Probing flow fluctuations through factorization breaking in heavy-ion collision

- based on P. Bozek, R. Samanta, PRC 105, 034904 (2022)

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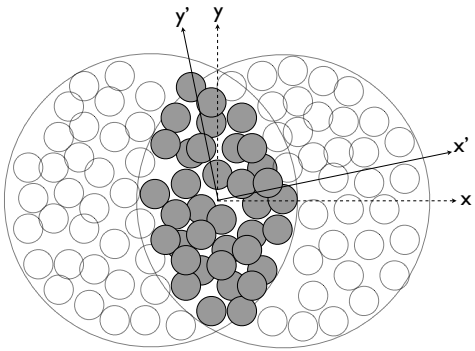
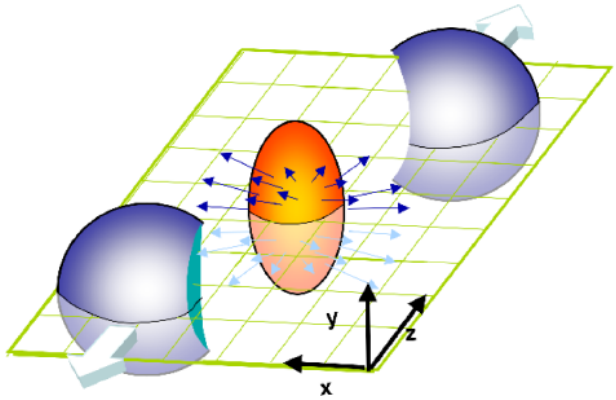
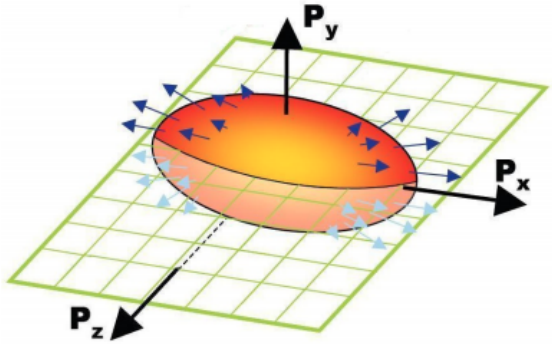
Collective flow in HI Collisions



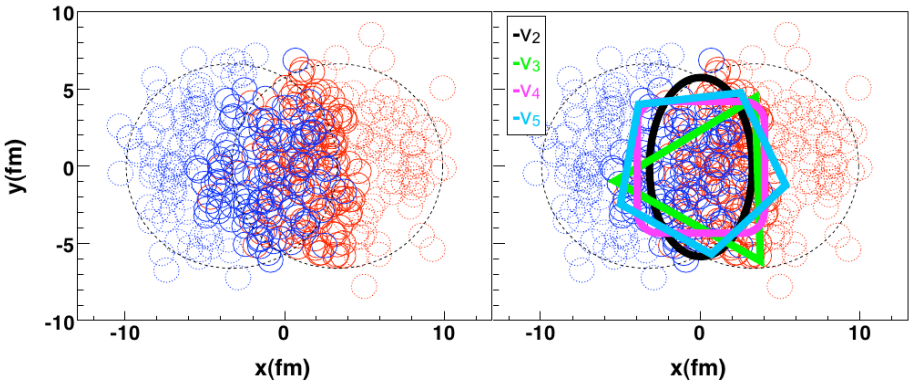
PHOBOS arXiv:0711.3724



U. Heinz, arXiv:0810.5529



BNL: RHIC



Alver, Baker, Loizides, Steinberg arXiv:0805.4411

Momentum anisotropy : Harmonic flow

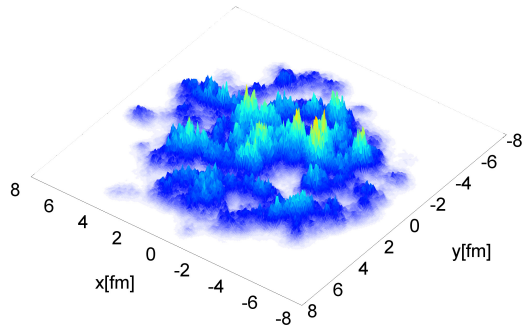
$$\frac{dN}{dp_T d\phi} = \frac{dN}{2\pi dp_T}\left(1 + 2\sum_{n=1}^{\infty} v_n(p_T)cos\left[n\left(\phi - \Psi_n\right)\right]\right)$$

$V_2 \rightarrow$ **Elliptic flow** & $V_3 \rightarrow$ **Triangular flow**

Flow fluctuation and mapping

- Flow vector, $V_n = |V_n|\ e^{i\ n\Psi_n}$
  $|V_n| \rightarrow$ Flow magnitude & $\Psi_n \rightarrow$ Flow angle
- Event by event fluctuation of initial state $\rightarrow$ event by event fluctuation of flow vectors V_n

lumpy structure of the initial density



Schenke, Tribedy, Venugopalan arXiv: 1206.6805

Mapping the flow fluctuations

- $$\frac{dN}{d\phi dp_T d\eta} \propto 1 + 2V_2(p_T,\eta)e^{-i\ 2\phi} + 3V_3(p_T,\eta)e^{-i\ 3\phi} + \ldots$$
- Can we map the flow $V_n(p_T,\eta)$ with the flow fluctuation **event by event ?** No !
- We could map the covariance : $\langle V_n(p_1,\eta_1)V_n^*(p_2,\eta_2)\rangle$

or the correlation : $$\frac{\langle V_n(p_1,\eta_1)V_n^*(p_2,\eta_2)\rangle}{\sqrt{\langle V_n(p_1,\eta_1)V_n^*(p_1,\eta_1)\rangle\langle V_n(p_2,\eta_2)V_n^*(p_2,\eta_2)\rangle}}$$

Flow correlation in transverse momentum (p_T) bin

1st moment

- Flow vector correlation or factorization coefficient:

$$r_n(p_1, p_2) = \frac{\langle V_n(p_1) V_n^*(p_2) \rangle}{\sqrt{\langle V_n(p_1) V_n^*(p_1) \rangle \langle V_n(p_2) V_n^*(p_2) \rangle}}$$

can be measured experimentally
$r_n(p_1, p_2) \leq 1$; correlation coefficient : $\rho(V_n(p1), V_n(p2))$

- Flow magnitude factorization coefficient :

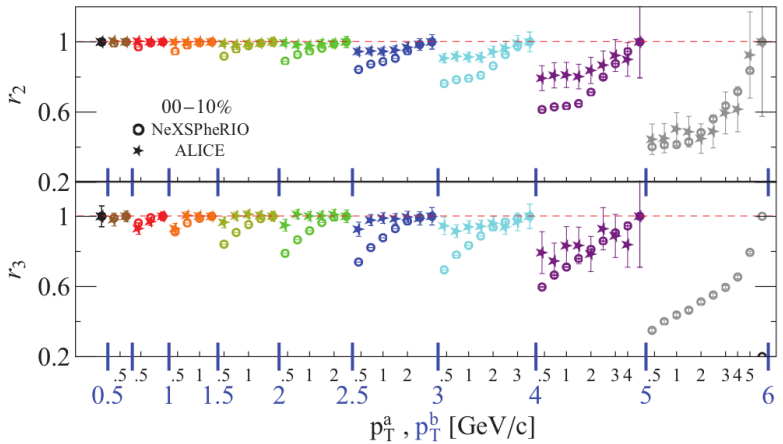
$$r_n^{v_n}(p_1, p_2) = \frac{\langle |V_n(p_1)||V_n(p_2)| \rangle}{\sqrt{\langle |V_n(p_1)|^2 \rangle \langle |V_n(p_2)|^2 \rangle}}$$

cannot be measured experimentally !

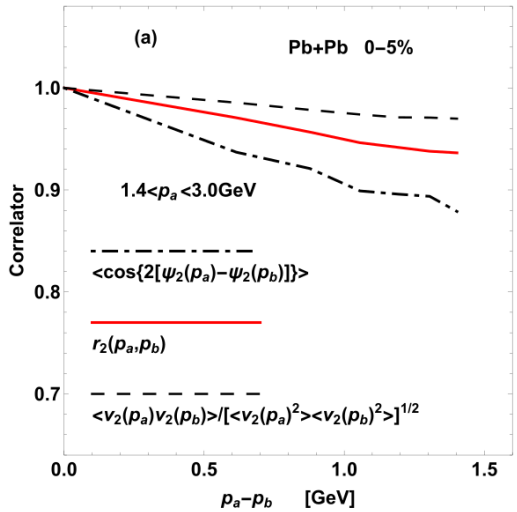
- Angle decorrelation:

$$r_n^{\Psi_n}(p_1, p_2) = \frac{\langle V_n(p_1) V_n^*(p_2) \rangle}{\langle |V_n(p_1)||V_n(p_2)| \rangle} = \frac{\langle |V_n(p_1)||V_n(p_2)| cos[n(\Psi_n(p1) - \Psi_n(p2))] \rangle}{\langle |V_n(p_1)||V_n(p_2)| \rangle}$$

which again cannot be measured experimentally !



