Fully resummed medium-induced emissions in dynamic media

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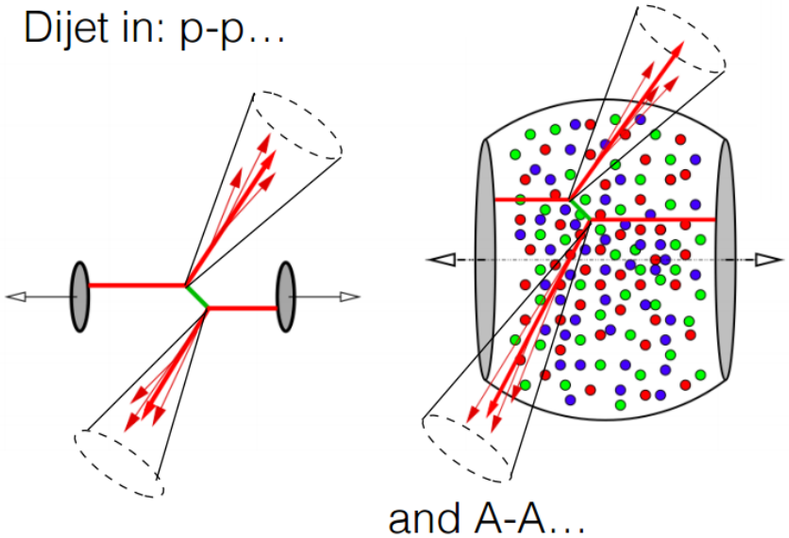
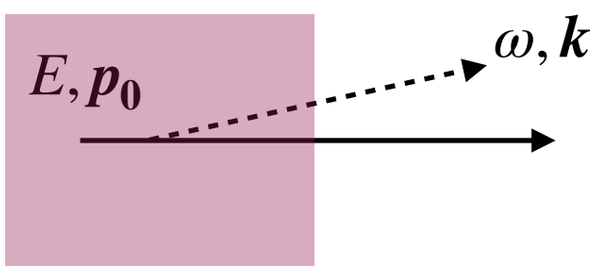
Energy loss

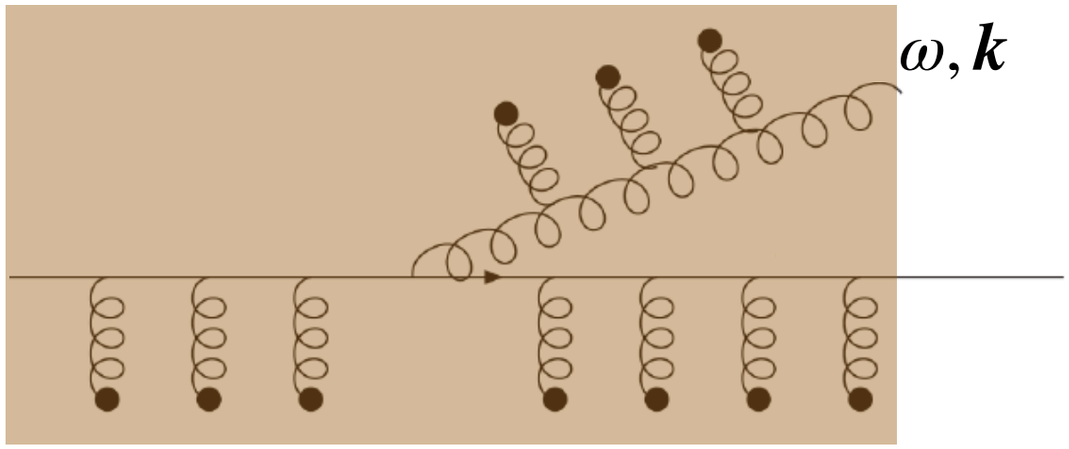


- Jet quenching: high energy partons interact with the QGP losing energy

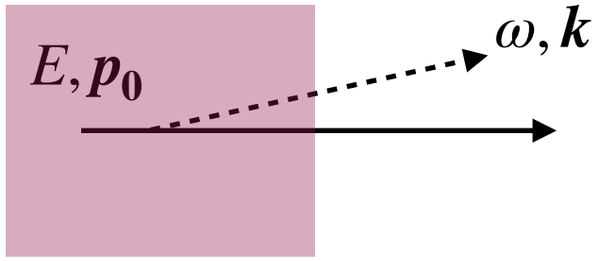
- How does a parton lose energy in a QCD medium?
  - Collisions - Important for heavy particles
  - **Radiation** - Extra gluon radiation induced by multiple scatterings with the medium

Dominant for light quarks and gluons (this talk)





The building block



- The in-medium spectrum is given by ($\omega \ll E$):

$$\omega \frac{dI}{d\omega d^2\boldsymbol{k}} = \frac{2\alpha_s C_R}{(2\pi)^2\omega^2} \mathrm{Re} \int_0^\infty dt' \int_0^{t'} dt \int_{\boldsymbol{pq}} \boldsymbol{p}\cdot\boldsymbol{q}\, \widetilde{\mathcal{K}}(t',\boldsymbol{q};t,\boldsymbol{p})\mathcal{P}(\infty,\boldsymbol{k};t',\boldsymbol{q})$$

BDMPS-Z

- It's numerical evaluation is difficult

- It has been traditionally evaluated in many approximations (GLV, AMY, HO…)

- Several **new approaches go beyond the usual approximations**

Finite length rates: Caron-Huot and Gale, 1006.2379

IOE (expansion around the HO): Mehtar-Tani, Barata, Soto-Ontoso, Tywoniuk, 1903.00506, 2106.07402

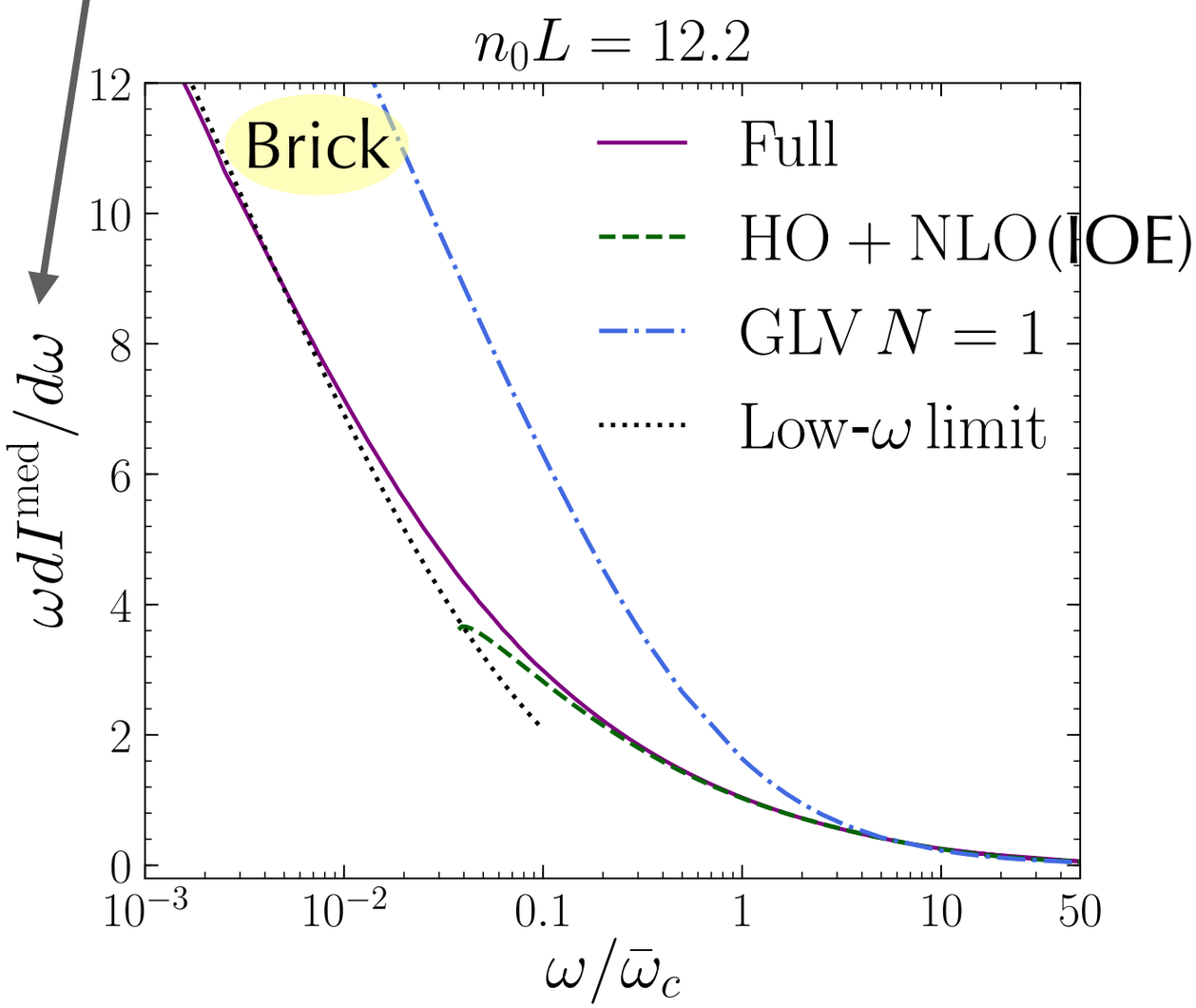
Fully resummed spectrum: CA, Apolinario, Martinez, Dominguez, 2002.01517, 2011.06522

Finite length rates + non-perturbative potential: Schlichting, Soudi, 2111.13731

The building block: in the brick

- Numerics done in the **brick**!

Emission spectrum

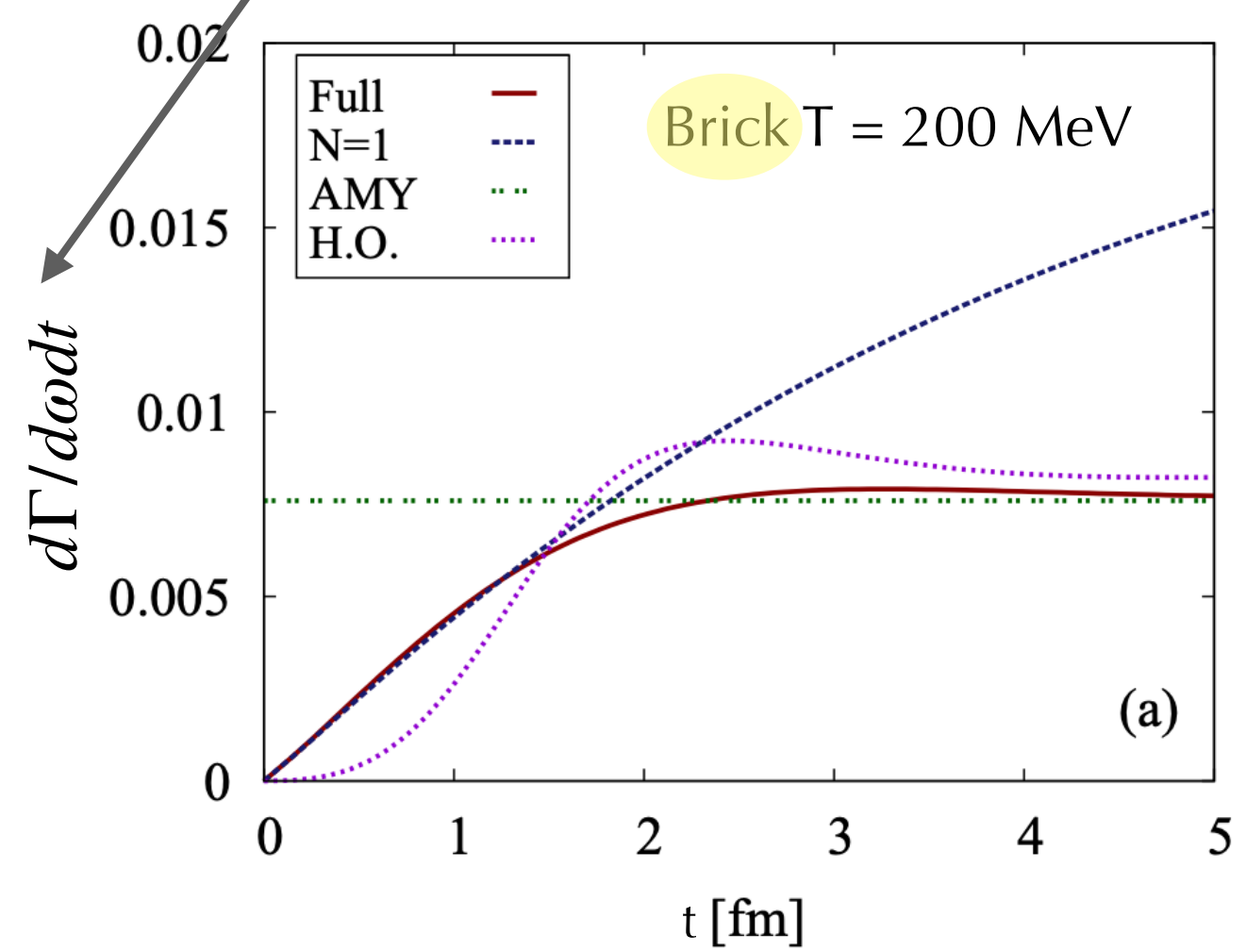


CA, Martinez, Dominguez, 2011.06522

$\bar{\omega}_c = \frac{\mu^2 L}{2}$

Emission rates

$$\omega \frac{dI}{d\omega} = \int_0^L dt\, \omega \frac{d\Gamma}{d\omega dt}$$

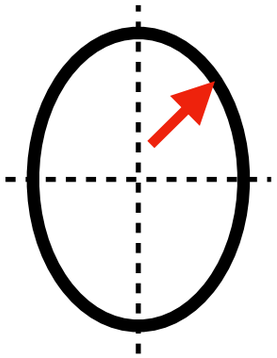


Caron-Huot and Gale, 1006.2379

How do we move to a realistic medium?

