Charmed hadron interactions and correlation functions

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• Introduction
• Charmed hadron interactions – \(DD^*\) and \(D\bar{D}^*\)
• Summary

Exotic Hadrons including $c\bar{c} / c\bar{c} / \bar{c}\bar{c}$

- Main playground of exotic hadron physics
  - $X(3872)$ *Belle (‘03)* $c\bar{c}q\bar{q}$
  - Many $X,Y,Z$ states
    *Belle, CDF, BaBar, LHCb, CMS, BESIII, ...*
  - Charmed pentaquark $Pc$ *LHCb (‘15, ‘19)*
  - Doubly charmed tetraquark state $Tcc$ *LHCb (‘21)* $ccq\bar{q}\bar{q}$

- Structure of exotic hadrons
  - Compact multiquark states
    → “good” [ud] diquark gains energy
  - Hadronic molecules
    → Many exotic states around thresholds
  - Their mixture...

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R. Aaji+ [LHCb], 2109.01038, 2109.01056

A. Ohnishi @ Quark Matter 2022, Apr.07, 2022, Online/ Krakow, Poland
Compact Tetraquarks or Hadronic Molecules

- Tcc = Compact Tetraquark?
  Good $[\bar{u}d]$ diquark gains energy
  S. Zouzou+(‘86), ZPC30,457.

- X(3872)
  - $c\bar{c}$ component? production
cross section Bignamini+ (0906.0882)
  - Large yield in Pb+Pb → Molecule?
    Sirunyan+ [CMS] (2102.13048)
c.f. $\Delta r/\Delta p$ is similar in HIC and molecule.
    ExHIC (‘11,’11,’17)

- Hadronic Molecule Conditions
  - Appears around the threshold → OK
  - Have large size $R \simeq 1/\sqrt{2\mu B}$ → Yield
  - Described by the $hh$ interaction

How can we access $hh$ int. with charm?
→ Femtoscopy
Femtoscopic study of hadron-hadron interaction

• How can we study interactions between short-lived particles? → Femtoscopy!

• Correlation function (CF)
  • Koonin-Pratt formula
    
    $$C(p_1, p_2) = \frac{N_{12}(p_1, p_2)}{N_1(p_1)N_2(p_2)} \approx \int dr S_{12}(r) |\varphi_q(r)|^2$$

  • Source size from quantum stat. + CF (Femtoscopy)
    
    *Hanbury Brown & Twiss (‘56); Goldhaber, Goldhaber, Lee, Pais (‘60)*

  • Hadron-hadron interaction from source size + CF
    
    • CF of non-identical pair from Gaussian source
      
      *R. Lednicky, V. L. Lyuboshits (‘82); K. Morita, T. Furumoto, AO (‘15)*
      
      $$C(q) = 1 + \int dr S(r) \left\{ |\varphi_0(r)|^2 - |j_0(qr)|^2 \right\} \quad (\varphi_0 = s\text{-wave w.f.})$$

**CF shows how much $|\varphi|^2$ is enhanced → $V_{hh}$ effects!**
Measured Correlation Functions (examples)

Adamczyk+[STAR], PRL114(‘15)022301

\[ \Lambda \Lambda \]

\[ C(Q) \]

\[ Q (\text{GeV/c}) \]

\[ 0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \]

\[ 0.6 \quad 0.7 \quad 0.8 \quad 0.9 \quad 1 \]

\[ \Lambda \Lambda \oplus \bar{\Lambda} \bar{\Lambda} \]

\[ \text{QS} \]

\[ \text{LL w/o residual} \]

\[ \text{LL with residual} \]

\[ 40-80\% \]

\[ \text{Model: } R_h = 2.5 \text{ fm} \]

\[ C(k^\prime) \]

\[ 0.8 \quad 0.9 \quad 1 \]

\[ k^\prime (\text{MeV/c}) \]

\[ 100 \quad 200 \quad 300 \]

\[ 1.2 \]

\[ 490 \quad [1808.02511] \]

\[ \text{c.f. talk by V. Mantovani Sarti, F. Grosa, N. Agrawal} \]

