Measuring the speed of sound using cumulants of baryon number

Agnieszka Sorensen

Institute for Nuclear Theory, University of Washington

In collaboration with
Volker Koch, Larry McLerran, Dima Oliinychenko

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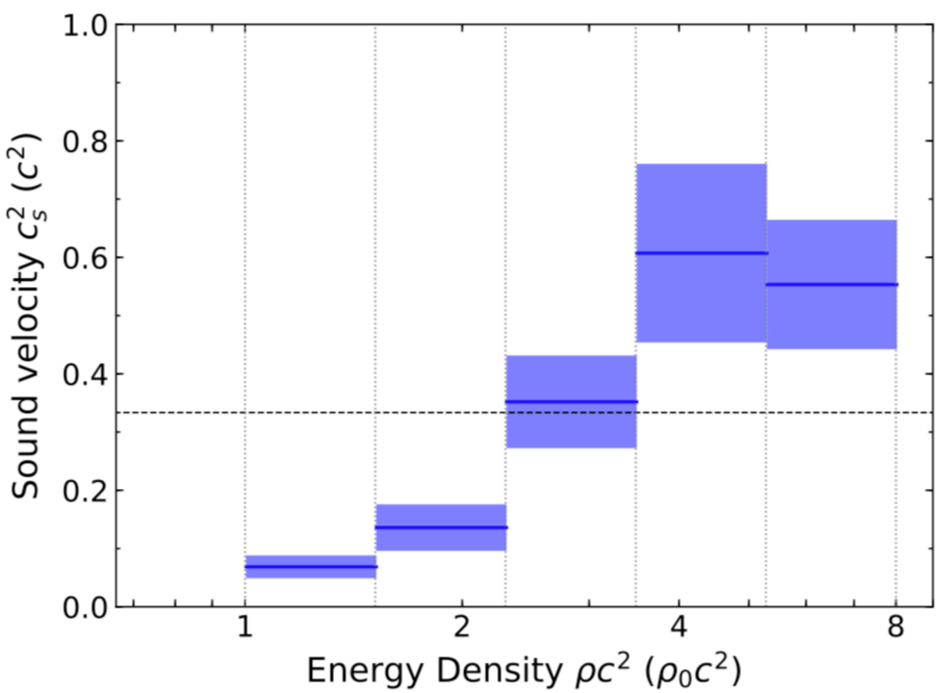
c_s^2 in neutron stars and heavy-ion collisions

In neutron stars: based on

- neutron stars with $M \gtrsim 2M_\odot$
- knowledge of the EOS of hadronic matter at "low" densities

c_s^2 may exceed the conformal limit of 1/3

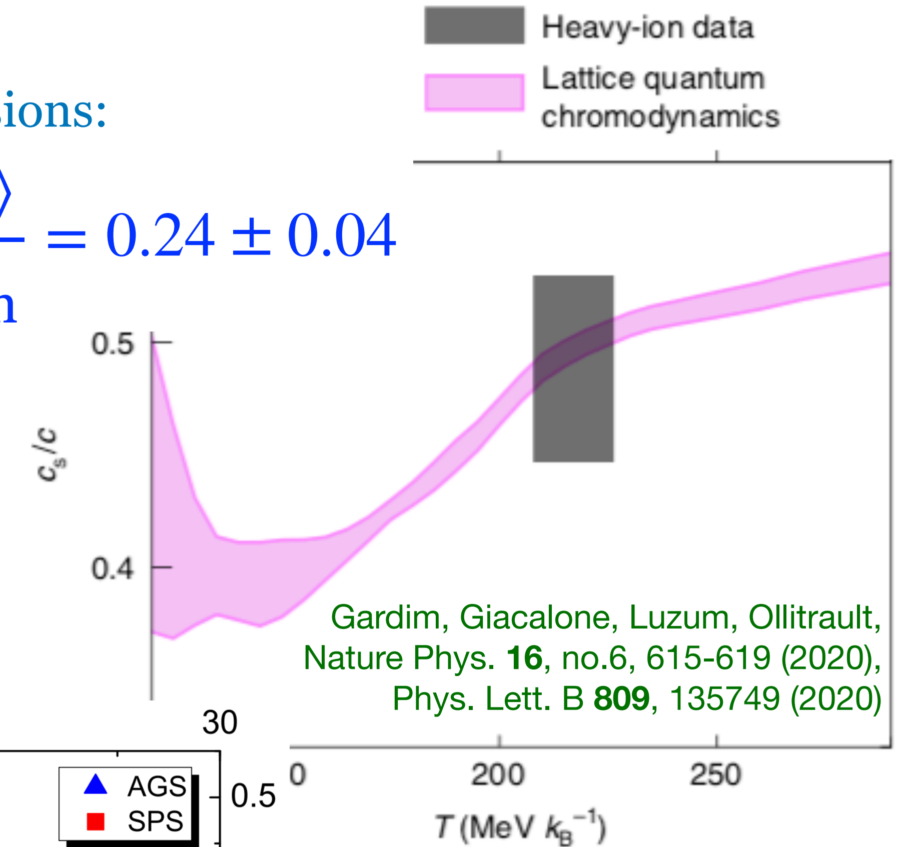
P. Bedaque and A. W. Steiner, Phys. Rev. Lett. **114**, no.3, 031103 (2015), arXiv: 1408.5116, Bedaque:2014sqa
I. Tews, J. Carlson, S. Gandolfi and S. Reddy, Astrophys. J. **860**, no.2, 149 (2018), arXiv:1801.01923, Tews:2018kmu



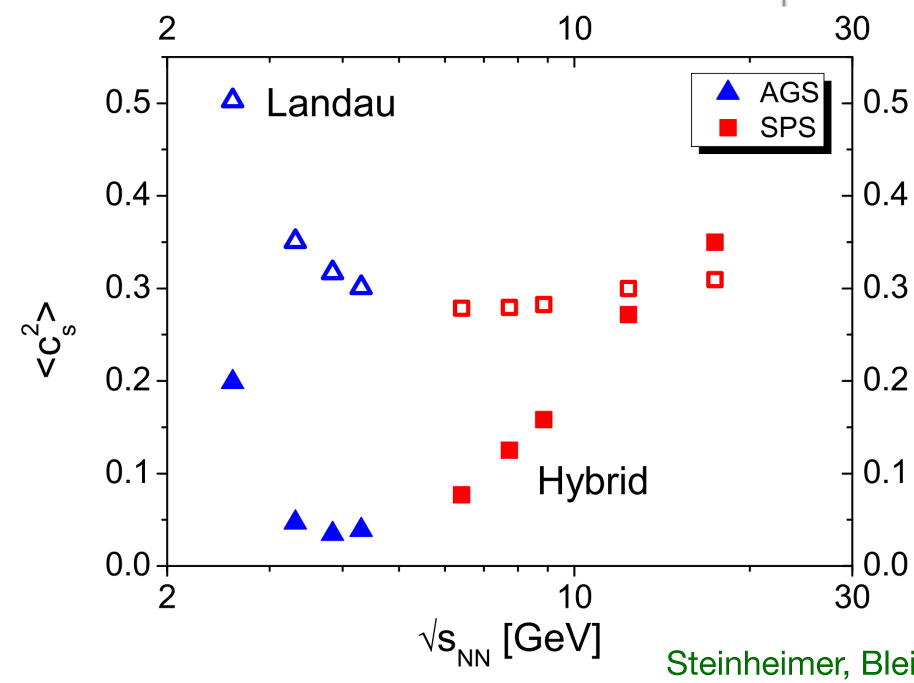
Fujimoto, Fukushima, Murase, Phys. Rev. D **101**, no.5, 054016 (2020), arXiv:1903.03400

In heavy-ion collisions:

$$c_s^2\Big|_V \approx \frac{d\ln\langle p_T\rangle}{d\ln N_{ch}} = 0.24 \pm 0.04$$



Gardim, Giacalone, Luzum, Ollitrault, Nature Phys. **16**, no.6, 615-619 (2020), Phys. Lett. B **809**, 135749 (2020)



Steinheimer, Bleicher, Eur. Phys. J. A 48, 100 (2012), arXiv:1207.2792

minimum around $\sqrt{s_{NN}} \in [4,\ 9]$ GeV

How to measure c_s^2 in heavy-ion collisions?

At finite T, one often considers the following two expressions for the speed of sound:

the isentropic speed of sound
($\sigma = s/n_B = const$):

$$c_\sigma^2 \equiv \left(\frac{dP}{d\mathcal{E}}\right)_\sigma = \frac{\frac{s}{n_B}\left(\frac{dP}{dT}\right)_{n_B} + \left(\frac{dP}{dn_B}\right)_T \left(\frac{ds}{dT}\right)_{n_B} - \left(\frac{dP}{dT}\right)_{n_B}\left(\frac{ds}{dn_B}\right)_T}{\left(\frac{\mathcal{E}+P}{n_B}\right)\left(\frac{ds}{dT}\right)_{n_B}}$$

the isothermal speed of sound
($T = const$):

$$c_T^2 \equiv \left(\frac{dP}{d\mathcal{E}}\right)_T = \frac{\left(\frac{dP}{dn_B}\right)_T}{T\left(\frac{ds}{dn_B}\right)_T + \mu_B}$$

common $T = 0$ limit:

$$c_T^2\Big|_{T=0} = c_\sigma^2\Big|_{T=0} = \frac{n_B}{\mu_B}\left(\frac{d\mu_B}{dn_B}\right)_T = \frac{1}{\mu_B}\left(\frac{dP}{dn_B}\right)_T = \frac{T}{\mu_B}\frac{\kappa_1}{\kappa_2}$$

$$\kappa_1 = Vn_B$$

$$\kappa_2 = VTn_B\left(\frac{dP}{dn_B}\right)_T^{-1}$$

Easy to notice: the speed of sound ~ derivatives of pressure

What "measures" derivatives of pressure? **Cumulants of baryon number!**

Can one connect the speed of sound with the cumulants?