Gardim, Grassi, Luzum, Ollitrault arXiv: 1211.0989



P. Bozek arXiv: 1808.04248

Flow vector, angle and magnitude decorrelation : General definitions

First moment

Let's consider two flow vectors : $\vec{X}$ and $\vec{Y}$

- flow vector correlation (factorization breaking coefficient)

$$r = \frac{\langle \vec{X}\vec{Y}^{*} \rangle}{\sqrt{\langle |\vec{X}|^2 \rangle \ \langle |\vec{Y}|^2 \rangle}} = \frac{\langle |\vec{X}| \ |\vec{Y}| \ cos(\Delta\Psi) \rangle}{\sqrt{\langle |\vec{X}|^2 \rangle \ \langle |\vec{Y}|^2 \rangle}}$$

measures both the magnitude and the angle decorrelation.
(can be measured experimentally)

- flow magnitude factorization coefficient :

$$= \frac{\langle |\vec{X}| \ |\vec{Y}| \rangle}{\sqrt{\langle |\vec{X}|^2 \rangle \ \langle |\vec{Y}|^2 \rangle}}$$

(cannot be measured experimentally !),
as in experiment we can only measure the scalar product of two vectors : $\vec{X} \cdot \vec{Y}^{*}$

- flow angle factorization coefficient

$$= \frac{flow \ vec. \ decor.}{flow \ magnitude \ decor.} = \frac{\langle |\vec{X}| \ |\vec{Y}| \ cos(\Delta\Psi) \rangle}{\langle |\vec{X}| \ |\vec{Y}| \rangle} \neq \langle cos(\Delta\Psi) \rangle$$

(cannot be measured experimentally !)

Second moment

- flow vector decorrelation(factorization breaking coefficient)

$$r^2 = \frac{\langle \vec{X}^2 \vec{Y}^{*2} \rangle}{\sqrt{\langle |\vec{X}|^4 \rangle \langle |\vec{Y}|^4 \rangle}} = \frac{\langle 2(\vec{X}\vec{Y}^*)^2 - |\vec{X}|^2 |\vec{Y}|^2 \rangle}{\sqrt{\langle |\vec{X}|^4 \rangle \langle |\vec{Y}|^4 \rangle}} = \frac{\langle |\vec{X}|^2 |\vec{Y}|^2 \, cos(2\Delta\Psi) \rangle}{\sqrt{\langle |\vec{X}|^4 \rangle \langle |\vec{Y}|^4 \rangle}}$$

measures both the magnitude and the angle decorrelation.
(can be measured experimentally)

- flow magnitude decorrelation :

$$= \frac{\langle |\vec{X}|^2 |\vec{Y}|^2 \rangle}{\sqrt{\langle |\vec{X}|^4 \rangle \langle |\vec{Y}|^4 \rangle}}$$

(can be measured experimentally !)

- flow angle decorrelation

$$= \frac{\textit{flow vec. decor.}}{\textit{flow magnitude decor.}} = \frac{\langle |\vec{X}|^2 |\vec{Y}|^2 \, cos(2\Delta\Psi) \rangle}{\langle |\vec{X}|^2 |\vec{Y}|^2 \rangle} \neq \langle cos(2\Delta\Psi) \rangle$$

(can be extracted from the two quantities above !)

Flow vector square decorrelation in η bins was first measured by the ATLAS collaboration. arXiv:1709.02301

Flow decorrelations in p_T : 2nd moment

- Flow vector **square** factorization coefficient:

$$\frac{\langle V_n(p_1)^2 V_n^*(p_2)^2 \rangle}{\sqrt{\langle |V_n(p_1)|^4 \rangle \langle |V_n(p_2)|^4 \rangle}}$$

could be measured experimentally

- Flow magnitude **square** factorization coefficient :

$$= \frac{\langle |V_n(p_1)|^2 |V_n(p_2)|^2 \rangle}{\sqrt{\langle |V_n(p_1)|^4 \rangle \langle |V_n(p_2)|^4 \rangle}}$$

Could be measured experimentally ! (unlike the corr. in first moment)

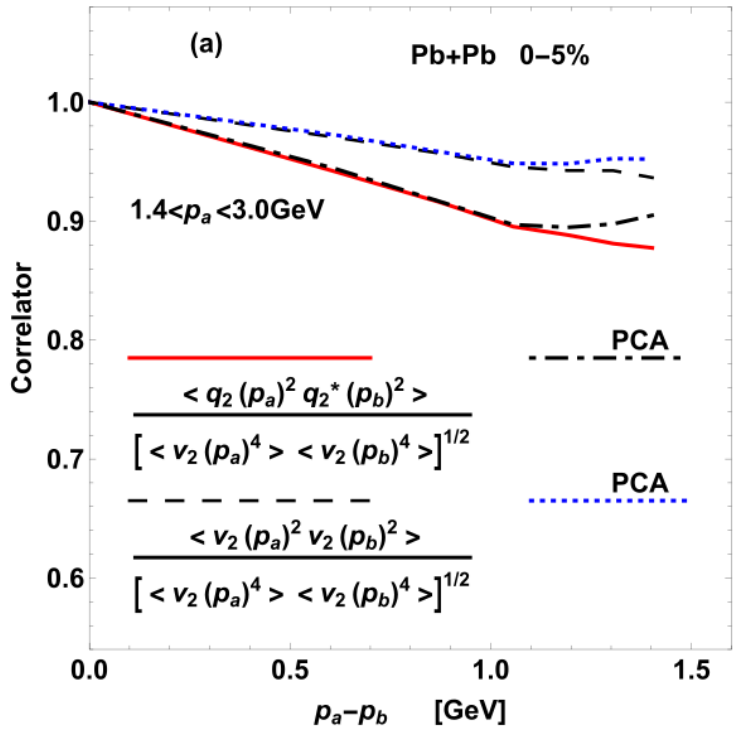
- Angle decorrelation :

$$\frac{\textit{flow vec. decor}}{\textit{flow mag. decorr}} = \frac{\langle V_n(p_1)^2 V_n^*(p_2)^2 \rangle}{\langle |V_n(p_1)|^2 |V_n(p_2)|^2 \rangle}$$

$$= \frac{\langle |V_n(p_1)|^2 |V_n(p_2)|^2 cos[2n(\Psi_n(p1) - \Psi_n(p2))] \rangle}{\langle |V_n(p_1)|^2 |V_n(p_2)|^2 \rangle}$$

Could be extracted from the above two !

Model results



P. Bozek arXiv: 1808.04248

In experiment it is difficult to use above formulas at high pT due to limited statistics !

Flow decorrelation in p_T with integrated flow

Correlations between integrated flow and the differential flow in pT

- One of the flow in one p_T bin ($V_n(p)$) and another as integrated flow (momentum averaged) (V_n) .
- Statistically preferable and accessible !

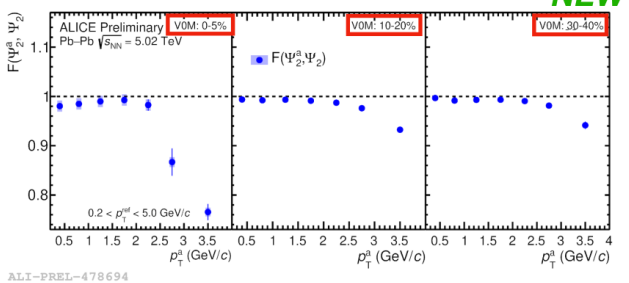
Flow angle and magnitude fluctuations

ALICE

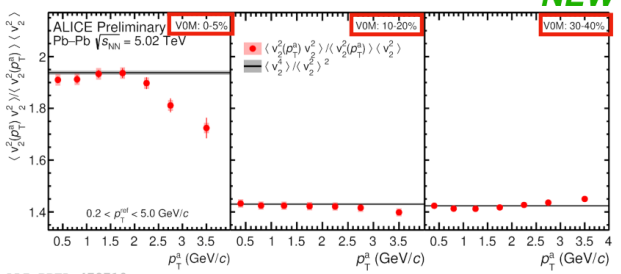
Flow angle fluctuations
$F(\Psi_2^a, \Psi_2) = \langle \cos 2n[\Psi_2(p_T^a) - \Psi_2] \rangle$

Flow magnitude fluctuations
$\langle v_n^2(p_T^a) v_n^2 \rangle / \langle v_n^2(p_T^a) \rangle \langle v_n^2 \rangle$

NEW



NEW



- Deviation from unity → p_T-dependent flow angle fluctuations
- $> 5\sigma$ significance in most centralities

- Deviation from $\langle v_n^4 \rangle / \langle v_n^2 \rangle^2$ → p_T-dependent flow magnitude fluctuations
- $\sim 5\sigma$ significance in most central collisions

Discovery of both flow angle and flow magnitude fluctuations in most central collisions!