Acharya+(ALICE), Nature (‘20)

\[ p \Xi^- \]

\[ p-\Xi^- \oplus p-\Xi^+ \]

\[ p-\Xi^- \] sideband background

\[ \text{Coulomb + HAL-QCD} \]

\[ \text{Coulomb} \]

\[ 2.2 \quad 2.4 \quad 2.6 \]

\[ 2.6 \quad 2.8 \quad 3.0 \]

\[ 0.8 \quad 1.0 \quad 1.2 \quad 1.4 \quad 1.6 \]

\[ k^\prime (\text{MeV/c}) \]

\[ 0 \quad 100 \quad 200 \quad 300 \]

ALICE, pp (\[\sqrt{s} = 5 \text{ TeV}\])

\[ r_0 = 1.13 \pm 0.02^{+0.17}_{-0.18} \text{ fm} \]

\[ \lambda = 0.68 \pm 0.07 \]

\[ p K^- \]

\[ p \phi \]

ALICE, 2105.05578

c.f. talk by V. Mantovani Sarti, F. Grosa, N. Agrawal

A. Ohnishi @ Quark Matter 2022, Apr.07, 2022, Online/ Krakow, Poland
Theoretical femtoscopic study of hh int. (examples)

Morita, Gongyo et al., (1908.05414),
Morita, AO, Etminan, Hatsuda (1605.06765)

Haidenbauer(1808.05049),
Morita+(1408.6682)

Mrówczyński, Słón (1904.08320, K−d),
Haidenbauer (2005.05012, Λd),
Etminan, Firoozabadi (1908.11484, Ωd),
K.Ogata+ (Ξ− d,2103.00100)

Y.Kamiya, K.Sasaki, et al., (2108.09644)

Z.-W. Liu, K.-W. Li,
L.-S. Geng (2201.04997)

A. Ohnishi @ Quark Matter 2022, Apr.07, 2022, Online/ Krakow, Poland
Femtoscopic study of charmed hadron int.

- “First study of the two-body scattering involving charm hadrons”
  
  Acharya+[ALICE] (2201.05352) [F. Grosa]

- $D^{-}p$ corr. func. is measured.
- Enhanced CF from Coulomb.
- One range gaussian potential with strength fitted to the $I=0$ scattering length of the model → attractive potentials are favored

<table>
<thead>
<tr>
<th>Model</th>
<th>$f_{0}$ ($I=0$)</th>
<th>$f_{0}$ ($I=1$)</th>
<th>$n_{\sigma}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coulomb</td>
<td></td>
<td></td>
<td>(1.1–1.5)</td>
</tr>
<tr>
<td>Haidenbauer et al. [21]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-\frac{g_{2}^{2}}{4\pi} = 1$</td>
<td>0.14</td>
<td>-0.28</td>
<td>(1.2–1.5)</td>
</tr>
<tr>
<td>$-\frac{g_{2}^{2}}{4\pi} = 2.25$</td>
<td>0.67</td>
<td>0.04</td>
<td>(0.8–1.3)</td>
</tr>
<tr>
<td>Hofmann and Lutz [22]</td>
<td>-0.16</td>
<td>-0.26</td>
<td>(1.3–1.6)</td>
</tr>
<tr>
<td>Yamaguchi et al. [24]</td>
<td>-4.38</td>
<td>-0.07</td>
<td>(0.6–1.1)</td>
</tr>
<tr>
<td>Fontoura et al. [23]</td>
<td>0.16</td>
<td>-0.25</td>
<td>(1.1–1.5)</td>
</tr>
</tbody>
</table>

[21] Haidenbauer+(0704.3668) (weakly / mildly attractive ($I=0$))
[22] Hofmann, Lutz (hep-ph/0507071) (repulsive ($I=0$))
[23] Fontoura+(1208.4058) (weakly attractive ($I=0$))
[24] Yamaguchi, Ohkoda, Yasui, Hosaka (1105.0734) (att., w/ bound state ($I=0$))
To be bound or not to be bound

- When there is a bound state, CF shows interesting dependence on the source size and relative momentum.
- $D^{-}p$ corr. func. shows the behavior with a bound state, and the best fit parameter set $(R, a_{0})$ is in the bound region. (If bound, it is the first weakly decaying pentaquark state.)

$$k \cot \delta = -\frac{1}{a_{0}} + \frac{1}{2} r_{\text{eff}} k^{2} + O(k^{3})$$

(Nuclear and atomic phys. convention.)

