Extracting Effective Viscosities from heavy ion collisions

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Based on 2010.11919, with J-Y. Ollitrault,

Krakow (online) – 7th April 2022
Relativistic Hydro and QGP

(1) **Initial Conditions (QGP)**

(2) **Hydro Evolution**

\[ \partial_\mu T^{\mu\nu} = 0 \quad T^{\mu\nu} = T_0^{\mu\nu} - \Pi^{\mu\nu} \]

**Ideal**

\[ T_0^{\mu\nu} = (e + P)u^\mu u^\nu - g^{\mu\nu}P \]

**Viscous**

\[ \Pi^{\mu\nu} = \eta\sigma^{\mu\nu} + \zeta \Delta^{\mu\nu} \nabla_\lambda u^\lambda + O(\partial^2) \]

(3) **Hadronization:**

When expanding, temperature and densities decrease, until the moment of emitting particles: Cooper-Frey, URQMD, AfterBurner .... then we have a spectrum of emitted particles

NeXus, IP-Glasma, Magma, Trento ....

Fig by L. Barbosa
Anisotropic flow: Experiment versus hydro

Nice agreement with data provided that viscous corrections are included.

One of the goals of heavy ion physics is to extract the viscosity from experimental data.

This Talk

(1) I show that from $v_2$ and $v_3$ data at a given energy, one cannot infer the whole temperature dependence of $\left(\frac{\eta}{s}\right)$ and $\left(\frac{\zeta}{s}\right)$ but only weighted averages, which I denote by effective viscosities.

(2) I then estimate the values of these effective viscosities using first-principles QCD calculations, and compare with the values inferred from data.
Effect of Constant $\frac{\eta}{s}$ on anisotropic flow

There is a decrease in the anisotropic flows, which depend on the intensity of viscosity.

The damping is approximately linear!
Effect of **NOT** **CONSTANT** $\frac{\eta}{s}$ on anisotropic flow

Different parametrizations, however, almost the same $v'$s
Probing viscous damping

For temperature-dependent viscosities, the condition that the damping is linear in the viscosity can be rewritten as

\[
\frac{v_n(\text{viscous})}{v_n(\text{ideal})} - 1 \approx \int dT \left( w_n^\eta(T) \frac{\eta(T)}{s(T)} + w_n^\zeta(T) \frac{\zeta(T)}{s(T)} \right)
\]

\[
w(T) = w_f \delta(T - T_f) + \frac{a_0 + a_1 T + a_2 T^2}{1 + b_1 T + b_2 T^2 + b_3 T^3}
\]
determined using viscous hydrodynamic calculations.

Freeze-out contribution: discrete  hydro contribution: smooth
Probing viscous damping

How to determine w’s?
Switch on shear/bulk viscosity only in a narrow temperature interval around a temperature $T_0$. In this way, we isolate the effect of viscosity around $T_0$.

Investigating the same IC (ideal and viscous), through a Gaussian parameterization.

**Computations:** boost invariant - MUSIC - hadrons at freeze-out after resonance decays @ LHC
Where is viscosity relevant?
Where is viscosity relevant?

\[
\frac{v_{n(\text{viscous})}}{v_{n(\text{ideal})}} - 1 \approx \int dT \left( w_{n}^{\eta}(T) \frac{\eta}{S}(T) + w_{n}^{\zeta}(T) \frac{\zeta}{S}(T) \right)
\]

**SHEAR \( v_2 \)**

- \( w<0 \): Suppression.
- Largest in absolute value at low temperature
Where is viscosity relevant?

\[ \frac{v_n^{(viscous)}}{v_n^{(ideal)}} - 1 \approx \int dT (w_n^\eta(T)\eta(T) + w_n^\zeta(T)\zeta(T)) \]

**SHEAR \( v_3 \)**

- \( w<0 \): Suppression.
- Similar temperature dependence
- Weight larger in absolute value
Where is viscosity relevant?

\[ \frac{v_n(\text{viscous})}{v_n(\text{ideal})} - 1 \approx \int dT (w_n^\eta(T) \eta_s(T) + w_n^\zeta(T) \zeta_s(T)) \]

