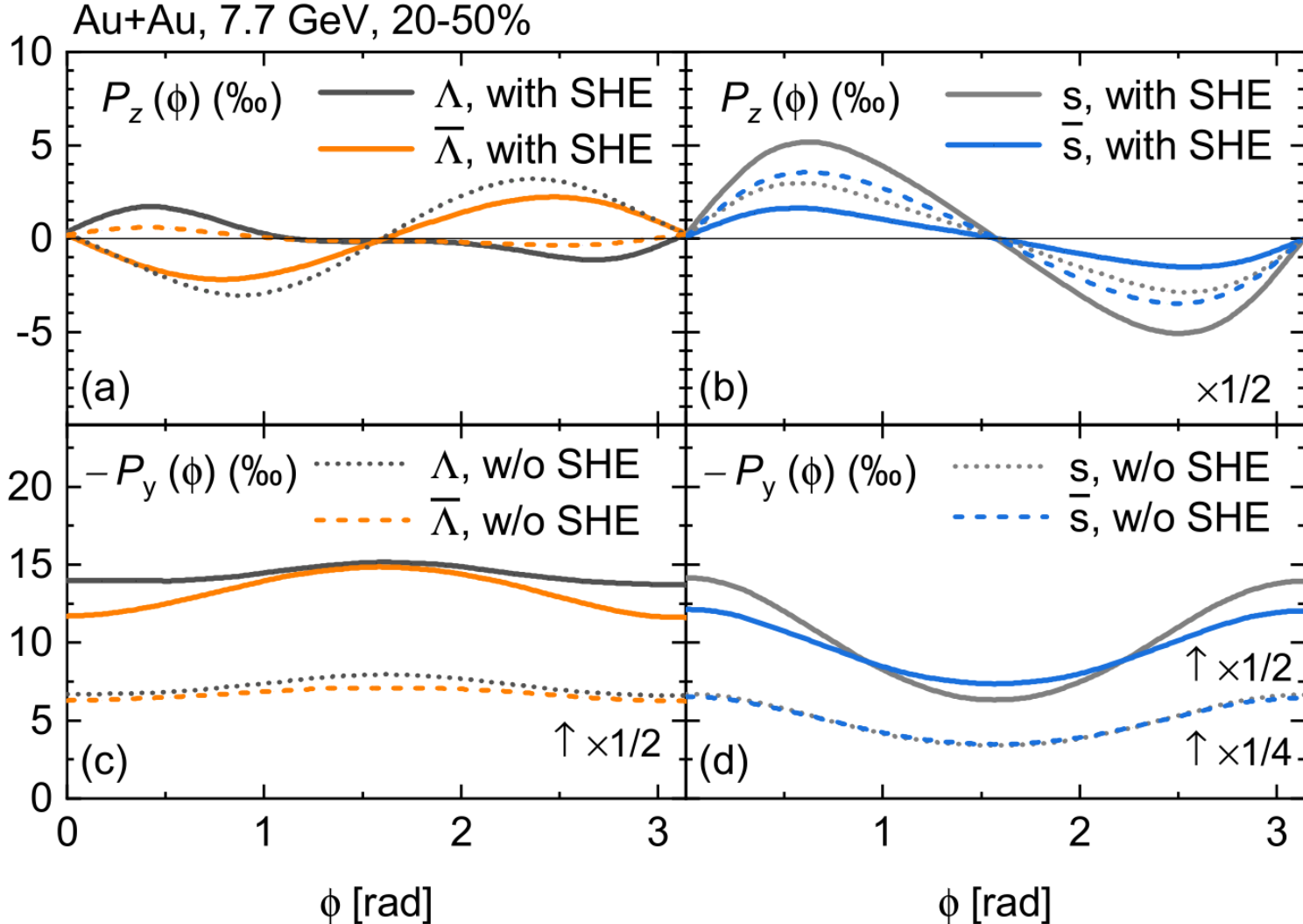
Kissikov, et al., Nature Comm. (2018) 9, 1058 25

Total $P_z(\phi)$ and $P_y(\phi)$ with SHE

BF, L.-G. Pang, H. Song and Y. Yin, arXiv: 2201.12970

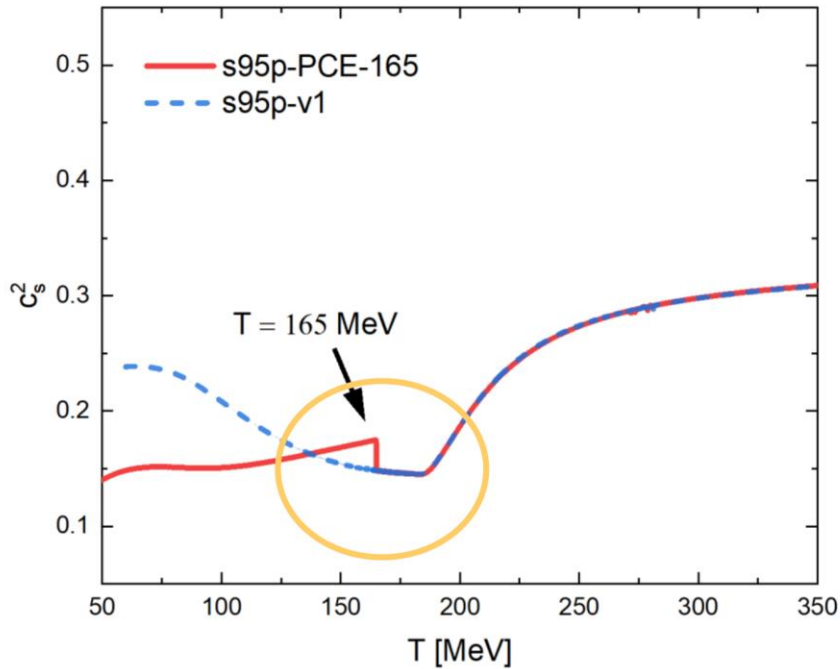
$$\text{Total } P^\mu = [\text{vorticity}] + [\text{Grad T}] + [\text{SIP}] + [\text{SHE}]$$

$$\vec{P}_{\text{SHE}} \propto \pm \vec{p} \times \vec{\nabla} \mu_B$$



- Separation between particles and anti-particles by SHE
 - Different local polarization w/o SHE:
 - Change the space-time of emitted particles
 - Pauli blocking
 - Scenario independent
- [O. Vitiuk, et al., PLB 2020](#)
[R-H. Fang, et al., PRC 2016](#)

Dependence on EoS



-Do not use EoS-s95p-PCE widely used in hydro calculations !

NEoS:

A. Monnai, B. Schenke, C. Shen, *Phys.Rev.C* 100 B. (2019) 2, 024907

S95p-v1:

P. Huovinen, P. Petreczky, *Nucl.Phys.A* 837 (2010) 26-53