Partial wave analysis

Unbound

Bound

Morita+(1908.05414) [ALICE] (2201.05352)
Femtoscopic study of charm hadron interactions
- $DD^*$ and $D\bar{D}^*$ correlation func. -
**Femtoscopic study of charmed hadron int. (2)**

- **$DD^*$ and $D\bar{D}^*$ correlation functions.** Kamiya, Hyodo, AO (2203.1381)
  - Related with Tcc and X(3872)
  - $DD^*$ and $D\bar{D}^*$ interactions
    \[
    V = \frac{1}{2} \begin{pmatrix}
    V_{I=0} + V_{I=1} & V_{I=0} - V_{I=1} \\
    V_{I=0} - V_{I=1} & V_{I=0} + V_{I=1}
    \end{pmatrix}
    \]
  - $I=0$: One range gaussian, strength fitted to the mass
  - $I=1$: ignored
  - Range = one pion exchange Yasui, Sudoh (0906.1452)
  - Strength is fitted to the pole mass.

\[
\begin{array}{c|cc|cc}
 & DD^* & \{D^0\bar{D}^*\} & \{D^+D^-\} \\
 & \{D\bar{D}^*\} & \{D^0\bar{D}^*\} & \{D^+D^-\} \\
\hline
DD^* & V_0 [\text{MeV}] & a_0^{D^0D^+} [\text{fm}] & a_0^{D^+D^*0} [\text{fm}] \\
& -36.569 - i1.243 & -7.16 + i1.85 & -1.75 + i1.82 \\
\hline
\{D\bar{D}^*\} & V_0 [\text{MeV}] & a_0^{D^0D^*0} [\text{fm}] & a_0^{D^+D^-} [\text{fm}] \\
& -43.265 - i6.091 & -4.23 + i3.95 & -0.41 + i1.47 \\
\end{array}
\]
We are sorry, but we use a Gaussian Source!

- Calculating HBT radius in dynamical models is not easy
  

- and a Gaussian source seems to work
  at the current precision of hh interaction studies.

  S. Acharya+[ALICE], PLB 811 (‘20) 135849.

- primary (universal ?)+ decay of short-lived resonances
  
  ~ eff. Gaussian

- Flow and source geometry effects are seen in CF,
  but the uncertainty of hh int. is the largest.
**$D^0 D^{*+}$ and $D^+ D^{*0}$ Correlation Functions**

- For small source ($R=1$ fm)
  
  \[ C(q) > 8 \text{ for the lower channel (}D^0 D^{*+}\text{)} \] (Very strong)
  
  \[ C(q) \sim 2.5 \text{ for upper channel } (D^+ D^{*0}) \] (strong)

- For large source ($R=5$ fm), CF show a dip

- Strong enhancement for small source, dip for large source
  
  $\rightarrow$ Characteristic dependence with a bound state ($T_{cc}$)

- Cusp is not significant
\( D^0 \bar{D}^{*0} \) and \( D^+ D^{*-} \) Correlation Functions

- \( C(D^0 \bar{D}^{*0}) \): Strong enh. for small source, dip for large source → Characteristic dependence with a bound state (X(3872))
- \( C(D^+ D^{*-}) \): Coulomb dominant
- Cusp may be observed for small size
Tcc and X(3872) structure

• Hadronic molecule structure is assumed
  → Eigenmomentum \( k \simeq -i/a_0 \), \( a_0 \simeq R = 1/\sqrt{2\mu B} \)

• What happens when multiquark state mixes?
  → Deviation from weak binding relation (X=compositeness)

