(Non-)perturbative jet dispersion in hot QCD

Philipp Schicho
philipp.schicho@helsinki.fi

Helsinki Institute of Physics, University of Helsinki

Quark Matter 2022, Kraków

J. Ghiglieri, G. D. Moore, P. Schicho, and N. Schlusser, The force-force-correlator in hot QCD perturbatively and from the lattice, JHEP 02 (2022) 58 [2112.01407]
Motivation
Heavy-ion collisions

Hard particles carry most of the stress-energy tensor $P \sim T$. Medium soft modes at scale $P \sim gT$.

Jet-medium interactions in the Quark-Gluon Plasma (QGP) can receive large non-perturbative IR contributions.¹

---

¹ S. Caron-Huot, $O(g)$ plasma effects in jet quenching, Phys. Rev. D 79 (2009) 065039 [0811.1603]

cf. talks by J. Brewer Fri 11:50 and J. Ghiglieri Fri 12:30
Hard particles carry most of the stress-energy tensor $P \sim T$. Medium soft modes at scale $P \sim gT$.

Jet-medium interactions in the Quark-Gluon Plasma (QGP) can receive large non-perturbative IR contributions.\(^1\)

---

\(^1\) S. Caron-Huot, *$O(g)$ plasma effects in jet quenching*, Phys. Rev. D 79 (2009) 065039 [0811.1603]

cf. talks by J. Brewer Fri 11:50 and J. Ghiglieri Fri 12:30
Important quantities

Collision kernel

\[ C(q_\perp) = \frac{d\Gamma}{d^2 q_\perp dL} \]

Wilson loop

Asymptotic masses

\[ m_\infty^2 = C_R(Z_g + Z_f) \]

Force-force-correlator

Time-independent and Euclidean Gluon zero modes. Calculate non-perturbative contributions in lattice “electrostatic QCD” (EQCD).

---


4 S. Caron-Huot, O(g) plasma effects in jet quenching, Phys. Rev. D 79 (2009) 065039 [0811.1603]
Important quantities

Collision kernel

\[ C(q_\perp) = \frac{d\Gamma}{d^2q_\perp dL} \]

Wilson loop\(^2\)

Asymptotic masses

\[ m_\infty^2 = C_R (Z_g + Z_f) \]

Force-force-correlator\(^3\)

Time-independent and Euclidean Gluon zero modes.\(^4\) Calculate non-perturbative contributions in lattice “electrostatic QCD” (EQCD).

---


**Dimensional Reduction (DR)**

*Integrate out* fast (hard) modes perturbatively $\rightarrow$ EFT for static modes.\(^5\)

All order thermal resummation to by-pass IR problem. Applied for thermodynamics of non-Abelian gauge theories such as (EW) phase transitions\(^6\) and QCD.

\[ \mathcal{L}_{\text{QCD}}, \text{hot QCD}, (3 + 1)\text{-dim} \]

\[ \mathcal{L}_{\text{EQCD}}, \text{SU}(N_c) + \text{adj. Higgs}, 3\text{-dim} \]

\[ \mathcal{L}_{\text{MQCD}}, \text{Yang Mills}, 3\text{-dim} \]

\[ \Phi \sim A_0 \text{ field} \]

\[ g^2 T / \pi \]

---


QCD described by 3-dimensional super-renormalisable theory

\[ S_{\text{EQCD}} = \frac{1}{T} \int_x \left\{ \mathcal{L}_{\text{EQCD}} + \sum_{n \geq 5} \frac{O_n}{(\pi T)^n} \right\} . \]

“Electrostatic QCD” (EQCD) at high \( T \) \((A_0^a \rightarrow \Phi^a)\)

\[ \mathcal{L}_{\text{EQCD}} \equiv \frac{1}{2} \text{Tr} \, F_{ij} F_{ij} + \text{Tr} \left[ D_i, \Phi \right] \left[ D_i, \Phi \right] + m_D^2 \text{Tr} \, \Phi^2 + \lambda_E (\text{Tr} \, \Phi^2)^2 , \]

\(D_i = \partial_i - ig_E A_i\). Developed to study high-\( T \) thermodynamics\(^7\), but also used for soft light-cone observables\(^8\).


