UNIVERSITY OF HELSINKI

(Non-)perturbative jet dispersion in hot QCD[☸]

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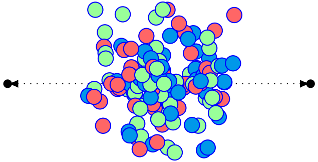
[☸] J. Ghiglieri, G. D. Moore, P. Schicho, and N. Schlusser, *The force-force-correlator in hot QCD perturbatively and from the lattice,* JHEP **02** (2022) 58 [2112.01407]

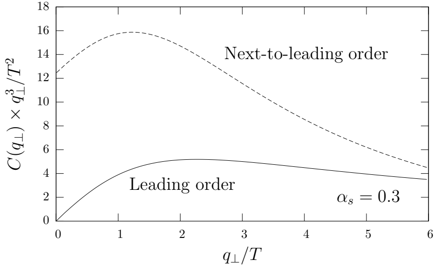
Motivation

Heavy-ion collisions

Hard particles carry most of the stress-energy tensor $P \sim T$.
Medium soft modes at scale $P \sim gT$.

Jet-medium interactions in the Quark-Gluon Plasma (QGP) can receive large non-perturbative IR contributions.[1]





[1] S. Caron-Huot, *O(g) plasma effects in jet quenching*, Phys. Rev. D **79** (2009) 065039 [0811.1603]
cf. **talks** by **J. Brewer Fri 11:50** and **J. Ghiglieri Fri 12:30**

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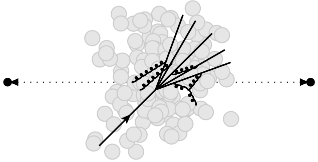

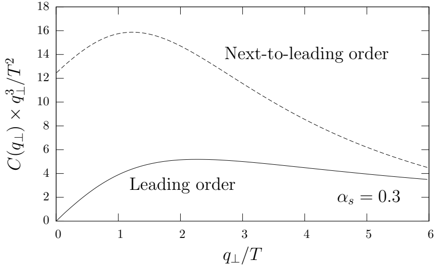

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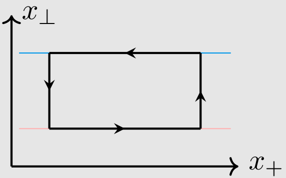
Important quantities

Collision kernel

$$C(q_\perp) = \frac{\mathrm{d}\Gamma}{\mathrm{d}^2 q_\perp \mathrm{d}L}$$

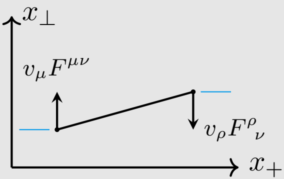
Wilson loop[2]



Asymptotic masses

$$m_\infty^2 = C_\mathrm{R}(Z_\mathrm{g} + Z_\mathrm{f})$$

Force-force-correlator[3]



Time-independent and Euclidean Gluon zero modes.[4] Calculate non-perturbative contributions in lattice "electrostatic QCD" (EQCD).

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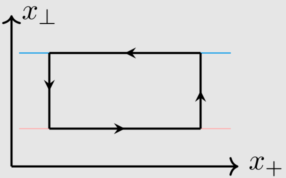

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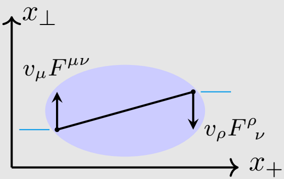

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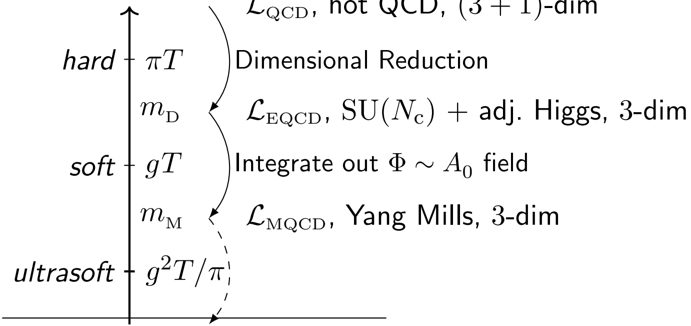
Dimensional Reduction (DR)

Integrate out fast (hard) modes perturbatively → EFT for static modes.[5]

All order thermal resummation to by-pass IR problem. Applied for thermodynamics of non-Abelian gauge theories such as (EW) phase transitions[6] and QCD.



