Equation of State of (2+1)-flavor QCD: An update based on high precision Taylor expansion results

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Based on [arXiv:2202.09184] and forthcoming

Outline

Motivation

- Bulk thermodynamic properties (Energy density ϵ, Pressure p, Entropy density s, number densities $n, \ldots$) of QCD are fundamental inputs for wide range of phenomena (HIC, early universe, compact stars, ...)
- Perturbative approaches face challenges.
- Widely used Hadron Resonance Gas shows deviations from QCD even below T_{pc} already in second order susceptibilities. [arxiv: 2107.10011]
- Non-perturbative, first-principle determination of QCD Equation of State necessary.

Equation of State at $\mu = 0$

- EoS at $\vec{\mu} = 0$ is accessed via trace anomaly $\Theta^{\mu\mu}$:

$$\frac{\Theta^{\mu\mu}(T)}{T^4} = -\frac{1}{VT^3}\frac{\mathrm{d}\ln\mathcal{Z}}{\mathrm{d}\ln a} = \frac{\epsilon - 3p}{T^4} = T\frac{\mathrm{d}}{\mathrm{d}T}\left(\frac{p}{T^4}\right).$$

- Integral method yields pressure:

$$\frac{p(T)}{T^4} = \frac{p_0}{T_0^4} + \int_{T_0}^{T} \mathrm{d}T' \frac{\Theta^{\mu\mu}(T)}{T^5}.$$

- “Solved” problem for (2+1)-flavor lattice QCD.

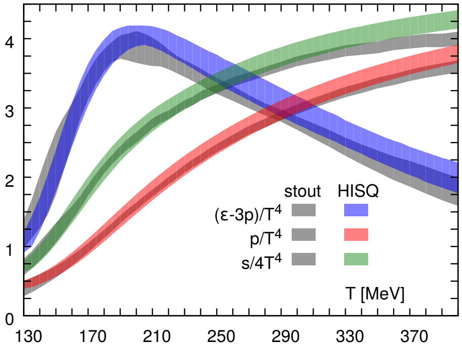


Figure: EoS calculated by WB (grey) and HotQCD (color) [arxiv:1407.6387]

Equation of State at $\mu > 0$

- Finite Density sign problem prevents direct simulations.
- "Production ready" approaches to reach $\mu > 0$:
  - Analytical continuation from $\mu^2 < 0$ (see talk by Jana Günther).
  - Taylor expansion around $\mu = 0$:

$$\frac{p(T,\mu)}{T^4} = \sum_{i,j,k=0} \frac{\chi_{ijk}^{BQS}(T)}{i!j!k!} \hat{\mu}_B^i \hat{\mu}_Q^j \hat{\mu}_S^k, \quad \hat{\mu}_x = \frac{\mu_x}{T},$$

$$\chi_{ijk}^{BQS}(T) = \frac{1}{VT^3} \left. \frac{\partial^{i+j+k} \ln \mathcal{Z}}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \right|_{\vec{\mu}=0} .$$

Equation of State at $\mu > 0$

- μ dependence of ϵ and s are extracted from expansion of trace anomaly:

$$\frac{\epsilon - 3p}{T^4} = T\frac{\partial p/T^4}{\partial T} = \sum_{i,j,k=0} \frac{\Xi_{ijk}^{BQS}(T)}{i!j!k!}\hat{\mu}_B^i\hat{\mu}_Q^j\hat{\mu}_S^k, \quad \Xi_{ijk}^{BQS}(T) = T\frac{\partial \chi_{ijk}^{BQS}(T)}{\partial T},$$

$$\frac{\epsilon}{T^3} = \sum_{i,j,k=0} \frac{\Xi_{ijk}^{BQS}(T) + 3\chi_{ijk}^{BQS}(T)}{i!j!k!}\hat{\mu}_B^i\hat{\mu}_Q^j\hat{\mu}_S^k,$$

$$\frac{s}{T^3} = \sum_{i,j,k=0} \frac{\Xi_{ijk}^{BQS}(T) + (4-i-j-k)\chi_{ijk}^{BQS}(T)}{i!j!k!}\hat{\mu}_B^i\hat{\mu}_Q^j\hat{\mu}_S^k.$$

