Soft pions near the QCD chiral critical point: transport and dynamics

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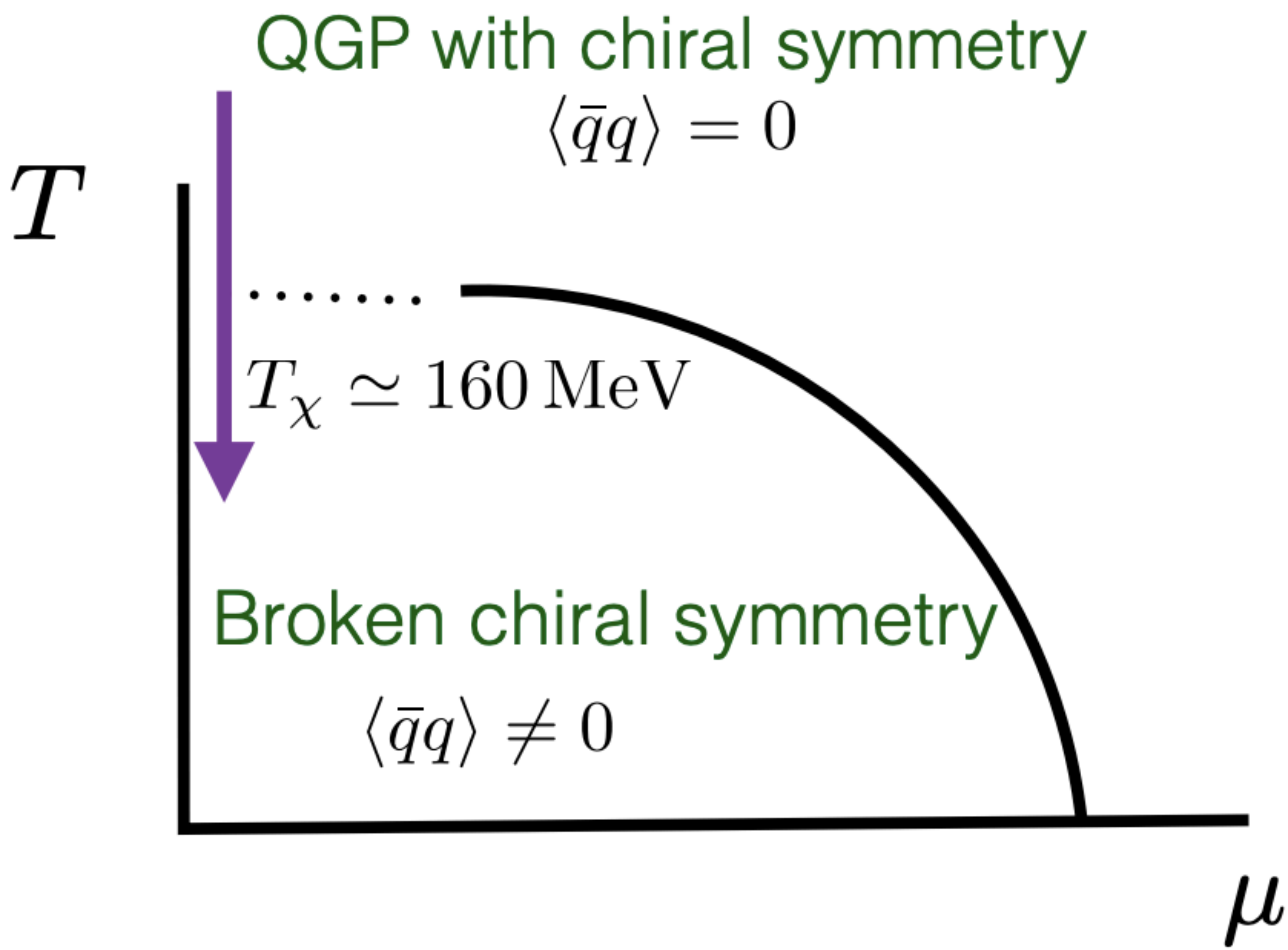
Motivation