[5] D. Bödeker, M. Sangel, and M. Wörmann, *Equilibration, particle production, and self-energy,* Phys. Rev. D **93** (2016) 045028 [1510.06742]

[6] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model,* Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379], K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *The Electroweak phase transition: A Nonperturbative analysis,* Nucl. Phys. B **466** (1996) 189 [hep-lat/9510020]

Dimensionally reduced effective theory for hot QCD

QCD described by 3-dimensional super-renormalisable theory

$$S_{\rm EQCD} = \frac{1}{T}\int_{\mathbf{x}} \left\{ \mathcal{L}_{\rm EQCD} + \sum_{n\geq 5} \frac{\mathcal{O}_n}{(\pi T)^n} \right\} .$$

"Electrostatic QCD" (EQCD) at high T ($A_0^a \to \Phi^a$)

$$\mathcal{L}_{\rm EQCD} \equiv \frac{1}{2}{\rm Tr}\, F_{ij}F_{ij} + {\rm Tr}\,[D_i,\Phi][D_i,\Phi] + m_{\rm D}^2\,{\rm Tr}\,\Phi^2 + \lambda_{\rm E}({\rm Tr}\,\Phi^2)^2 \ ,$$

$D_i = \partial_i - i g_{\rm E} A_i$. Developed to study high-T thermodynamics[7], but also used for soft light-cone observables[8].

[7] P. Ginsparg, *First and second order phase transitions in gauge theories at finite temperature,* Nucl. Phys. B **170** (1980) 388, T. Appelquist and R. D. Pisarski, *High-temperature Yang-Mills theories and three-dimensional quantum chromodynamics,* Phys. Rev. D **23** (1981) 2305

[8] S. Caron-Huot, *O(g) plasma effects in jet quenching,* Phys. Rev. D **79** (2009) 065039 [0811.1603], J. Ghiglieri, J. Hong, A. Kurkela, E. Lu, G. D. Moore, and D. Teaney, *Next-to-leading order thermal photon production in a weakly coupled quark-gluon plasma,* JHEP **2013** (2013) 10 [1302.5970]

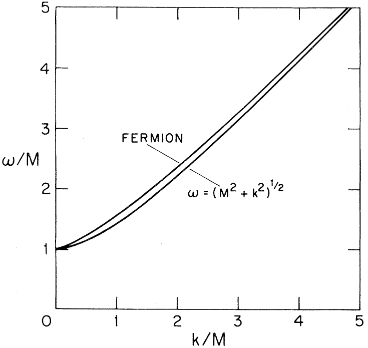
Asymptotic masses

Integrate out jet energy scale $E \gg T$. Truncate $\frac{T}{E}$-series: LO correlators[9]

$$m_\infty^2 = C_\mathrm{R}(Z_\mathrm{g} + Z_\mathrm{f})$$

$$Z_\mathrm{f} \equiv \frac{1}{2d_\mathrm{R}} \left\langle \overline{\psi} \frac{v_\mu \gamma^\mu}{v \cdot D} \psi \right\rangle$$

$$Z_\mathrm{g} \equiv -\frac{1}{d_\mathrm{A}} \left\langle v_\mu F^{\mu\nu} \frac{1}{(v \cdot D)^2} v_\rho F^\rho{}_\nu \right\rangle$$