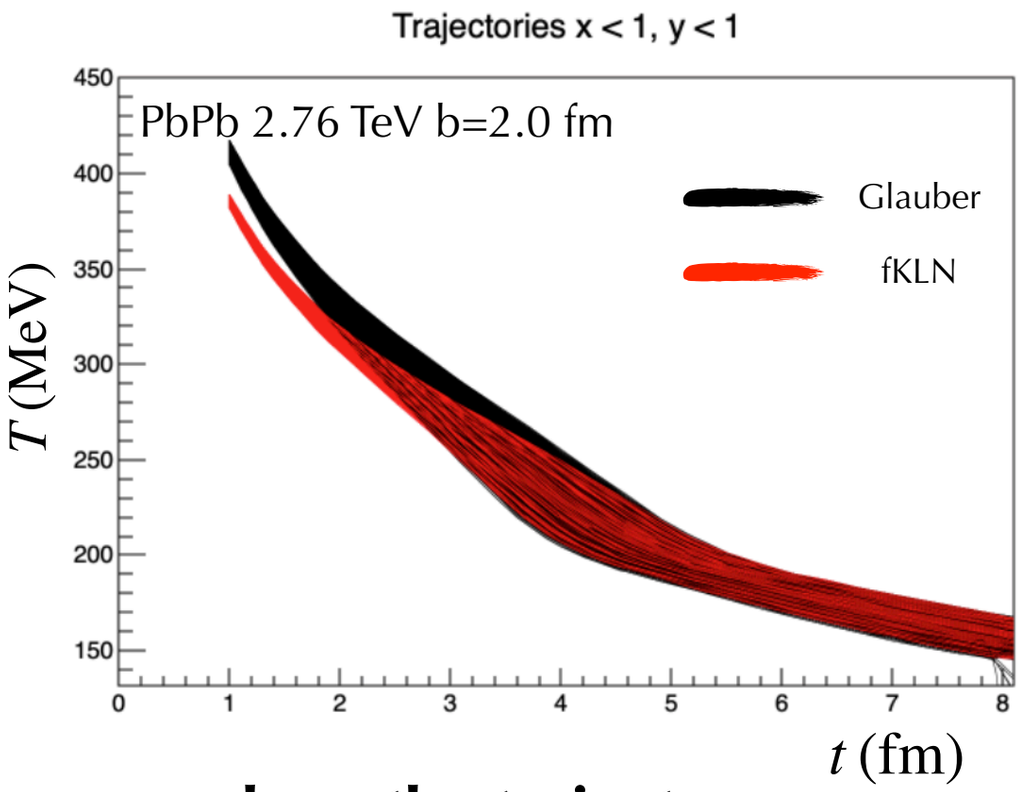
Beyond the brick

*with multiple scatterings



- The goal is to compute the energy lost by a hard parton along **its trajectory** within an **evolving** media

- One should read the medium properties (for instance, the temperature T) from the hydro at each point of the path



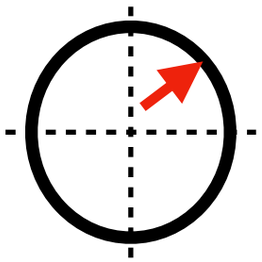
- Then, obtain the medium parameters entering the spectrum: $n(T(t))\,,\ \mu(T(t))\ldots$

- And feed them to the code and compute the spectrum **along the trajectory**

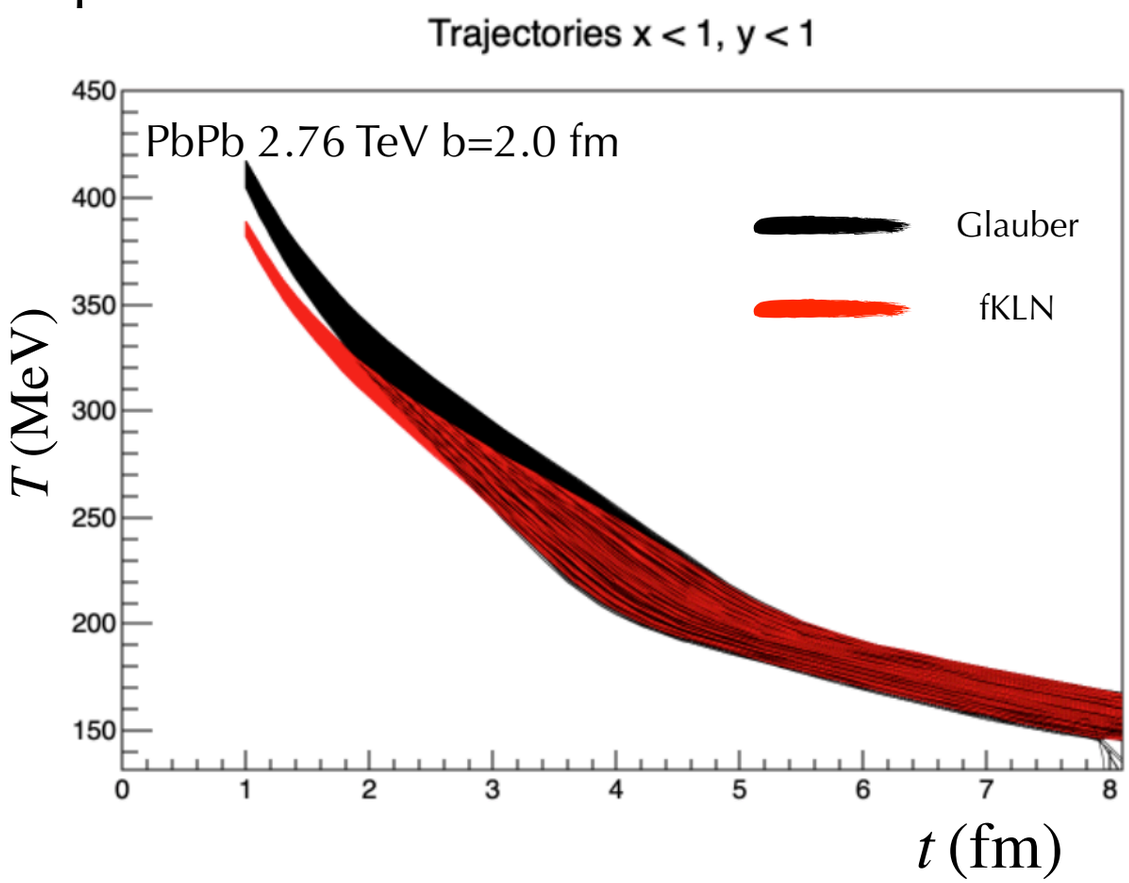
The spectrum depends on the full trajectory, there is no "per-point spectrum"

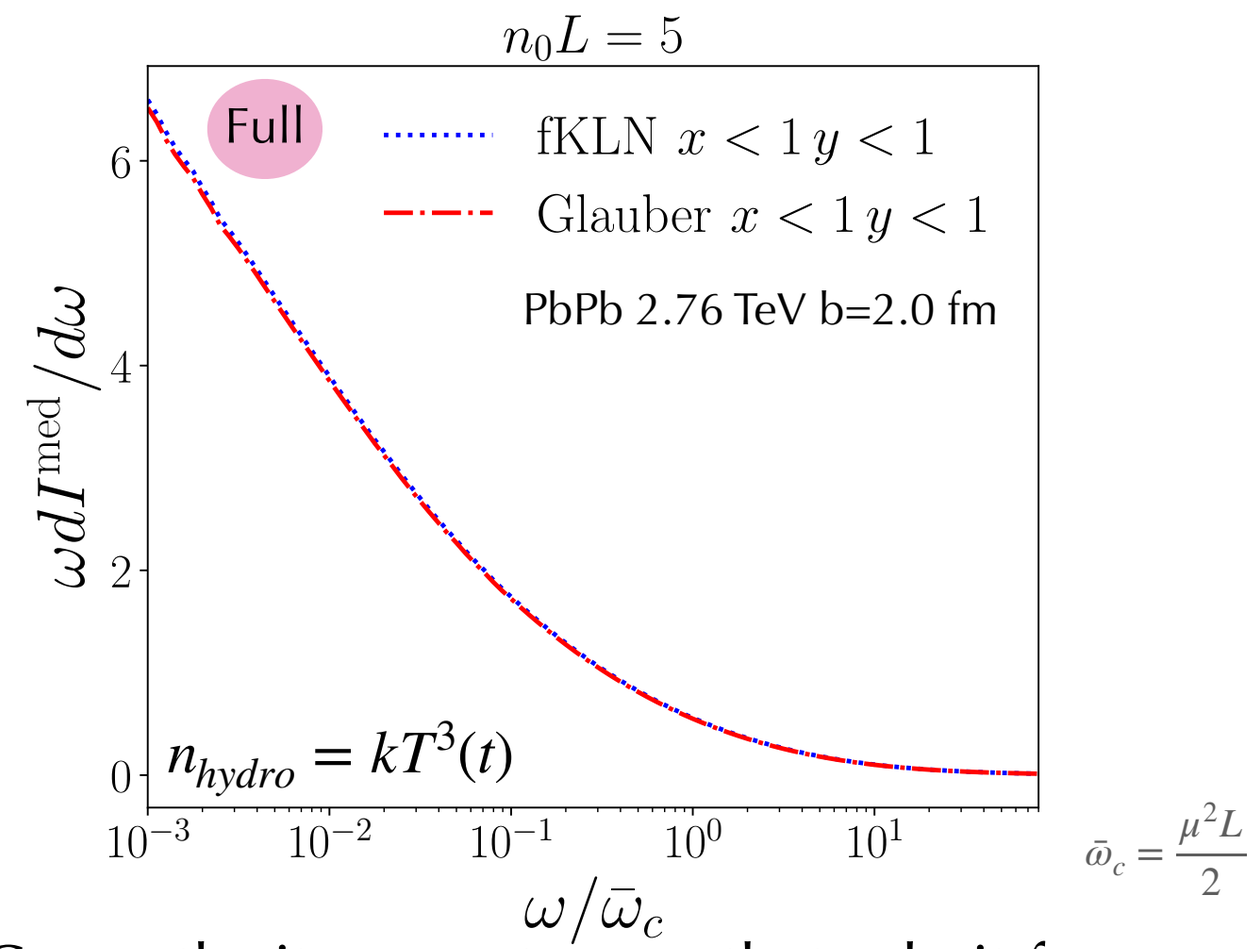
Beyond the brick II

***with multiple scatterings**



- We can compute the spectrum with time-dependent variables along a path

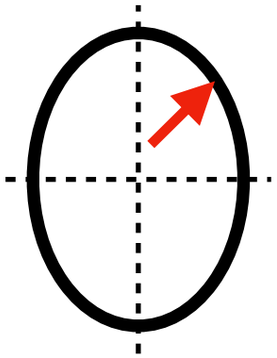




- But this is computationally demanding. Currently, it seems too costly to do it for every trajectory *on the fly*

- Pre-tabulate it? **How do we know a priori how the medium parameters will behave along all possible paths**?

Average values?



- Using average values

  Use a static spectrum whose parameters are given by their average along the path

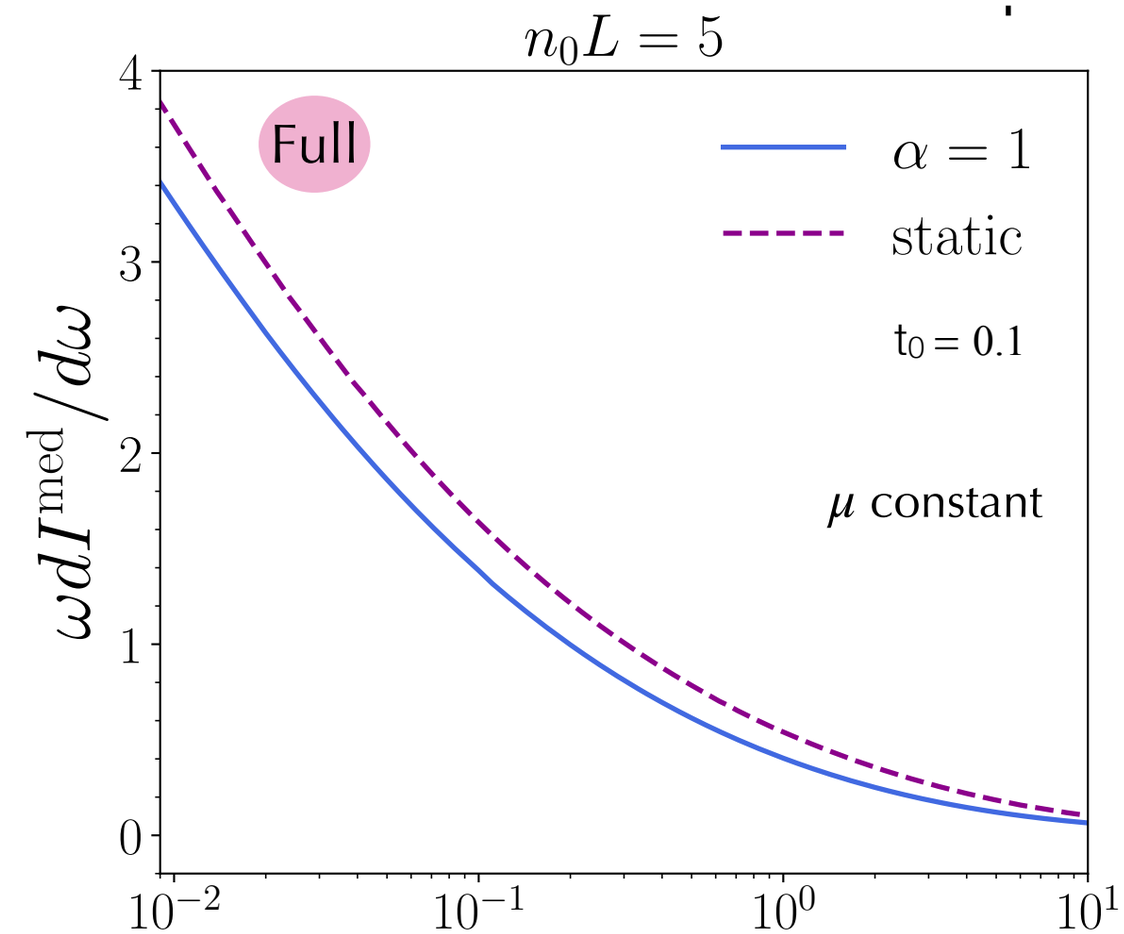
- Let's say that along the path:

