Embedding a Critical Point in a Hadron to Quark-Gluon Crossover Equation of State

Joseph Kapusta[1], Tom Welle[1], Christopher Plumberg[2]

[1]University of Minnesota
[2]University of Illinois at Urbana-Champaign

Quark Matter 2022 (Krakow)

Purpose and Goals

• Many model calculations predict the existence of a critical point in the QCD phase diagram at a value of the chemical potential where current lattice simulations are unreliable.

• How to combine or merge a critical equation of state with a smooth background is a long-standing problem in statistical physics with no unique solution.

• Our goal is to construct an equation of state in the same universality class as the liquid–gas phase transition and the 3D Ising model. It should have parameters which may be inferred by hydrodynamic modeling of heavy ion collisions in the Beam Energy Scan II at the Relativistic Heavy Ion Collider or in experiments at other accelerators.

• Such an equation of state is also needed for modeling neutron star mergers and closely related to the cold dense matter comprising neutron stars.

Construction

- Motivated by S-shaped curves in first order phase transitions and the cubic equation

$$Q_{\pm}(T,\mu) = \left\{ \left[(\Delta^2(T))^2 + r^2(T,\mu) \right]^{1/2} \pm r(T,\mu) \right\}^k$$

$$r(T,\mu) = \frac{\mu^4 - \mu_x^4(T)}{\mu^4 + \mu_x^4(T)} \qquad \Delta^2(T) \sim d_{\pm}|T/T_c - 1|^p \ \text{ for } \ T \to T_c^{\pm}$$

- Only two exponents k and p.

$$P(T,\mu) = P_{BG}(T,\mu) R(T,\mu)$$

- For $T \geq T_c$

$$R(T,\mu) = 1 - a(T)\left(\sqrt{\Delta^4 + 1} + 1\right)^k - a(T)\left(\sqrt{\Delta^4 + 1} - 1\right)^k + a(T)(Q_+ + Q_-)$$

- For $T \leq T_c$ and $\mu \leq \mu_x(T)$

$$R_H = 1 + a(T)Q_-(T,\mu) - a(T)\left(\sqrt{\Delta^4 + 1} + 1\right)^k$$

- For $T \leq T_c$ and $\mu \geq \mu_x(T)$

$$R_Q = 1 + a(T)Q_+(T,\mu) - a(T)\left(\sqrt{\Delta^4 + 1} + 1\right)^k$$

Critical Behavior

- As $n \to n_c$ along the critical isotherm

$$P - P_c \sim \mathrm{sgn}(n - n_c)|n - n_c|^\delta\,, \quad \delta = 1/(k-1)$$

- As $t = (T - T_c)/T_c \to 0^+$ the susceptibility and heat capacity are

$$\chi_B \to \chi_+ t^{-\gamma}\,, \quad \gamma = (2-k)p$$

$$c_V \to c_+ t^{-\alpha}\,, \quad \alpha = 2 - kp$$

- As $t \to 0^-$ the susceptibility, heat capacity and density difference along the coexistence curve are

$$\chi_B \to \chi_-(-t)^{-\gamma}$$

$$c_V \to c_-(-t)^{-\alpha}$$

$$\Delta n \sim (-t)^\beta\,, \quad \beta = (k-1)p$$

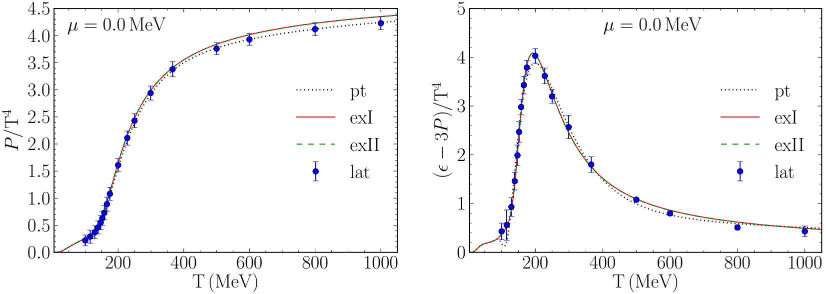
- The critical exponents automatically satisfy the known relations $\alpha + 2\beta + \gamma = 2$ and $\gamma = \beta(\delta - 1)$.

- Predicts relation between universal ratios of critical amplitudes

$$\left(\frac{c_+}{c_-}\right)^{2-k} = 4\left(\frac{\chi_-}{\chi_+}\right)^k = 2^{2-k}\left(\frac{d_+}{d_-}\right)^{(2-k)k}$$

Background Equation of State

The background equation of state uses a switching function to transition smoothly from a hadron resonance gas, with excluded volume interactions, to a perturbative quark–gluon plasma. Two parameters in the QCD running coupling, two in the switching function, and an excluded volume parameter are adjusted and fixed by fitting to lattice QCD at $\mu = 0$.



M. Albright, J. Kapusta, and C. Young, Phys. Rev. C (2014)

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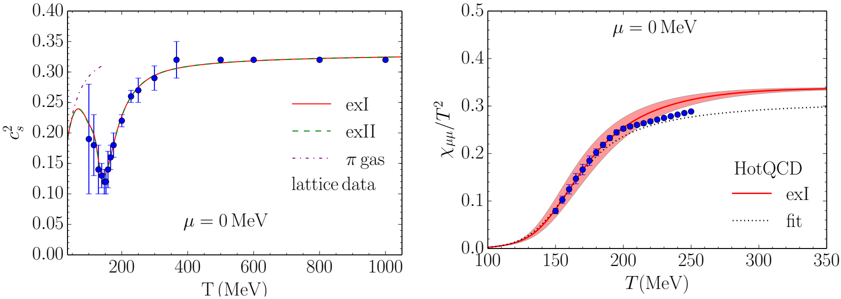

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Illustrative Parameter Choices

- In order to have an inverted U-shaped coexistence curve in the $T - n$ plane, as seen in the argon and carbon dioxide liquid-gas phase transitions, the function $\mu_x(T)$ is determined by $R(T, \mu_x(T)) n_{BG}(T, \mu_x(T)) = n_c$.