Emil Gorm Nielsen (NBI) IS2021 - Fluctuations and correlations of flow 10

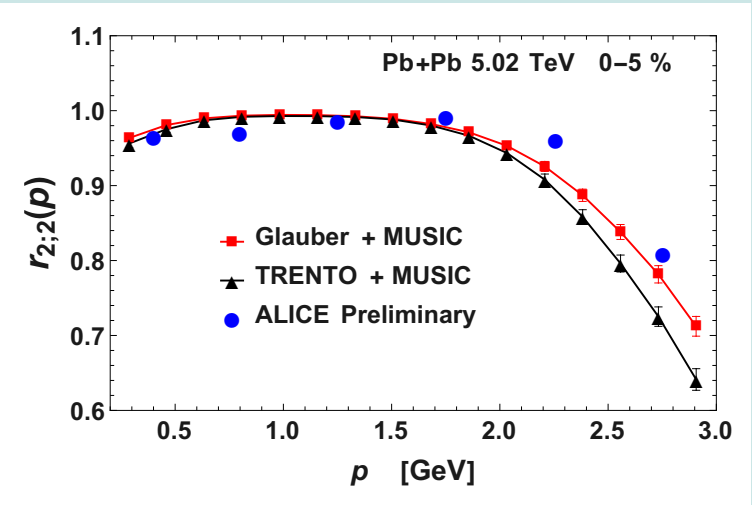
factorization breaking between V_n^2 and $V_n^2(p)$ can be measured

Model study

Bozek, Samanta arXiv: 2109.07781

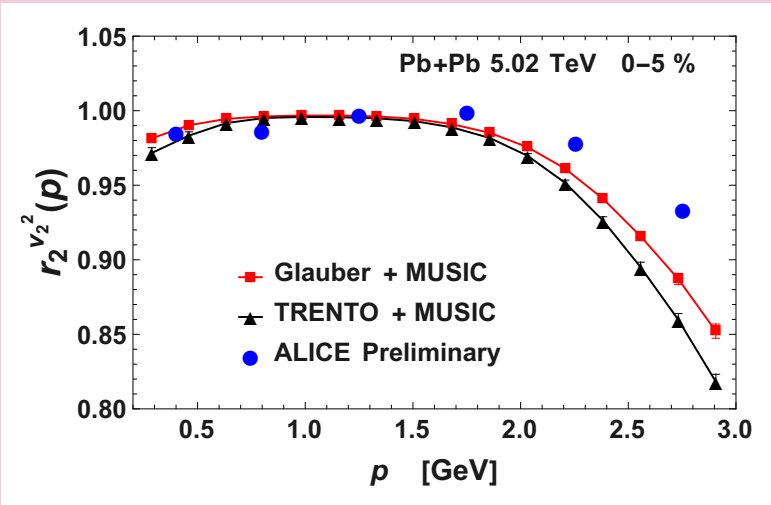
(flow vector)^2-(flow vector)^2 decor.

$$r_{n;2}(p) = \frac{\langle V_n^2 V_n^*(p)^2 \rangle}{\sqrt{\langle |V_n|^4 \rangle \ \langle |V_n(p)|^4 \rangle}}$$



(flow magnitude)^2-(flow magnitude)^2

$$r_n^{v_n^2}(p) = \frac{\langle |V_n|^2 |V_n(p)|^2 \rangle}{\sqrt{\langle |V_n|^4 \rangle \ \langle |V_n(p)|^4 \rangle}}$$



Observations

- The flow magnitude decorrelation is **approximately one half** of the flow vector decorrelation:

$$[1 - r_n^{v_n^2}(p)] \simeq \frac{1}{2}[1 - r_{n;2}(p)]$$

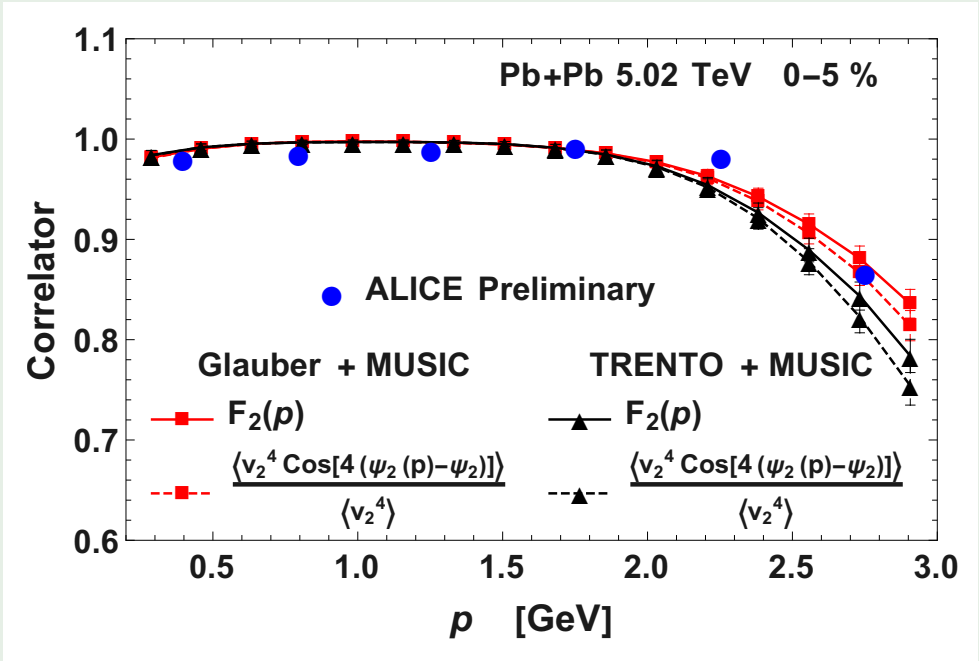
- for central collision our model results reproduce the data.

flow angle decorrelation

- Flow angle factorization coefficient,

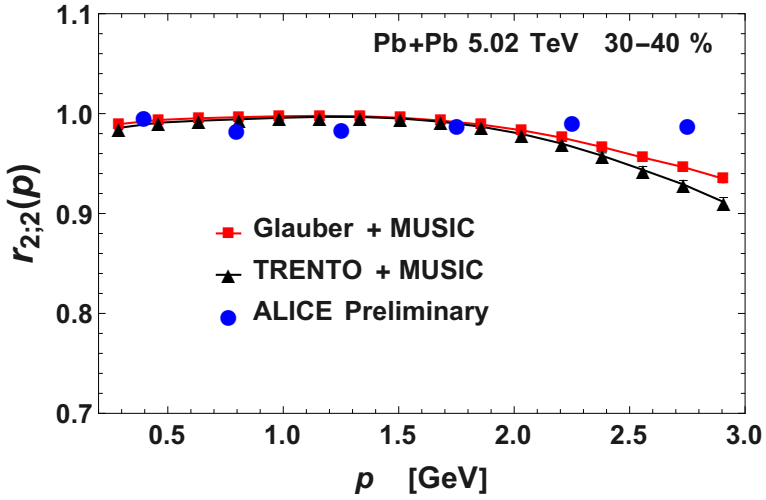
$$F_n(p) = \frac{\textit{flow vec. decor}}{\textit{flow mag. decor}} = \frac{\langle V_n^2 V_n^*(p)^2 \rangle}{\langle |V_n|^2 |V_n(p)|^2 \rangle}$$
$$= \frac{\langle |V_n|^2 |V_n(p)|^2 cos[2n(\Psi_n - \Psi_n(p))] \rangle}{\langle |V_n|^2 |V_n(p)|^2 \rangle} \simeq \frac{\langle |V_n|^4 cos[2n(\Psi_n - \Psi_n(p))] \rangle}{\langle |V_n|^4 \rangle}$$

- Strong correlation between flow angle and flow magnitude $\rightarrow$ can't be factorized.

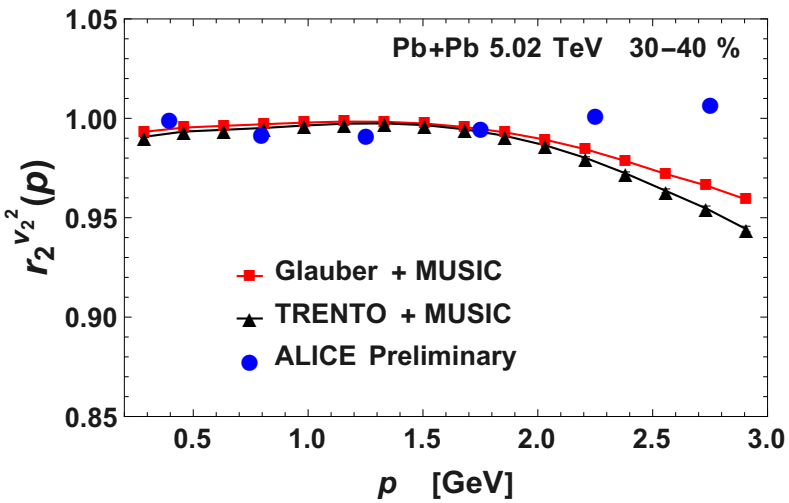


Semi-peripheral collision (30-40 %)

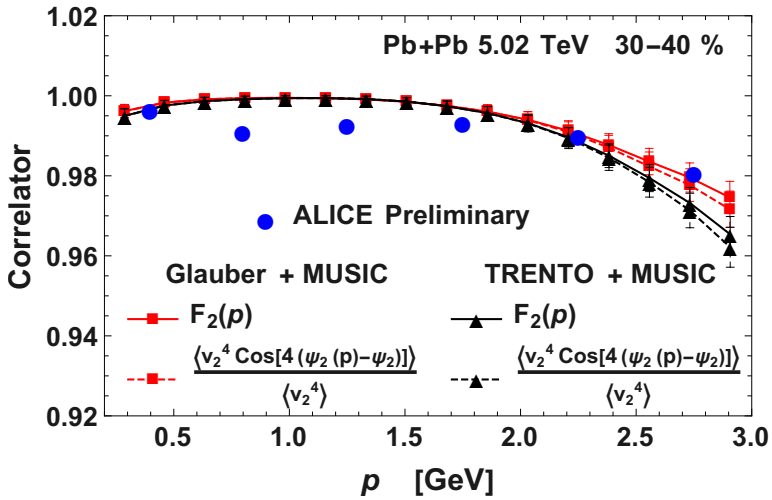
flow vec. decor.



flow mag. decor.



flow angle decor.