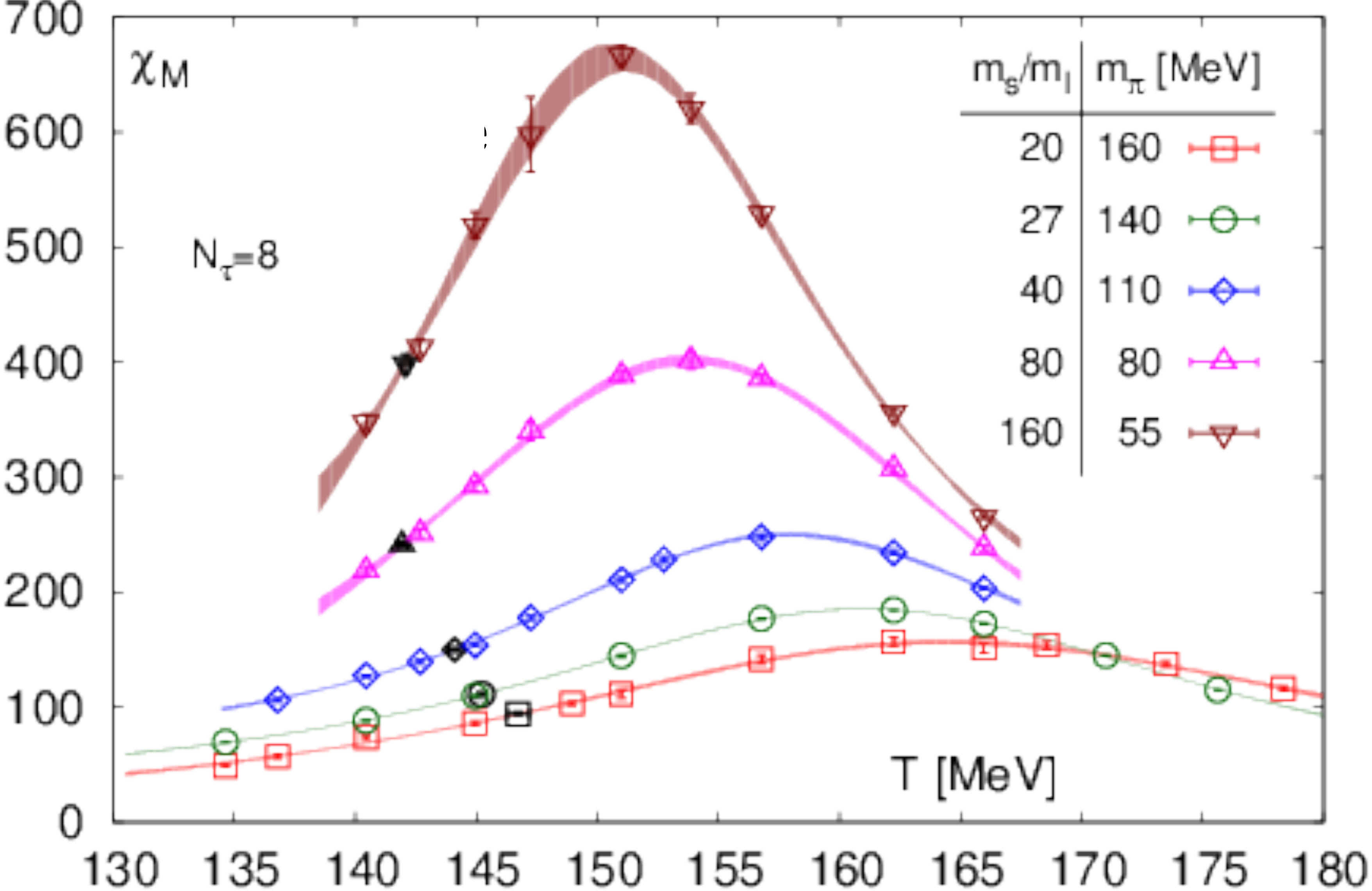


We are neglecting any hydro-dynamics of the chiral condensate !

Motivation 2

$$\chi_M = \frac{\partial \langle \bar{q} q \rangle}{\partial m_l}$$

[Ding et al PRL 2019]



The independent left and right rotation of the flavour of the quark is spontaneous broken.
The Universality class seems to be the same of O(4) scalar field

The chiral susceptibility seems to respect the scaling as predicted from O(4) universality class in d=3

Setup: the O(4) phase transition

The (approximated) conserved quantities of 2 flavour QCD are

$T^{\mu\nu}$	J^μ_V	J^μ_A
Stress	Iso-vector (isospin)	Iso-axial
(T, u^μ)	μ_V	μ_A
	$\bar{q}\gamma^0 t_I q$	$\bar{q}\gamma^0\gamma_5 t_I q$

The approximate flavour symmetry $SU(2)_L \times SU(2)_R \sim O(4)$

The order parameter is the chiral condensate

$$\langle \bar{q}q \rangle \sim \phi_\alpha = (\sigma, \varphi_\alpha) = (\text{sigma}, \text{pions})$$

We need the hydrodynamic theory of the charge and the order parameter

Rajagopal Wilczek (93)
Son and Stephanov (02)

Equation of motion (Model G)

Rajagopal Wilczek (93)

Chiral condensate ϕ_a + Axial and Vector charge $n_{ab} = \chi_0 \mu_{ab}$

$$\partial_t \phi_a + g_0\, \mu_{ab} \phi_b = \Gamma_0 \nabla^2 \phi_a - \Gamma_0 (m_0^2 + \lambda \phi^2) \phi_a + \Gamma_0 H_a + \theta_a \,,$$

$$\partial_t n_{ab} + g_0\, \nabla \cdot (\nabla \phi_{[a} \phi_{b]}) + H_{[a} \phi_{b]} = D_0 \nabla^2 n_{ab} + \partial_i \Xi^i_{ab} \,.$$