Problematic: difficult to estimate T derivatives of cumulants from experiment

$$c_T^2 = \left[\frac{T}{\kappa_1} \left(\frac{d\kappa_1}{dT} \right)_{\mu_B} + \frac{\mu_B}{T} \frac{\kappa_2}{\kappa_1} \right]^{-1}$$

$T/\mu_B \ll 1$

$$c_T^2 \approx \frac{T\kappa_1}{\mu_B \kappa_2}$$

an upper limit to c_T^2

$$\left(\frac{d \ln c_T^2}{d \ln n_B} \right)_T + c_T^2 \approx 1 - \frac{\kappa_3 \kappa_1}{\kappa_2^2}$$

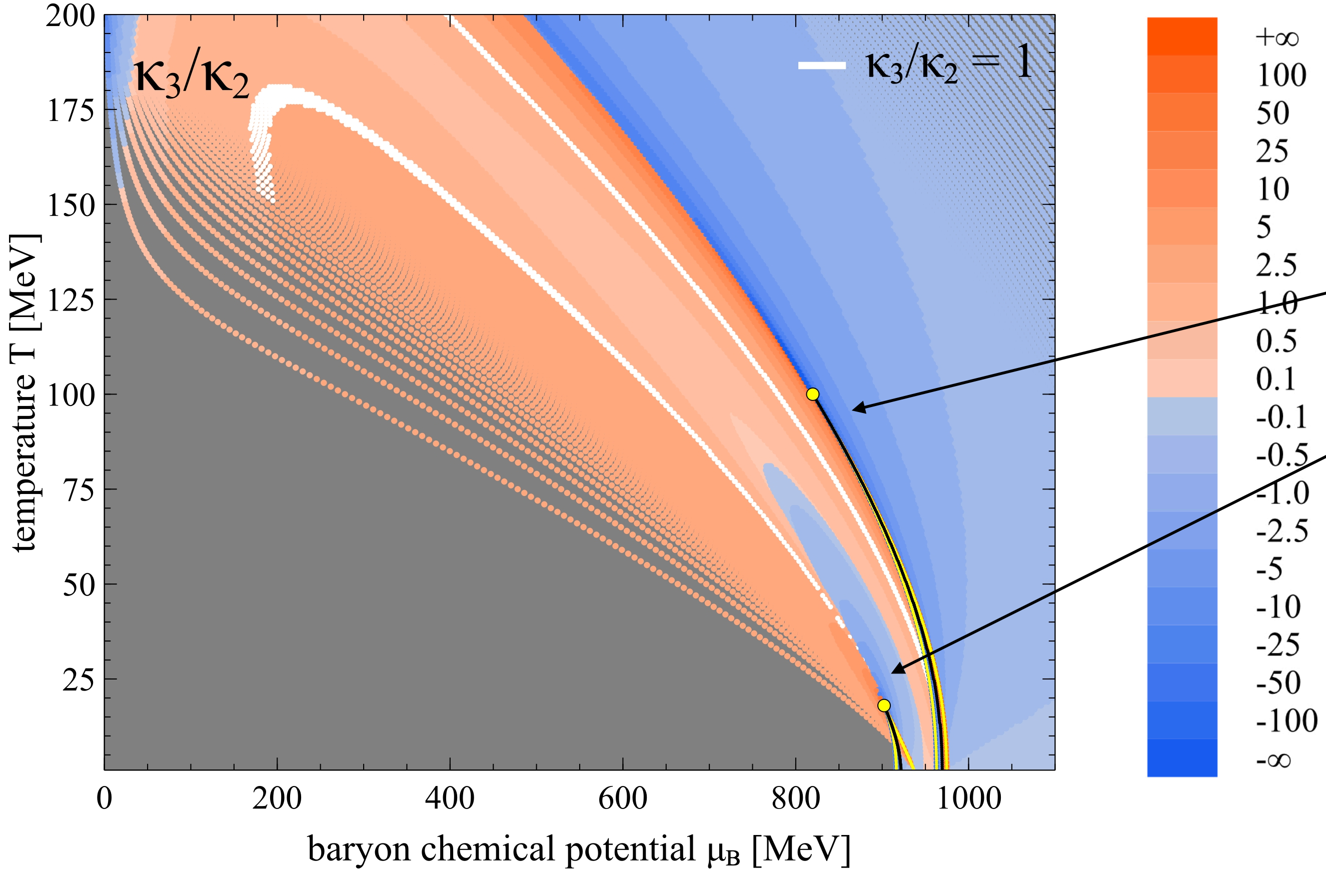
AS, D. Oliinychenko, V. Koch, L. McLerran, Phys. Rev. Lett. **127** (2021) 042303, arXiv:2103.07365, Sorensen:2021zme

From this arose a somewhat audacious idea:
to measure c_s^2 using not only the qualitative,
but also the quantitative behavior of the cumulants

Tests in model(s)

AS, V. Koch, Phys. Rev. C **104** no. 3 (2021) 034904, arXiv:2011.06635, Sorensen:2020ygf

the VDF (vector density functional) model: $$P = g\int \frac{d^3p}{(2\pi)^3}\,\frac{p^2}{3\sqrt{p^2+m^2}}\,f_{\mathbf{p}} + \sum_{i=1}^{4} C_i \frac{b_i - 1}{b_i} n_B^{b_i}$$



two phase transitions:
the known nuclear liquid-gas PT
and a “QGP-like” PT

orange = positive values, blue = negative values, white lines for values = 1 (Poisson limit), yellow lines = spinodal regions, black lines for coexistence regions

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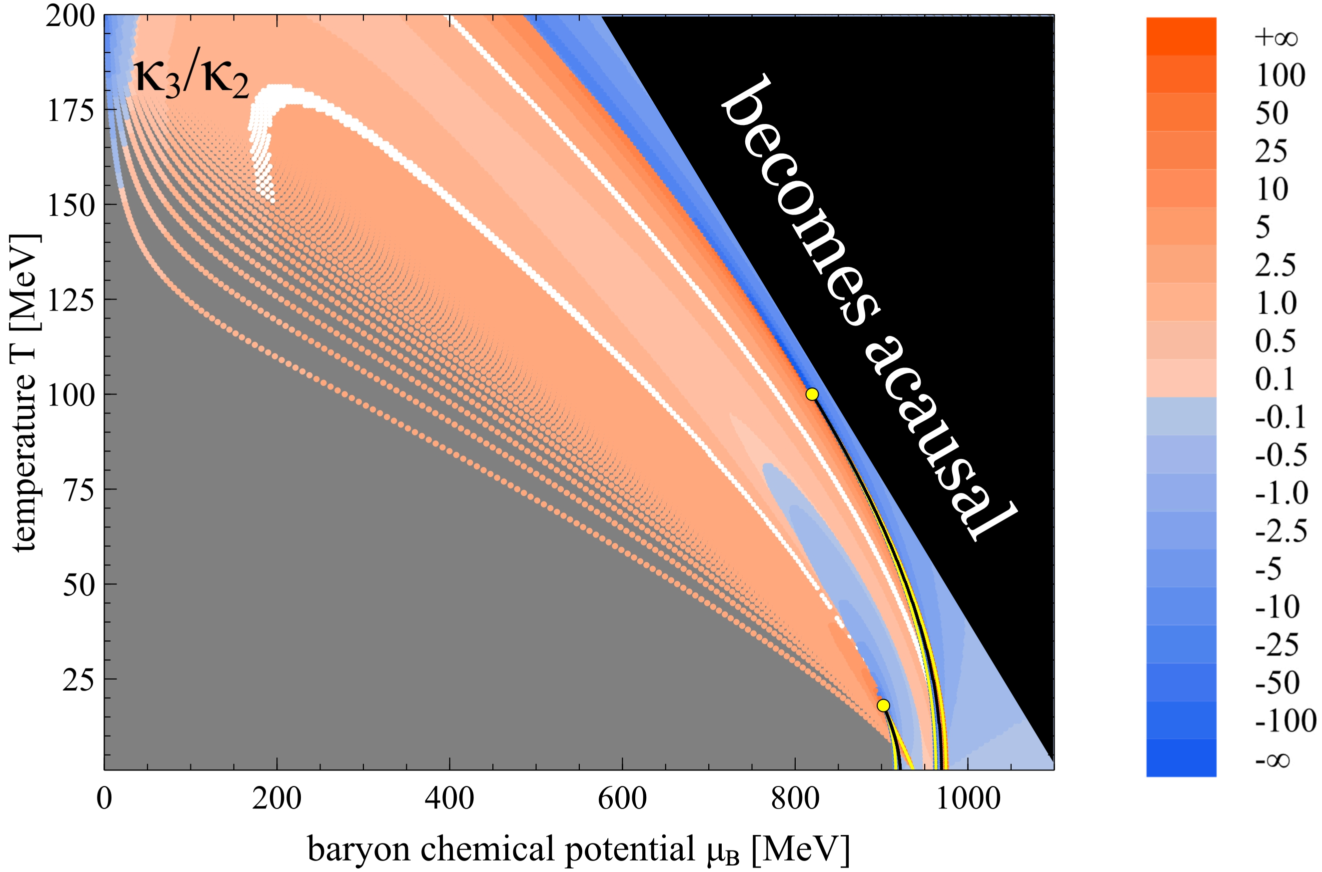
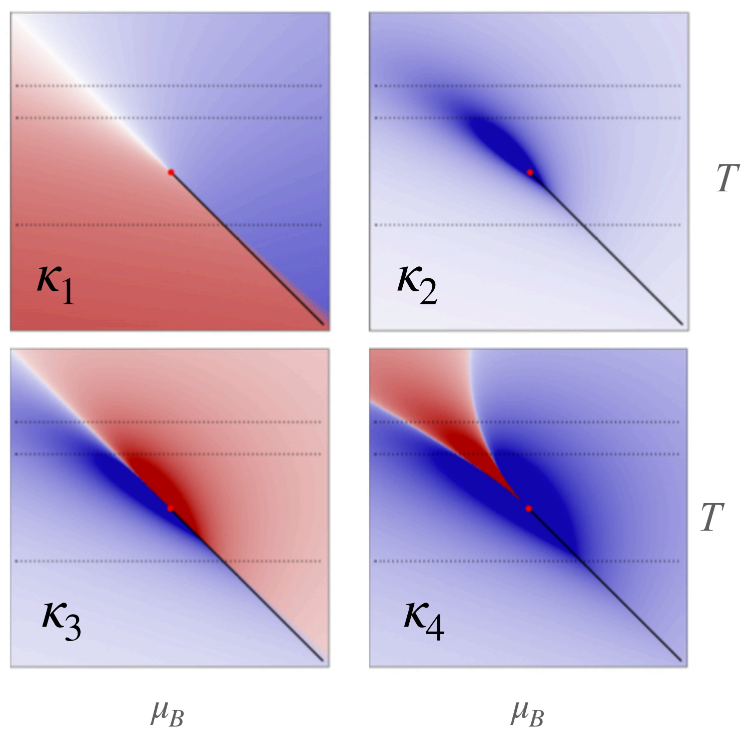




A. Bzdak *et al.*, Physics Reports **853** (2020) 1-87, arXiv:1906.00936, Bzdak:2019pkr

orange = positive values, blue = negative values, white lines for values = 1 (Poisson limit), yellow lines = spinodal regions, black lines for coexistence regions