Observations

- For semi-peripheral collision our model results do not reproduce the data
- For 30-40 % the data go slightly above 1 at high p_T $\longrightarrow$ may indicate a significant non-flow contribution.

What we measure in flow angle decorrelation ?

Measuring flow angle decorrelation

- Flow angle factorization coefficient,

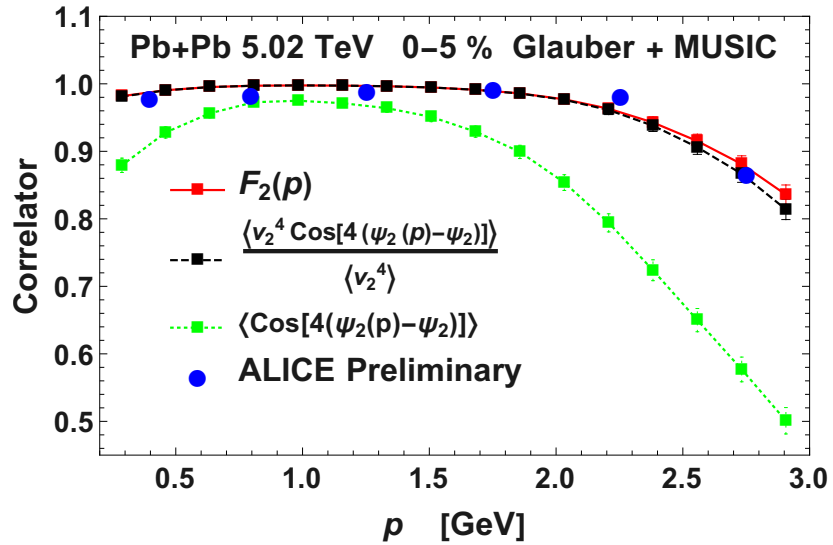
$$F_n(p) = \frac{\langle |V_n|^2 |V_n(p)|^2 cos[2n(\Psi_n - \Psi_n(p))]\rangle}{\langle |V_n|^2 |V_n(p)|^2\rangle}$$

- We can measure magnitude weighted angle decorrelation :

$$\frac{\langle |V_n|^2 |V_n(p)|^2 cos[2n(\Psi_n - \Psi_n(p))]\rangle}{\langle |V_n|^2 |V_n(p)|^2\rangle} \simeq \frac{\langle |V_n|^4 cos[2n(\Delta\Psi)]\rangle}{\langle |V_n|^4\rangle}$$

- But we cannot measure simple angle decorrelation :

$$\frac{\langle |V_n|^4 cos[2n(\Delta\Psi)]\rangle}{\langle |V_n|^4\rangle} \neq \langle cos[2n(\Delta\Psi)]\rangle$$



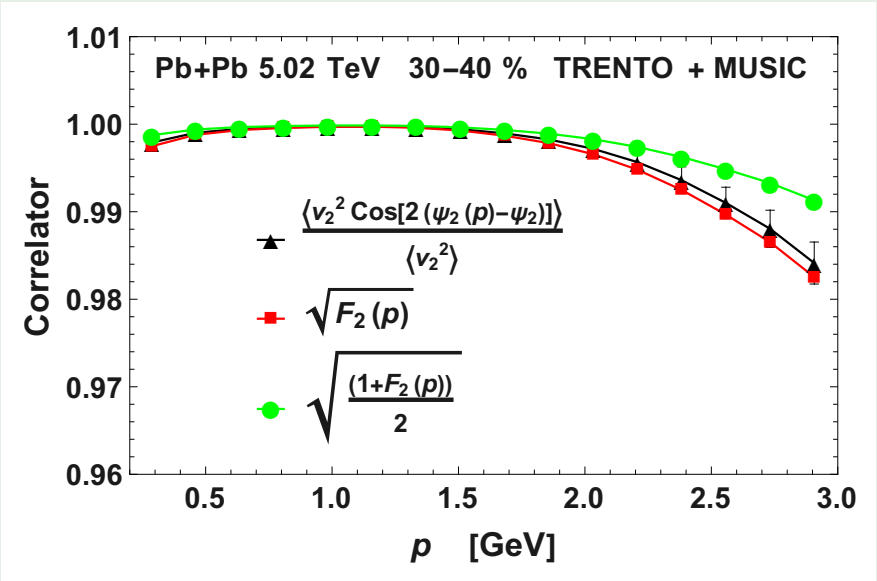
Extracting 1st order flow decorrelation

- It follows a simple approximation from the 2nd order flow angle decorrelation :

$$\frac{\langle |V_n|^2 cos[n(\Delta\Psi)]\rangle}{\langle |V_n|^2\rangle} \simeq \sqrt{\frac{\langle |V_n|^4 cos[2n(\Delta\Psi)]\rangle}{\langle |V_n|^4\rangle}}$$

$$= \sqrt{F_n(p)}$$

- Upper limit $\rightarrow \sqrt{(1+F_n(p))/2}$



Factorization breaking between mixed harmonics (Event-plane correlation)

- Serves as a measure of non linear response of the hydrodynamic expansion
- General definition (1st order):

$$\frac{\langle V_m^*(p) V_k V_n \rangle}{\sqrt{\langle |V_m(p)|^2 \rangle \langle |V_k|^2 |V_n|^2 \rangle}}$$

with the constraint: $m = k + n$

- For example, we have,

$$V_4 = V_4^L + V_4^{NL} \ , \ \textit{where} \ V_4^{NL} \propto V_2^2$$

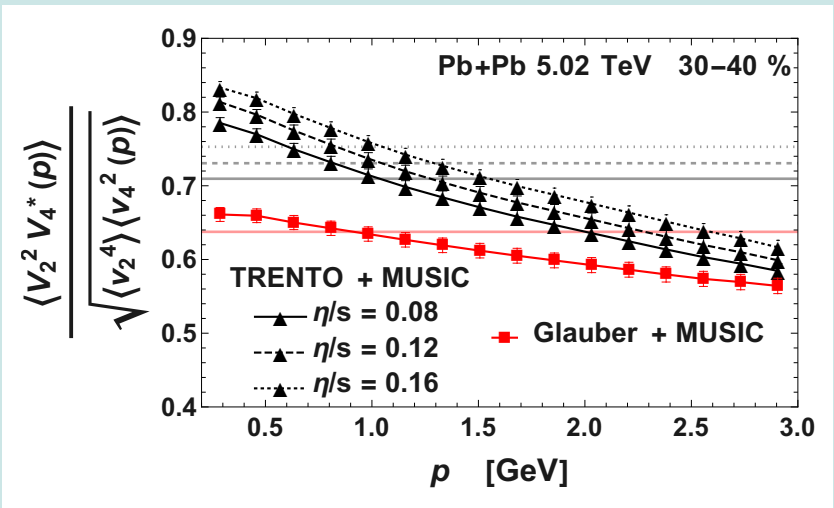
$$V_5 = V_5^L + V_5^{NL} \ , \ \textit{where} \ V_5^{NL} \propto V_2 V_3$$

So, V_2^2-$V_4(p)$ and $V_5(p)$-$V_2 V_3$ correlations measures the **non-linear coupling**

First order mixed correlations of flow harmonics (New !)