[9] E. Braaten and R. D. Pisarski, *Simple effective Lagrangian for hard thermal loops,* Phys. Rev. D **45** (1992) R1827, S. Caron-Huot, *O(g) plasma effects in jet quenching,* Phys. Rev. D **79** (2009) 065039 [0811.1603]

figure by H. A. Weldon, *Effective Fermion Masses of Order gT in High Temperature Gauge Theories with Exact Chiral Invariance,* Phys. Rev. D **26** (1982) 2789

Condensates of the asymptotic masses

In QCD rewrite detour through the medium as[10]

$$Z_{\rm g} = -\frac{1}{d_{\rm A}} \int_0^\infty {\rm d}x^+ x^+ \Big\langle v_\mu F_a^{\mu\nu}(x^+)\, U_{\rm A}^{ab}(x^+;0)\, v_\rho F^{\rho}_{b\,\nu}(0) \Big\rangle \;,$$

and match also operator onto EQCD

$$Z_{\rm g}^{\rm 3d} = -\frac{4T}{d_{\rm A}} \int_0^\infty {\rm d}L\, L \Big(-\langle EE \rangle + \langle BB \rangle + i\langle EB \rangle \Big) \;.$$

Correlator splits into electro- and magneto-static contributions:

$$\langle EE \rangle \equiv \frac{1}{2}\Big\langle (D_x\Phi(L))^a\, \tilde{U}_{\rm A}^{ab}(L,0)\, (D_x\Phi(0))^b \Big\rangle \;,$$

$$\langle BB \rangle \equiv \frac{1}{2}\Big\langle F^a_{xz}(L)\, \tilde{U}_{\rm A}^{ab}(L,0)\, F^b_{xz}(0) \Big\rangle \;,$$

$$i\langle EB \rangle \equiv \frac{i}{2}\Big\langle (D_x\Phi(L))^a\, \tilde{U}_{\rm A}^{ab}(L,0)\, F^b_{xz}(0) \Big\rangle + [BE] \;.$$

[10] $U_{\rm A}(x^+;0)$ is an adjoint, light-like Wilson line.

EFT matching with full QCD

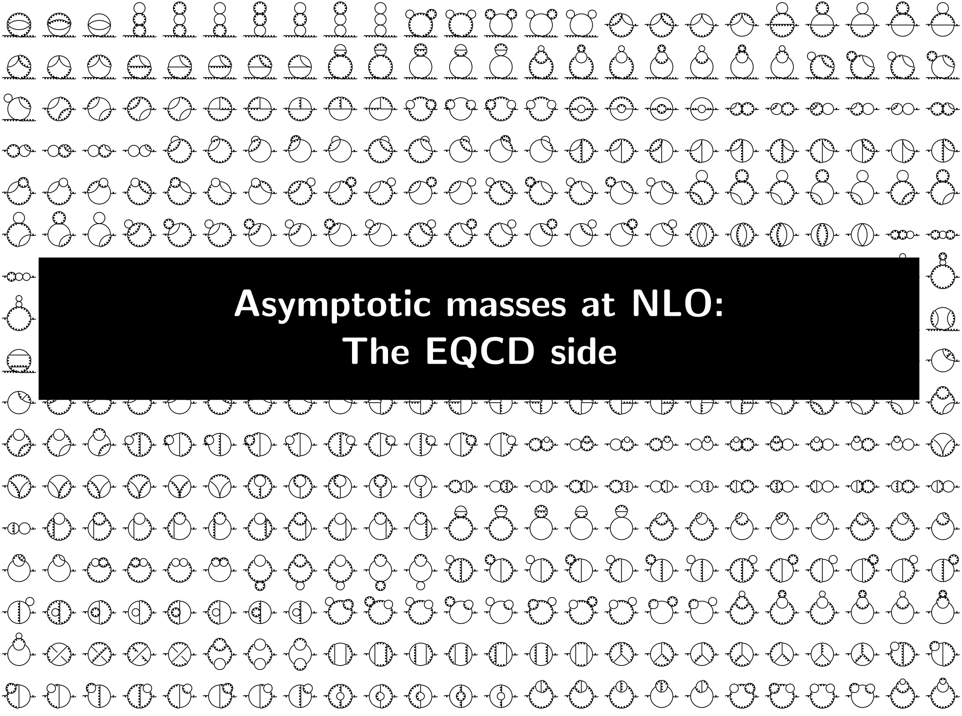
Strategy:

$$C_{\mathrm{QCD}}(x) = \underbrace{\left(C_{\mathrm{QCD}}(x) - C_{\mathrm{EQCD}}(x)\right)}_{\text{UV dominated}} + \underbrace{C_{\mathrm{EQCD}}(x)}_{\text{lattice}}$$

- Done[11] for $C(q_\perp)$.
- Partially done[12] for m_∞^2. Missing full QCD contribution.

[11] cf. **talk** by **I. Soudi Thu 12:50**,
P. Arnold and W. Xiao, *High-energy jet quenching in weakly coupled quark-gluon plasmas,* Phys. Rev. D **78** (2008) 125008 [0810.1026], J. Ghiglieri and H. Kim, *Transverse momentum broadening and collinear radiation at NLO in the $\mathcal{N} = 4$ SYM plasma,* JHEP **2018** (2018) 49 [1809.01349], G. D. Moore, S. Schlichting, N. Schlusser, and I. Soudi, *Non-perturbative determination of collisional broadening and medium induced radiation in QCD plasmas,* JHEP **10** (2021) 059 [2105.01679], S. Schlichting and I. Soudi, *Splitting rates in QCD plasmas from a non-perturbative determination of the momentum broadening kernel $C(q_\perp)$,* [2111.13731]

[12] J. Ghiglieri, G. D. Moore, P. Schicho, and N. Schlusser, *The force-force-correlator in hot QCD perturbatively and from the lattice,* JHEP **02** (2022) 58 [2112.01407]



Asymptotic masses at NLO:
The EQCD side

Contributions to $Z_{\rm g}$

$$
\begin{aligned}
& \qquad \text{scale } T \qquad\qquad\qquad \text{scale } gT \qquad\qquad\qquad\qquad\qquad\qquad \text{scale } g^2T \\
Z_{\rm g} = & \Big[\frac{T^2}{6} - \frac{T\mu_{\mathsf{h}}}{\pi^2} \Big] \\
& + \Big[- \frac{Tm_{\rm D}}{2\pi} + \frac{T\mu_{\mathsf{h}}}{\pi^2} \Big] \\
& + \Big[c^{\ln}_{\mathsf{hard}} \ln \frac{T}{\mu_{\mathsf{h}}} + c_T + c^{\ln}_{\mathsf{hard}} \ln \frac{\mu_{\mathsf{h}}}{m_{\rm D}} + c^{\ln}_{\mathsf{soft}} \ln \frac{m_{\rm D}}{\mu_{\mathsf{s}}} + c_{gT} + c^{\ln}_{\mathsf{soft}} \ln \frac{\mu_{\mathsf{s}}}{g^2T} + c_{gT^2} \Big] \\
& + \mathcal{O}(g^3) \,,
\end{aligned}
$$

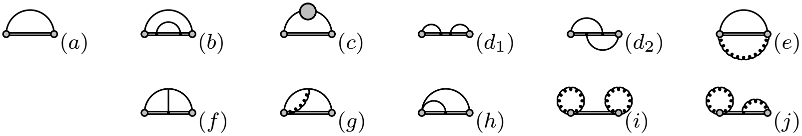
$Z_{\rm g}$ receives IR contributions already at $\mathcal{O}(g)$.[13]

Scheme-dependent at NLO; use intermediate regulators $T \gg \mu_{\mathsf{h}} \gg gT$ and $gT \gg \mu_{\mathsf{s}} \gg g^2T$.