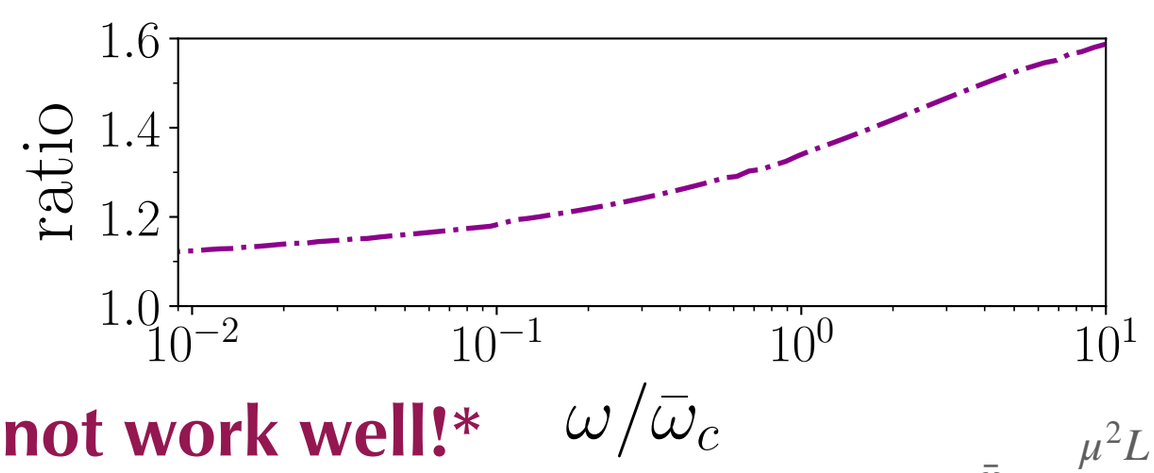
$$n(t) = \frac{n_0'}{(t+t_0)^\alpha}$$

- Obtain the full dynamic solution for n(t)

- Compare to the static case where n_0L is given by the average of n(t) along the path:

$$n_0 L = \int_0^L dt\, n(t)$$



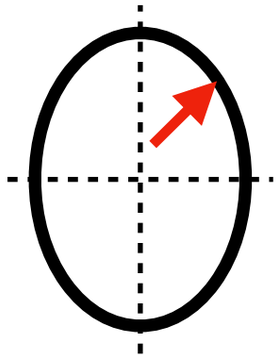


$$\bar{\omega}_c = \frac{\mu^2 L}{2}$$

Using average values does not work well!*

* And this is something we know since 2003

Scaling laws?



- The idea is to **find an equivalent static scenario**

  Find the values of the parameters that best approximate the dynamic spectrum along the path

- Example:

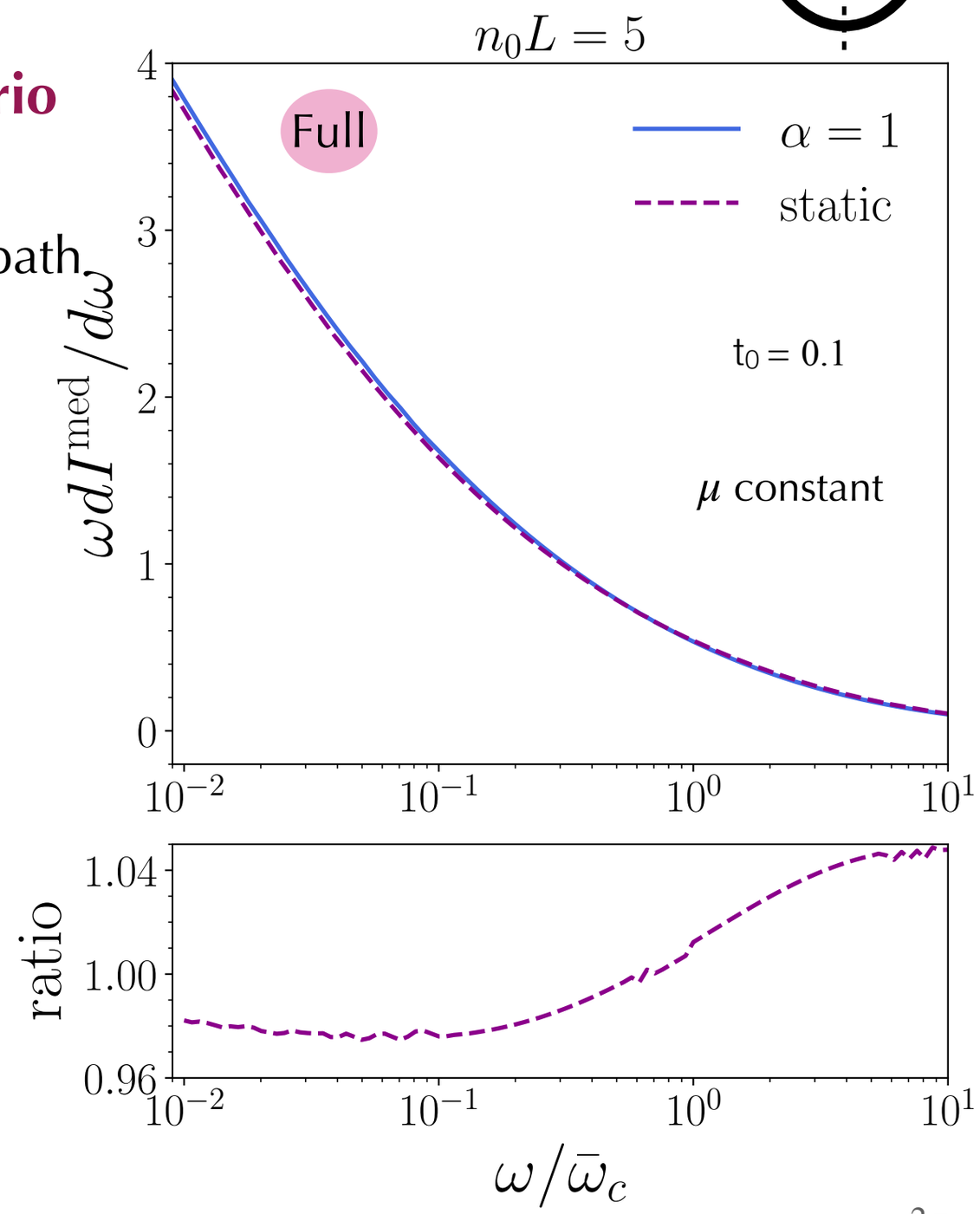
  - Compute the spectrum for a dynamic media where

$$n(t) = \frac{n_0'}{(t+t_0)^\alpha}$$

  - Compare to the static scenario given by

$$n_0 L = \int_0^{L'} dt\, n(t) \qquad \frac{n_0 L^2}{2} = \int_0^{L'} dt\, t\, n(t)$$

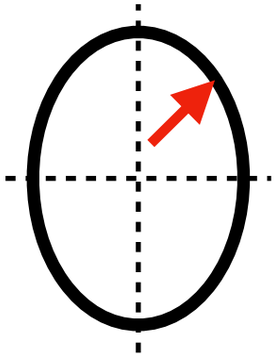
Scaling laws



$$\bar{\omega}_c = \frac{\mu^2 L}{2}$$

Scaling laws similar to these ones have been used in the HO. Salgado, Wiedemann, 0302184

Scaling laws?



- Compute the full dynamic solution using

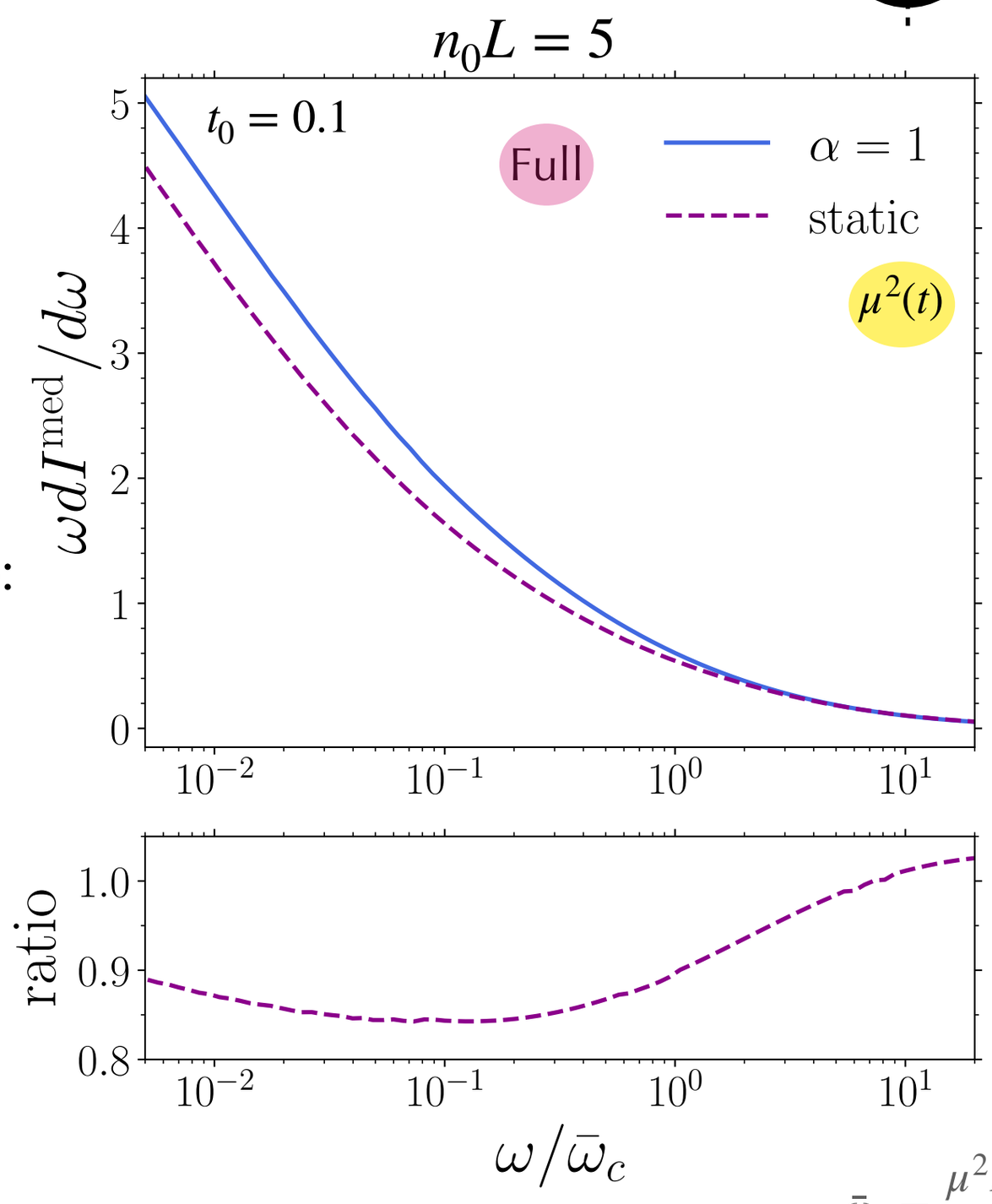
$$n(t) = \frac{n_0'}{(t+t_0)^{\alpha}} \qquad \mu^2(t) = \frac{\mu'^2}{(t+t_0)^{2\alpha}}$$

- Find the values of the parameters of the static scenario we want to compare to:

$$n_0 L = \int_0^{L'} dt\, n(t)$$

$$\frac{n_0 \mu^2 L^2}{2} = \int_0^{L'} dt\, t\, n(t)\, \mu^2(t)$$

- It works better than using average values, but the **errors go up to ~20%**



$$\bar{\omega}_c = \frac{\mu^2 L}{2}$$

Scaling laws? Hydro

- Compute the full solution along a path thorough a hydro

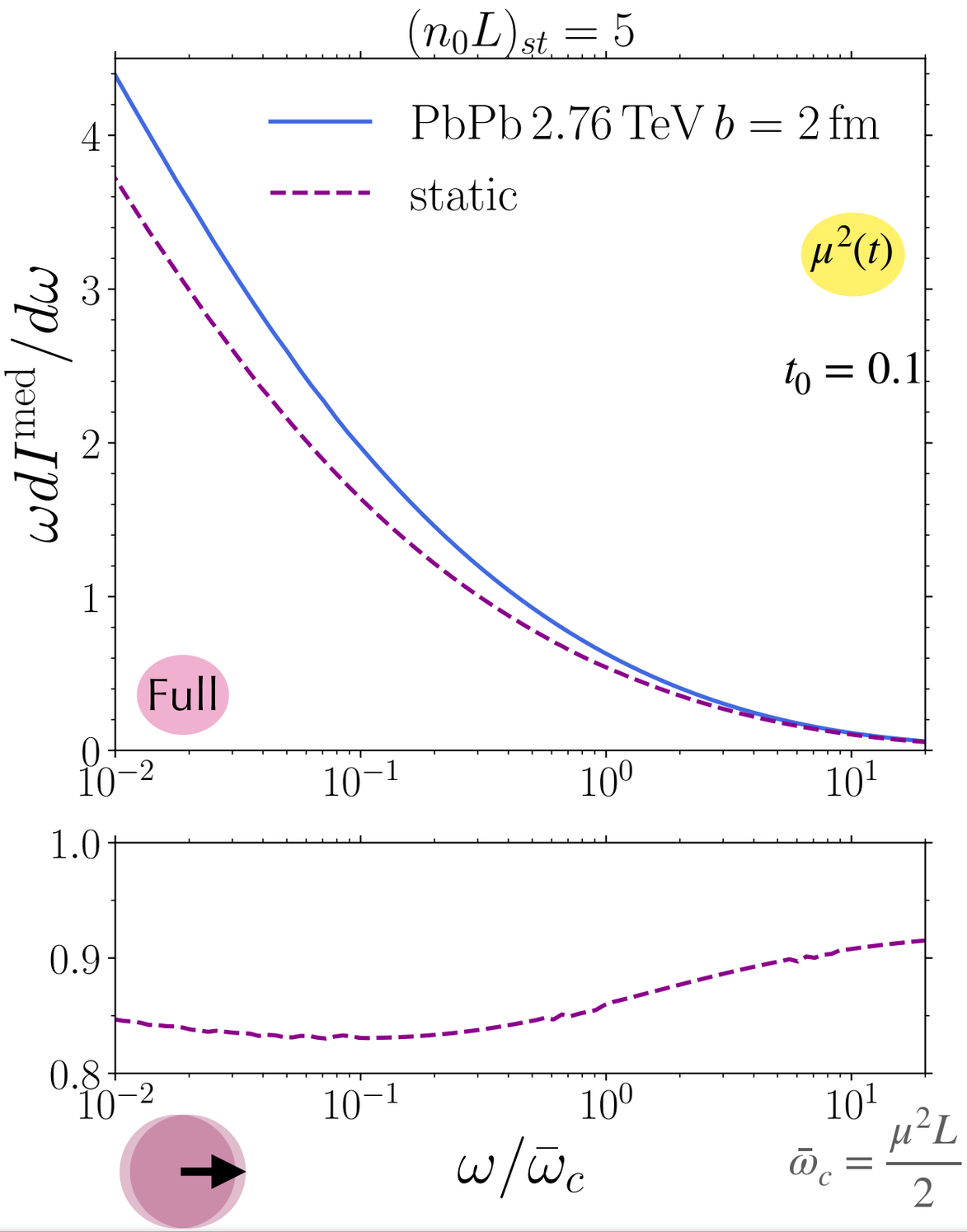
$$n_{hydro}(t) = k_1 T(t) \qquad \mu^2_{hydro}(t) = k_2 T^2(t)$$