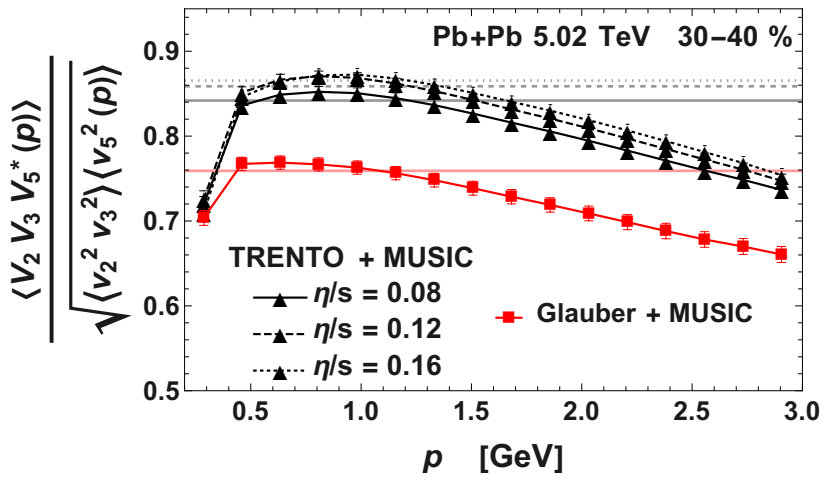
$V_2^2 - V_4(p)$ corr.

$$\frac{\langle V_2^2 V_4^*(p)\rangle}{\sqrt{\langle |V_2|^4\rangle \langle |V_4(p)|^2\rangle}}$$



$V_2 V_3 - V_5(p)$ corr.

$$\frac{\langle V_2 V_3 V_5^*(p)\rangle}{\sqrt{\langle |V_2|^2 |V_3^2|\rangle \langle |V_5(p)|^2\rangle}}$$



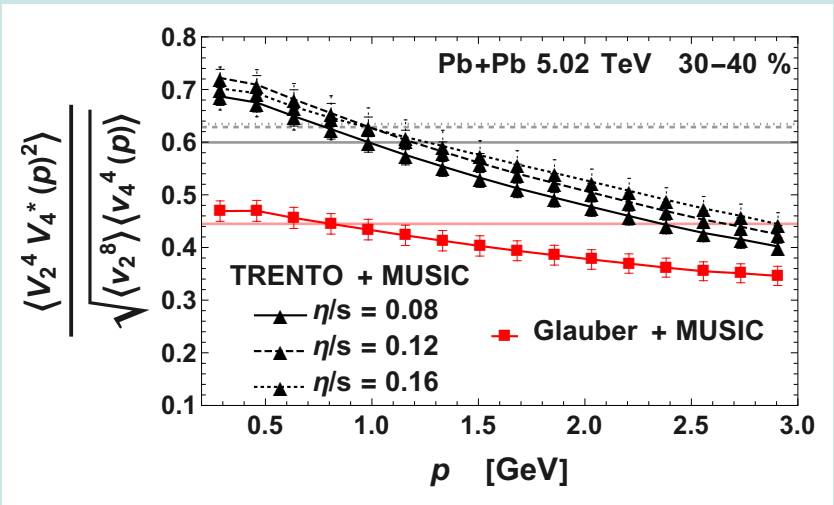
Comment

- 1st moment $\rightarrow$ (flow vector)^2 - (flow vector) decorrelation
- For flow magnitude and flow angle correlations $\rightarrow$ 2nd order correlation : V_2^4-$V_4(p)^2$ or $V_2^2 V_3^2$ - $V_5^2(p)$ correlations

2nd moment : $V_2^4 - V_4^2(p)$ correlations (New !)

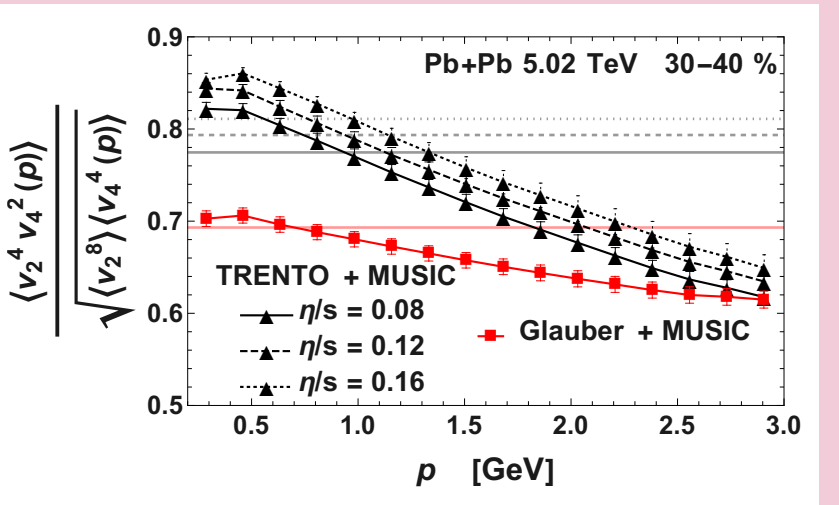
(flow vector)^4-(flow vector)^2 decor.

$$\frac{\langle V_2^4 V_4^*(p)^2\rangle}{\sqrt{\langle |V_2|^8\rangle \langle |V_4(p)|^4\rangle}}$$



(flow magnitude)^4-(flow magnitude)^2

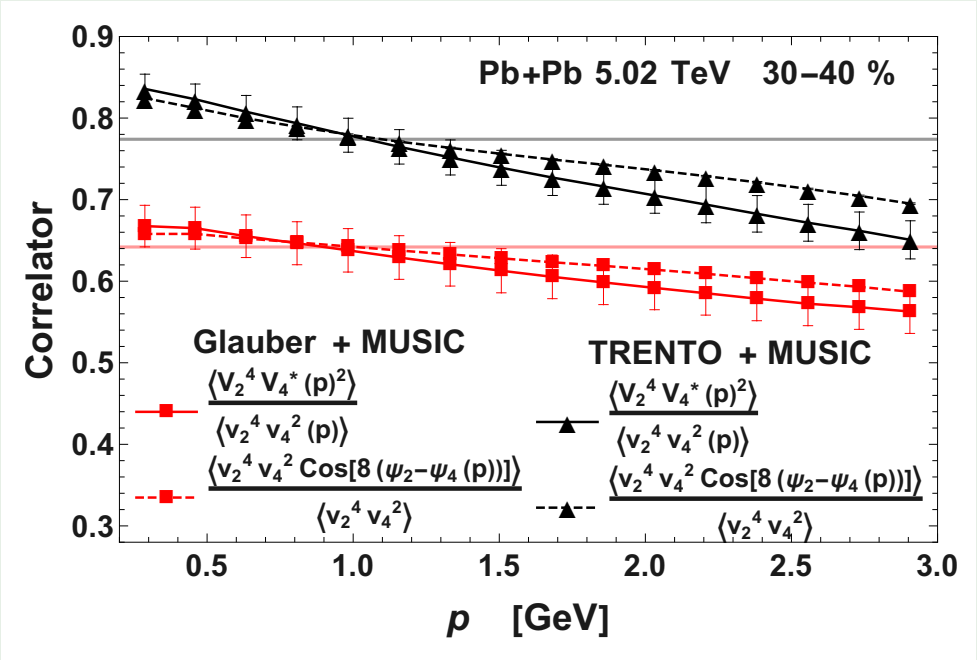
$$\frac{\langle |V_2|^4 |V_4(p)|^2\rangle}{\sqrt{\langle |V_2(p)|^8\rangle \langle |V_4(p)|^4\rangle}}$$



Flow angle decorrelation

Separate measurements of flow vector and flow magnitude correlation lead to the extraction of flow angle decorrelation :

$$\frac{\langle V_2^4 V_4^*(p)^2\rangle}{\langle |V_2|^4 |V_4(p)|^2\rangle} = \frac{\langle |V_2|^4 |V_4(p)|^2 cos[8(\Psi_2 - \Psi_4(p))]\rangle}{\langle |V_2|^4 |V_4(p)|^2\rangle} \simeq \frac{\langle |V_2|^4 |V_4|^2 cos[8(\Psi_2 - \Psi_4(p))]\rangle}{\langle |V_2|^4 |V_4|^2\rangle}$$



Study of the factorization breaking of mixed flow puts additional constraints on the initial state models !

Conclusions and outlook

- Flow fluctuations $\rightarrow$ flow decorrelation in transverse momentum (p_T)
- Extraction of flow angle decorrelation $\rightarrow$ 2nd moment of flow decorrelation:
  - (flow vector)^2- (flow vector)^2 $= \frac{\langle V_n(p_1)^2 V_n^*(p_2)^2 \rangle}{\sqrt{\langle |V_n(p_1)|^4 \rangle \ \langle |V_n(p_2)|^4 \rangle}}$
  - (flow magnitude)^2- (flow magnitude)^2 $= \frac{\langle |V_n(p_1)|^2 |V_n(p_2)|^2 \rangle}{\sqrt{\langle |V_n(p_1)|^4 \rangle \ \langle |V_n(p_2)|^4 \rangle}}$
  - flow angle decor. $= \frac{flow \ \ vec. \ \ decor}{flow \ \ mag. \ \ decor} = \frac{\langle V_n(p_1)^2 V_n^*(p_2)^2 \rangle}{\langle |V_n(p_1)|^2 |V_n(p_2)|^2 \rangle}$
- We measure the angle decor : $\frac{\langle |V_n|^4 cos[2n(\Delta\Psi)] \rangle}{\langle |V_n|^4 \rangle}$ not $\langle cos[2n(\Delta\Psi)] \rangle$
- We propose similar approach for non linear coupling between mixed flows (New !)