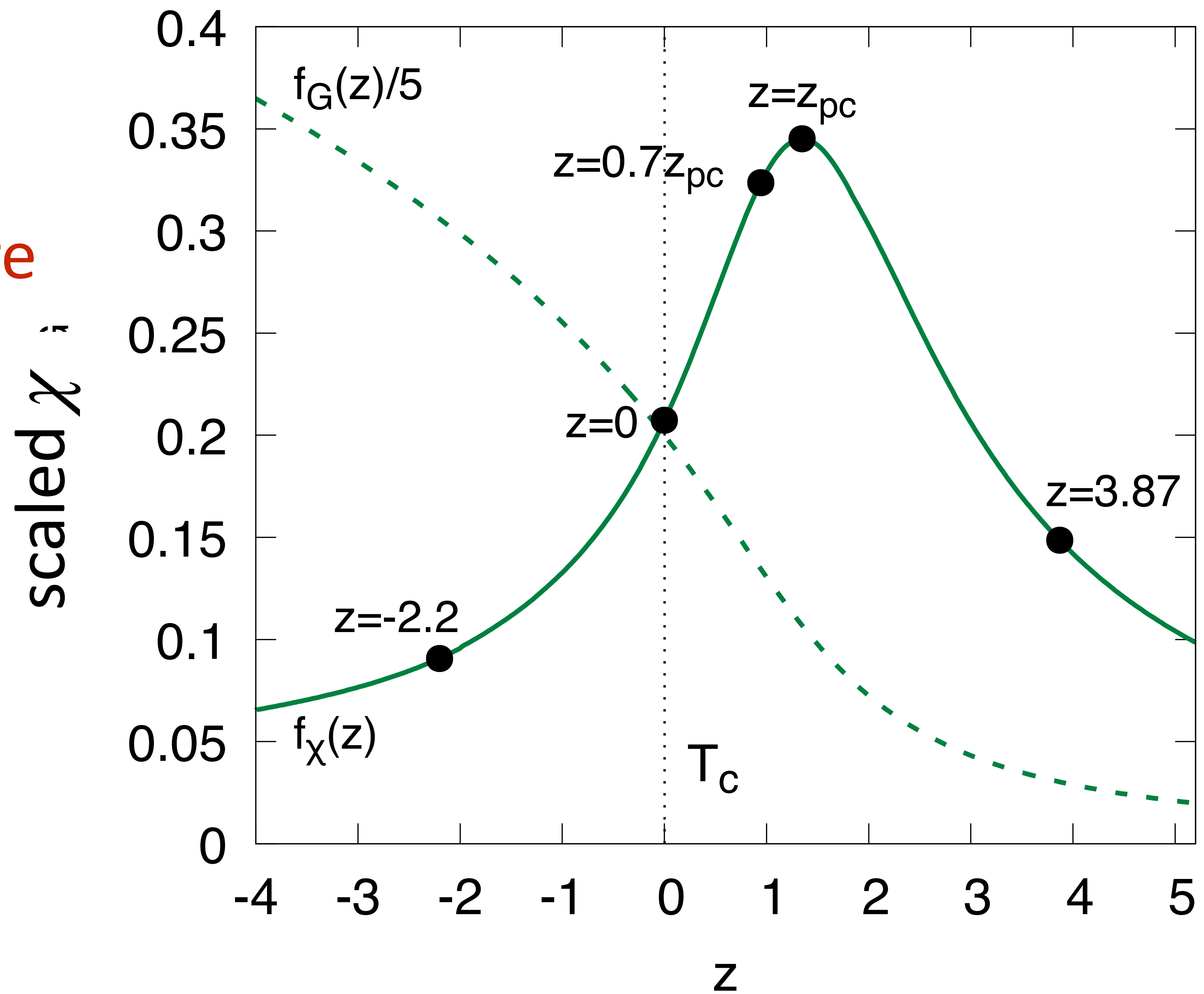
Ideal part — Dissipative part — Gaussian Noise

- The ideal part is charge conservation and Josephson constraint
- Two dissipative coefficient Γ_0 and D_0 and noise
- The simulation of the stochastic process is done with an ideal step and metropolis update.