- Find the values of the parameters of the static scenario we want to compare to:

$$n_0 L = \int_0^{L'} dt\, n_{hydro}(t)$$

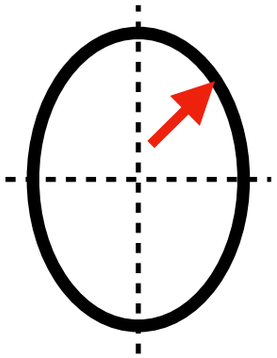
$$\frac{n_0 \mu^2 L^2}{2} = \int_0^{L'} dt\, t\, n_{hydro}(t)\, \mu^2_{hydro}(t)$$

- It works better than using average values, but the **errors go up to ~20%**



Hydro: Luzum and Romatschke, 0901.4588

Scaling laws w.r.t. a power-law case?



- Why the pre-tabulated spectrum needs to be the **static** one?

- The idea is to **find an equivalent scenario given by a power-law**

  Goal: find matching relations between the spectrum in the real world (along a path throughout a hydro) and a pre-tabulated power-law spectrum

- For instance:

  Compute the **spectrum along a path** throughout a hydro:

$$n_{hydro}(t) = k_1 T(t) \qquad \mu^2_{hydro}(t) = k_2 T^2(t)$$

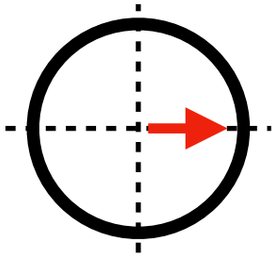
  Compare to a **power-law** spectrum for a profile given by:

$$n(t) = \frac{n'_0}{(t+t_0)^\alpha} \qquad \mu^2(t) = \frac{\mu'^2}{(t+t_0)^{2\alpha}}$$

  with a **scaling law** given by

$$\int_0^{L_1} dt\, n(t) = \int_0^{L_2} dt\, n_{hydro}(t) \qquad \int_0^{L_1} dt\, t\, n(t)\, \mu^2(t) = \int_0^{L_2} dt\, t\, n_{hydro}(t)\, \mu^2_{hydro}(t)$$

Scaling laws w.r.t. a power-law case



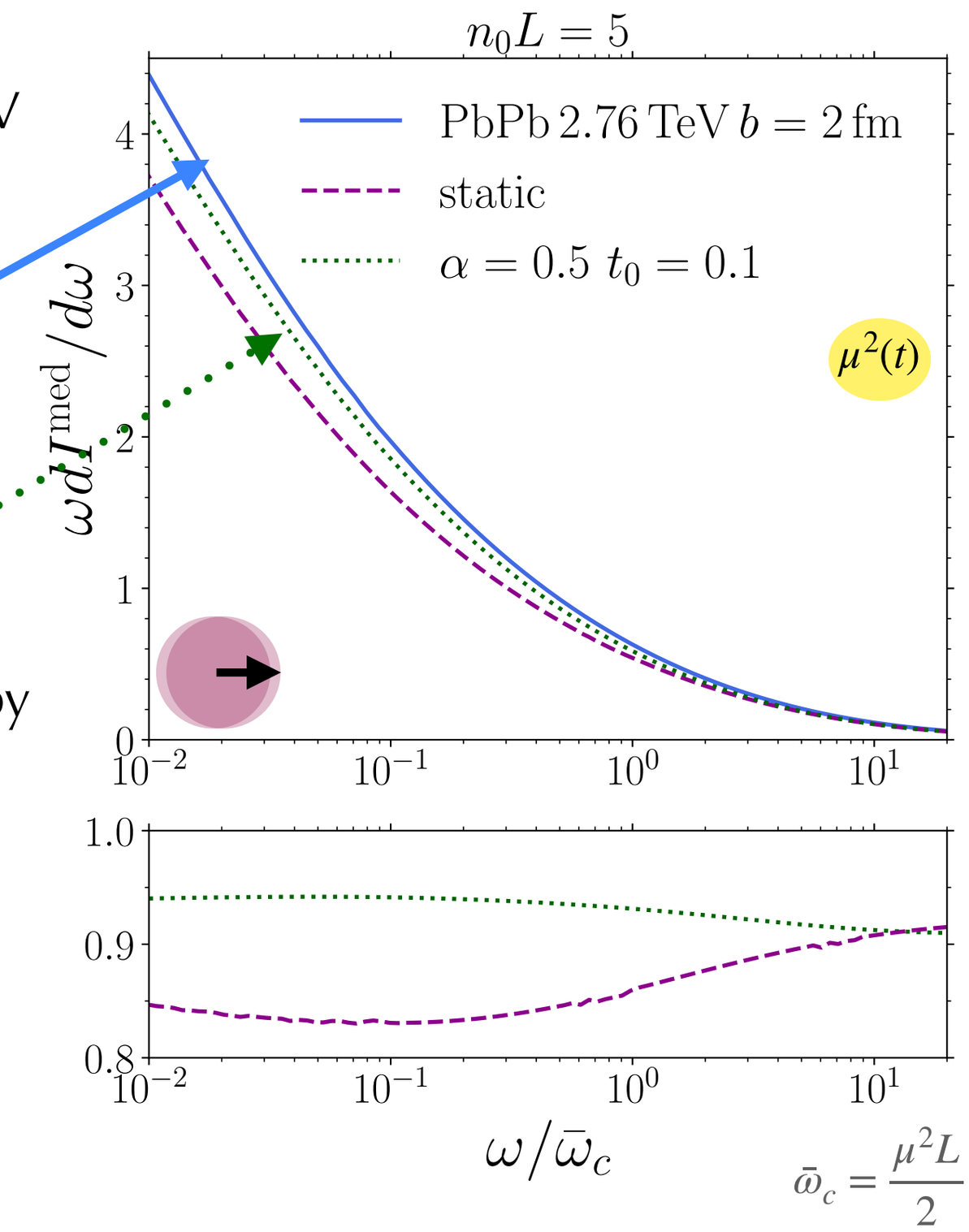
- Select a trajectory in central PbPb 2.76 TeV collisions

- Compute the full spectrum along this path

- Compare to a power-law spectrum given by

$$n(t) = \frac{n_0'}{(t+t_0)^{\alpha}} \qquad \mu^2(t) = \frac{\mu'^2}{(t+t_0)^{2\alpha}}$$

Errors below the 10%



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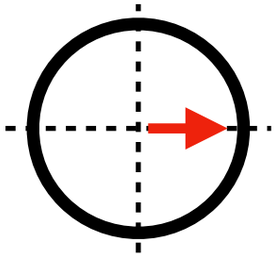
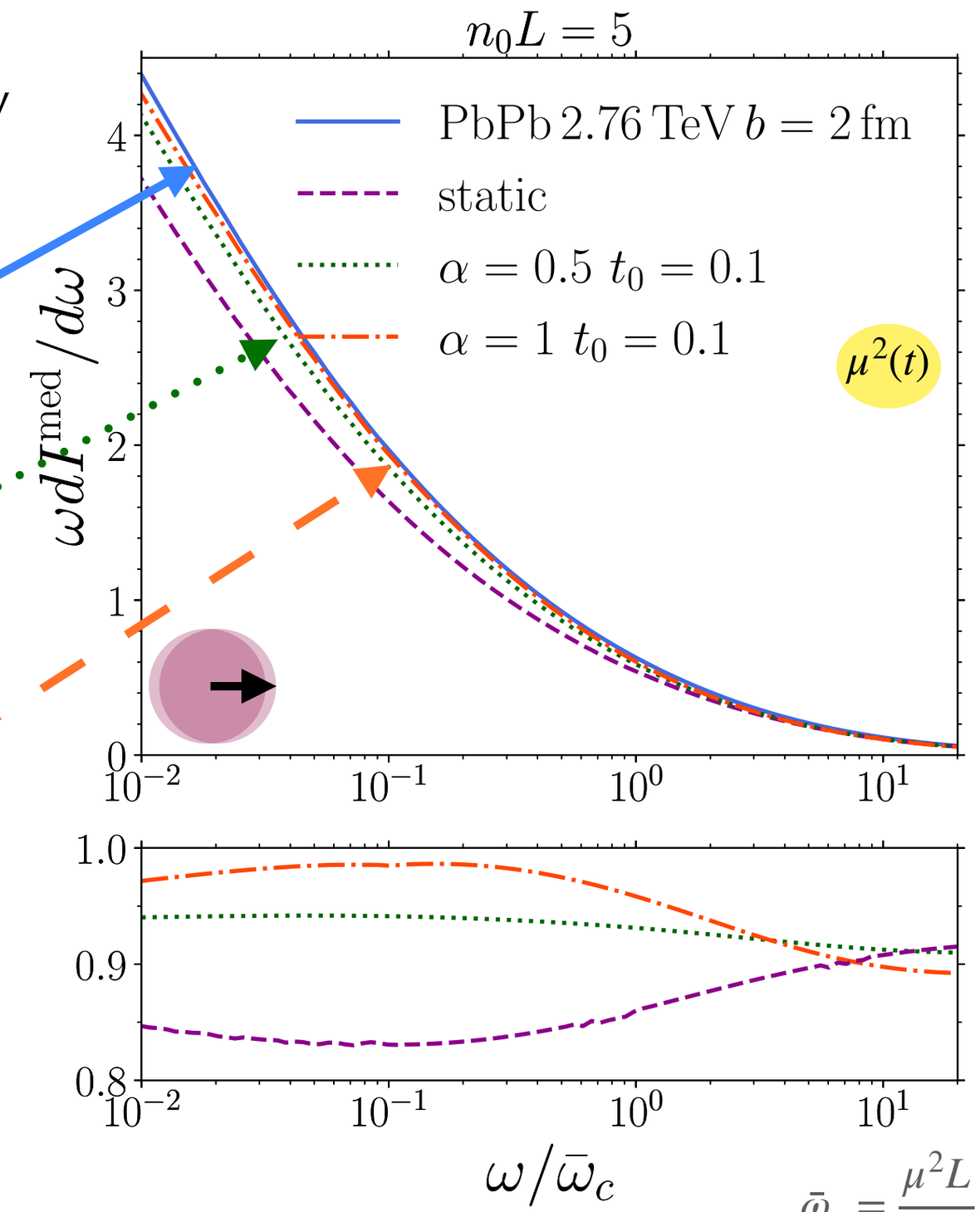
Scaling laws w.r.t. a power-law case



- Select a trajectory in central PbPb 2.76 TeV collisions
- Compute the full spectrum along this path
- Compare to a power-law with $\alpha = 0.5$
- Compare to a power-law with $\alpha = 1$