Studies of flow vector, flow magnitude , flow angle decorrelation bring new information on initial correlations, initial flow, dynamical fluctuations, . . .

Thank you !

for your attention

Back up

Removing non-flow correlation

Forming correlation in η bins along with p_T

- four well-separated pseudorapidty bins; $-\eta_F$,$-\eta$,η and η_F
- Flow vector factorization coefficient :

$$r_{n;2}(p) \simeq \frac{\langle V_n(-\eta_F)V_n^*(-\eta,p)V_n^*(\eta,p)V_n(\eta_F)\rangle\langle V_n^*(-\eta)V_n(\eta)\rangle}{\langle V_n(-\eta_F)V_n^*(-\eta)V_n^*(\eta)V_n(\eta_F)\rangle\langle V_n^*(-\eta,p)V_n(\eta,p)\rangle}$$

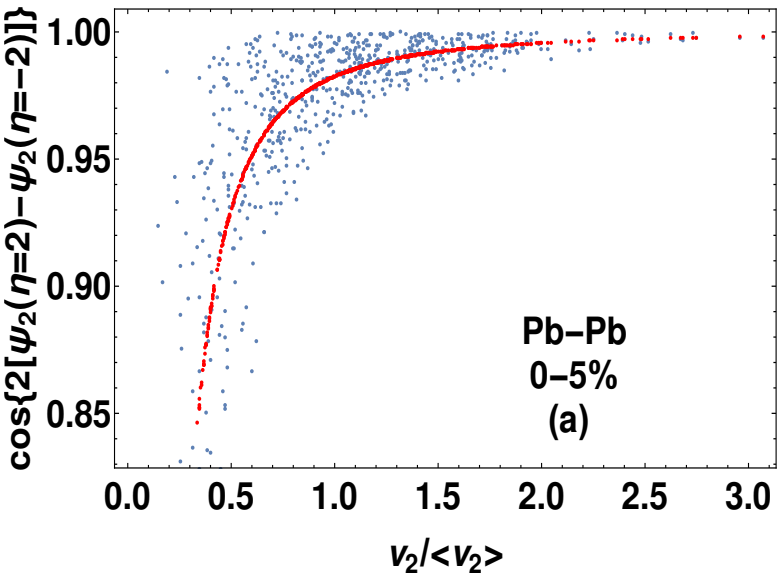
- Flow magnitude factorization coefficient :

$$r_{n;2}^{v_n^2}(p) \simeq \frac{\langle V_n(-\eta_F)V_n(-\eta,p)V_n^*(\eta,p)V_n^*(\eta_F)\rangle\langle V_n^*(-\eta)V_n(\eta)\rangle}{\langle V_n(-\eta_F)V_n(-\eta)V_n^*(\eta)V_n^*(\eta_F)\rangle\langle V_n^*(-\eta,p)V_n(\eta,p)\rangle}$$

- Flow angle decorrelation (ratio of the above two) :

$$F_n(p) \simeq \frac{\langle V_n(-\eta_F)V_n^*(-\eta,p)V_n^*(\eta,p)V_n(\eta_F)\rangle\langle V_n(-\eta_F)V_n(-\eta)V_n^*(\eta)V_n^*(\eta_F)\rangle}{\langle V_n(-\eta_F)V_n(-\eta,p)V_n^*(\eta,p)V_n^*(\eta_F)\rangle\langle V_n(-\eta_F)V_n^*(-\eta)V_n^*(\eta)V_n(\eta_F)\rangle}$$

Twist angle - flow magnitude correlation in η Bozek, Broniowski arXiv: 1711.03325



Observations

- **Strong correlation** exists between flow magnitude and twist angle
- Correct measure of angle decor.:

$$ang\ decor. = \frac{flow\ vec.\ decor.}{flow\ mag.\ decor.} = \frac{\langle v_n^2 cos(n(\Delta\Psi))\rangle}{\langle v_n^2\rangle} \neq \langle cos(n(\Delta\Psi)\rangle$$

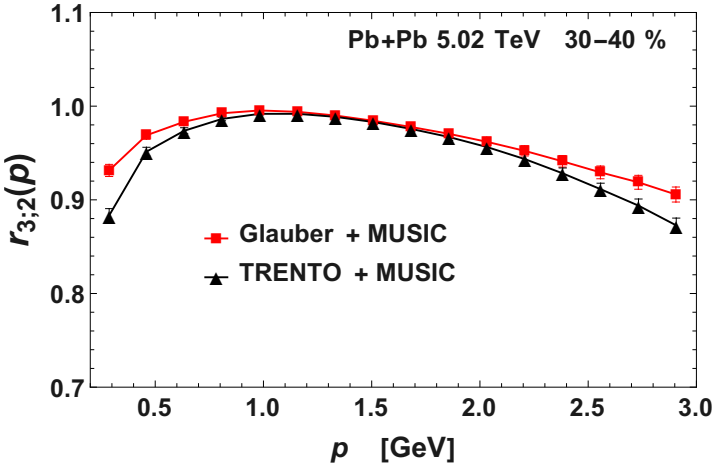
Equivalence of different normalization

Scaling of data

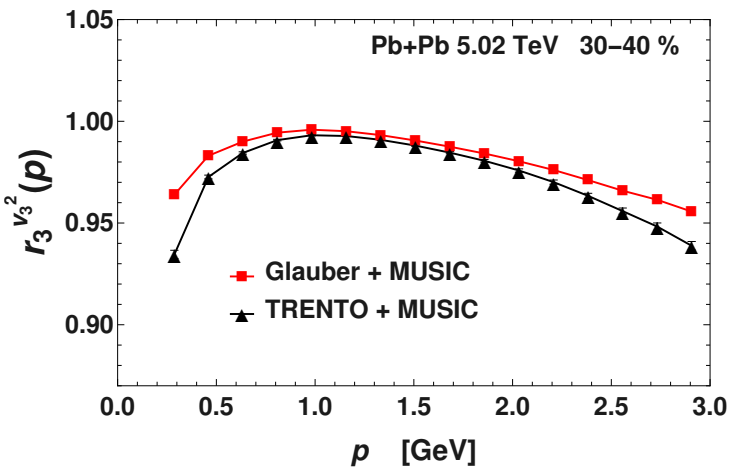
- The ALICE collaboration use different normalization in their data, namely for vector and magnitude correlation :
$\frac{\langle V_n^2 V_n^*(p)^2\rangle}{\langle|V_n|^2\rangle\ \langle|V_n(p)|^2\rangle}$ and $\frac{\langle|V_n|^2|V_n(p)|^2\rangle}{\langle|V_n|^2\rangle\ \langle|V_n(p)|^2\rangle}$ respectively
- We divide them by a factor $\frac{\langle|V_n^4|\rangle}{\langle|V_n^2|\rangle^2}$, the **baseline** of the plots, and we have:
$\frac{\langle V_n^2 V_n^*(p)^2\rangle\ \langle|V_n^2|\rangle}{\langle|V_n|^4\rangle\ \langle|V_n(p)|^2\rangle}$ and $\frac{\langle|V_n|^2|V_n(p)|^2\rangle\ \langle|V_n^2|\rangle}{\langle|V_n|^4\rangle\ \langle|V_n(p)|^2\rangle}$
- But we use the definitions: $\frac{\langle V_n^2 V_n^*(p)^2\rangle}{\sqrt{\langle|V_n|^4\rangle\ \langle|V_n(p)|^4\rangle}}$ and $\frac{\langle|V_n|^2|V_n(p)|^2\rangle}{\sqrt{\langle|V_n|^4\rangle\ \langle|V_n(p)|^4\rangle}}$
- The difference between the two normalization is a factor :
$\sqrt{\frac{\langle|V_n^4(p)|\rangle\langle|V_n^2|\rangle^2}{\langle|V_n^4|\rangle\langle|V_n^2(p)|\rangle^2}} \simeq 1 \implies \frac{\sqrt{\langle|V_n^4(p)|\rangle}}{\langle|V_n^2(p)|\rangle} \simeq \frac{\sqrt{\langle|V_n^4|\rangle}}{\langle|V_n^2|\rangle}$

Same for triangular flow (v_3)

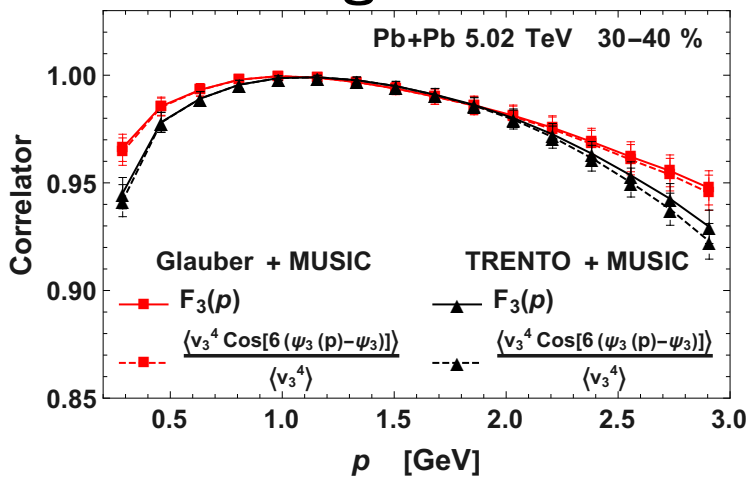
flow vec. decor.



flow mag. decor.



flow angle decor.