  *Weinberg, Phys. Rev. 137, B672 (1965), Hyodo, Jido, Hosaka (1108.5524),
  Kunigawa, Hyodo (2112.00249)*

\[
a_0 = R \left[ \frac{2X}{1 + X} \right] + \mathcal{O}(R_{\text{typ}}) \\
\left[ R_{\text{typ}} = \max(m_{\pi}^{-1}, r_{\text{eff}}), R = 1/\sqrt{2\mu B} \right]
\]

• Smaller scattering length in DD* may signal the **genuine** tetraquark nature of Tcc.

\[
f = \frac{1}{k \cot \delta - ik} \simeq \frac{1}{1/a_0 - ik} \\
\text{(high-energy phys. convention)}
\]
Summary

• Two-particle correlation functions are useful to deduce
  • Scattering length
  • Existence of a bound state
  • and hopefully the compositeness
• Charm hadron interactions are within the reach.
  • $D^- p$ correlation function has been measured, and the data favor attractive interaction. *ALICE (2201.05352)*
  • $D D^*$ and $D \bar{D}^*$ correlation functions are predicted to reflect the existence of bound states (Tcc and X(3872)) by using simple potentials fitting to the mass and width. *Y. Kamiya, T. Hyodo, AO (2203.13814)*
  • Precise measurement of the correlation function will constrain the scattering length, which may tell us the structure of exotic hadrons.
Thank you for attention!

Y. Kamiya

T. Hyodo

AO
• Source size (R) dependence of C(q) is helpful to deduce the existence of a bound state.

Morita+(‘16, ‘20), Kamiya+(‘20), Kamiya+(2108.09644)

• With a bound state, C(q) is suppressed at small q when R \sim |a_0|.
  (w.f. has a node at r \sim |a_0| with a bound state.)

• Qualitative understanding by the analytic model (LL formula)
  [Lednickey, Lyuboshits (‘82)] with the zero range approx. (r_{\text{eff}}=0)

Kamiya+(2108.09644)
Coupled-Channel Correlation Function

- Correlation function with CC effects (KPLLL formula)
  \[ C(q) = \sum_j \omega_j \int dr S_j(r) |\Psi_j^-(r)|^2 \]
  \[ \Psi_j^-(r) = [e^{iq\cdot r} - j_0(qr)]\delta_{1j} + \psi_j^-(r) \]
  \[ \psi_j^-(q) \propto e^{-iqr}/r \text{ or } e^{-\kappa r}/r \quad (r \to \infty) \]

(No Coulomb case)

- Effects of coupled-channel, strong & Coulomb pot., and threshold difference are taken into account in the charge base, \( p\Xi^-, n\Xi^0, \Lambda\Lambda \).
- Source size (R) and source weight \( (\omega_j) \) need to be determined.
A New Insight from CMS: Exotic/Normal Ratio

- **ExHIC index = Coalescence / Statistical Ratio**
  \[ R_{CS}^{h} = \frac{\text{Yields in Coalescence}}{\text{Yields in Statistical model}} \]

- **CMS index = Exotic / Normal Ratio**
  \[ \rho_{exo/nor} = \frac{N(\text{Exotic hadron candidate})}{N(\text{Normal hadron})} \]

  - X(3872) / ψ(2S) ratio in pp and PbPb collisions.
    \[ \rho_{X/\psi}(\text{PbPb}) = 1.08 \pm 0.49(\text{stat.}) \pm 0.52(\text{syst.}) \]
    \[ \rho_{X/\psi}(pp) \simeq 0.1 \]

*ExHIC prediction is found to be (qualitatively) true!*
State-of-the-art Femtoscopy of radii

- Systematic measurement of 3D HBT radii (side, out, long)


Figure 3: because particles with heavier masses have smaller thermal velocities, their source volumes are more strongly confined by collective flow. For longitudinal flow (left panel) this results in smaller values of $R_{\text{long}}$ for particles with higher $m_T = \sqrt{m^2 + p_T^2}$. For radial flow (right panel) this confines heavier particles toward the surface, which results in both a reduced volume and an offset $\Delta r$ in the outward direction.

S. Acharya+[ALICE], PLB811(‘20)135849
We are sorry for using a Gaussian Source!

• Calculating HBT radius in dynamical models is not easy (HBT puzzle).