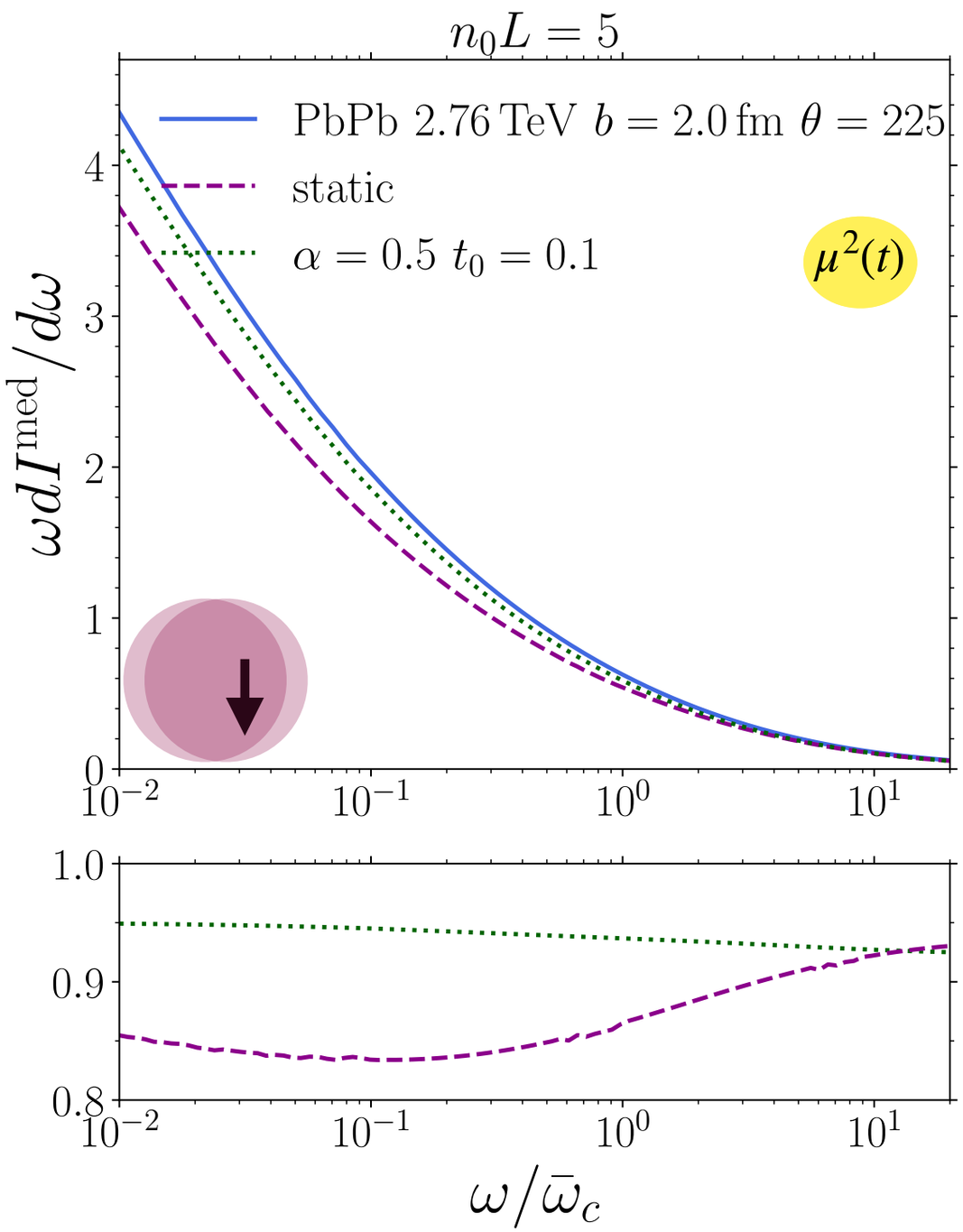
Errors below the 10%



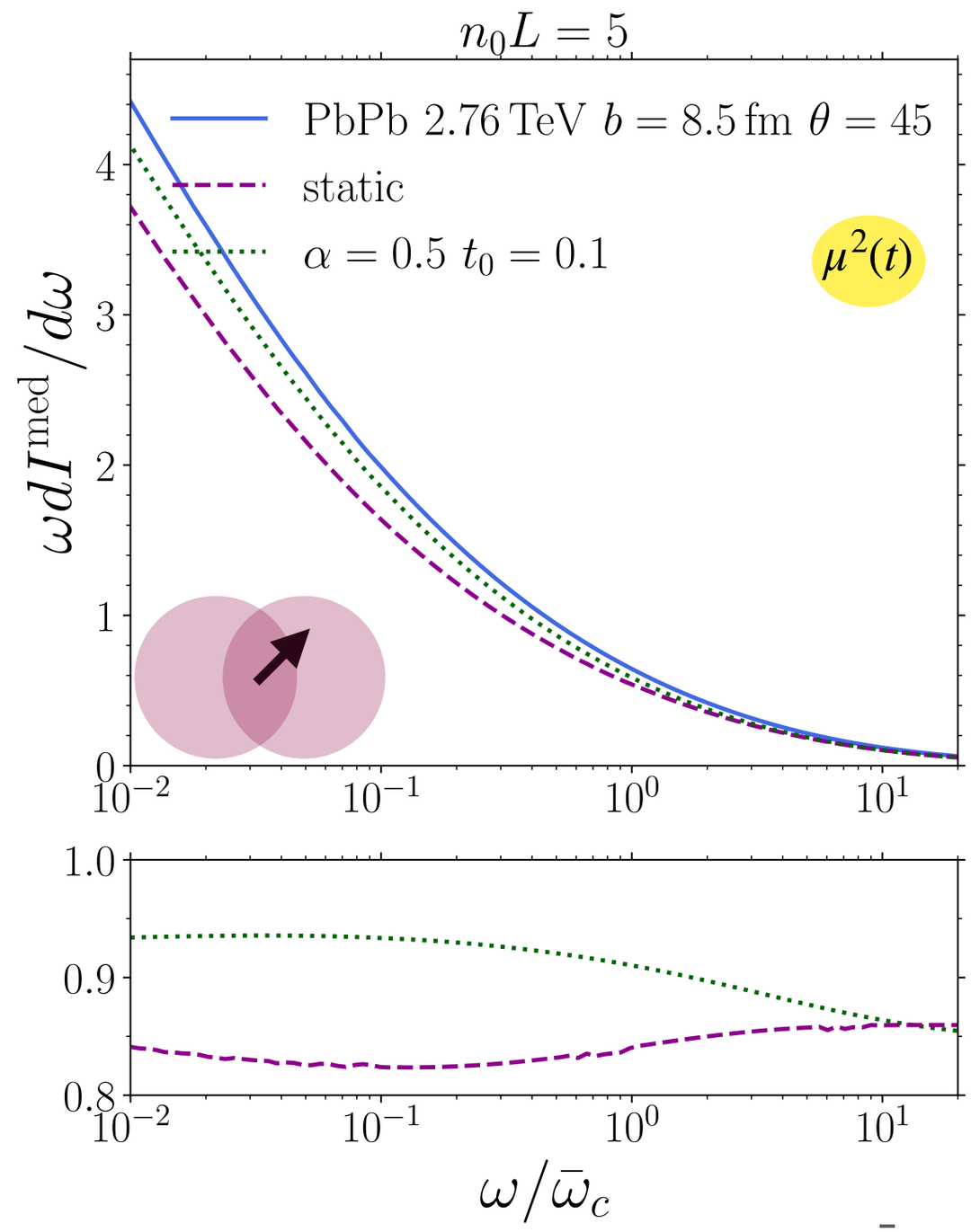
Hydro: Luzum and Romatschke, 0901.4588

Scaling laws w.r.t. a power-law case

- PbPb 2.76 TeV 0-5%



- PbPb 2.76 TeV 30-40%

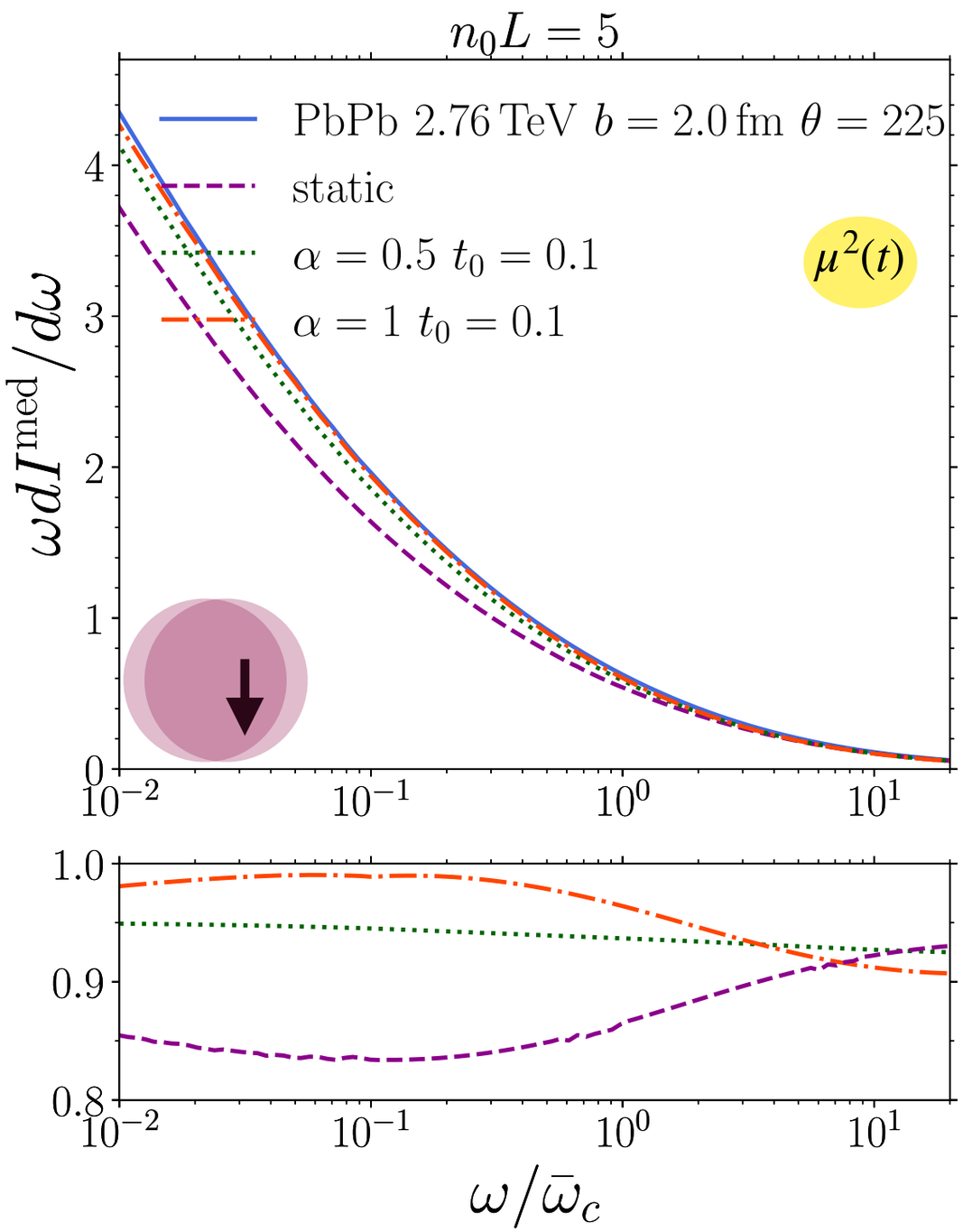


Hydro: Luzum and Romatschke, 0901.4588

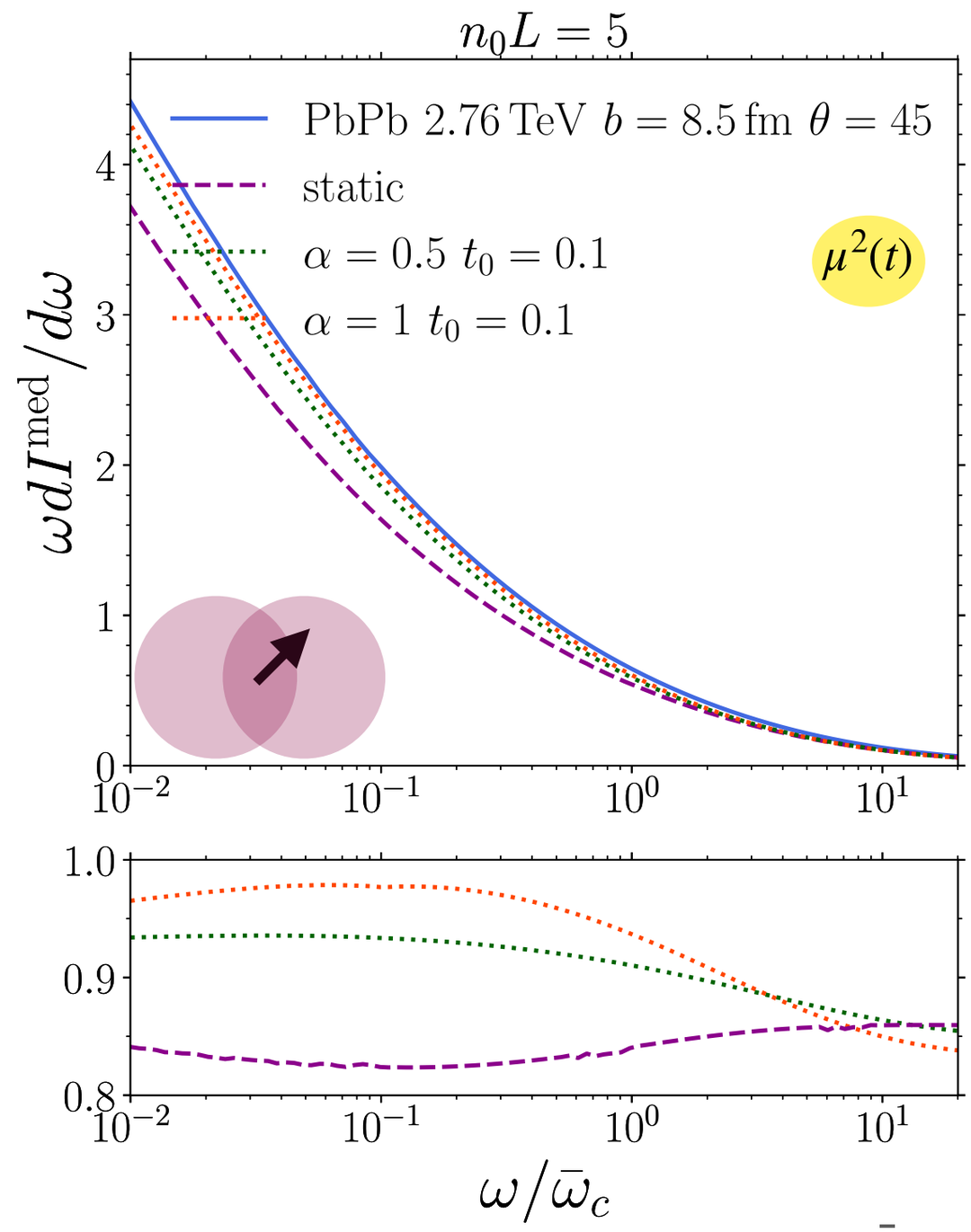
$$\bar{\omega}_c = \frac{\mu^2 L}{2}$$

Scaling laws w.r.t. a power-law case

- PbPb 2.76 TeV 0-5%



- PbPb 2.76 TeV 30-40%



$$\bar{\omega}_c = \frac{\mu^2 L}{2}$$

Hydro: Luzum and Romatschke, 0901.4588

Conclusions

- Many **new developments** in the computation of the **medium-induced radiation** spectrum (in the **brick**)

- These numerical approaches **allow to compute the spectrum along a path in realistic media** (given by a hydro)

  - But it is computationally demanding

  - So we want to use **pre-compute the spectra** for a set of given profiles approximating realistic conditions (scaling laws)