Asymptotic masses

Integrate out jet energy scale $E \gg T$. Truncate $\frac{T}{E}$-series: LO correlators

\[ m_\infty^2 = C_R(Z_g + Z_f) \]

\[ Z_f \equiv \frac{1}{2d_R} \left\langle \bar{\psi} \frac{v_\mu \gamma^\mu}{v \cdot D} \psi \right\rangle \]

\[ Z_g \equiv -\frac{1}{d_A} \left\langle v_\mu F_{\mu\nu} \frac{1}{(v \cdot D)^2} v_\rho F_{\rho\nu} \right\rangle \]

---


Condensates of the asymptotic masses

In QCD rewrite detour through the medium as

\[ Z_g = -\frac{1}{d_A} \int_0^\infty dx^+ x^+ \left\langle \nu_\mu F_\mu^\nu(x^+) U_A^{ab}(x^+; 0) \nu_\rho F_\rho^\nu(0) \right\rangle, \]

and match also operator onto EQCD

\[ Z_{g}^{3d} = -\frac{4T}{d_A} \int_0^\infty dL L \left( -\langle EE \rangle + \langle BB \rangle + i\langle EB \rangle \right). \]

Correlator splits into electro- and magneto-static contributions:

\[ \langle EE \rangle \equiv \frac{1}{2} \left\langle (D_x \Phi(L))^a \tilde{U}_A^{ab}(L, 0) (D_x \Phi(0))^b \right\rangle, \]

\[ \langle BB \rangle \equiv \frac{1}{2} \left\langle F_{xz}^a(L) \tilde{U}_A^{ab}(L, 0) F_{xz}^b(0) \right\rangle, \]

\[ i\langle EB \rangle \equiv \frac{i}{2} \left\langle (D_x \Phi(L))^a \tilde{U}_A^{ab}(L, 0) F_{xz}^b(0) \right\rangle + [BE]. \]

\(^{10}U_A(x^+; 0)\) is an adjoint, light-like Wilson line.
EFT matching with full QCD

Strategy:

\[
C_{QCD}(x) = (C_{QCD}(x) - C_{EQCD}(x)) + C_{EQCD}(x)
\]

- **Done**\(^{11}\) for \(C(q_\perp)\).
- **Partially done**\(^{12}\) for \(m_\infty^2\). Missing full QCD contribution.

---

\(^{11}\)cf. talk by I. Soudi Thu 12:50,


\(^{12}\)J. Ghiglieri, G. D. Moore, P. Schicho, and N. Schlusser, *The force-force-correlator in hot QCD perturbatively and from the lattice*, JHEP 02 (2022) 58 [2112.01407]
Asymptotic masses at NLO: The EQCD side
Contributions to $Z_g$

$$Z_g = \begin{bmatrix}
\text{scale } T \\
\frac{T^2}{6} - \frac{T\mu_h}{\pi^2} \\
\text{scale } gT \\
- \frac{Tm_D}{2\pi} + \frac{T\mu_h}{\pi^2} \\
\text{scale } g^2T \\
c_{\text{hard}} \ln \frac{T}{\mu_h} + c_T + c_{\text{hard}} \ln \frac{\mu_h}{m_D} + c_{\text{soft}} \ln \frac{m_D}{\mu_s} + c_{gT} + c_{\text{soft}} \ln \frac{\mu_s}{g^2T} + c_{gT^2}
\end{bmatrix} + \mathcal{O}(g^3),$$

$Z_g$ receives IR contributions already at $\mathcal{O}(g)$.\(^\text{13}\)

Scheme-dependent at NLO; use intermediate regulators $T \gg \mu_h \gg gT$ and $gT \gg \mu_s \gg g^2T$.