Diffusion at high temperature, pion propagation at low temperature as the vev develops

Simulating the O(4) crossover

Low temperature



High temperature

Susceptibility

$$\chi = h^{1/\delta - 1} f_\chi(z)$$

Vev

$$\bar{\sigma} = h^{1/\delta} f_G(z)$$

Scaling variabile

$$z = h^{-1/\beta\delta}\left(\frac{T - T_c}{T_c}\right)$$

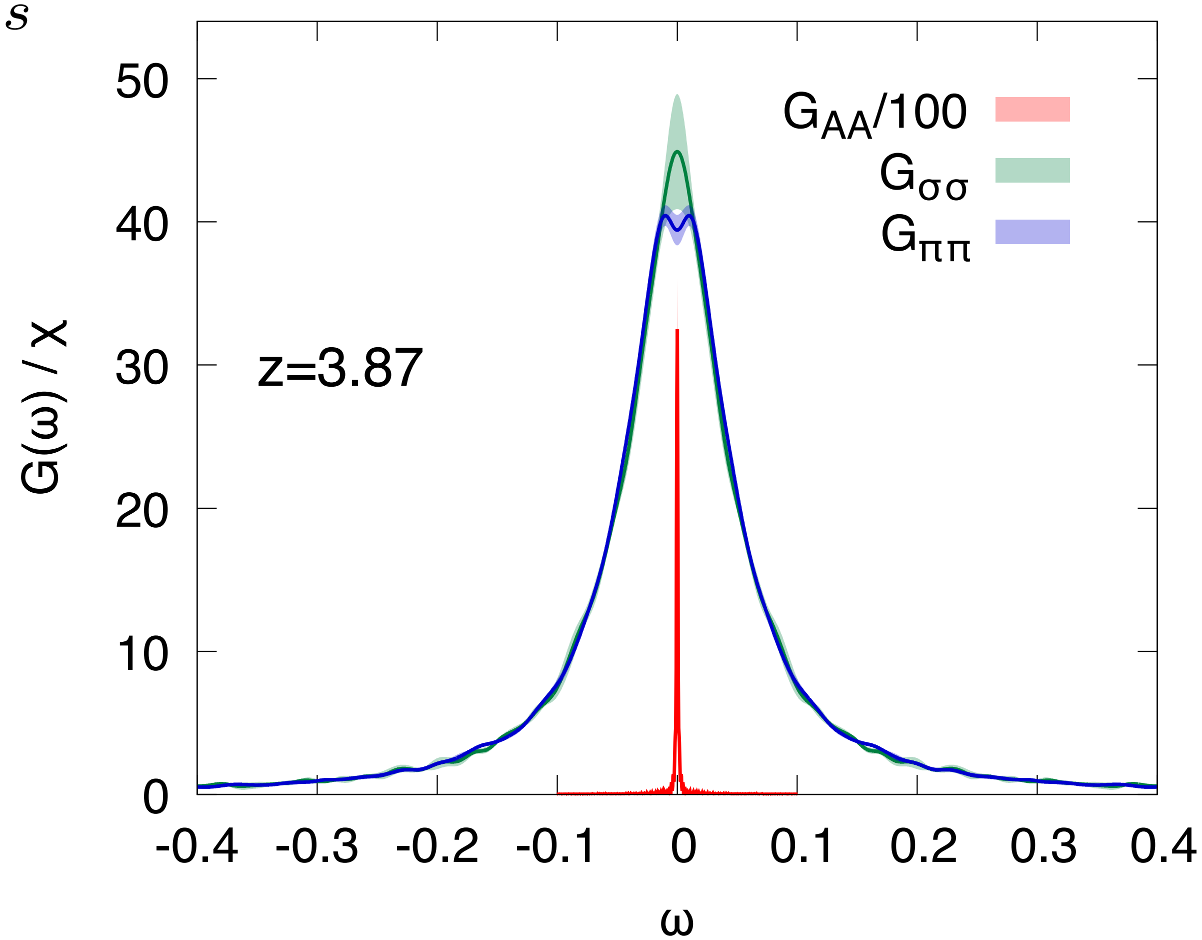
High temperature

$$G_{\sigma\sigma}(t,k) \equiv \frac{1}{V}\langle \sigma(t,\mathbf{k})\sigma(0,-\mathbf{k})\rangle_c\,,$$

$$G_{\pi\pi}(t,k) \equiv \frac{1}{3V}\sum_s \langle \pi_s(t,\mathbf{k})\pi_s(0,-\mathbf{k})\rangle_c\,,$$

$$G_{AA}(t,k) \equiv \frac{1}{3V}\sum_s \langle n_A^s(t,\mathbf{k})n_A^s(0,-\mathbf{k})\rangle_c\,,$$