    - Using **power-law profiles reduces the errors substantially**

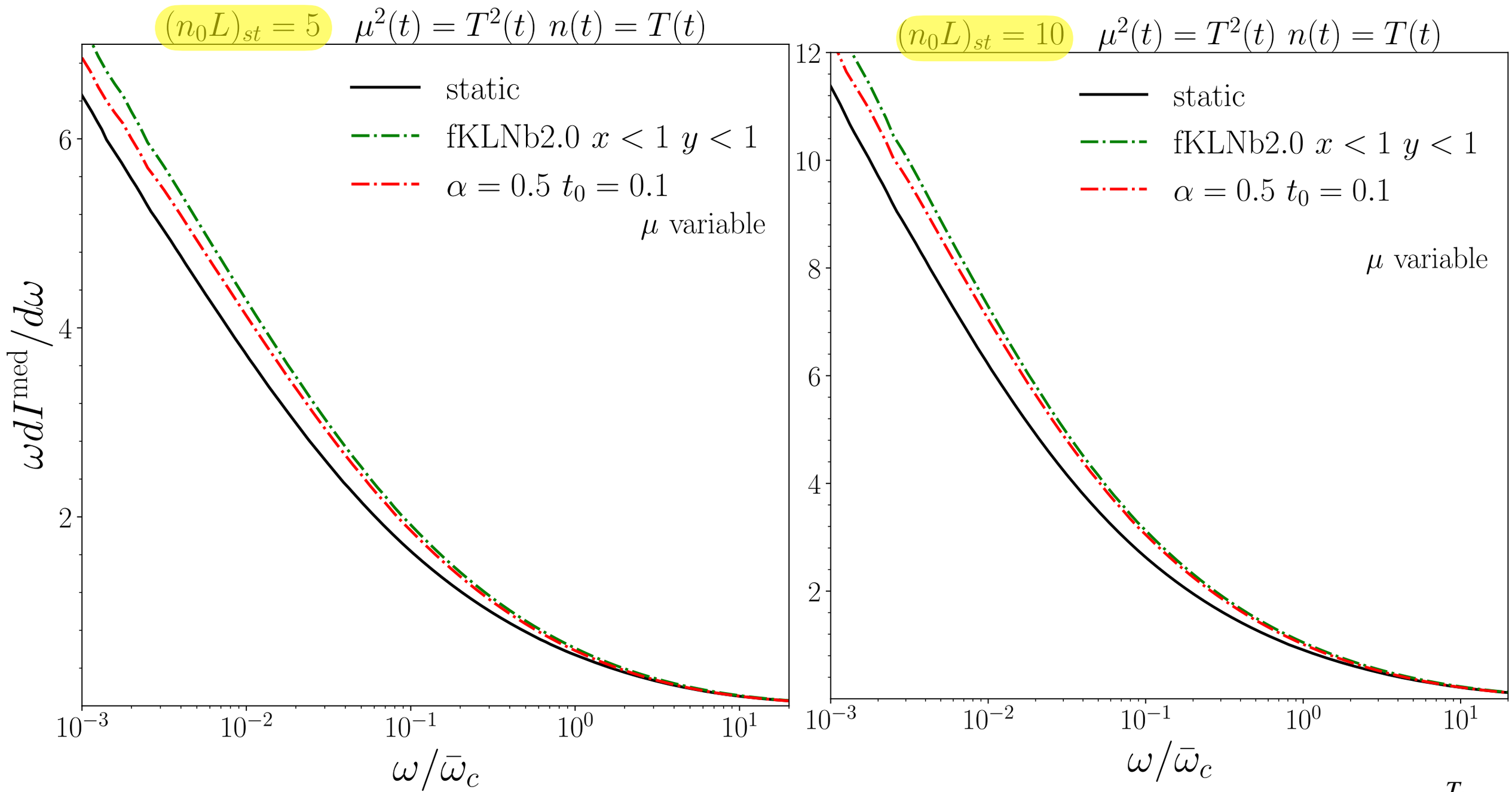
- We can think of other approaches

  For instance, MC approach to mimic the Caron-Huot rates

  Park et al. HP2016 proceedings 1612.06754

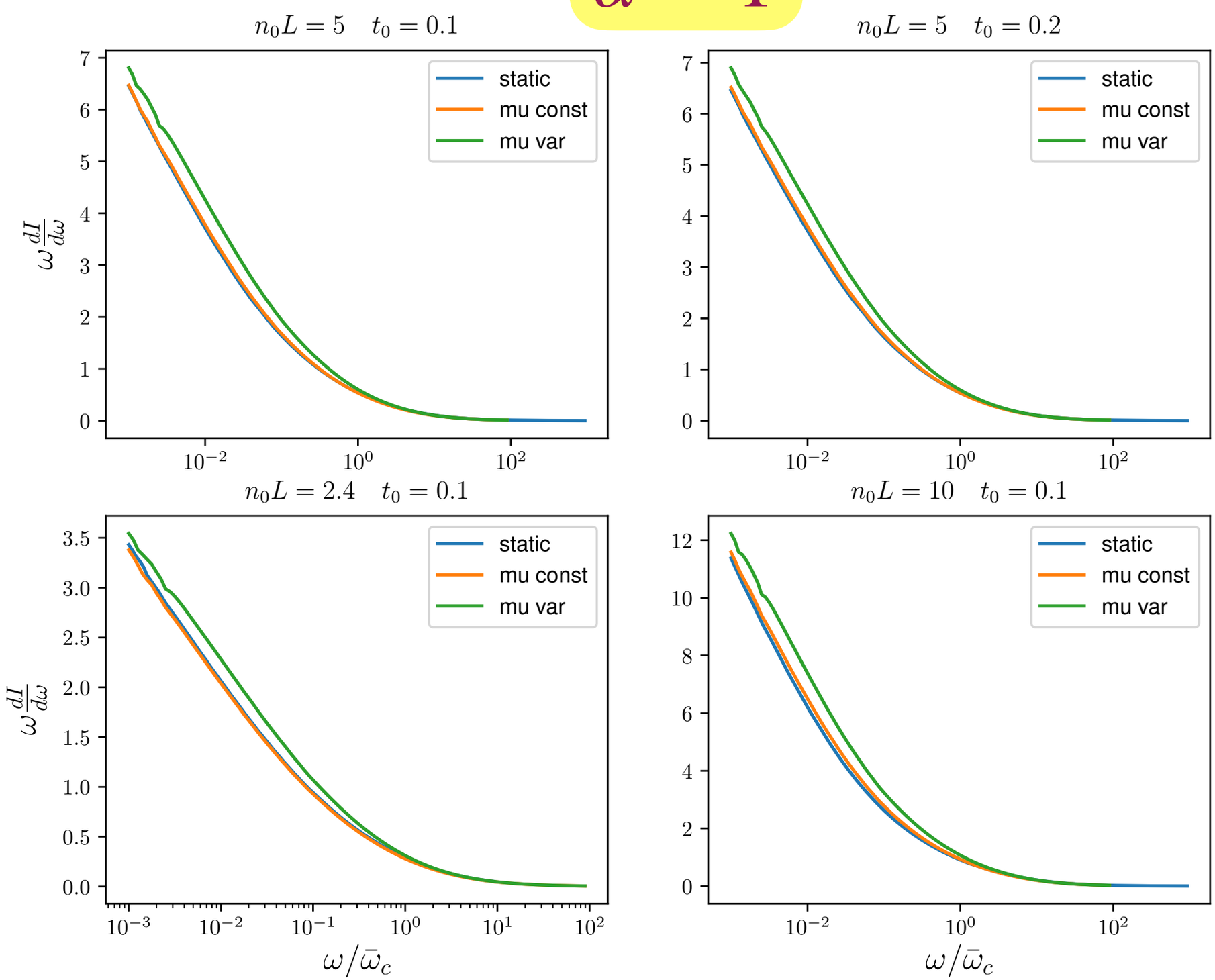
Whatever the approach/approximation used, we can quantify the errors!

Thanks



$$T(t) = \frac{T_0}{(t+t_0)^\alpha}$$

$$\alpha = 1$$

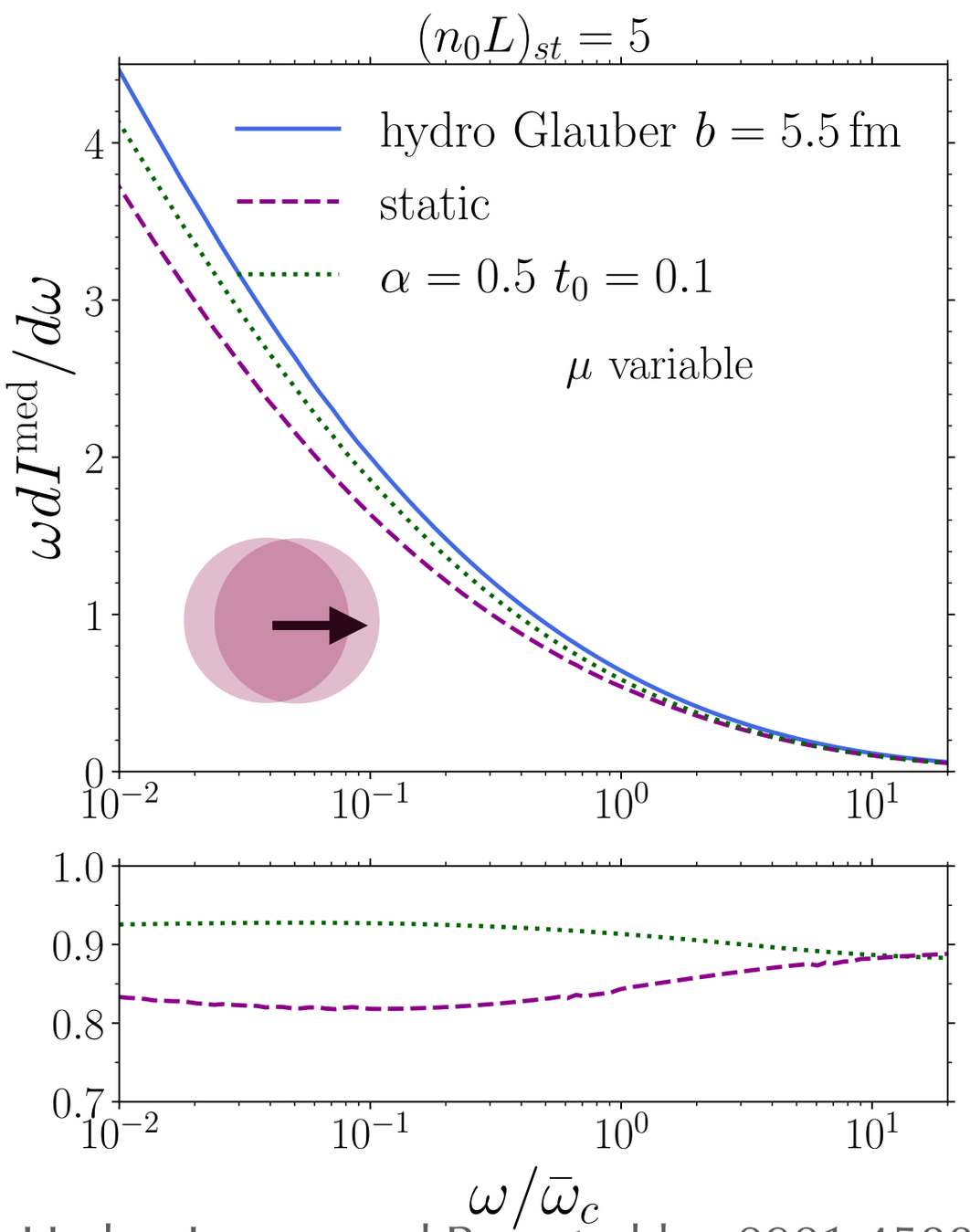


$$n(t) = \frac{n_0'}{(t+t_0)^\alpha}$$

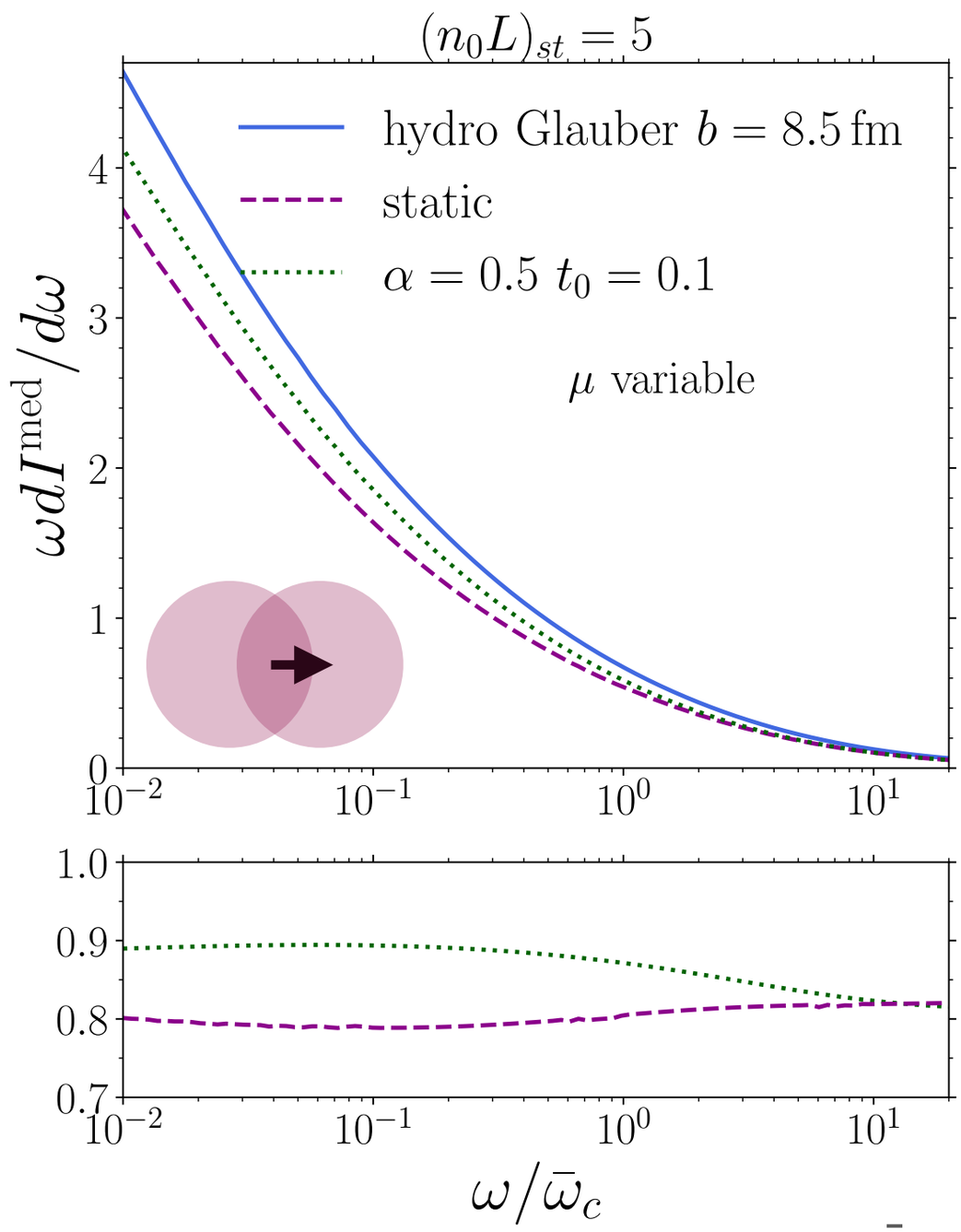
$$\mu'^2(t) = \frac{\mu_0'^2}{(t+t_0)^{2\alpha}}$$