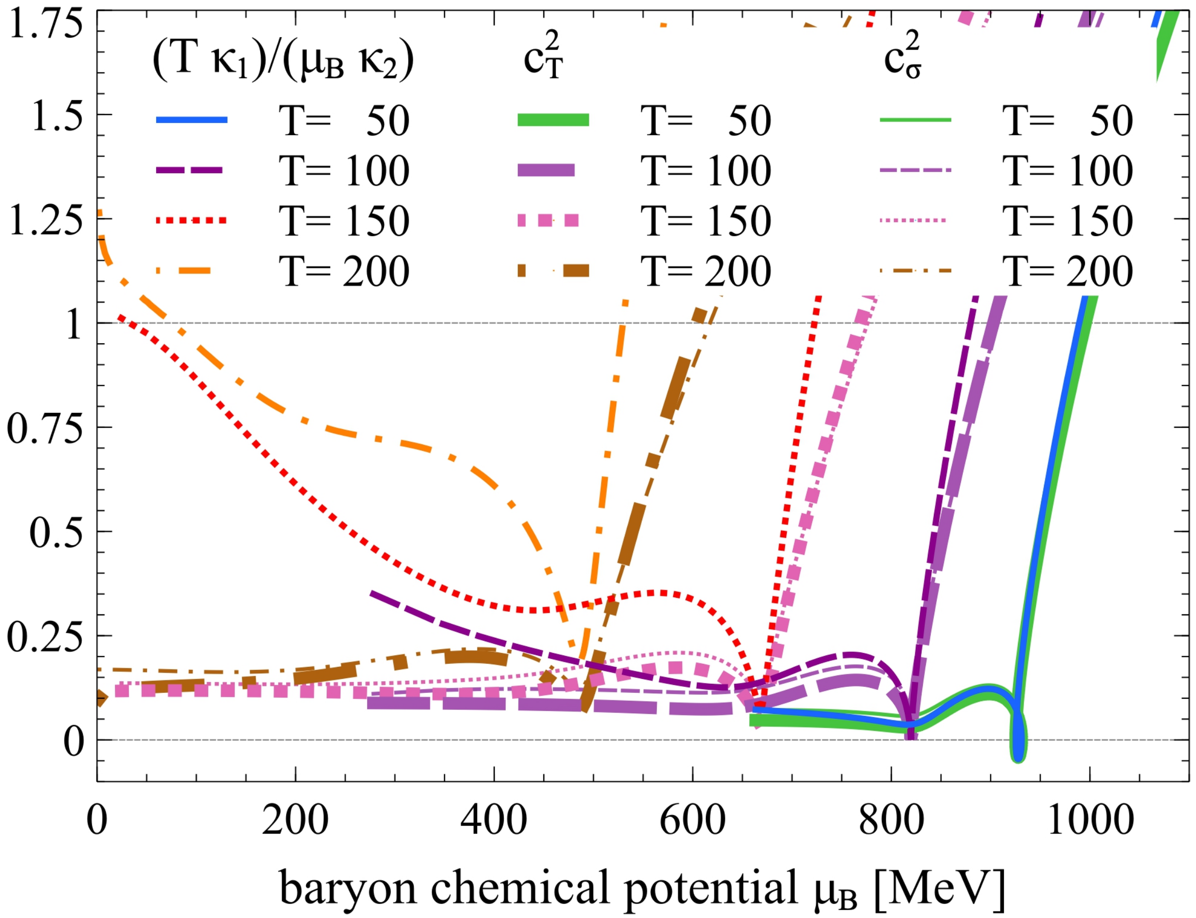
Tests in the VDF model

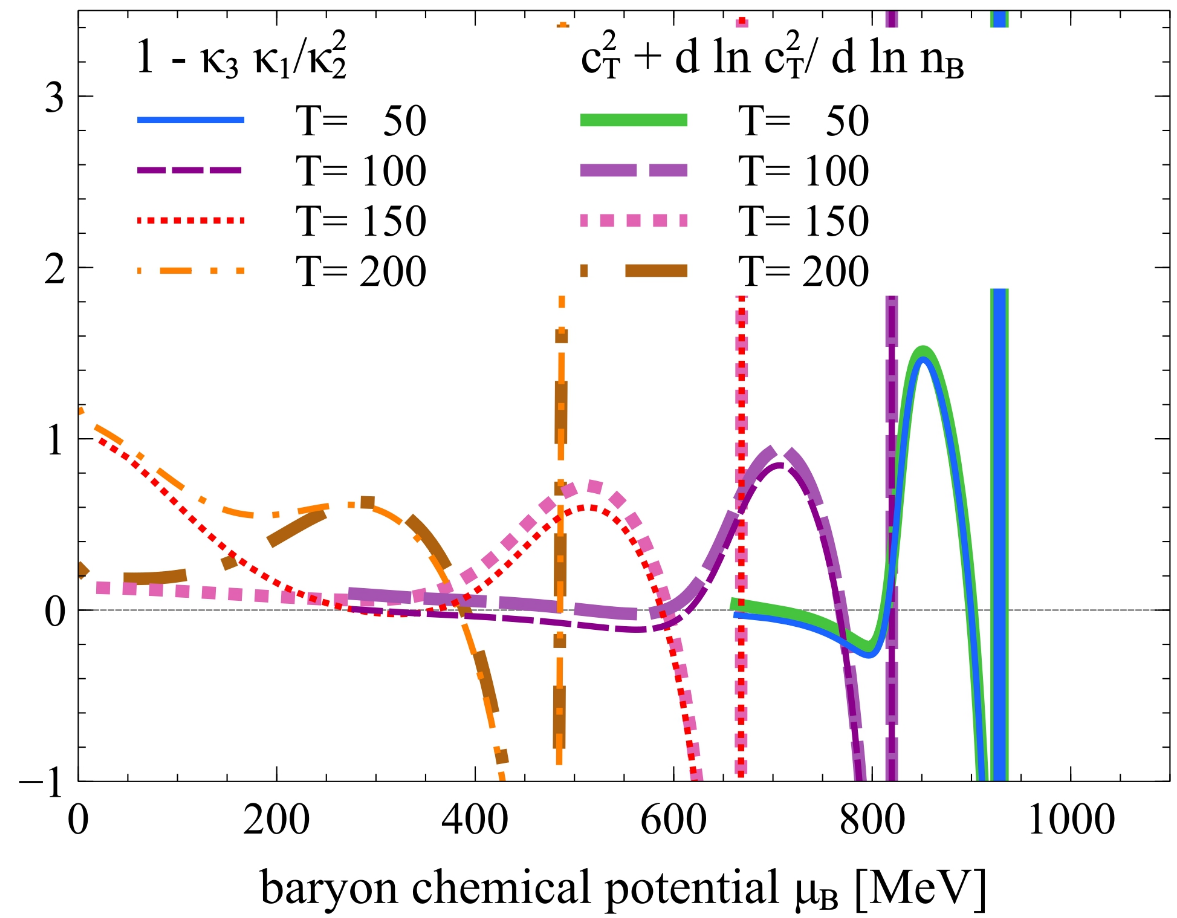
$$c_T^2 \approx \frac{T\kappa_1}{\mu_B \kappa_2}$$

works for $\mu_B \gtrsim 600$ MeV and $T < 150$

$$\left(\frac{d \ln c_T^2}{d \ln n_B}\right)_T + c_T^2 \approx 1 - \frac{\kappa_3 \kappa_1}{\kappa_2^2}$$

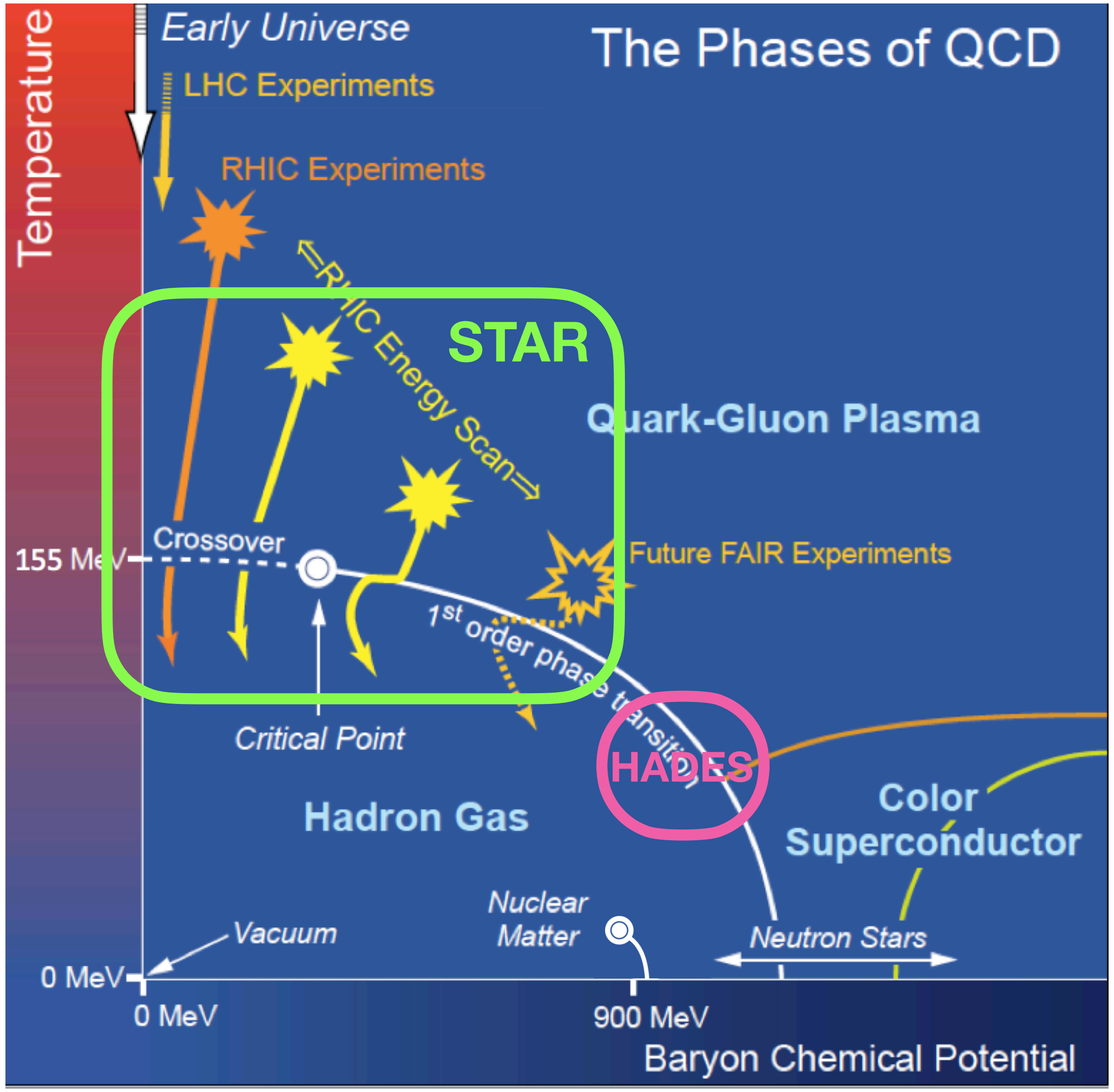
works for $\mu_B \gtrsim 200$ MeV





AS, D. Oliinychenko, V. Koch, L. McLerran, Phys. Rev. Lett. **127** (2021) 042303
arXiv:2103.07365, Sorensen:2021zme

Experimental data



The freeze-out parameters (T_{fo}, μ_{fo}) are obtained from particle yields:

$\sqrt{s}$ [GeV]	$T_{\rm fo}$ [MeV]	$\mu_{\rm fo}$ [MeV]
200	164.3	28
62.4	160.3	70
54.4	160.0	83
39	156.4	103
27	155.0	144
19.6	153.9	188
14.5	151.6	264
11.5	149.4	287
7.7	144.3	398
2.4	65	784

Cumulants κ_2, κ_3, ... are fluctuations around the means

M. Abdallah *et al.* (STAR), Phys. Rev. C **104** (2021) no. 2 024902, arXiv:2101.12413, STAR:2021iop
M. L. for the HADES collaboration (2019), 3rd EMMI Workshop

Experimental data

AS, D. Oliinychenko, V. Koch, L. McLerran, Phys. Rev. Lett. **127** (2021) 042303
arXiv:2103.07365, Sorensen:2021zme