The other definition

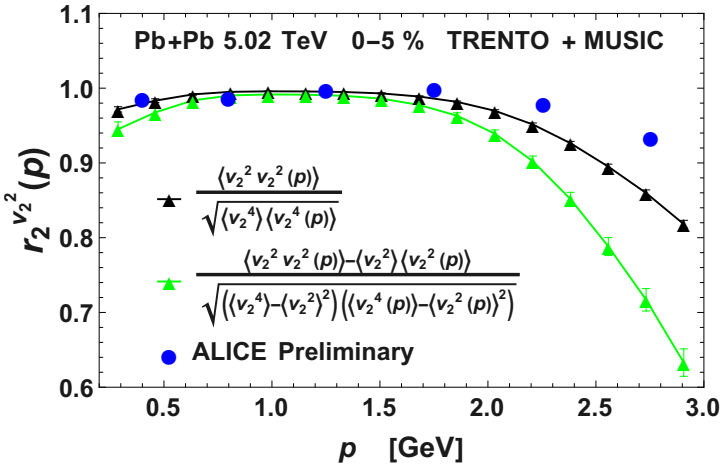
The usual correlation (ρ)

One could use a normalization :

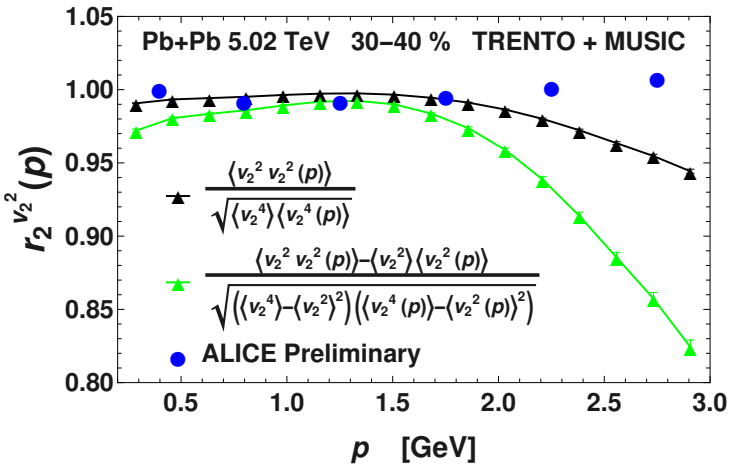
$$r_n^{v_n^2}(p) = \frac{\langle |V_n|^2 |V_n(p)|^2 \rangle - \langle |V_n|^2 \rangle \langle |V_n(p)|^2 \rangle}{\sqrt{\left(\langle |V_n|^4 \rangle - \langle |V_n|^2 \rangle^2\right)\left(\langle |V_n(p)|^4 \rangle - \langle |V_n(p)|^2 \rangle^2\right)}}$$

But that gives quite different result !

flow vec. decor.



flow mag. decor.



Measuring moments of the flow harmonics : Cumulant method

Measuring moments of the flow harmonics : Cumulant method

- Experimentally, $q_n = \frac{1}{N}\sum_{i=1}^{N} e^{i\ n\phi_i}$, N = the number of particles

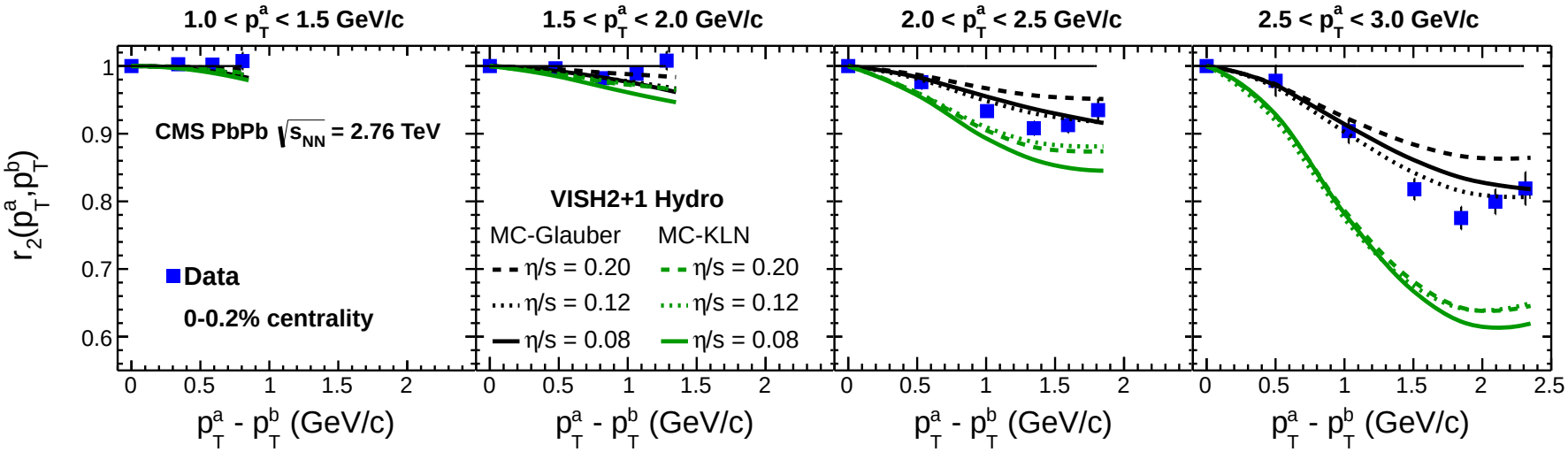
  ϕ_i = the azimuth of i^{th} particle
- The two particle cumulant, $v_n\{2\}^2 = \langle q_n q_n^* \rangle_{without\ \ self-correlation}$ where,

$$\langle q_n q_n^* \rangle_{without\ \ self-corr.} = \langle \frac{1}{N(N-1)} \sum_{i \neq j} e^{i\ n(\phi_i - \phi_j)} \rangle$$

- **Scalar product** of the flows is used to measure the cumulants.

Only even orders of the cumulants can be measured !

Experimental vs theoretical results

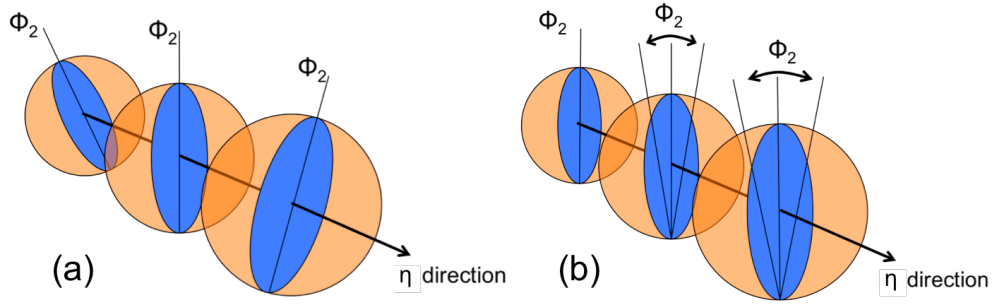


CMS arXiv: 1503.01692

Observations

- Deviation of the correlations or the factorization coefficients from 1 is identified as the **flow decorrelation**.
- flow decorrelation increases with the increase in the difference between the p_T bins.
- Results are qualitatively described by models
- Such study can provide constraints on initial conditions

Flow decorrelation in longitudinal direction : correlation in η



Jia, Huo arXiv: 1402.6680

Correlations in η bins

- Flow correlation in η can be constructed using 3-bin or 4-bin correlator
- Flow vector decorrelation (angle + magnitude):

$$r_{n,k}(\eta) = \frac{\langle V_n^k(-\eta) V_n^{*k}(\eta_{ref})\rangle}{\langle V_n^k(\eta) V_n^{*k}(\eta_{ref})\rangle} \quad (3-bin\ corr.)$$

$$= \frac{\langle v_n^k(-\eta) v_n^k(\eta_{ref}) cos[nk(\Psi_n(-\eta) - \Psi_n(\eta_{ref}))]\rangle}{\langle v_n^k(\eta) v_n^k(\eta_{ref}) cos[nk(\Psi_n(\eta) - \Psi_n(\eta_{ref}))]\rangle}$$

$$\simeq 1 - 2F_{n,k}^{asy} - 2F_{n,k}^{twi}$$

- $F_{n,k}^{asy} \longrightarrow$ FB v_n asymmetry $\longrightarrow$ magnitude decorrelation