---

\(^\text{13}\) S. Caron-Huot, $O(g)$ plasma effects in jet quenching, Phys. Rev. D 79 (2009) 065039 [0811.1603]
Diagrams contributing at LO and NLO to the EQCD force-force correlator $Z_g$:

Example: LO colour-electric condensate $\langle EE \rangle$ – free solution

$$Z_g \text{ in EQCD perturbatively}$$

$$Z_g = 2 \times (a)^{EE} = \partial_x \partial_{x'} \text{Tr} \left\langle \Phi^a(x, L) \Phi^a(x', 0) \right\rangle \bigg|_{x, x' \to 0}$$

$$= \frac{2C_A C_F}{4\pi L^3} \epsilon^{-m_D L} (1 + m_D L)$$
Three different correlators contribute to $Z_g \subset m_\infty^2$ in EQCD:

- **small-$L$:** NLO perturbative estimate
- **large-$L$:** Fit long $L$-tail to model\(^{14}\)

Asymptotic masses (non-)perturbatively

Three different correlators contribute to \( Z_g \subset m_\infty^2 \) in EQCD:

\[
\langle \langle \langle EE(L) \rangle \rangle \rangle L^3
\]

\[
T = 100 \text{ GeV} \\
T = 1 \text{ GeV} \\
T = 500 \text{ MeV} \\
T = 250 \text{ MeV}
\]

\[
\frac{i\langle EB(L) \rangle + \langle BE(L) \rangle}{L/g_{3d}^5}
\]

\[
0.00 \quad 0.05 \quad 0.10 \quad 0.15
\]

\[
0.00 \quad 0.05 \quad 0.10 \quad 0.15
\]

\[
g_{3d}^2 L
\]

\[
g_{3d}^2 L
\]

▷ small-\( L \): NLO perturbative estimate

▷ large-\( L \): Fit long \( L \)-tail to model\(^{14}\)

Asymptotic masses (non-)perturbatively

Three different correlators contribute to $Z_g \subset m_\infty^2$ in EQCD:

\[ \langle EE(L) \rangle L^3 \]

\[ \langle (EE) - (BB)/g_{3d}^2 - i(EB)/g_{3d}^3 \rangle L^3 \]

- small-$L$: NLO perturbative estimate
- large-$L$: Fit long $L$-tail to model\(^{14}\)

Asymptotic masses (non-)perturbatively

For $T = 100$ GeV and $N_f = 5$, strong agreement between perturbative and non-perturbative $Z_g$. 

![Graph showing lattice data, LO PT, and NLO PT for $\langle EE(L) \rangle L^3$ and $-\langle BB(L) \rangle L^3 / g_{3d}^2$.]
Conclusions

- Jet modifications (+other transport) involves soft IR QCD $\rightarrow$ (lattice) QCD
- Key quantities are $C(b_\perp)$ and asymptotic mass $m^2_\infty$ from lattice EQCD

What’s next for $m^2_\infty$?

- Finalise matching computation to full QCD
- Input to effective kinetic theory AMY$^{15}$ $\rightarrow$ GMT$^{16}$
- Ingredients for NNLO-transport
- Feed into event generator

---


Backup
Replace the lightlike Wilson line $U_A$ with its EQCD counterpart\textsuperscript{17}

\[ \tilde{U}_A(L; 0) = \text{P} \exp \left( ig_E \int_0^L \text{d} z \left( A^a_z(z) + i\Phi^a(z) \right) T^a_A \right). \]

\textsuperscript{17} S. Caron-Huot, $O(g)$ plasma effects in jet quenching, Phys. Rev. D 79 (2009) 065039 [0811.1603]
As elaborated in,\cite{18} it is necessary to model the large-$g_{EL}^2 L$ tail of the correlators in order to perform the $dL L$ integration up to $\infty$. For $\langle EE \rangle$ and $\langle BB \rangle$, their functional form\cite{19} is

$$
\frac{A}{(g_{EL}^2 L)^2} \exp(-B \cdot g_{EL}^2 L),
$$

with the fitting constants $A$ and $B$. Considering $i\langle EB \rangle$, we find that the data rather follows

$$
A' \exp(-B' \cdot g_{EL}^2 L),
$$

with the respective fitting constants $A'$ and $B'$. As already argued above, the impact of $i\langle EB \rangle$ on $Z_g$ is small.

\begin{flushleft}
\footnotesize

\end{flushleft}