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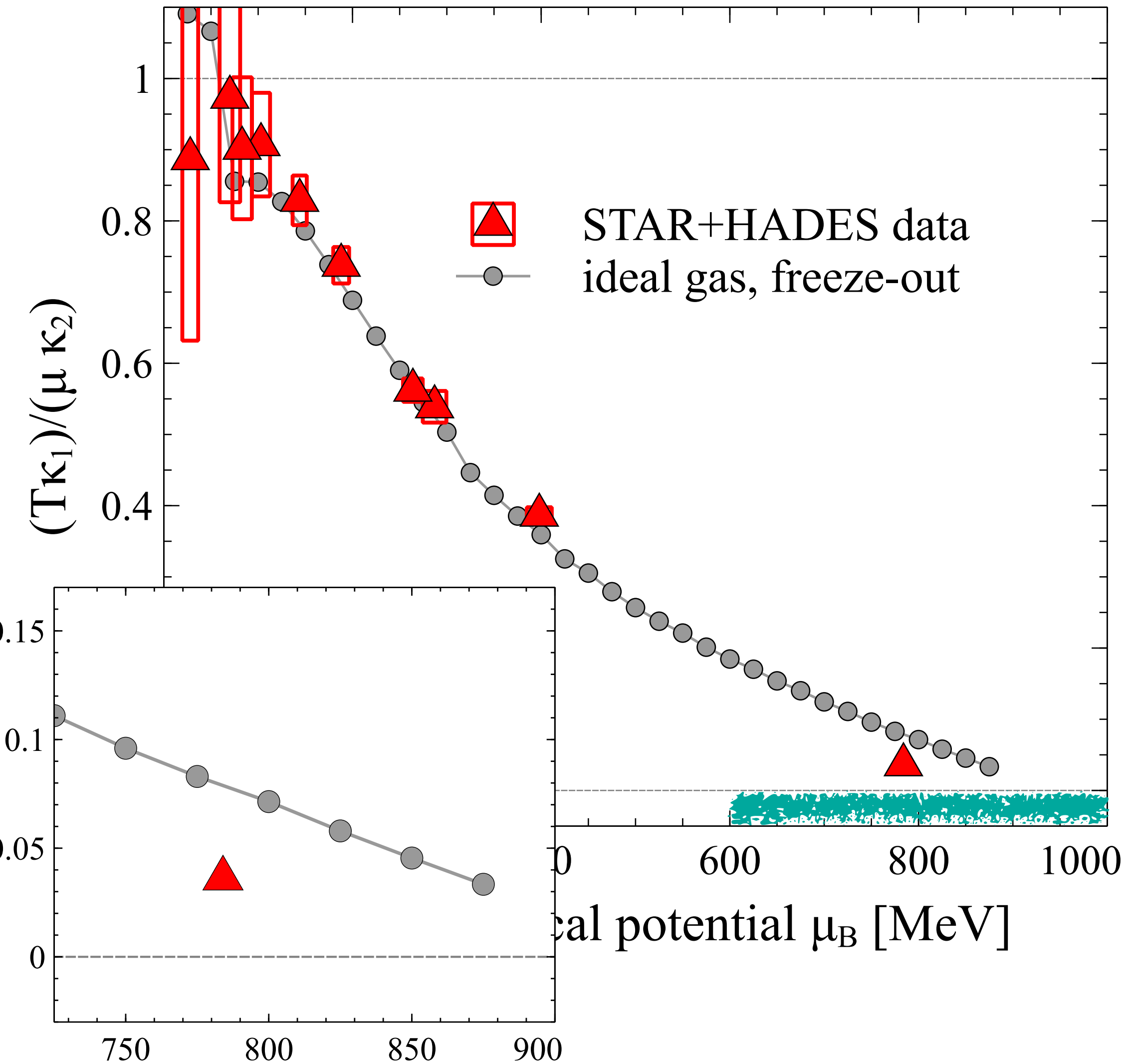
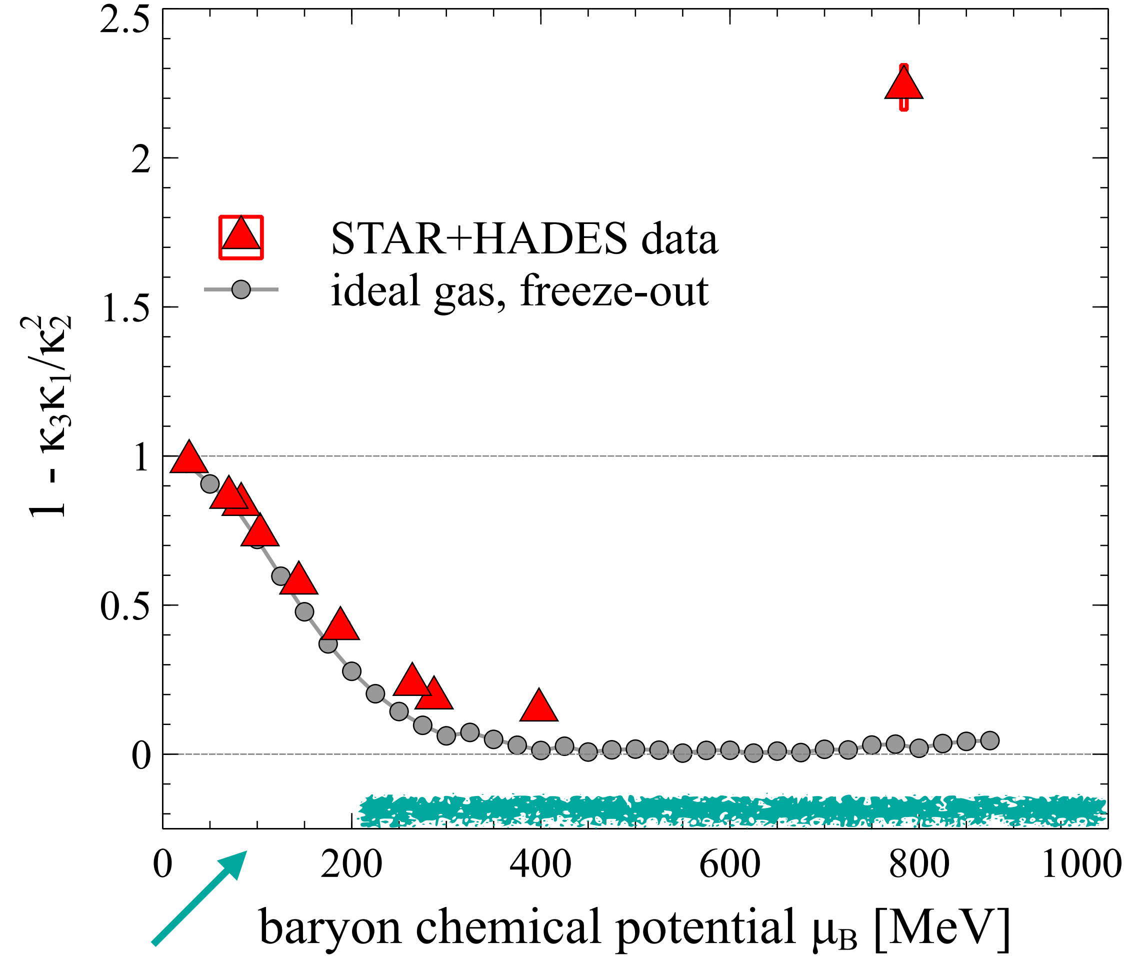
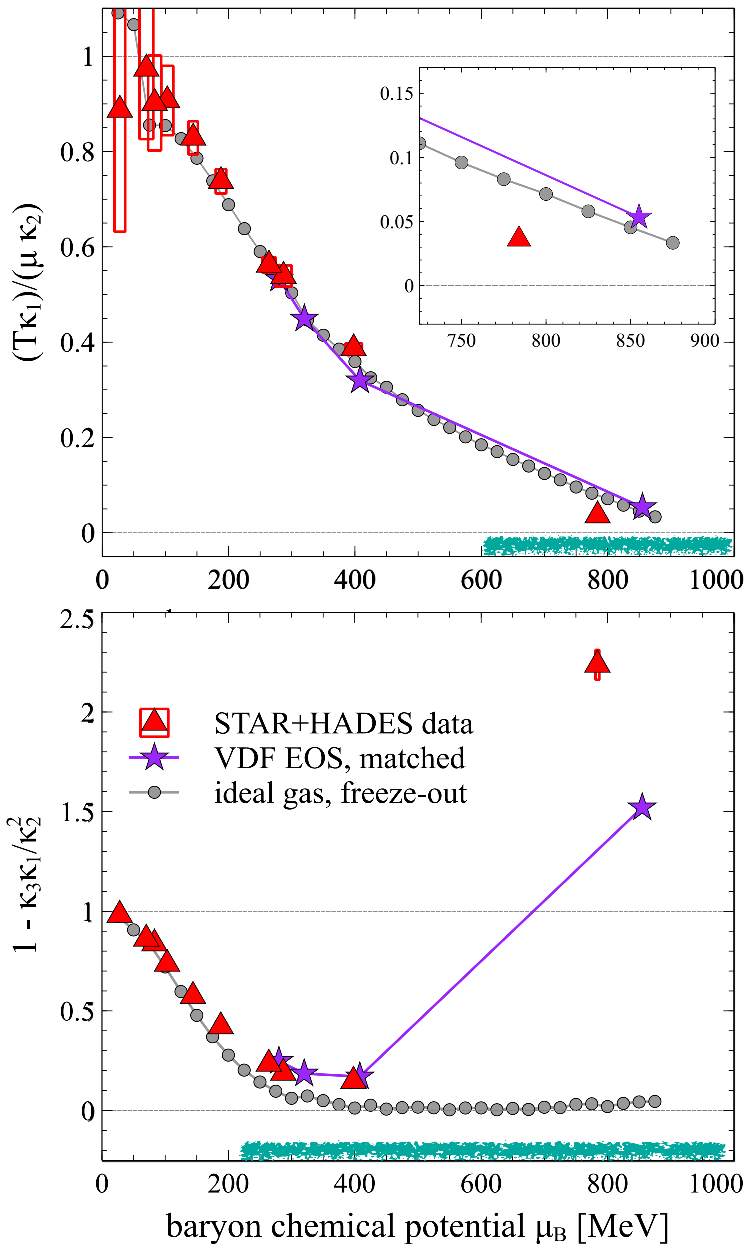




region of validity

M. Abdallah *et al.* (STAR), Phys. Rev. C **104** (2021) 2, 024902,
arXiv:2101.12413 ,STAR:2021iop
J. Adamczewski-Musch *et al.* (HADES), Phys. Rev. C **102** (2020),
arXiv:2002.08701, HADES:2020wpc

Experimental data: can we understand what is happening?