- The critical parameters T_c, μ_c, n_c are related by $R(T_c, \mu_c) n_{BG}(T_c, \mu_c) = n_c$.

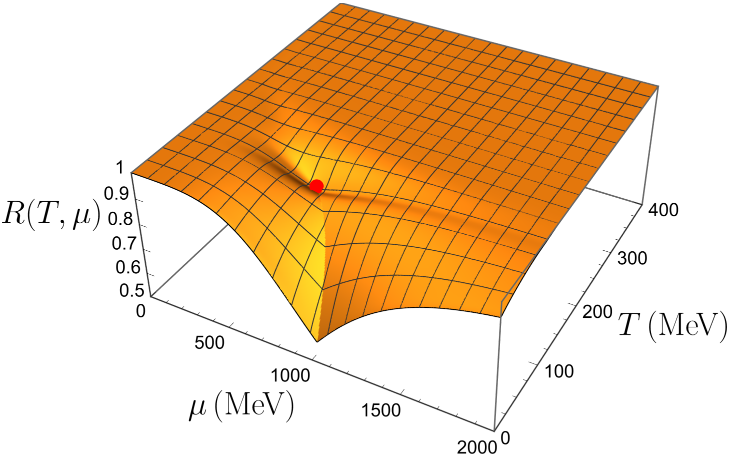
- 3D Ising model exponents give $k = 1.209, p = 1.564$ (mean field values are $k = 4/3, p = 3/2$). Then ratios of critical amplitudes give $d_+/d_- \approx 1/3$ (mean field value is $d_+/d_- = 1$).

$$
\begin{aligned}
a(T) &= a_0 \exp(-T/T_a) \\
\Delta^2(T) &= d_+ (T/T_c - 1)^p \exp(-T/T_d) \quad T \geq T_c \\
\Delta^2(T) &= d_- (1 - T/T_c)^p \exp(-T/T_d) \quad T \leq T_c
\end{aligned}
$$

- $T_a = 80$ and $T_d = 200$ MeV determine how fast $R \to 1$ above the critical point.

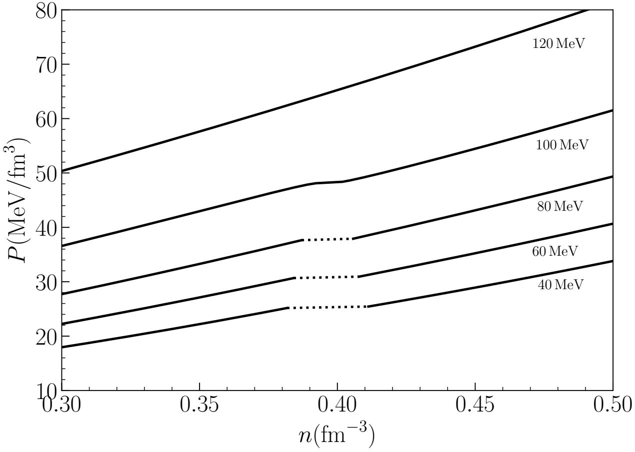
- $a_0 = 0.15$ and $d_- = 50$ are just reasonable guesses.

$$P(T,\mu) = P_{BG}(T,\mu)R(T,\mu)$$



$T_c = 100$ MeV, $\mu_c = 750$ MeV, $n_c \approx 0.4\ \mathrm{fm}^{-3}$

Isotherms of Pressure versus Density



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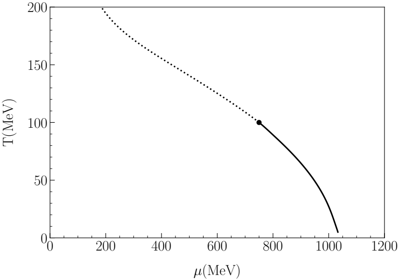
Coexistence Curve

$$R(T_c, \mu_c) n_{BG}(T_c, \mu_c) = n_c$$



$T_c = 100$ MeV, $\mu_c = 750$ MeV, $n_c \approx 0.4$ fm^{-3}

Conclusion

Lattice QCD simulations have shown unequivocally that the transition from hadrons to quarks and gluons is a crossover when the baryon chemical potential is zero or small. Many model calculations predict the existence of a critical point at a value of the chemical potential where current lattice simulations are unreliable. We show how to embed a critical point in a smooth background equation of state so as to yield the critical exponents and critical amplitude ratios expected of a transition in the same universality class as the liquid–gas phase transition and the 3D Ising model. There are only two independent critical exponents; the relations $\alpha + 2\beta + \gamma = 2$ and $\beta(\delta - 1) = \gamma$ arise automatically, as does a relation between the two critical amplitudes. The resulting equation of state has parameters which may be inferred by hydrodynamic modeling of heavy ion collisions in the Beam Energy Scan II at the Relativistic Heavy Ion Collider or in experiments at other accelerators. See poster 410 (Mayank Singh presenter) for hydrodynamic modeling using this background equation of state.

This work was supported by the U.S. DOE Grants No. DE-FG02-87ER40328 and DE-SC0020633.