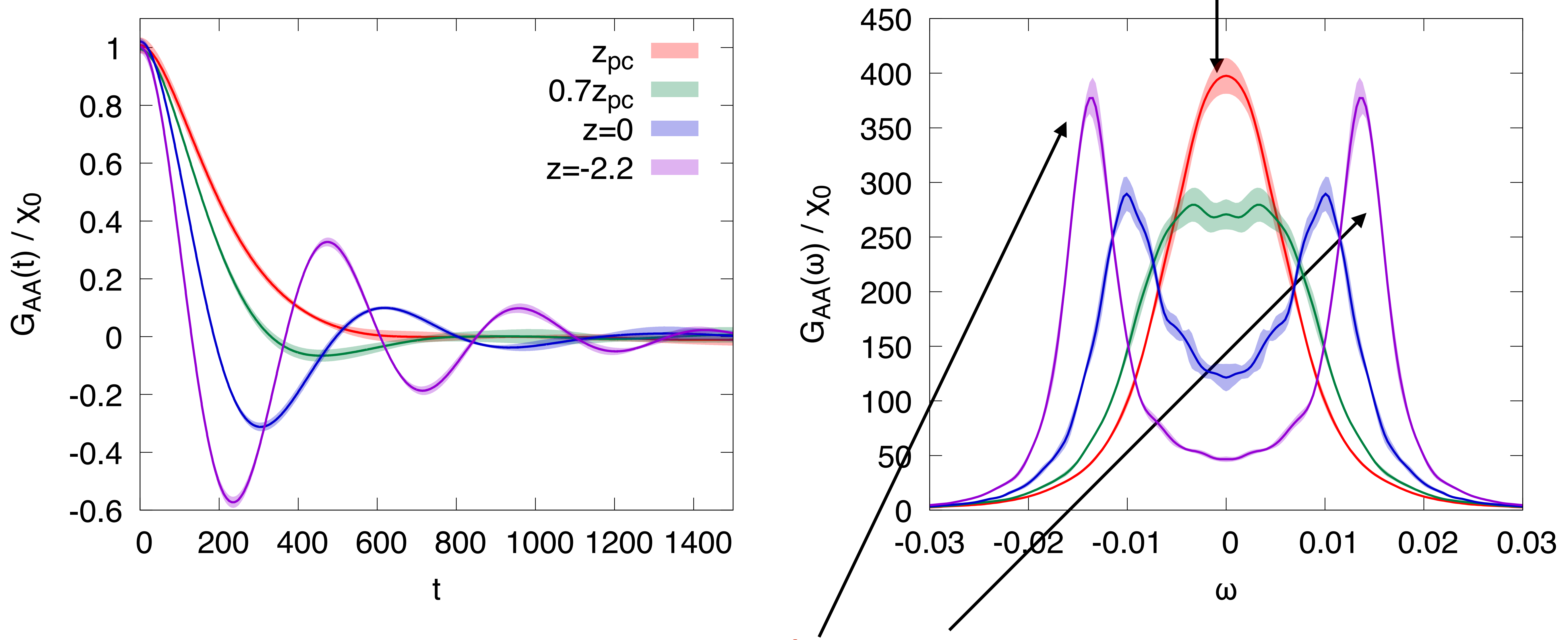
We focus on the statistical correlator at $k = 0$



The axial charge is almost conserved the O(4) field are simply dissipate with a broad width