**BULK \( v_2 \)**

- \( w<0 \): Suppression.
- Magnitude is of the same order as for shear.
- Different temperature dependence. Peak around 220 MeV
Where is viscosity relevant?

\[
\frac{v_n(\text{viscous})}{v_n(\text{ideal})} - 1 \approx \int dT \left( w_n^\eta(T) \frac{\eta(T)}{s(T)} + w_n^\zeta(T) \frac{\zeta(T)}{s(T)} \right)
\]

**BULK \( v_3 \)**

- \( w<0 \): Suppression.
- similar temperature dependence as bulk \( v_2 \)
- Peak around 200 MeV
Effective Viscosities

\[ v_2(\text{viscous}) = v_2(\text{ideal}) \left( 1 - 1.34 \left( \frac{\eta}{s} \right)_{2,\text{eff}} - 1.30 \left( \frac{\zeta}{s} \right)_{2,\text{eff}} \right) \]

\[ v_3(\text{viscous}) = v_3(\text{ideal}) \left( 1 - 2.33 \left( \frac{\eta}{s} \right)_{3,\text{eff}} - 2.61 \left( \frac{\zeta}{s} \right)_{3,\text{eff}} \right) \]

\[ \left( \frac{\eta}{s} \right)_{n,\text{eff}} = \frac{\int_{T_f}^{\infty} (\eta/s)(T)w_n^{(\eta)}(T)dT}{\int_{T_f}^{\infty} w_n^{(\eta)}(T)dT} \]

\[ \left( \frac{\zeta}{s} \right)_{n,\text{eff}} = \frac{\int_{T_f}^{\infty} (\zeta/s)(T)w_n^{(\zeta)}(T)dT}{\int_{T_f}^{\infty} w_n^{(\zeta)}(T)dT} \]
Effective Viscosities

\[ v_2(\text{viscous}) = v_2(\text{ideal}) \left( 1 - 1.34 \left( \frac{\eta}{s} \right)_{2,\text{eff}} - 1.30 \left( \frac{\zeta}{s} \right)_{2,\text{eff}} \right) \]

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\[ \left( \frac{\eta}{s} \right)_{n,\text{eff}} = \frac{\int_{T_f}^{\infty} (\eta/s)(T) w_n^{(\eta)}(T) dT}{\int_{T_f}^{\infty} w^{(\eta)}(T) dT} \]

\[ \left( \frac{\zeta}{s} \right)_{n,\text{eff}} = \frac{\int_{T_f}^{\infty} (\zeta/s)(T) w_n^{(\zeta)}(T) dT}{\int_{T_f}^{\infty} w^{(\zeta)}(T) dT} \]
Effective Viscosities

\[ v_2(\text{viscous}) = v_2(\text{ideal}) \left( 1 - 1.34 \left( \frac{\eta}{s} \right)_{2,\text{eff}} - 1.30 \left( \frac{\zeta}{s} \right)_{2,\text{eff}} \right) \]

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The variation of \( U_n \) due to viscosity is proportional to the sum of effective shear and bulk viscosities.

Viscous suppression is a factor \( \sim 2 \) larger for \( v_3 \) than \( v_2 \).

Viscous suppression for bulk is of same order as for shear.

\( U_n \) data can only constrain the effective viscosity, not the whole temperature dependence.
Effective Viscosity: An Example

H. Niemi et al PRC 93, 024907 (2016)

\[ \frac{\eta}{s} \mid _{2,\text{effec}} = S \]

\[ \begin{align*}
\eta &= 0.175 \\
\frac{\eta}{s} &= 0.206 \\
\frac{\eta}{s} &= 0.184
\end{align*} \]

Similar Effective Viscosity

ALICE \( v_n \{2\} \)
LHC 2.76 TeV Pb + Pb
\( p_T = [0.2...5.0] \) GeV

\[ \begin{align*}
v_n \{2\} &= 0.175 \\
\frac{\eta}{s} &= 0.206
\end{align*} \]

SAME ANISOTROPIC FLOW
What is the expected value of the effective SHEAR viscosity in QCD?

from QCD calculations

from heavy ion data
What is the expected value of the effective SHEAR viscosity in QCD?