Equation of State at $\mu > 0$

Constrain Taylor expansion to lines parameterized by μ_B:

$$\Delta p/T^4 = \sum_n P_{2n}\hat{\mu}_B^{2n},\ n_B/T^3 = \sum_n N^B_{2n-1}\hat{\mu}_B^{2n-1},$$
$$\Delta\epsilon/T^3 = \sum_n \epsilon_{2n}\hat{\mu}_B^{2n},\ \Delta\sigma/T^3 = \sum_n \sigma_{2n}\hat{\mu}_B^{2n}.$$

- Simplest case: $\mu_Q = \mu_S = 0$ $\rightarrow$ EoS can be calculated from diagonal χ^B_{2n}'s alone ($P_{2n} \sim N_{2n-1} \sim \chi^B_{2n}$).
- Heavy Ion Collision case: $n_Q/n_B = 0.4$, $n_S = 0$ $\rightarrow$ express μ_Q and μ_S via baryon chemical potential μ_B and fulfill constraints order by order.

$$\hat{\mu}_Q = q_1(T)\hat{\mu}_B + q_3(T)\hat{\mu}_B^3 + q_5(T)\hat{\mu}_B^5 + \cdots,$$
$$\hat{\mu}_S = s_1(T)\hat{\mu}_B + s_3(T)\hat{\mu}_B^3 + s_5(T)\hat{\mu}_B^5 + \cdots,$$

EoS then involves off-diagonal χ^{BQS}_{ijk}'s as well.

Generalized Susceptibilities via lattice QCD

$$\chi_{ijk}^{BQS} \equiv \frac{1}{VT^3} \frac{\partial^{i+j+k} \log \mathcal{Z}}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k}, \quad \hat{\mu}_X \equiv \frac{\mu_X}{T}$$

To compute χ_{ijk}^{BQS}, we need to solve integrals of the form

$$\frac{1}{\mathcal{Z}} \int \prod_{x,\nu} \mathrm{d}U_{x,\nu} \, \mathrm{Tr}\left(M_f^{-1} M'_f \cdots \right) \mathrm{e}^{-S_{\mathrm{eff}}}.$$

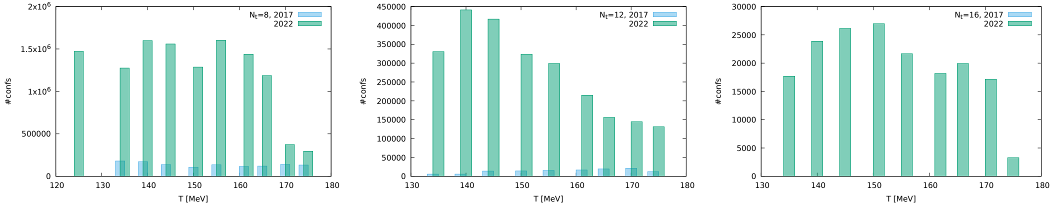
These are calculated via Markov-Chain Monte-Carlo:

1. Generate $\{U_{x,\nu}\}$-ensembles via RHMC algorithm[1].
2. Evaluate $\mathrm{Tr}\left(M_f^{-1} M'_f \cdots \right)$ on $\{U_{x,\nu}\}$-ensembles using random noise method: $\mathrm{Tr}\left(\hat{\mathcal{M}}_f \right) \sim \frac{1}{N} \sum_{i=0}^{N} \eta_i^* \hat{M}_f \eta_i$. $\rightarrow$ Sparse matrix inversions with 500-2000 right-hand sides η_i for each trace. (Optimizations: Multi-RHS CG + TR-Lanczos with spectrum filter)

[1] https://github.com/LatticeQCD/SIMULATeQCD

HotQCD Setup & Statistics

- Dynamical Fermions (HISQ) with $N_f = 2+1$, physical quark masses ($\frac{m_s}{m_l} = 27$), $T \in [135, 175]$ MeV and lattice sizes $N_\tau = 6, 8, 12, 16, N_\sigma = 4N_\tau$.
- For Temperatures $T > 180$ MeV: $\frac{m_s}{m_l} = 20$ and $N_\tau = 6, 8, 12$[2].