Propagation of axial charge across the transition

Quark diffusion T>>Tc

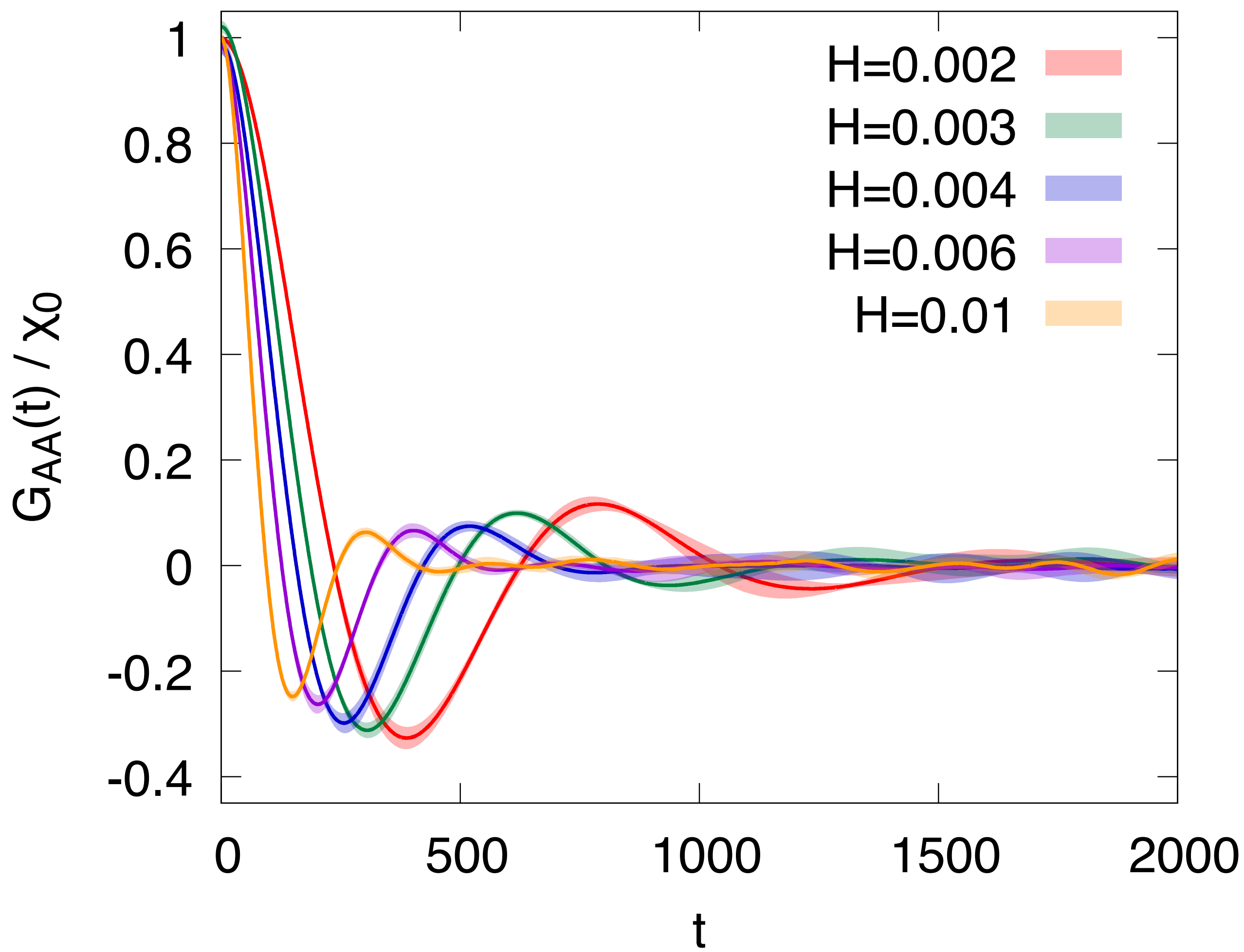


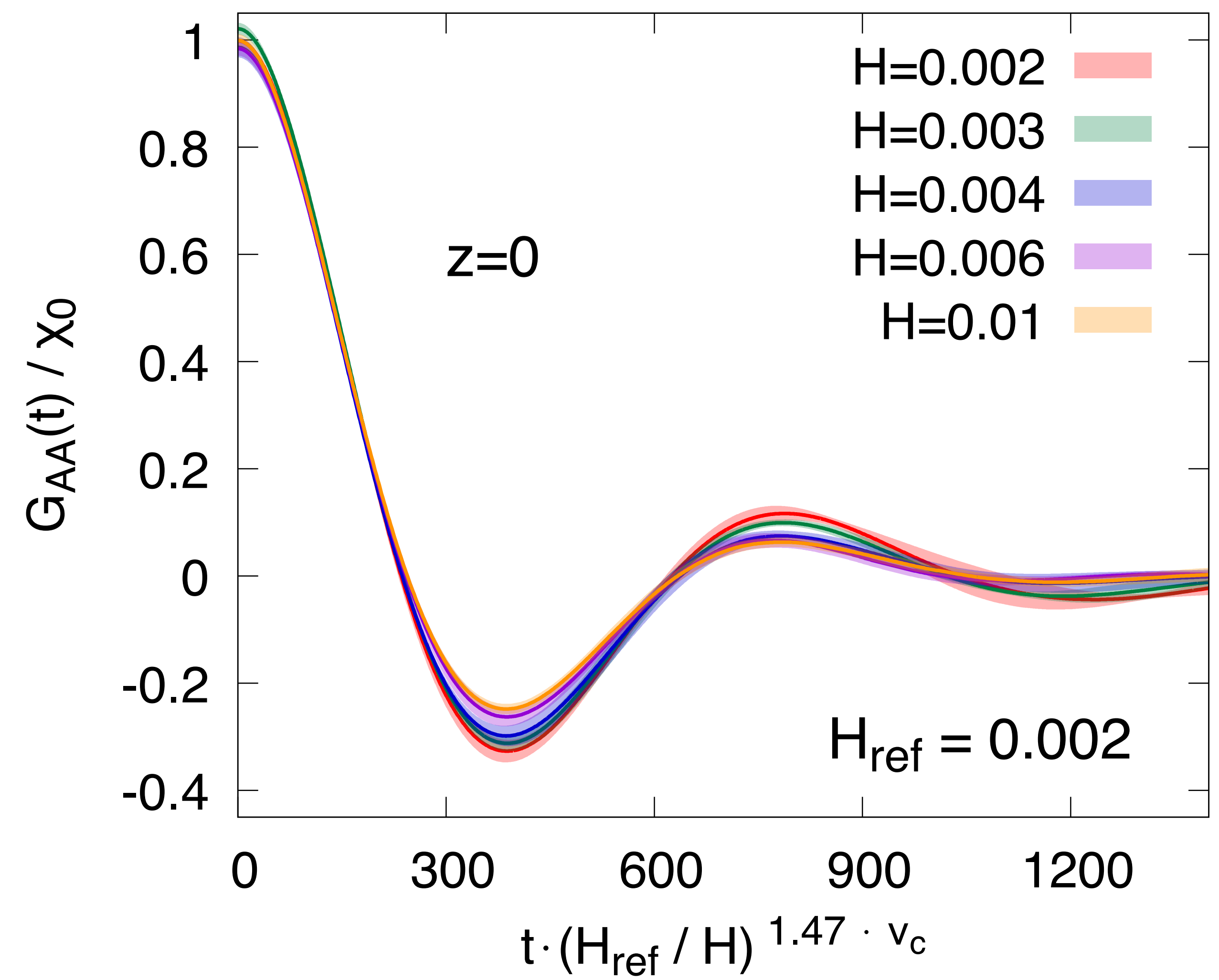
Quasiparticle Pions T<<Tc

Around T_{pc} the axial charge start changing form a diffusive form to a quasiparticle one