- From QCD calculations, one expects that the effective shear is essentially identical for $v_2$ and $v_3$
- Effective shear viscosity extracted from Bayesian analyses of heavy ion data is compatible with that from QCD calculations, but somewhat smaller
What is the expected value of the effective **BULK** viscosity in QCD?

from QCD calculations

from heavy ion data
What is the expected value of the effective BULK viscosity in QCD?

- From QCD calculations, one expects that the effective bulk is essentially identical for $v_2$ and $v_3$.
- Effective bulk viscosity extracted from Bayesian analyses of heavy ion data is compatible with that from QCD calculations.

Error bars: 90% posterior
What we learn from effective viscosities in heavy ion physics?

- Data can determine only the sum of effective shear and effective bulk.
- Effective bulk viscosity is much smaller than effective shear viscosity.

We can “only” extract the effective shear from data.
Summary

- At each energy, one can, at most, extract an effective viscosity, which is a weighted average of the temperature-dependent viscosities.
- One cannot separate bulk from shear from $v_2$ and $v_3$ alone.
- The decrease of $v_2$ and $v_3$ in viscous hydrodynamics is determined by an effective viscosity, which encapsulates the detailed information contained in $(\eta / s) (T)$ and $(\zeta / s) (T)$.
- The effective viscosity is likely to be almost identical for $v_2$ and $v_3$.
- We can “only” extract the effective shear from data.
Thanks

T ≈ 2.6 \times 10^{12} K

-14 fm
Backup Slides
Viscosity in QCD

Small $\eta$ typically imply strong interactions

Small for a dilute gas

Small at high $T$ (massless quarks and gluons)
What do we know about viscosity from calculations

Minimum close to $T_c$ as expected

Uncertainties are large.

NOT YET
What do we know about viscosity from data

Model-to-data comparison with Bayesian inference

Uncertainties on shear and bulk are similar in absolute value.

Almost-perfect fluid, but how viscosity affects the observable?
Results on viscosity

\[
\ln \frac{v_n(\text{viscous})}{v_n(\text{ideal})} = \int dT \left( w_n^\eta(T) \frac{\eta}{s}(T) + w_n^\zeta(T) \frac{\zeta}{s}(T) \right)
\]
Effective Shear Viscosity

Different parametrizations

hydro

\[ \ln \frac{v_n(\text{viscous})}{v_n(\text{ideal})} = \int dT \left( w_n^\beta(T) \frac{\eta}{s}(T) \right) \]

\( T \) (MeV) \( T \) (MeV)

ln \( v_2^{\text{viscous}}/v_2^{\text{ideal}} \) calculated

ln \( v_3^{\text{viscous}}/v_3^{\text{ideal}} \) calculated

ln \( v_2^{\text{viscous}}/v_2^{\text{ideal}} \) predicted

ln \( v_3^{\text{viscous}}/v_3^{\text{ideal}} \) predicted

Our method

Effective viscosity is an excellent predictor !!!
Centrality

\[
\frac{\zeta}{s}(T) \quad \text{vs} \quad \ln\left(\frac{v_2^\text{viscous}}{v_2^\text{ideal}}\right)
\]

\[
\ln\left(\frac{v_2}{v_2^\text{ideal}}\right) \quad \text{vs} \quad \ln\left(\frac{v_3^\text{viscous}}{v_3^\text{ideal}}\right)
\]

\[
\ln\left(\frac{v_3}{v_3^\text{ideal}}\right) \quad \text{vs} \quad \ln\left(\frac{v_3^\text{viscous}}{v_3^\text{ideal}}\right)
\]

- 0.5% value rescaled by Re
- const T
- Duke shear+bulk
- 0-5% 20-30%

\[T\ (\text{MeV})\]

\[0.0 \quad 0.1 \quad 0.2\]

\[-0.4 \quad -0.2 \quad -0.1 \quad 0.0\]
The case of RHIC

Results turn out to be different in the end. The main difference is that freeze-out is no longer a small contribution to the viscous damping. And it is the non-robust part. This implies that it will be harder to constrain the viscosity from RHIC data.
RHIC