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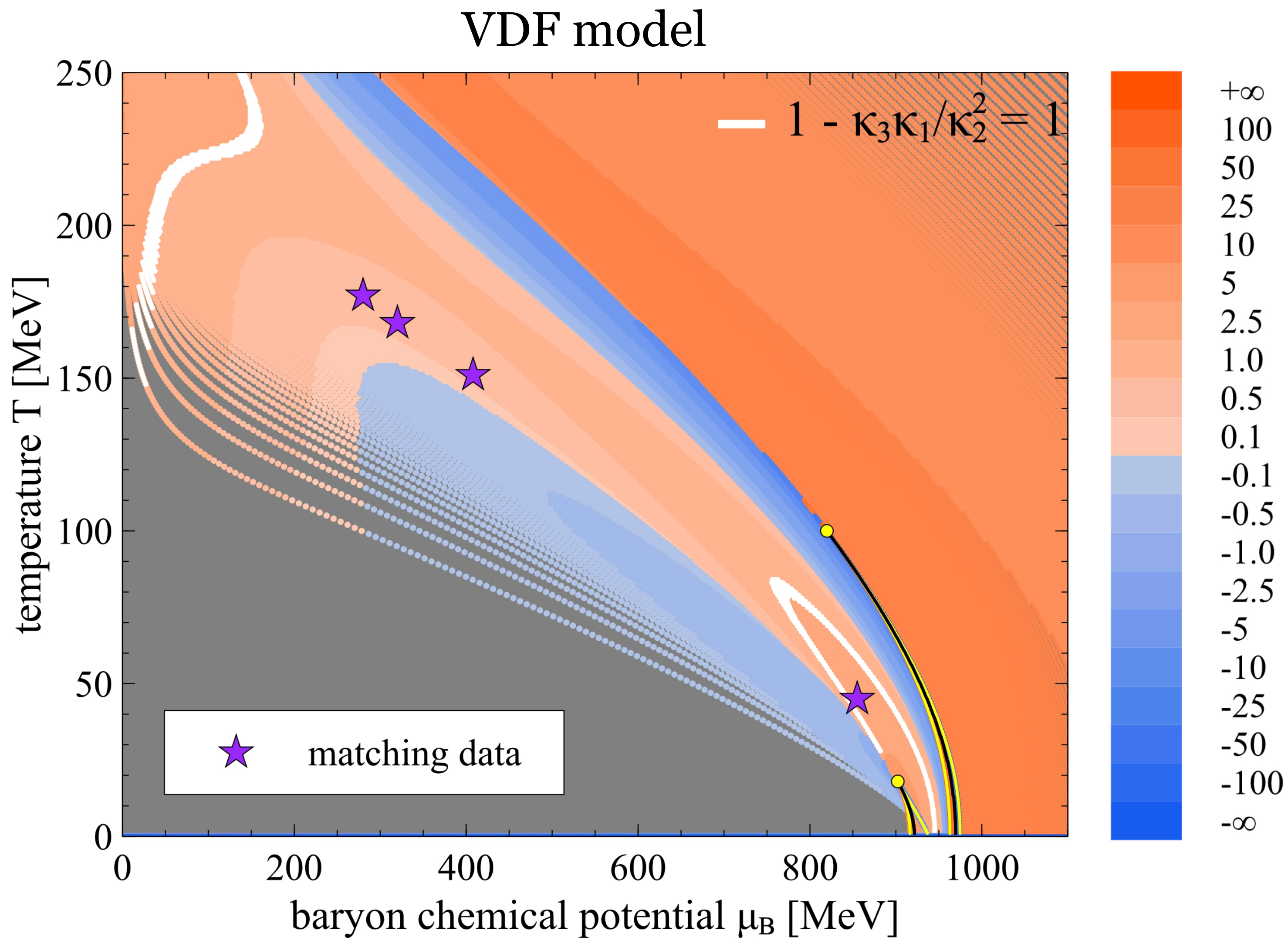

Agnieszka Sorensen (INT)

AS, D. Oliinychenko, V. Koch, L. McLerran, Phys. Rev. Lett. **127** (2021) 042303
arXiv:2103.07365, Sorensen:2021zme

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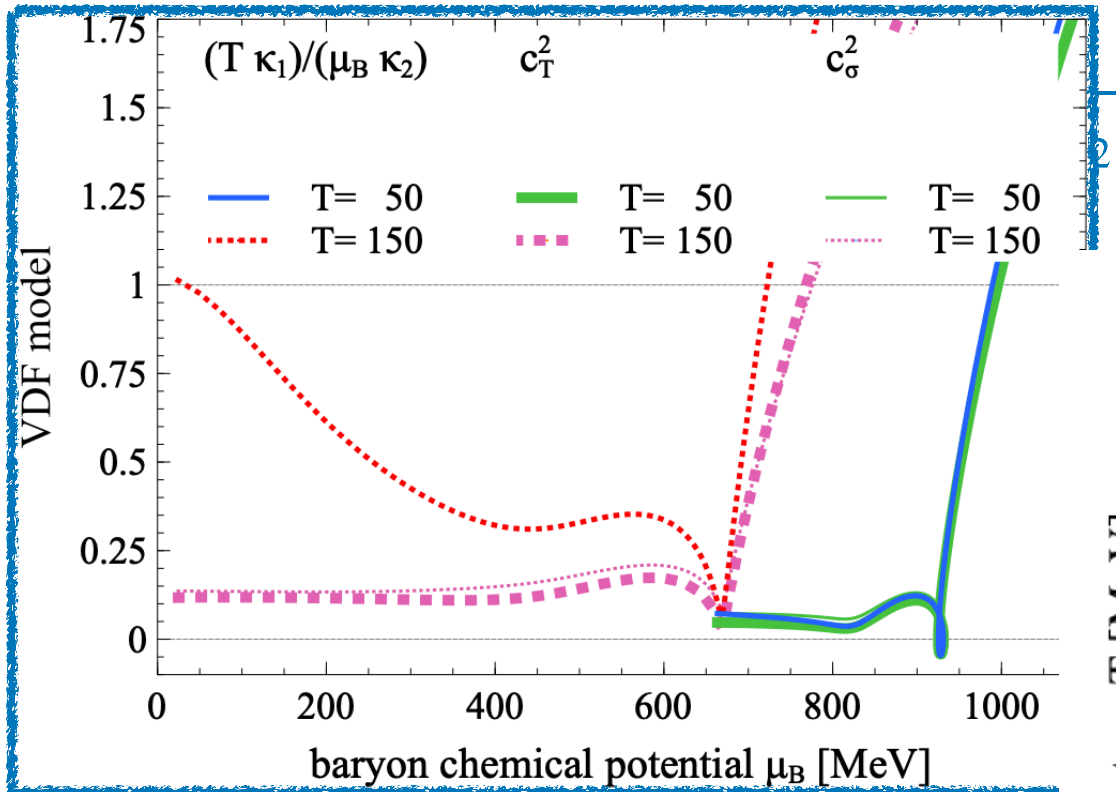
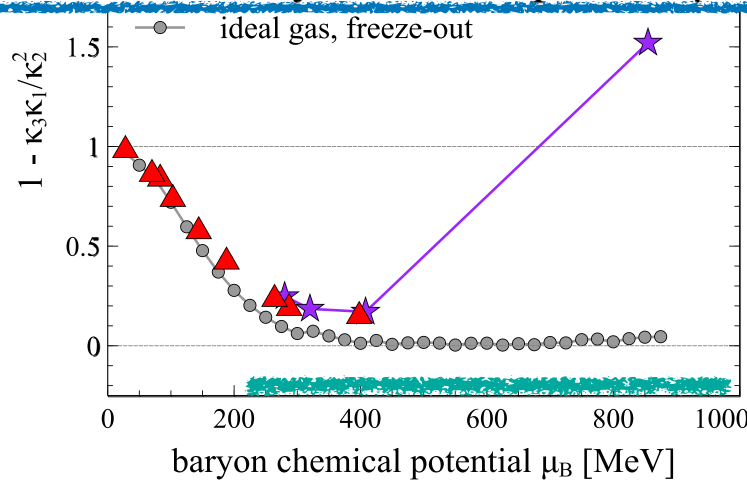
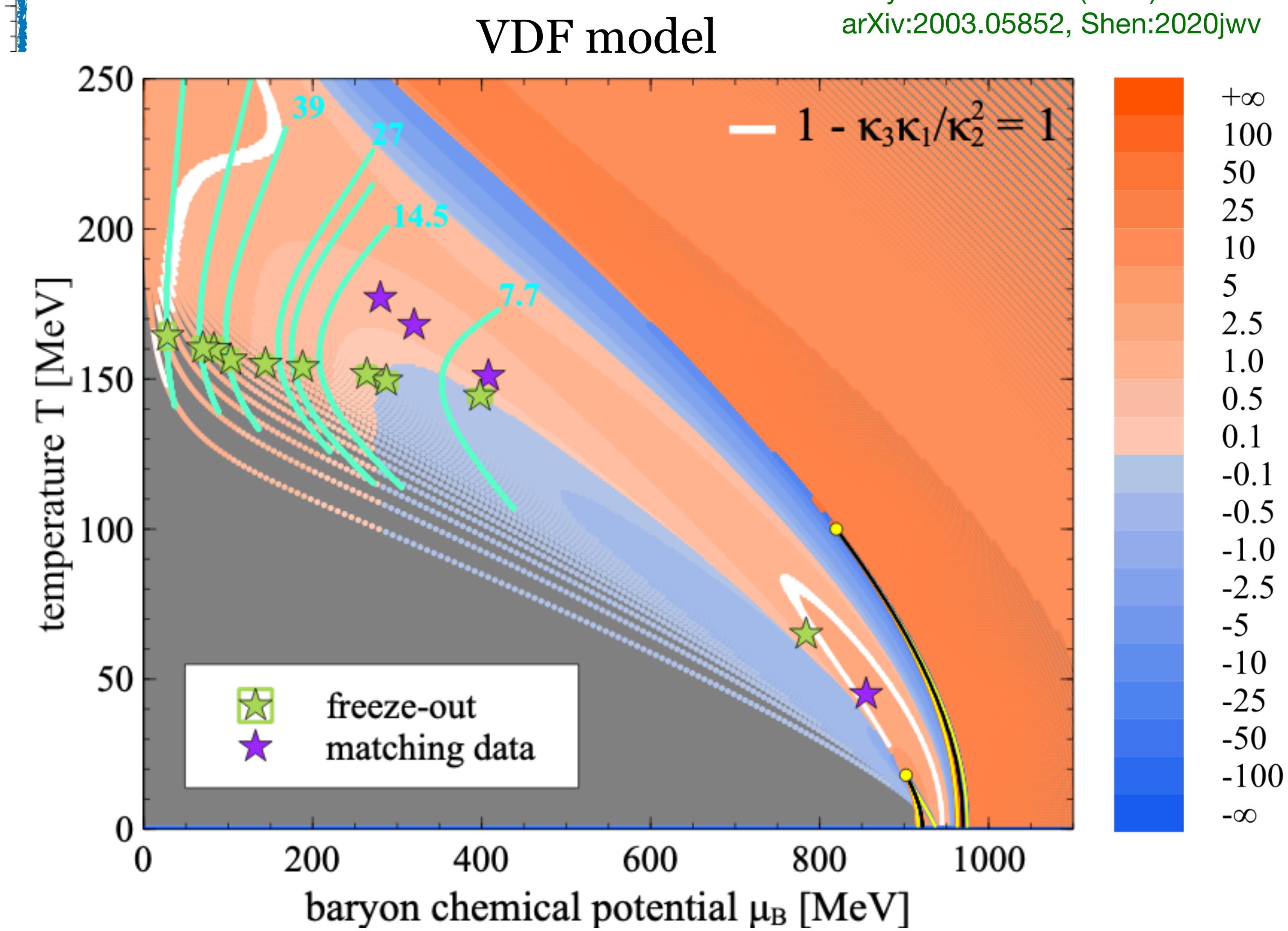




$$\left(\frac{d\ln c_T^2}{d\ln n_B}\right)_T + c_T^2 \approx 1 - \frac{\kappa_3\kappa_1}{\kappa_2^2}$$