Data set 2022 [arXiv:2202.09184] vs 2017 [arxiv:1701.04325]

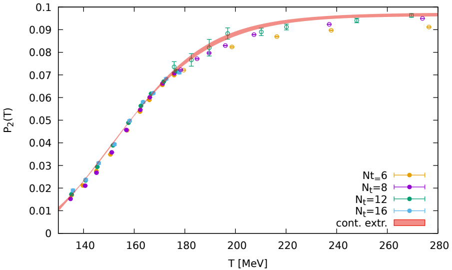
[2]Only lowest orders available for $N_\tau = 12$ at $T > 180$ MeV

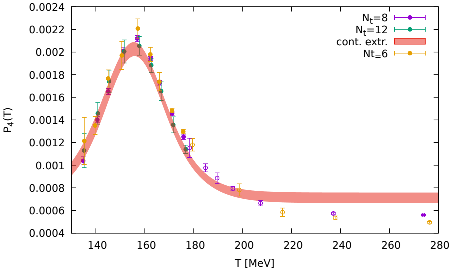
Continuum extrapolation strategy

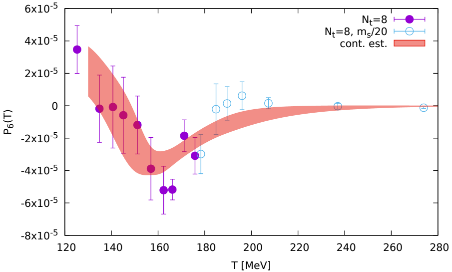
- For $\mu_Q = \mu_S = 0$ and isospin symmetric data analysis details: Talk by Jishnu Goswami & [arxiv:2202.09184].
- For $n_Q/n_B = 0.4$: need continuum extrapolations of $P_{2n}(T)$, $N_{2n-1}(T)$, $q_{2n-1}(T)$ and their T-derivatives.
- Errors in data points at individual T and N_τ are normally distributed and independent.
- Fit $P_{2n}(T)$, $N_{2n-1}(T)$, $q_{2n-1}(T)$ on each bootstrap sample generated from joint $N_\tau = 6, 8, 12, 16$[3] data set using $1/N_\tau^2$ corrections.
- Error bands are given by 1σ spread of bootstrap values at given T.

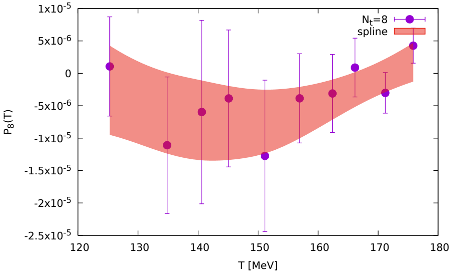
[3] 4th order: $N_\tau = 6, 8, 12$, 6th order: $N_\tau = 8$ fit, 8th order $N_\tau = 8$ spline

Continuum extrapolations: P_{2n} for $n_Q/n_B = 0.4, n_S = 0$ (preliminary)