[13] S. Caron-Huot, *O(g) plasma effects in jet quenching*, Phys. Rev. D **79** (2009) 065039 [0811.1603]

Z_{g} in EQCD perturbatively

Diagrams contributing at LO and NLO to the EQCD force-force correlator Z_{g}:



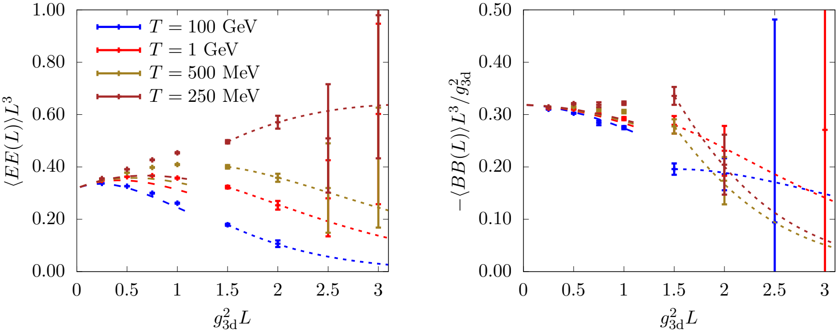
Example: LO colour-electric condensate $\langle EE \rangle$ – free solution

$$= 2 \times (a)^{\mathrm{EE}} = \partial_x \partial_{x'} \mathrm{Tr} \left\langle \Phi^a(x, L) \Phi^a(x', 0) \right\rangle \Big|_{x,x' \to 0}$$

$$= \frac{2 C_{\mathrm{A}} C_{\mathrm{F}}}{4\pi L^3} \epsilon^{-m_{\mathrm{D}} L} \left(1 + m_{\mathrm{D}} L\right)$$

Asymptotic masses (non-)perturbatively

Three different correlators contribute to $Z_{\mathrm{g}} \subset m_\infty^2$ in EQCD:



- small-L: NLO perturbative estimate
- large-L: Fit long L-tail to model[14]

[14] M. Laine and O. Philipsen, *Gauge-invariant scalar and field strength correlators in three dimensions*, Nucl. Phys. B **523** (1998) 267 [9711022]

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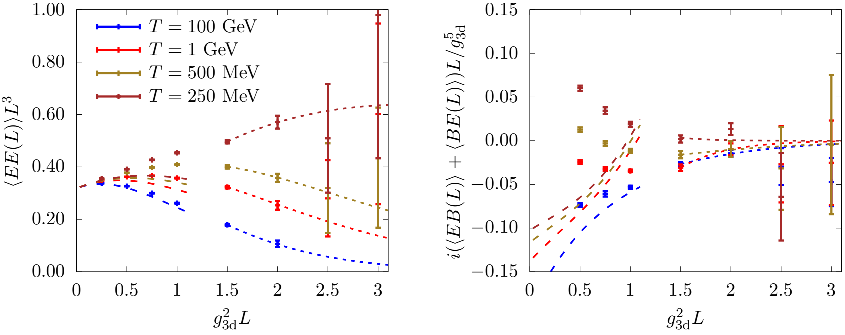

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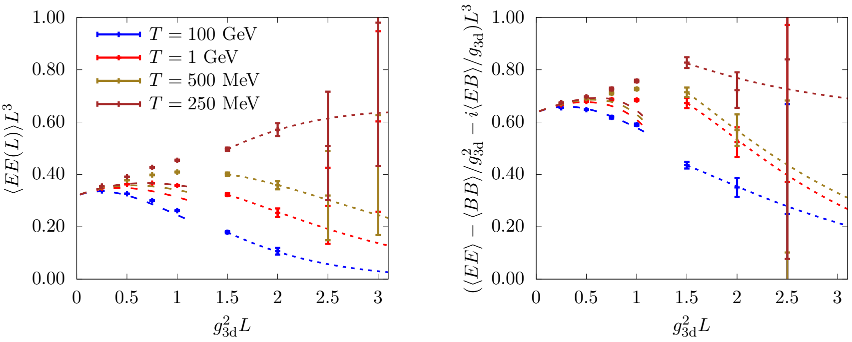