Dynamical scaling on the critical line

On the Critical line z=0 we should have scaling with a dynamical critical exponent ζ





$$\frac{G_{AA}(t,H)}{\chi_0} = Y_A^c\left(H^{\zeta\nu_c}t\right) ,$$

Mesure: $\zeta = 1.47 \pm 0.01$

Expected: $\zeta = d/2 = 1.5$

Rajagopal Wilczek (93)

Effective Boltzmann equation

E.G., A.Soloviev, D. Teaney, F. Yan PRD (2020)

From the linear propagator we can define (using the Wigner transform) an effective kinetic description of the soft pions distribution function

$$\partial_t f_\pi + \frac{\partial E_{\mathbf{p}}}{\partial \mathbf{q}} \frac{\partial f_\pi}{\partial \mathbf{x}} - \frac{\partial E_{\mathbf{p}}}{\partial \mathbf{x}} \frac{\partial f_\pi}{\partial \mathbf{q}} = \text{interaction terms}$$

Well below the phase transition the pions propagate like quasipartilce with a modified energy dispersion from the medium

$$E_{\mathbf{p}} = v^2(p^2 + m^2)$$

Depends on $\bar{\sigma}$

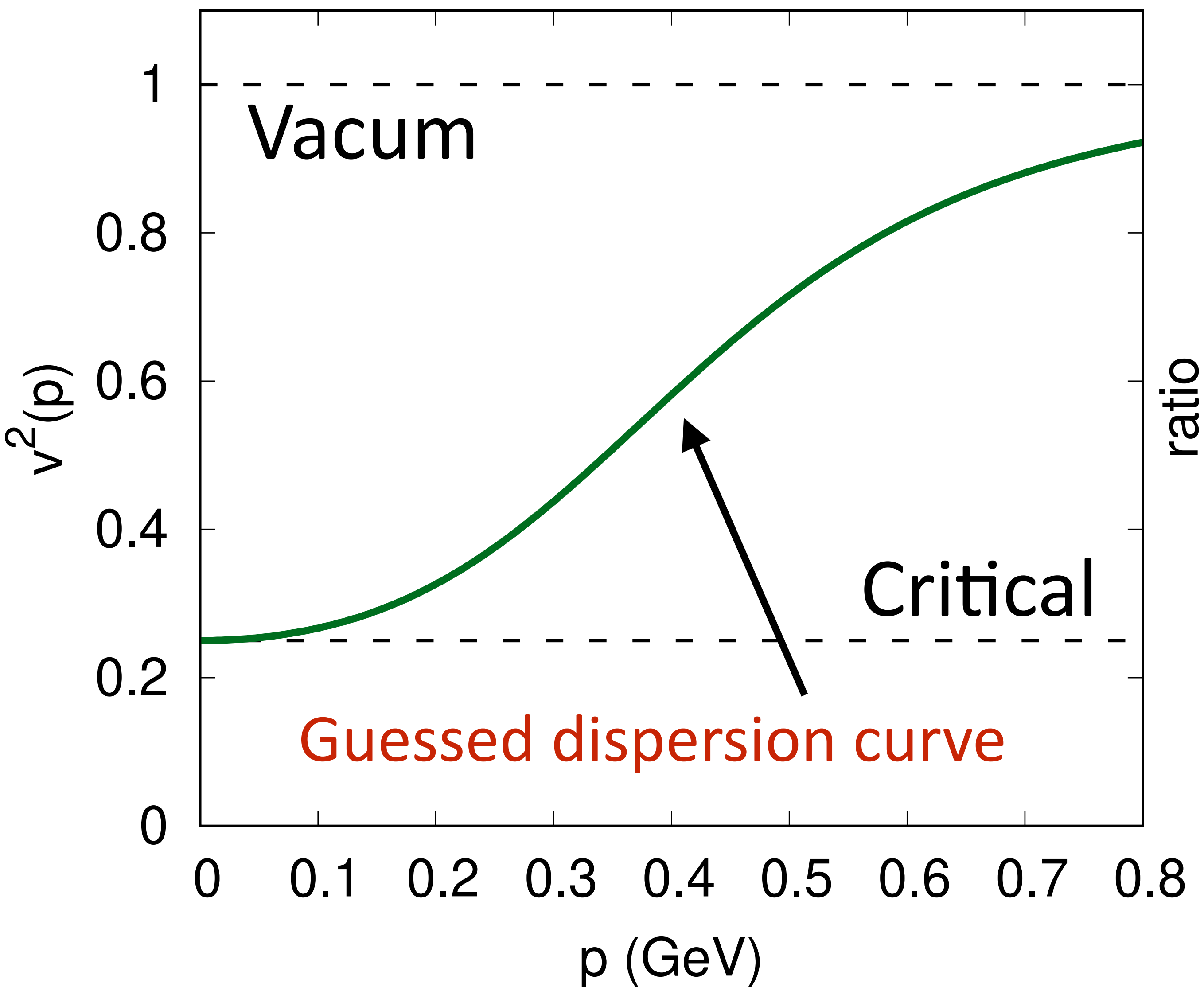
Soft pion enhancement

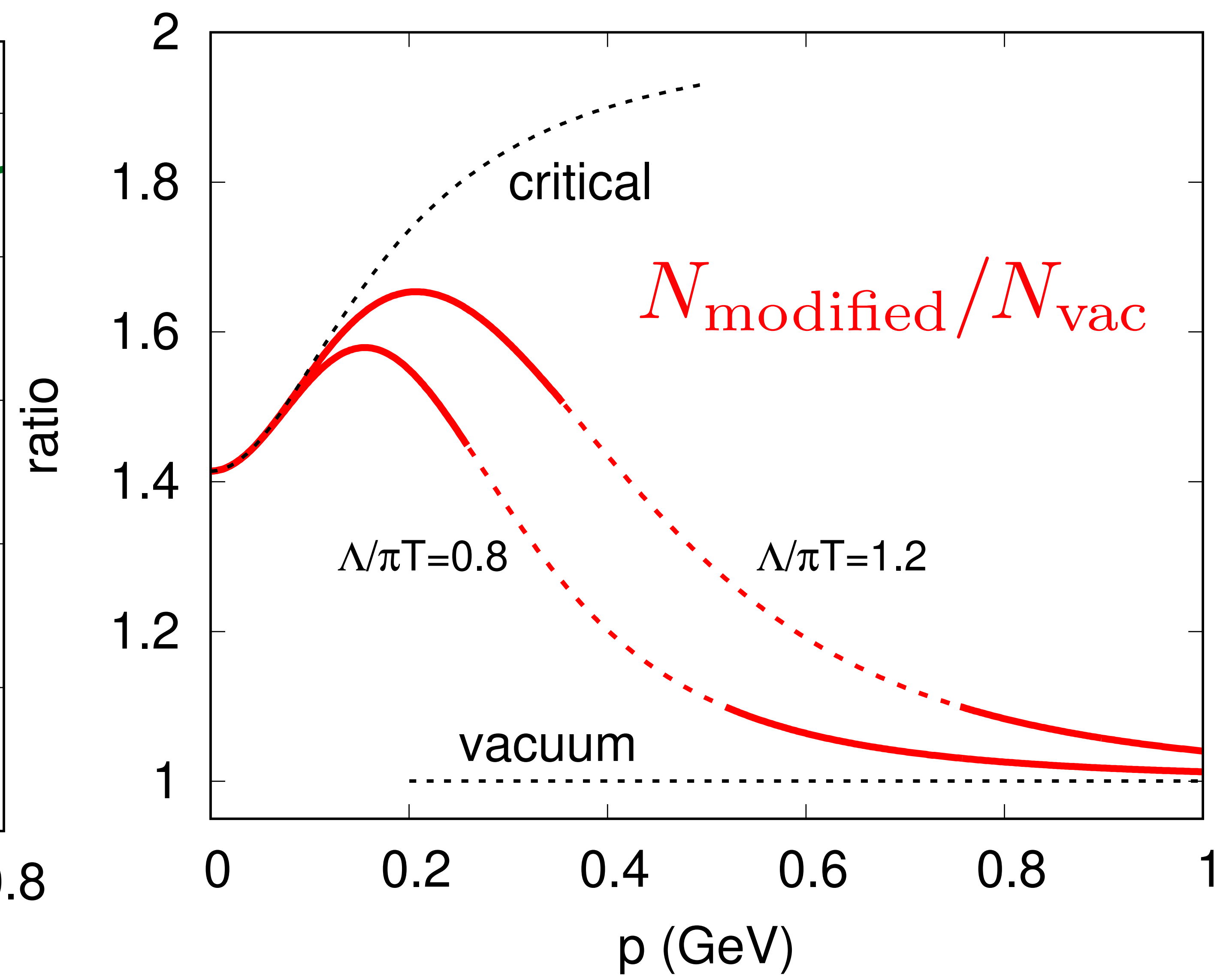
E.G., A. Soloviev, D. Teaney, F. Yan PRD (2021)

The dispersion curve get modified form the phase transition

$$E_{\mathbf{p}} = v^2(p)(p^2 + m^2)$$

$$n(E_p) = \frac{1}{e^{E_p/T} - 1}$$





Pion enhanced $p < 0.5$ GeV

Thank you!