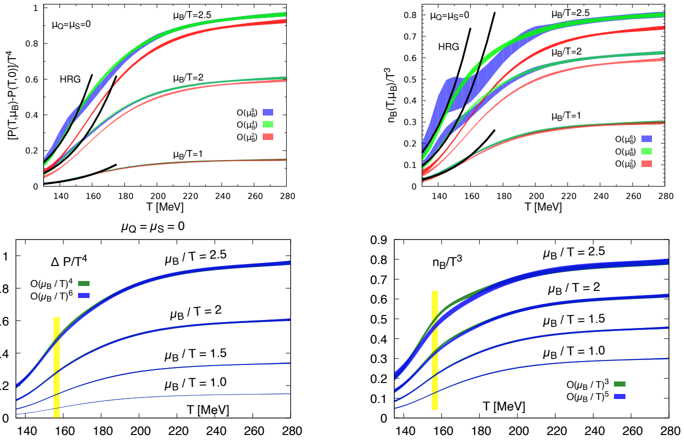








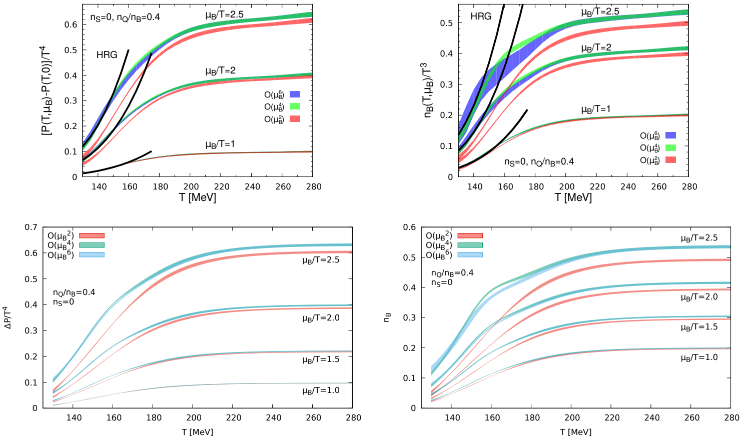
Equation of State for $\mu_Q = \mu_S = 0$



- Better control over $\mathcal{O}(\mu_B^6)$ significantly reduces spurious "wiggles" at high μ_B/T.

EoS 2017 (top) vs 2022 (bottom)

Equation of State for $n_Q/n_B = 0.4, n_S = 0$ (preliminary)



EoS 2017 (top) vs 2022 (bottom)

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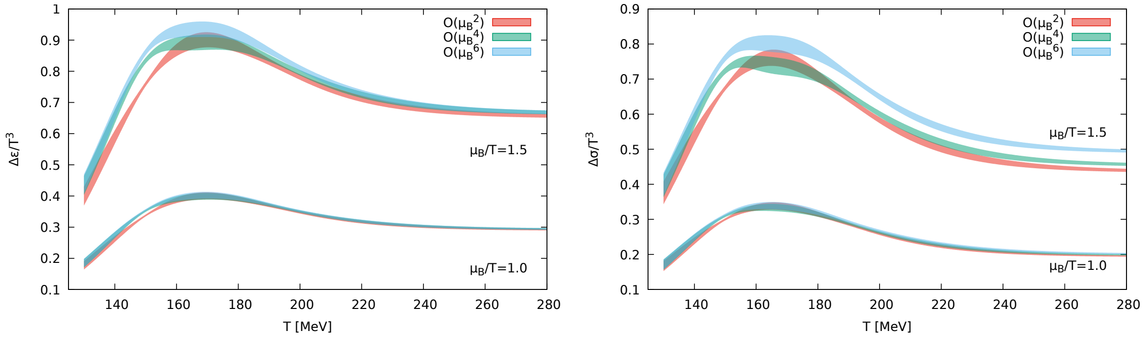
Equation of State for $n_Q/n_B = 0.4, n_S = 0$ (preliminary)



- $\mathcal{O}(\mu_B^6)$ coefficients that contain derivatives (ϵ_6, σ_6) remain hard to compute!
- Smaller range of reliability compared to P and n_B at $\mathcal{O}(\mu_B^6)$.

Summary & Outlook

- Multi-year computation campaign generating high statistics data set of (2+1)-flavor HISQ configurations.
- Extension of Equation of State Taylor expansion coefficients up to 8th order.
- Significantly improved control over 6th order coefficients removes spurious "wiggles" of earlier study.
- No evidence for a breakdown of convergence of the Taylor Series for $\mu_B/T < 2.5$ in the entire temperature range explored in this study.

Upcoming:

- Taylor series resummation using Pade method (see talk by Jishnu Goswami).
- Updated parametrization of Equation of State for strangeness neutral systems.
- Updated calculations of isothermal & isoentropic speed-of-sound.