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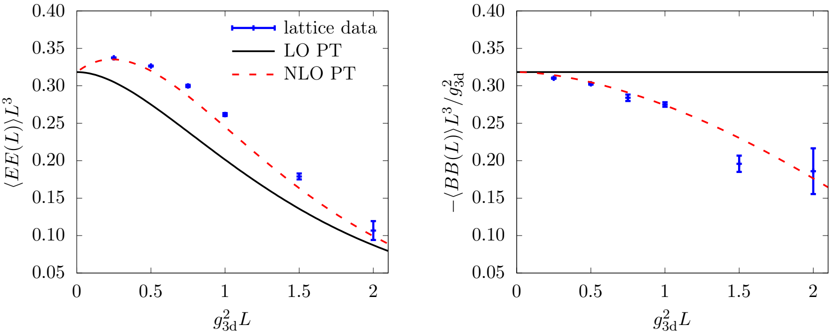
Asymptotic masses (non-)perturbatively

For $T = 100$ GeV and $N_\mathrm{f} = 5$, strong agreement between perturbative and non-perturbative Z_g.



Conclusions

- Jet modifications (+other transport) involves soft IR QCD $\rightarrow$ (lattice) QCD
- Key quantities are $C(b_\perp)$ and asymptotic mass m_∞^2 from lattice EQCD

What's next for m_∞^2?

- ☆ Finalise matching computation to full QCD
- ☆ Input to effective kinetic theory AMY[15] $\rightarrow$ GMT[16]
- ☆ Ingredients for NNLO-transport
- ☆ Feed into event generator

[15] P. B. Arnold, G. D. Moore, and L. G. Yaffe, *Effective kinetic theory for high temperature gauge theories*, JHEP **01** (2003) 030 [hep-ph/0209353]

[16] J. Ghiglieri, G. D. Moore, and D. Teaney, *Jet-medium interactions at NLO in a weakly-coupled quark-gluon plasma*, JHEP **2016** (2016) 95 [1509.07773]

Backup

Wilson line

Replace the lightlike Wilson line U_{A} with its EQCD counterpart[17]

$$\tilde{U}_{\mathrm{A}}(L;0) = \mathsf{P}\exp\Big(ig_{\mathrm{E}}\int_0^L \mathrm{d}z\,\big(A_z^a(z) + i\Phi^a(z)\big)T_{\mathrm{A}}^a\Big)\,.$$

[17] S. Caron-Huot, *O(g) plasma effects in jet quenching,* Phys. Rev. D **79** (2009) 065039 [0811.1603]

Fitting estimates

As elaborated in,[18] it is necessary to model the large-$g_{\rm E}^2 L$ tail of the correlators in order to perform the $\mathrm{d}L\, L$ integration up to ∞. For $\langle EE \rangle$ and $\langle BB \rangle$, their functional form[19] is

$$\frac{A}{\left(g_{\rm E}^2 L\right)^2} \exp(-B \cdot g_{\rm E}^2 L) \ ,$$

with the fitting constants A and B. Considering $i\langle EB \rangle$, we find that the data rather follows

$$A' \exp(-B' \cdot g_{\rm E}^2 L) \ ,$$

with the respective fitting constants A' and B'. As already argued above, the impact of $i\langle EB \rangle$ on $Z_{\rm g}$ is small.

[18] G. D. Moore and N. Schlusser, *The nonperturbative contribution to asymptotic masses,* Phys. Rev. D **102** (2020) 094512 [2009.06614]

[19] M. Laine and O. Philipsen, *Gauge-invariant scalar and field strength correlators in three dimensions,* Nucl. Phys. B **523** (1998) 267 [9711022]