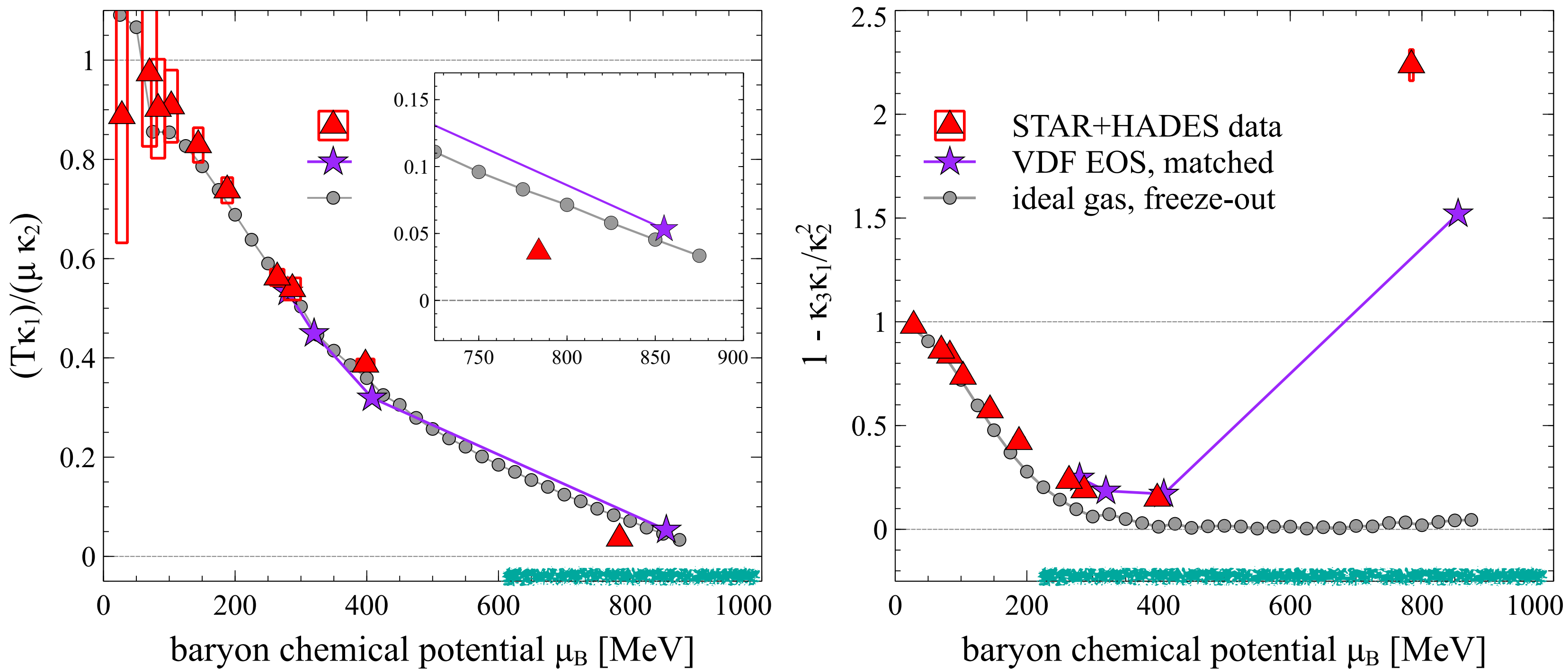
C. Chen, S. Alzhrani, Phys. Rev. C **102** (2020) 014909
arXiv:2003.05852, Shen:2020jwv



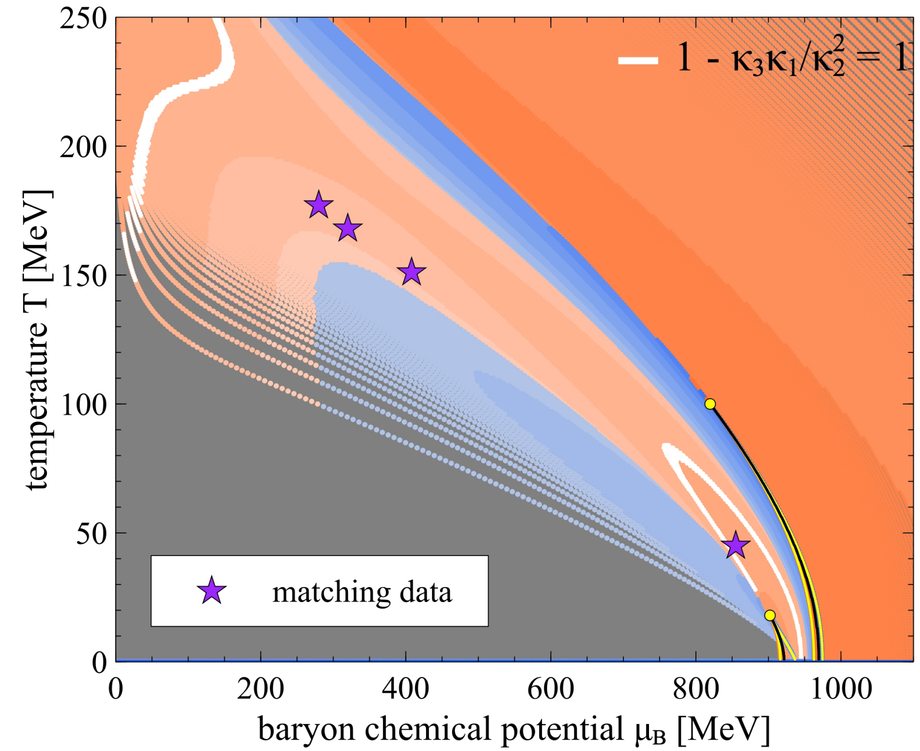
AS, D. Oliinychenko, V. Koch, L. McLerran, Phys. Rev. Lett. **127** (2021) 042303
arXiv:2103.07365, Sorensen:2021zme

The results prompt more questions

STAR + HADES data



VDF model



- *something* significant is happening: κ_3 changes sign!
- results suggest vastly different behavior of c_s^2 at energies probed by STAR (collider) and HADES
- is behavior of the cumulants at low energies dominated by hadronic effects and the nuclear liquid-gas phase transition???

Need more experimental data and comparisons with simulations!

AS, D. Oliinychenko, V. Koch, L. McLerran, Phys. Rev. Lett. **127** (2021) 042303
arXiv:2103.07365, Sorensen:2021zme

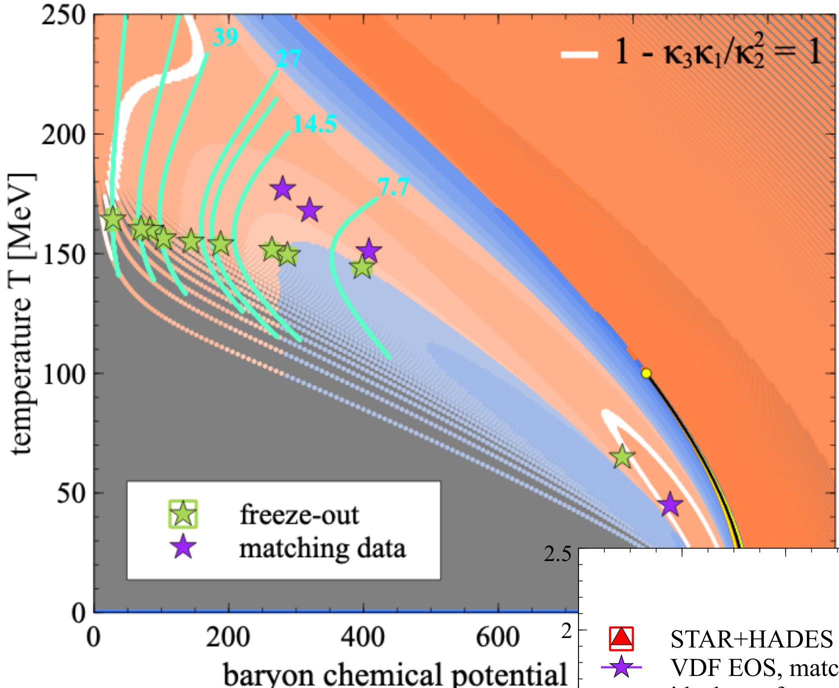
Issue 1: Proton vs. baryon cumulants

Y. Hatta, Y. and M. A. Stephanov, Phys. Rev. Lett. **91** (2003) 102003, arXiv:0302002, Hatta:2003wn

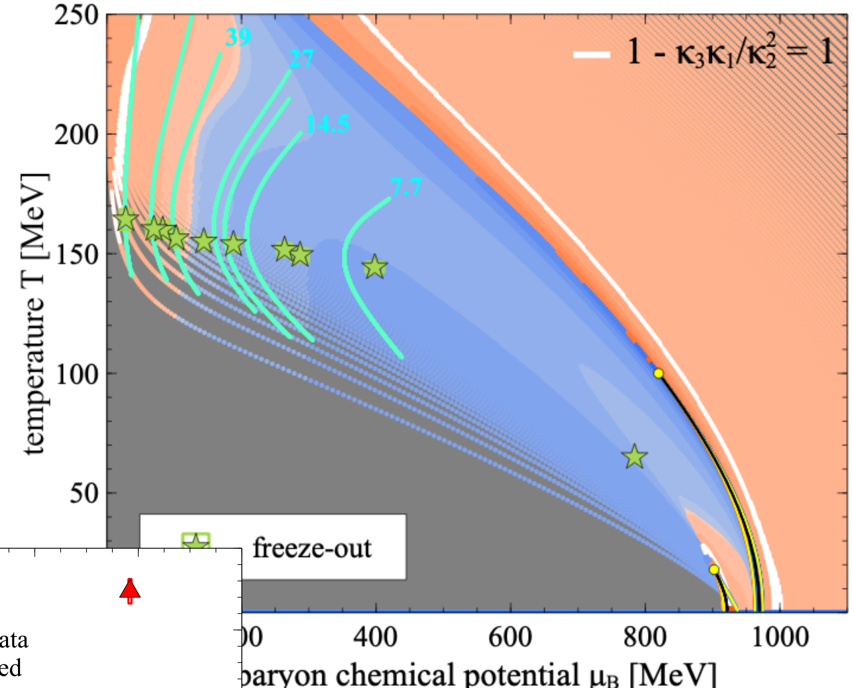
Proton cumulants said to be a good proxy for baryon cumulants.
Are they?