Correlations in η bins

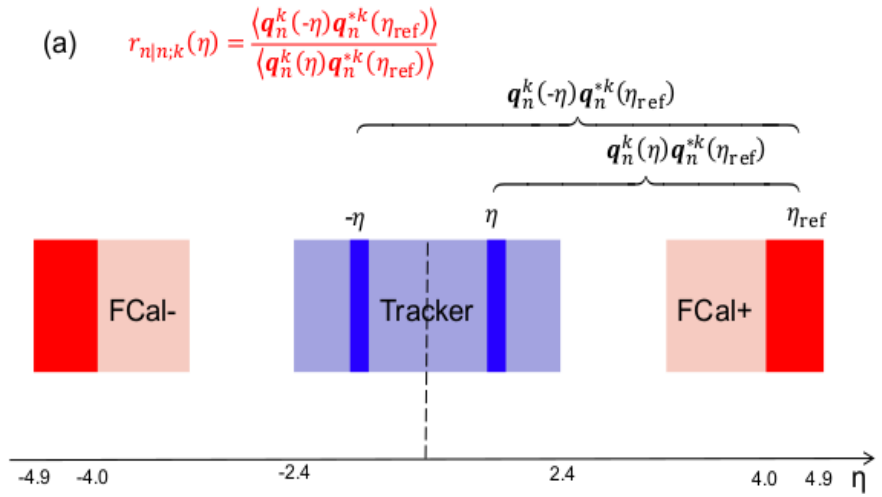
- $2F_{n,k}^{twi} \longrightarrow$ FB event plane twist $\longrightarrow$ angle decorrelation
- Also, one can construct

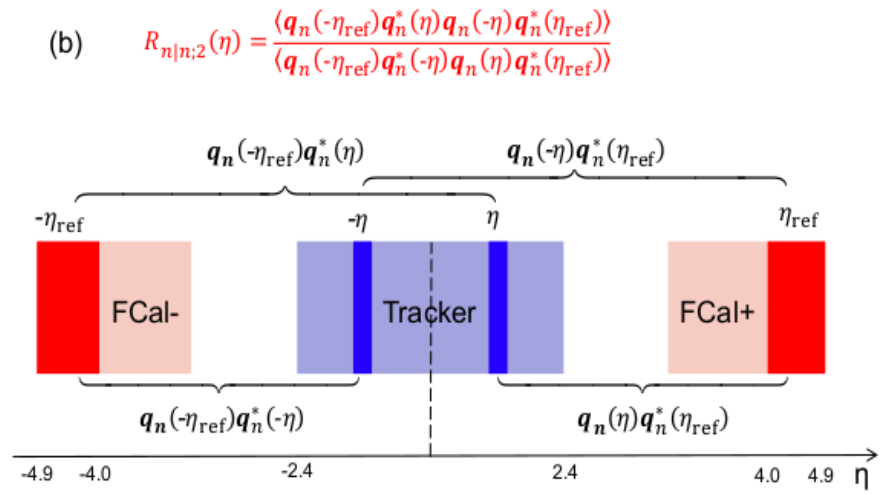
$$R_{n,2}(\eta) = \frac{\langle V_n(-\eta_{ref}) V_n^*(\eta) V_n(-\eta) V_n^*(\eta_{ref})\rangle}{\langle V_n(-\eta_{ref}) V_n^*(-\eta) V_n(\eta) V_n^*(\eta_{ref})\rangle} \quad (4-bin\ corr.)$$

$$= \frac{\langle v_n(-\eta_{ref}) v_n(\eta) v_n(-\eta) v_n(\eta_{ref}) cos[n(\Psi(-\eta_{ref}) - \Psi(\eta) + \Psi(-\eta) - \Psi(\eta_{ref}))]\rangle}{\langle v_n(-\eta_{ref}) v_n(-\eta) v_n(\eta) v_n(\eta_{ref}) cos[n(\Psi(-\eta_{ref}) - \Psi(-\eta) + \Psi(\eta) - \Psi(\eta_{ref}))]\rangle}$$

$$\simeq 1 - 2F_{n,2}^{twi}$$

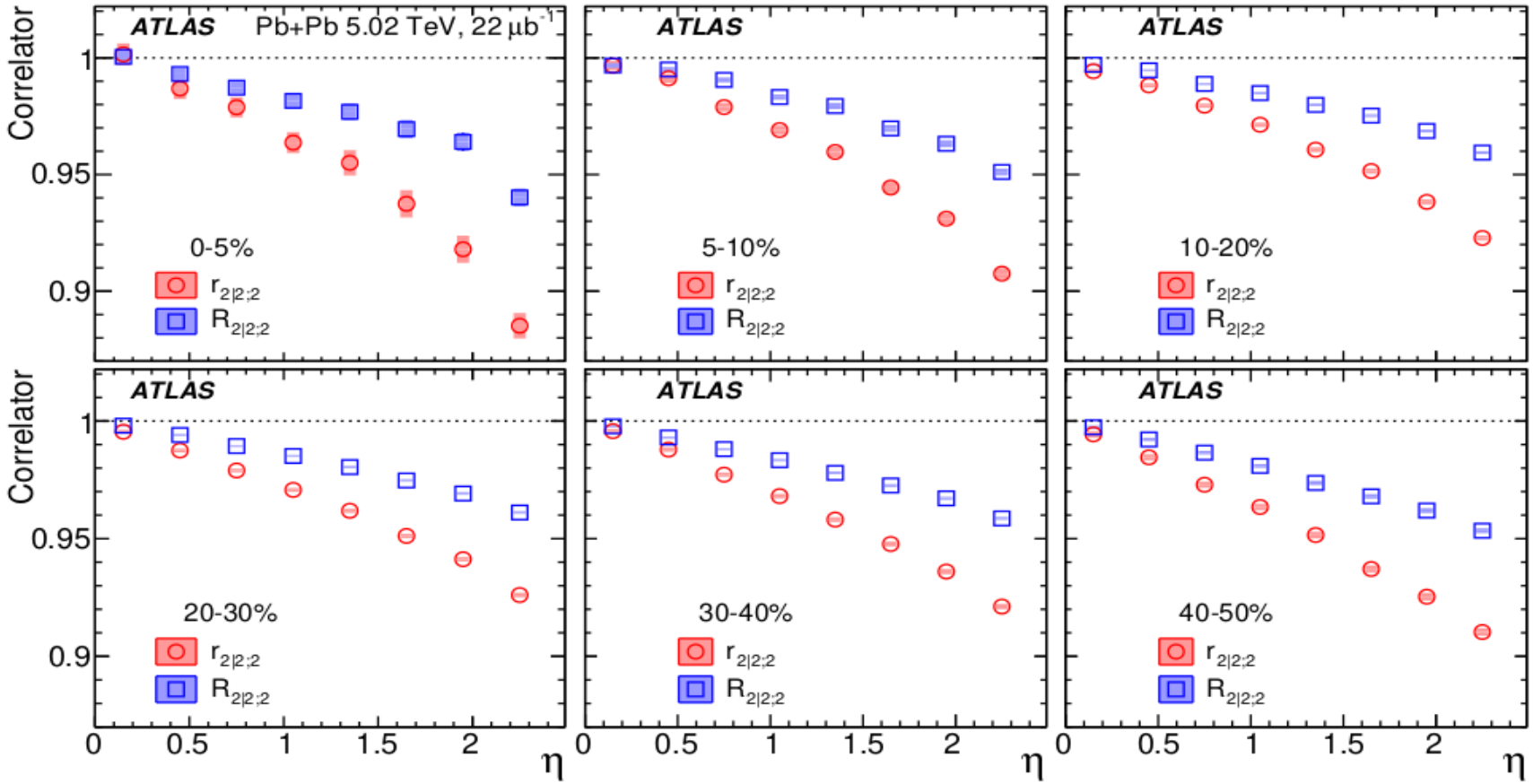
  - effect of asymmetry is same in both numerator and denominator
  - only angle decorrelation due to twist





Correlations in η bins

- Measuring $r_{n,k}(\eta)$ and $R_{n,2}(\eta)$, the magnitude and angle decorrelation can be separated.
- Moreover we can see, **Magnitude decor. $\simeq$ Angle decor.**



ATLAS arXiv:1709.02301

Why magnitude decorrelation $\simeq$ angle direction ?

Simple model of vector decorrelation Bozek, Mehrabpour

Let's consider two vector: $\vec{X}_n = \vec{V}_n + \vec{\delta}_n$ and $\vec{Y}_n = \vec{V}_n - \vec{\delta}_n$

- It can be shown that, factorization breaking of flow vector:

$$\frac{\langle X_n Y_n^{\star} \rangle}{\sqrt{\langle X_n^2 \rangle \langle Y_n^2 \rangle}} \simeq 1 - 2\frac{\langle \delta_n^2 \rangle}{\langle V_n^2 \rangle}$$

- factorization breaking of flow magnitude:

$$\frac{\langle X_n Y_n \rangle}{\sqrt{\langle X_n^2 \rangle \langle Y_n^2 \rangle}} \simeq 1 - \frac{\langle \delta_n^2 \rangle}{\langle V_n^2 \rangle}$$

- and flow angle decorrelation :

$$\frac{\langle V_n^2 \cos\left(n\Delta\Psi\right)) \rangle}{\langle V_n^2 \rangle} \simeq 1 - \frac{\langle \delta_n^2 \rangle}{\langle V_n^2 \rangle}$$