Scaling laws w.r.t. a power-law case

- PbPb 2.76 TeV 20-30%



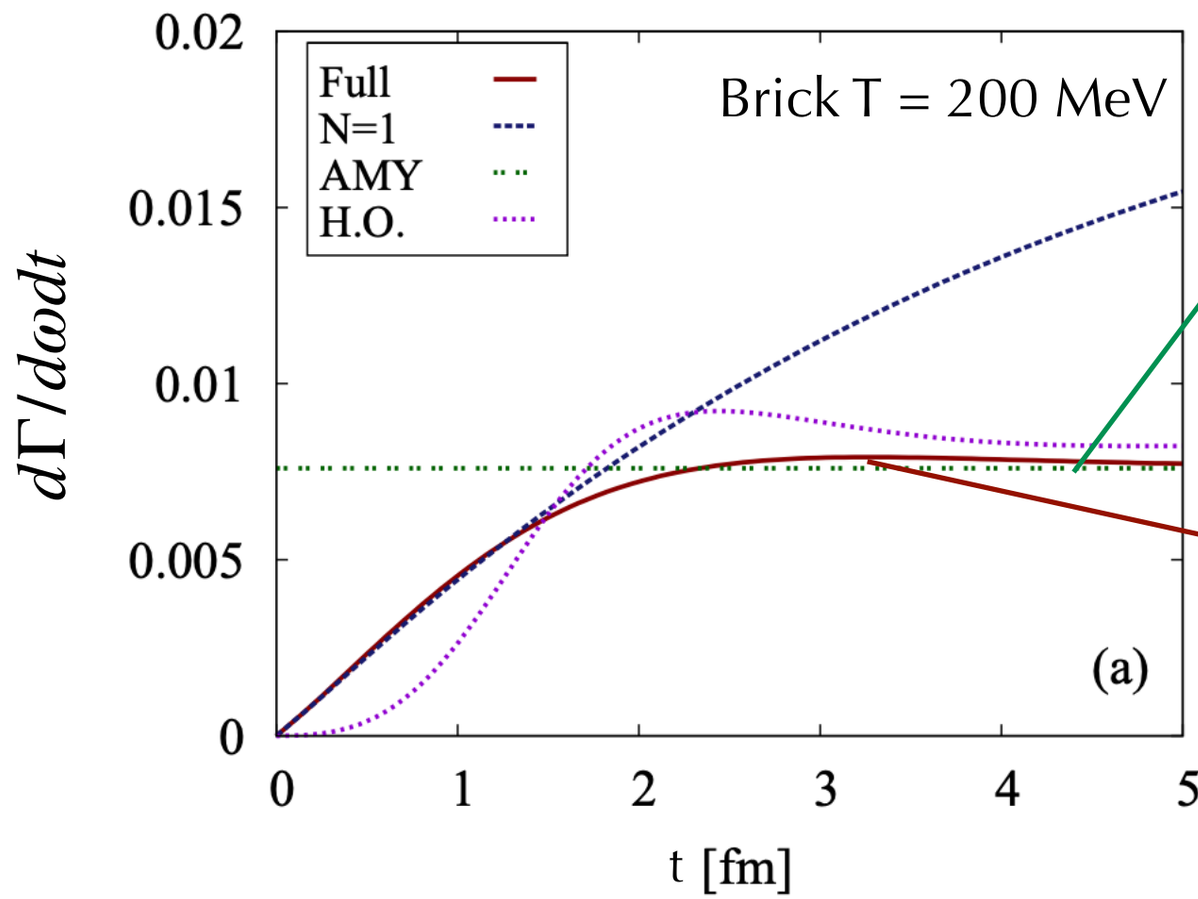
- PbPb 2.76 TeV 30-40%



$$\bar{\omega}_c = \frac{\mu^2 L}{2}$$

Hydro: Luzum and Romatschke, 0901.4588

Why don't we use the rates?

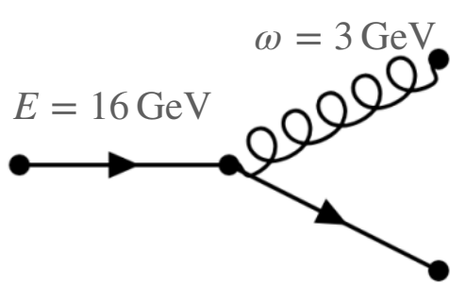


Asymptotic rates

$$\lim_{t\to\infty} \frac{d\Gamma}{d\omega dt}$$

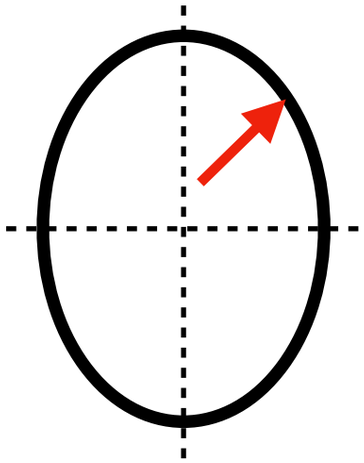
Finite length (full) rates

They depend on the time



Caron-Huot and Gale, 1006.2379

Using rates



- The spectrum is the integral of the rates over the trajectory

$$\omega \frac{dI}{d\omega} = \int_0^L dt\, \omega \, \frac{d\Gamma}{d\omega dt}$$

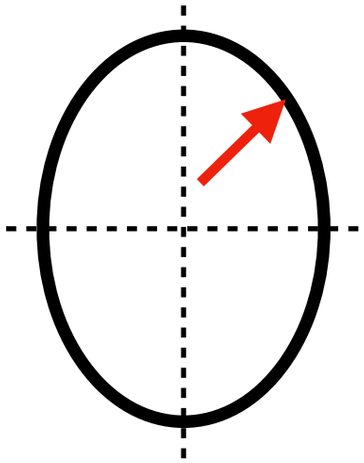
- The rates are only sensitive to a region of the medium of the size of the formation time

- If the formation time is small, computing the rates for a brick is a good approximation (i.e. we can assume a constant temperature during the emission)

- So, one can pre-compute the static rates for all medium temperatures and at each point of the path just read the rate corresponding to the temperature at that point

This is how MARTINI implements the AMY (infinite length) rates

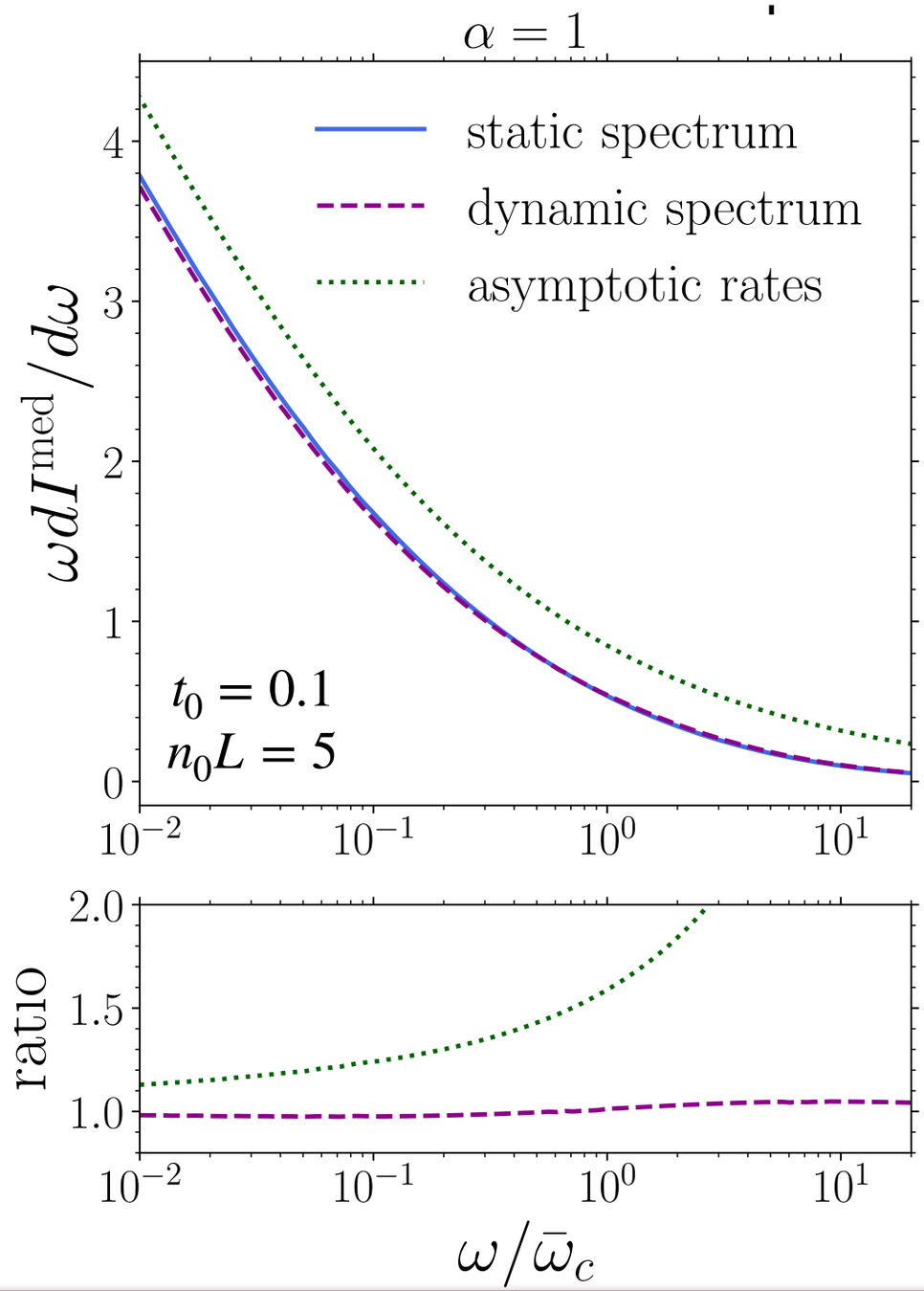
Use of asymptotic rates



Dynamic spectrum: spectrum along the path
for $n(t) = \dfrac{n_0'}{(t+t_0)^\alpha}$

Asymptotic rates:
spectrum obtained from integrating
the asymptotic (static) rates

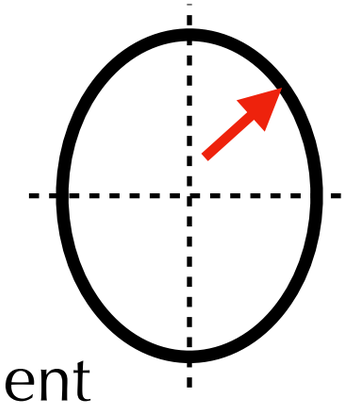
Static spectrum:
static spectrum where the
values of the parameters are set by the scaling laws



- Asymptotic rates do not work well, especially at large energies

$$t_f \propto \frac{\omega}{\boldsymbol{k}^2}$$

Use of full rates



- For each point of the trajectory read the value of n(t) and find the equivalent full static rate

- But the full rates depend on the time

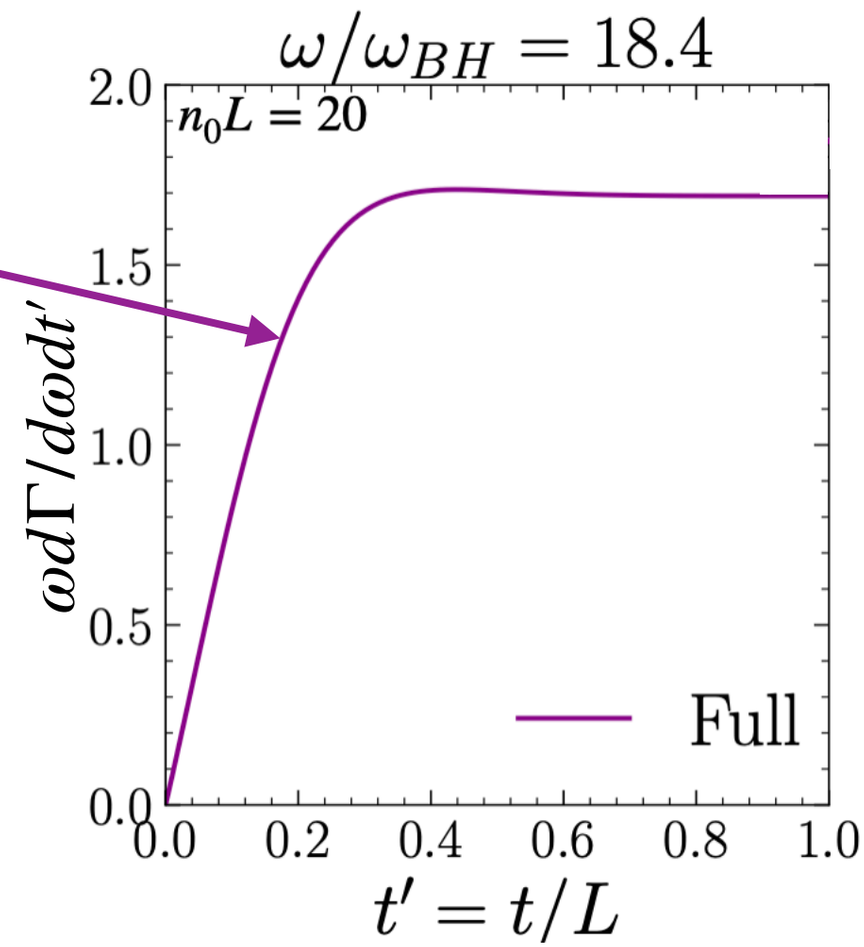
At each point of the trajectory we need to know which time to look at