Definitely NOT in the VDF model:

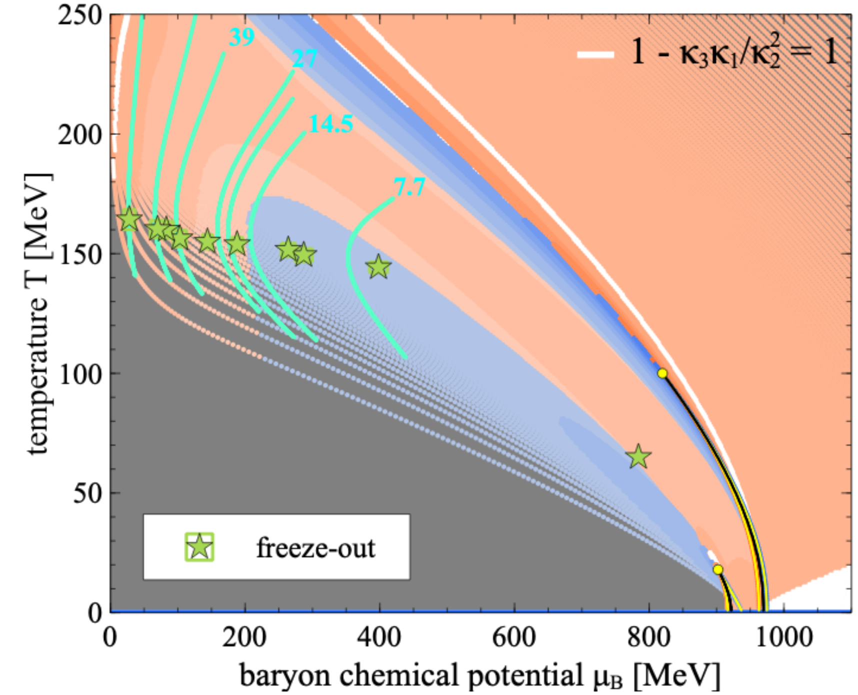
baryon cumulants

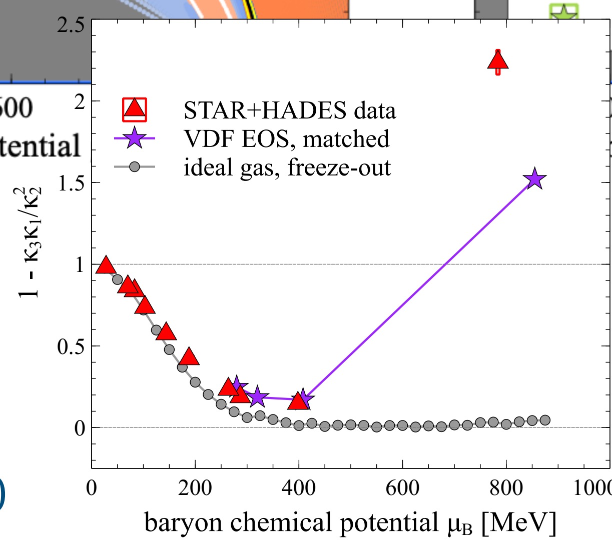


proton cumulants



binomial correction
to baryon cumulants





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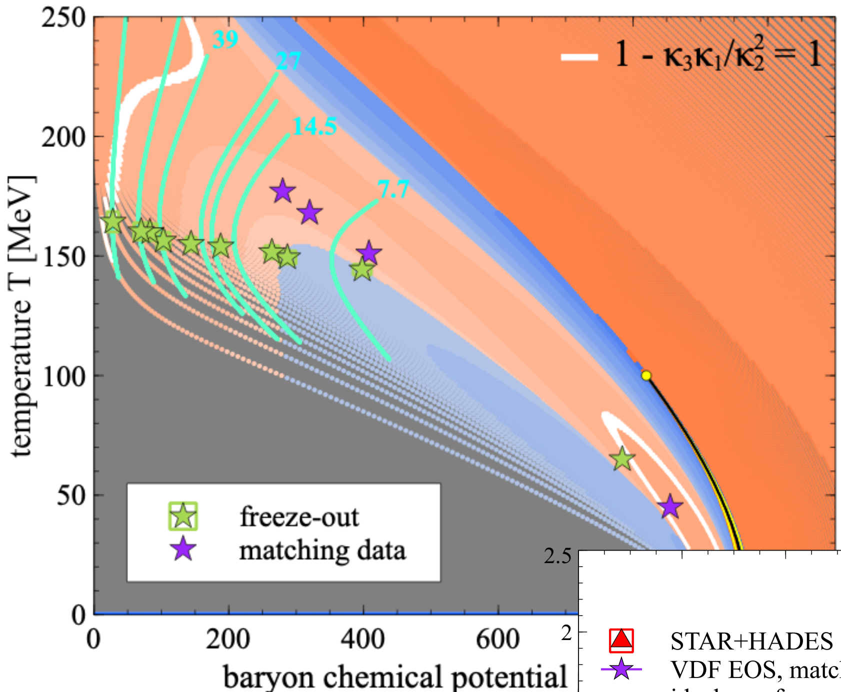
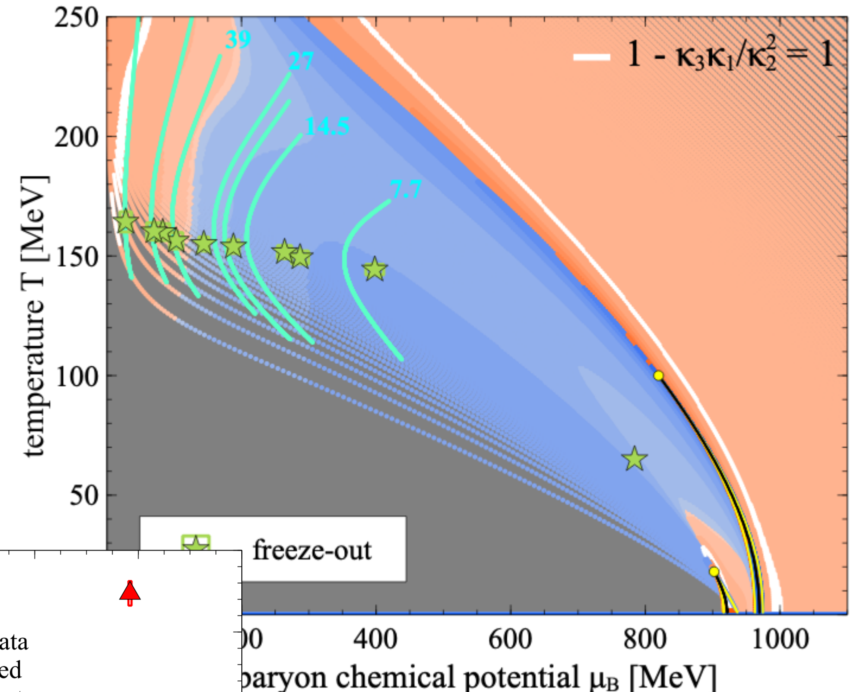
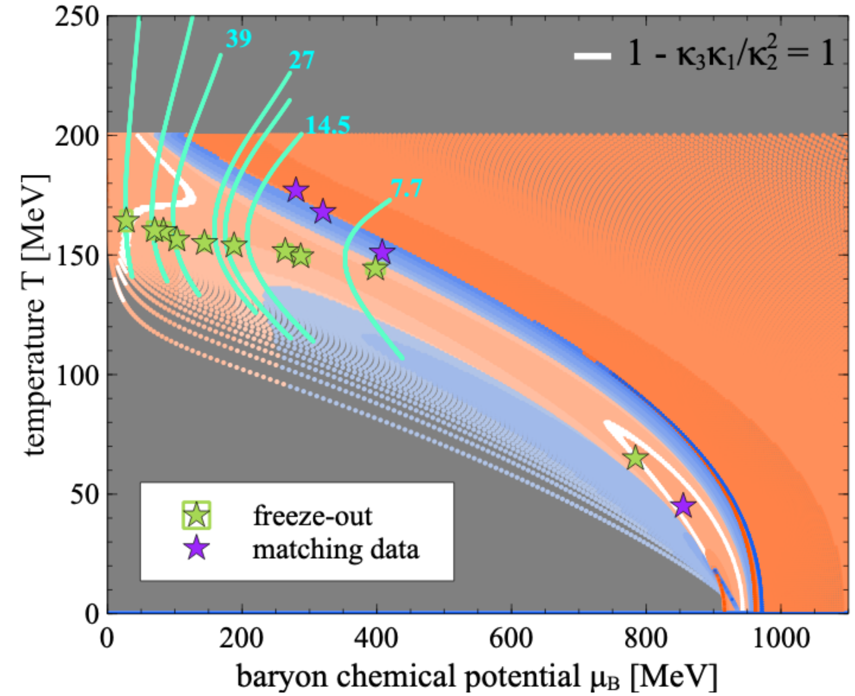
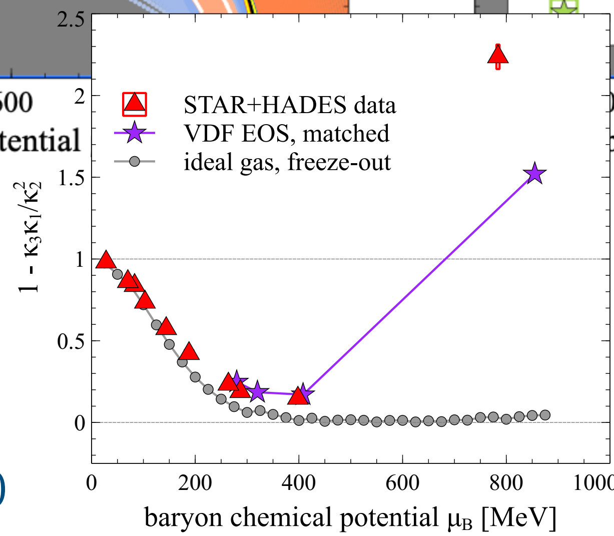


proton cumulants



baryon cumulants
(63 baryon species)



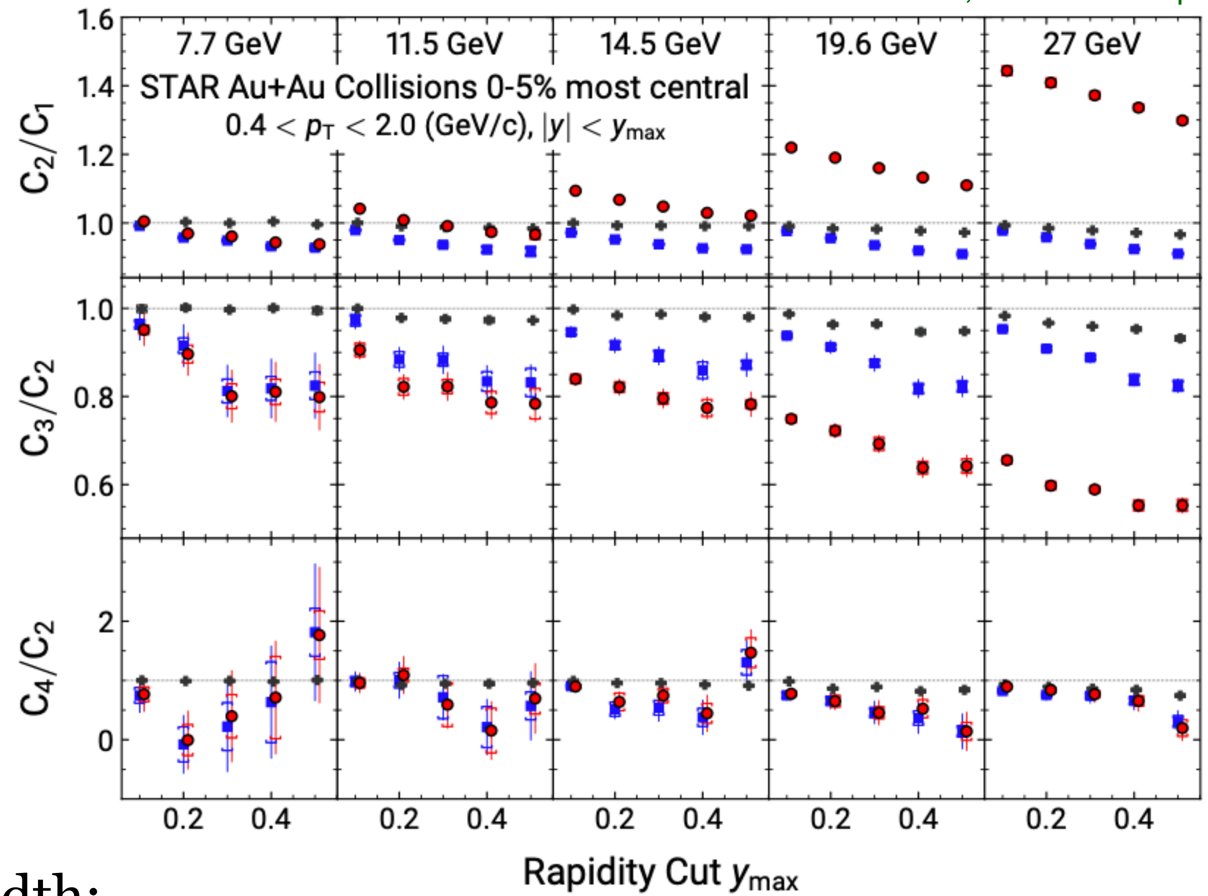


Also: momentum space vs coordinate space cumulants

V. Vovchenko and V. Koch,
"Thermodynamic approach to proton number fluctuations in baryon-rich heavy-ion matter created at moderate collision energies",
arXiv:2204.00137, Vovchenko:2022szk

Issue 2: Rapidity window dependence vs. theory calculations

STAR, Phys. Rev. C **104** (2021) no. 2, 024902,
arXiv:2101.12413, STAR:2021iop



Changing the rapidity width:

⇒ changes the probed scale (what bin width will "capture" the correlations?)

⇒ can increase/reduce baryon number conservation effects

V. Vovchenko, O. Savchuk, R. Poberezhnyuk, M. Gorenstein, V. Koch,
Phys. Lett. B **811** (2020) 135868, arXiv:2003.13905, Vovchenko:2020tsr

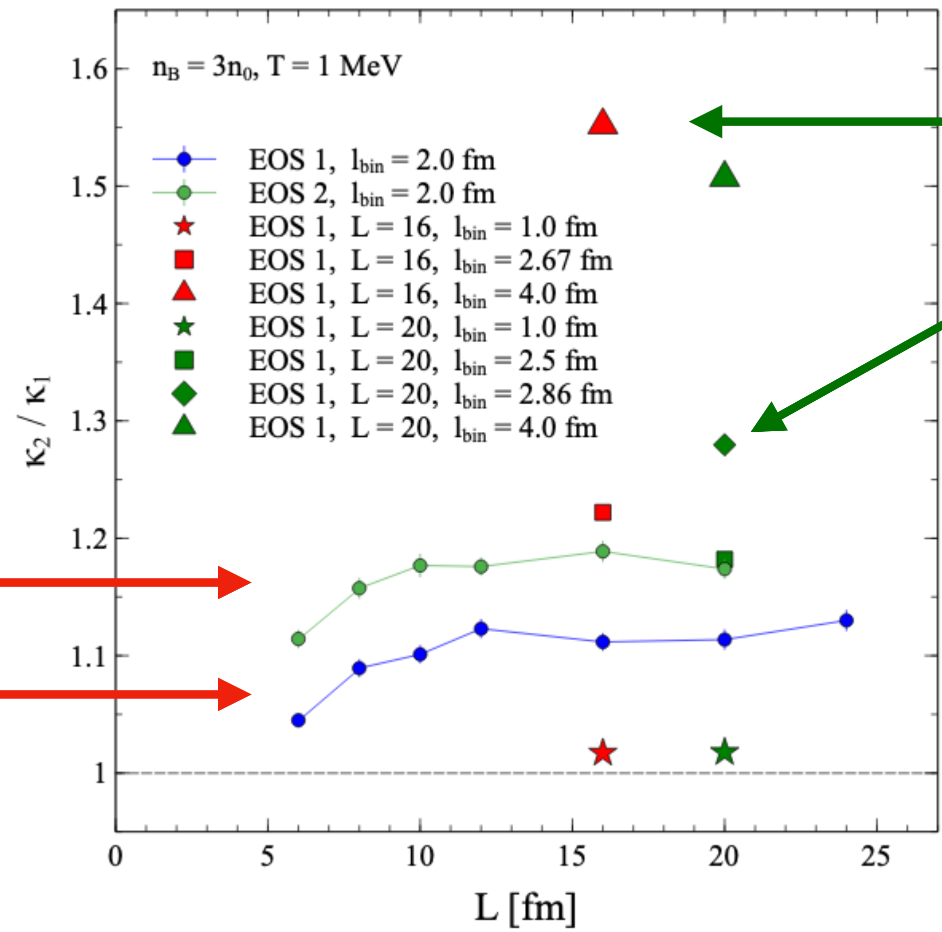
Results at which bin width should be compared with theory?

V. Kuznietsov, O. Savchuk, M. Gorenstein, V. Koch, V. Vovchenko,
"Critical point particle number fluctuations from molecular dynamics", arXiv:2201.08486, Kuznietsov:2022pcn

Issue 3: Comparison with hadronic transport simulations

Heavy-ion collisions are dynamical, messy phenomena occurring in a finite volume and within a finite time span. At low energies hadronic transport is the appropriate tool to use.

- Representing a single baryon with many ($N_T \gg 1$) test particles with fractional baryon number ($Q_B = 1/N_T$) to calculate mean-fields: has to be accounted for when comparing to experiment
- Connecting values of cumulants from simulations (e.g. SMASH) to values from the theory (e.g. VDF EOS)



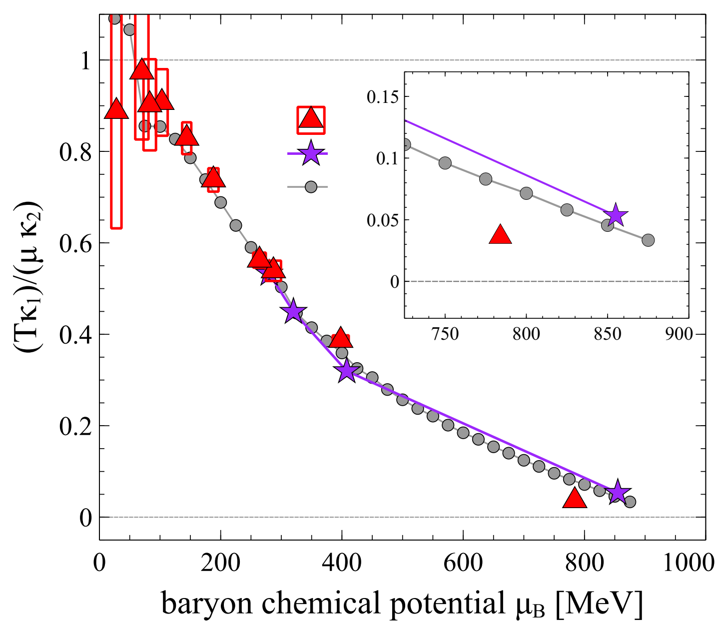
bin width dependence at a given box length

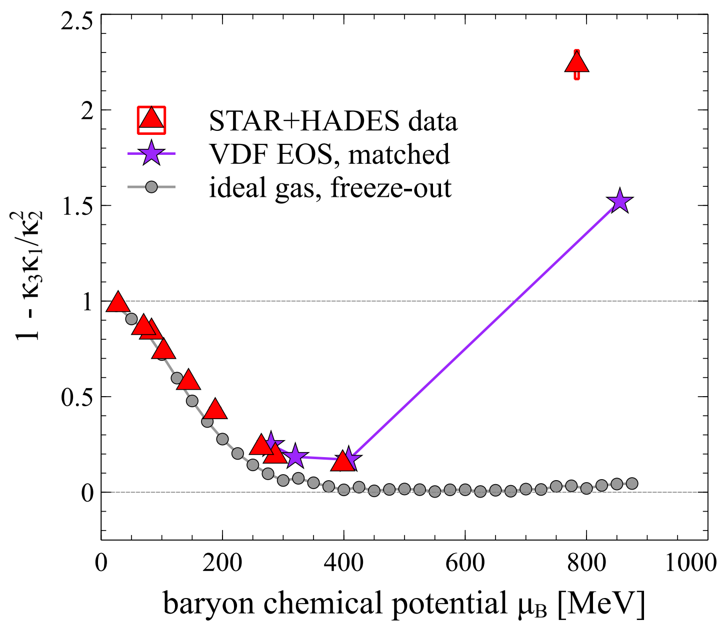
saturation of results at a given bin width with increased size of the box (two different EOSs)

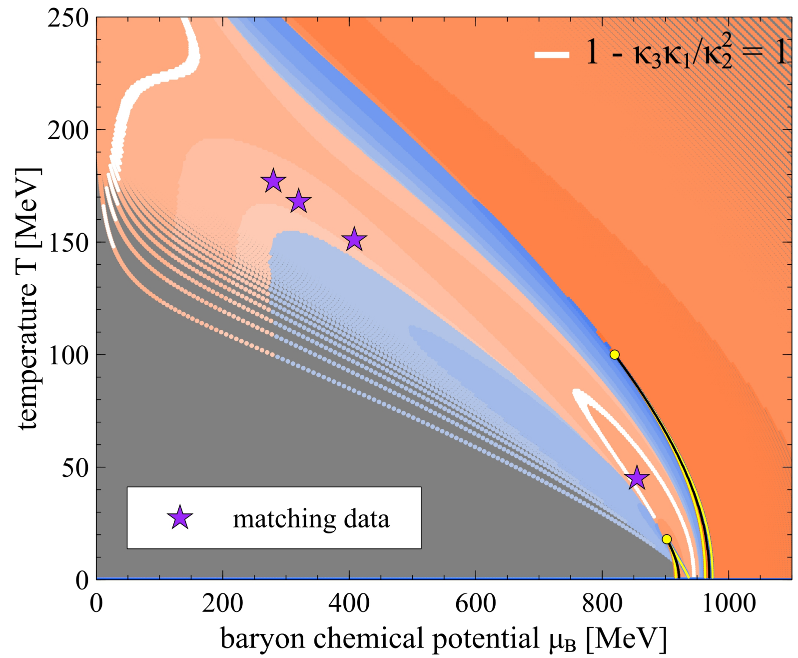
Summary

- Cumulants reimagined: if we understand them quantitatively, they can reveal the speed of sound
- Independently of the model interpretation, the data points towards interesting behavior
- The path forward is to understand finite number, binning, conservation, ... effects in fluctuation observables and connect results from measurements to theory

Thank you for your attention







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