- How fast the rates reach the asymptote depends on how dense the medium is

The static rate must be evaluated at an *effective* time

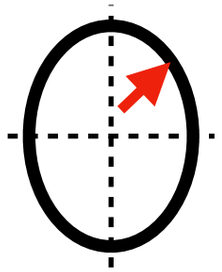
$$n_0\tau(t) = \int_0^t dt'\, n(t')$$

Scaling laws at the level of the rates (at each point of the path)



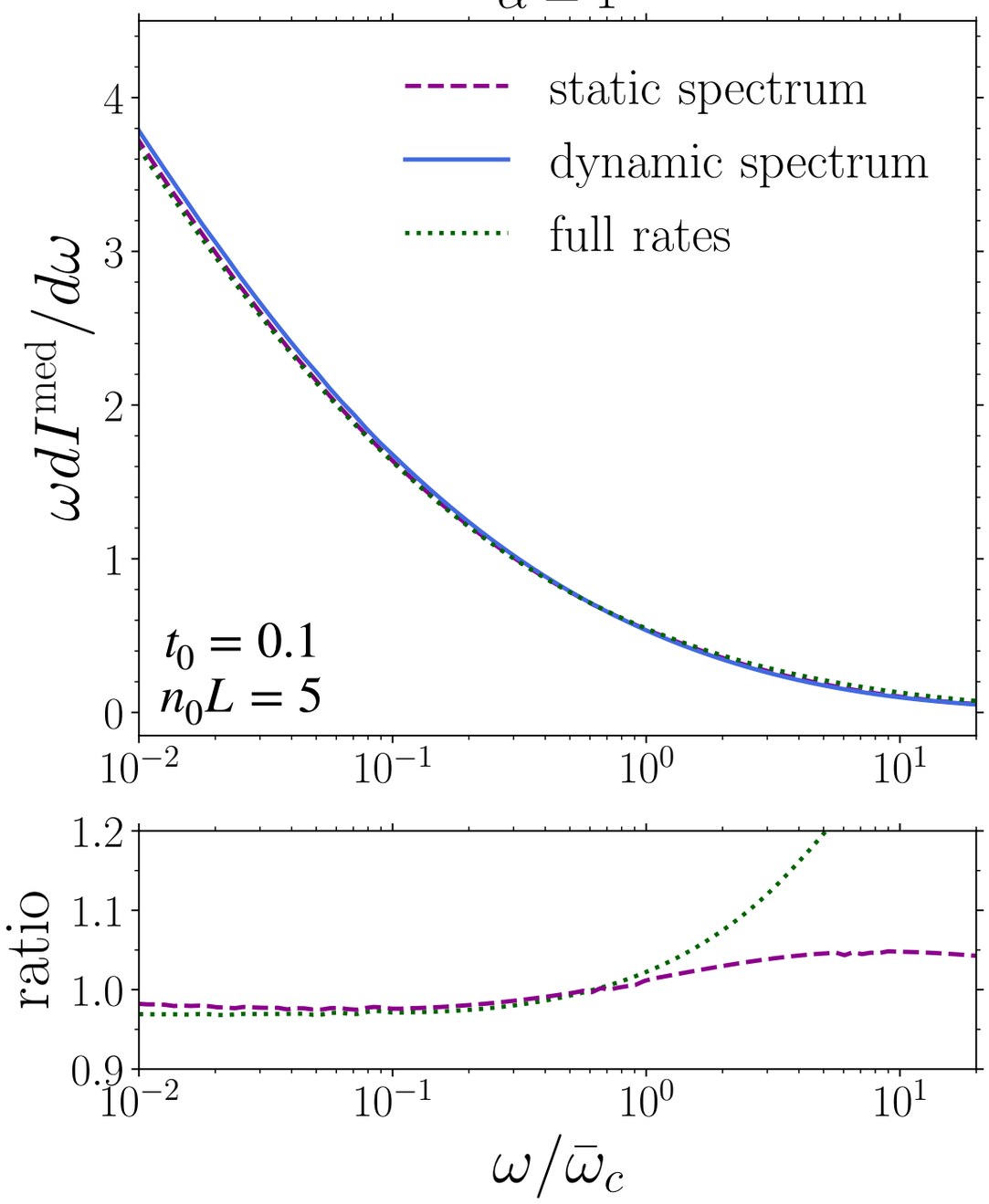
Full Rates

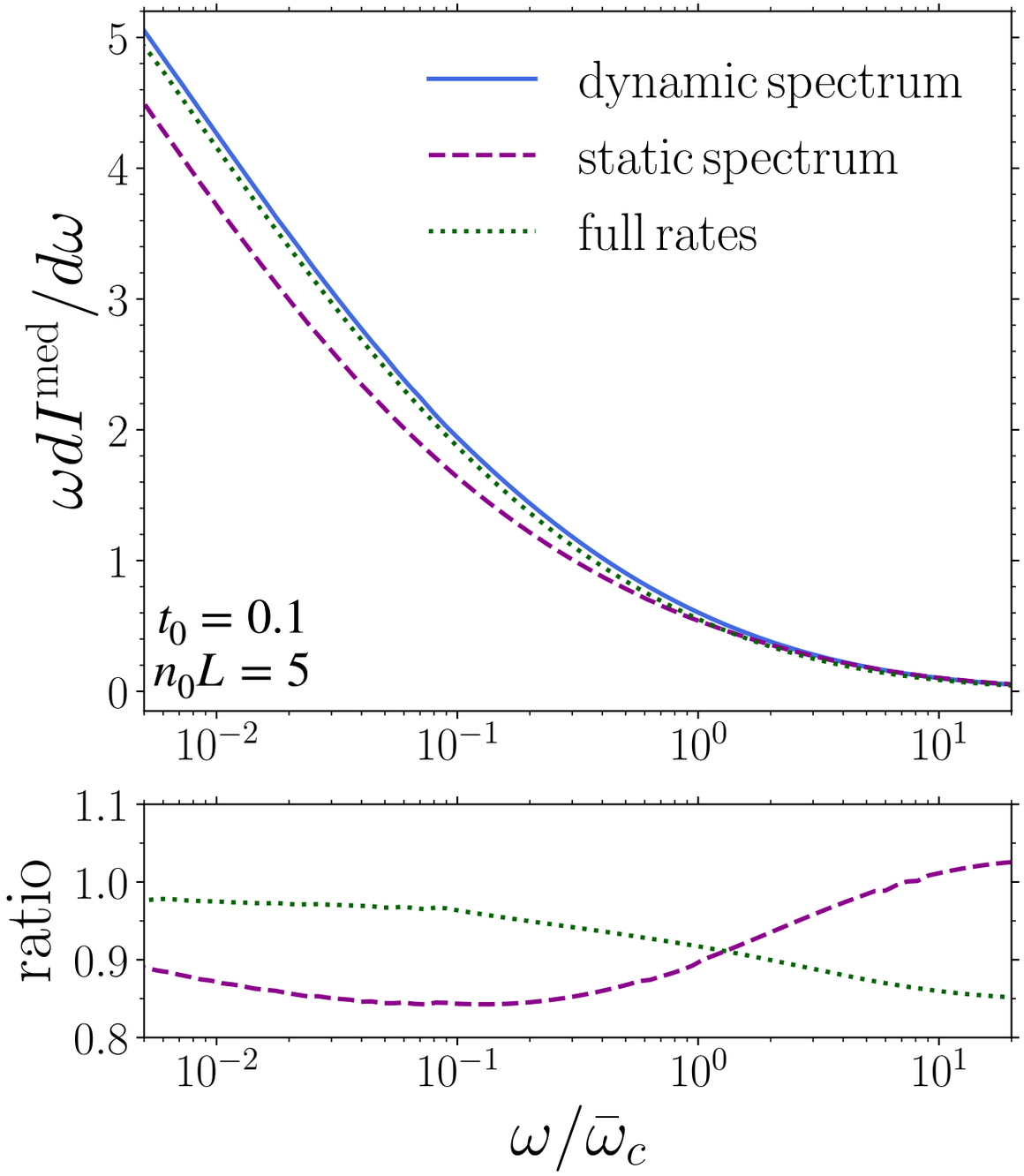
$$n(t) = \frac{n_0'}{(t+t_0)^\alpha} \qquad \mu^2(t) = \frac{\mu'^2}{(t+t_0)^{2\alpha}}$$



Debye mass constant

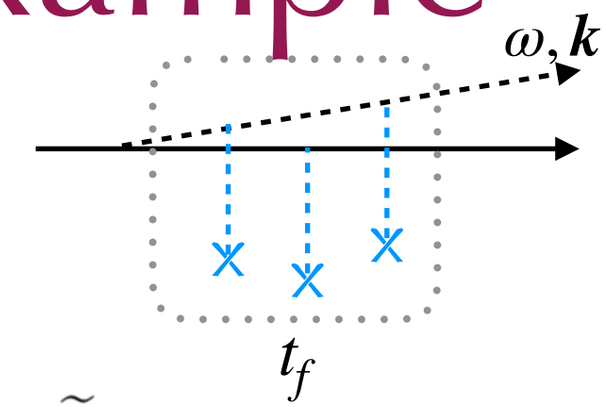
$\alpha = 1$





- It works similar to the scaling laws for the spectrum: errors go up to ~15%

The full solution as an example



- Full one (soft) gluon emission in-medium calculation

$$\omega \frac{dI^{\text{med}}}{d\omega d^2\boldsymbol{k}} = \frac{2\alpha_s C_R}{(2\pi)^2\omega} \text{Re} \int_0^L ds\, n(s) \int_0^s dt \int_{\boldsymbol{pql}} i\boldsymbol{p} \cdot \left(\frac{\boldsymbol{l}}{\boldsymbol{l}^2} - \frac{\boldsymbol{q}}{\boldsymbol{q}^2} \right) \sigma(\boldsymbol{l} - \boldsymbol{q}) \widetilde{\mathcal{K}}(s, \boldsymbol{q}; t, \boldsymbol{p}) \mathcal{P}(L, \boldsymbol{k}; s, \boldsymbol{l})$$

BDMPS-Z
(Vacuum subtracted)

- Broadening

$$\partial_\tau \mathcal{P}(\tau, \boldsymbol{k}; s, \boldsymbol{l}) = -\frac{1}{2} n(\tau) \int_{\boldsymbol{k}'} \sigma(\boldsymbol{k} - \boldsymbol{k}') \mathcal{P}(\tau, \boldsymbol{k}'; s, \boldsymbol{l})$$

- Emission Kernel

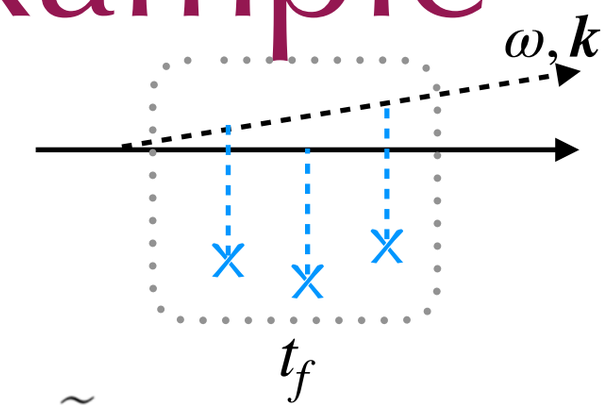
$$\partial_t \widetilde{\mathcal{K}}(s, \boldsymbol{q}; t, \boldsymbol{p}) = \frac{i\boldsymbol{p}^2}{2\omega} \widetilde{\mathcal{K}}(s, \boldsymbol{q}; t, \boldsymbol{p}) + \frac{1}{2} n(t) \int_{\boldsymbol{k}'} \sigma(\boldsymbol{k}' - \boldsymbol{p}) \widetilde{\mathcal{K}}(s, \boldsymbol{q}; t, \boldsymbol{k}')$$

Screening mass

$$\sigma(\boldsymbol{q}) \equiv -V(\boldsymbol{q}) + (2\pi)^2 \delta^2(\boldsymbol{q}) \int_{\boldsymbol{l}} V(\boldsymbol{l}) \qquad V(\boldsymbol{q}) = \frac{8\pi\mu^2(t)}{\left(\boldsymbol{q}^2 + \mu^2(t)\right)^2}$$

CA, L. Apolinário, F. Dominguez, 2002.01517

The full solution as an example



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BDMPS-Z
(Vacuum subtracted)

- Broadening

$$\partial_\tau \mathcal{P}(\tau, \boldsymbol{k}; s, \boldsymbol{l}) = -\frac{1}{2} n(\tau) \int_{\boldsymbol{k}'} \boxed{\sigma(\boldsymbol{k} - \boldsymbol{k}')} \mathcal{P}(\tau, \boldsymbol{k}'; s, \boldsymbol{l})$$

Medium information

- Emission Kernel

$$\partial_t \widetilde{\mathcal{K}}(s, \boldsymbol{q}; t, \boldsymbol{p}) = \frac{i\boldsymbol{p}^2}{2\omega} \widetilde{\mathcal{K}}(s, \boldsymbol{q}; t, \boldsymbol{p}) + \frac{1}{2} n(t) \int_{\boldsymbol{k}'} \boxed{\sigma(\boldsymbol{k}' - \boldsymbol{p})} \widetilde{\mathcal{K}}(s, \boldsymbol{q}; t, \boldsymbol{k}')$$

Screening mass

$$\sigma(\boldsymbol{q}) \equiv -V(\boldsymbol{q}) + (2\pi)^2 \delta^2(\boldsymbol{q}) \int_{\boldsymbol{l}} V(\boldsymbol{l}) \qquad V(\boldsymbol{q}) = \frac{8\pi\mu^2(t)}{\left(\boldsymbol{q}^2 + \mu^2(t)\right)^2}$$

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