  • M.A.Lisa, S.Pratt, R.Soltz, U.Wiedemann, Ann.Rev.Nucl.Part.Sci.55(‘05)357 [nucl-ex/0505014];

choices then tends to exceed the number of experimental constraints. In fact, all the model results that we review in the current subsection remain unsatisfactory with this respect: They either deviate significantly from femtoscoopic data, or they reproduce these data at the price of missing other important experimental information. In particular, there is so far no dynamically consistent model that reproduces quantitatively both the systematic trends discussed in Section 4 and the corresponding single inclusive spectra. In this situation, the scope of this subsection is

  • S. Pratt, PRL102(‘09)232301 [0811.3363].

Two particle correlation data from the BNL Relativistic Heavy Ion Collider have provided detailed femtoscoopic information describing pion emission. In contrast with the success of hydrodynamics in reproducing other classes of observables, these data had avoided description with hydrodynamic-based approaches. This failure has inspired the term “HBT puzzle,” where HBT refers to femtoscoopic studies which were originally based on Hanbury Brown–Twiss interferometry. Here, the puzzle is shown to originate not from a single shortcoming of hydrodynamic models, but the combination of several effects: mainly prethermalized acceleration, using a stiffer equation of state, and adding viscosity.

How about afterburner effects?
Impact of $S=-2$ Baryon-Baryon Interactions (1)

- Is “H(uuddss)” bound, unbound, or quasi-bound?

- It is plausible not to be bound below $\Lambda\Lambda$.
  - Bound $H$ in the SU(3)$_f$ limit.
    - Bag model: Jaffe, PRL38(1977)195.
    - LQCD: HALQCD(‘11), NPLQCD(‘11,’13), Mainz (‘19).
  - But no discovery of bound $H$.
    - No $M(\Lambda p\pi^-)$ peak; $\Lambda\Lambda$ hypernucl.: Takahashi+ (‘01);
      Femtoscopy: STAR(‘15); ALICE(‘19); Morita+ (‘15).

- Quasi-bound state below $N\Xi$ or Unbound?
  - Resonance “H” from $(K^-, K^+)$?
    - KEK-E522 (‘07)
  - LQCD at almost physical $m_q$ → Unbound
    - HAL QCD(‘20).

\[
\begin{align*}
\text{Unbound} & \quad \text{HAL (’20)} \\
\text{Quasi-bound} & \quad 4\text{He}+\Lambda\Lambda \\
\text{Bound} & \quad \Lambda\Lambda \sim 6.91 \text{ MeV} \\
& \quad \Lambda\Lambda \sim 80 \text{ MeV} \\
& \quad Jaffe (’77)
\end{align*}
\]
**Impact of S= -2 Baryon-Baryon Interactions (2)**

- \(\Lambda\Lambda\) and \(N\Xi\) interactions are relevant to “Hyperon Puzzle”

- \(\Lambda\) and \(\Xi\) are predicted to appear at \((2-4)\rho_0\),
  and softened EOS cannot support 2 \(M_\odot\) neutron stars.
  → Repulsive YNN interactions, Quark Matter, Modified Gravity?

- Precise \(\Lambda N\), \(\Lambda\Lambda\), \(N\Xi\), and \(\Lambda\NN\) interactions need to be known.
  - Repulsive \(\Xi N\) interaction (I=1) may help support 2 \(M_\odot\) NS

*Weissborn et al., NPA881 (‘12) 62.*

*Ishizuka, AO, Tsubakihara, Sumiyoshi, Yamada (‘08)*
S=-2 Baryon-Baryon Interactions

• Theoretical Approaches
  • Phenomenological (Nijmegen, Jülich, Ehime, Quark model, …)
  • Chiral EFT [Haidenbauer, Meissner, Petschauer (‘16); Li, Hyodo, Geng (‘18)]
  • Lattice QCD [Sasaki+ [HAL QCD] (‘20)]

• Experimental Information
  • Double Λ and Ξ hypernuclei
    Takahashi+(‘01); Nakazawa+(‘15); Hayakawa+[E07](‘21); Yoshimoto+[E07](‘21).
  • Femtoscopic study of hadron-hadron interactions
    [See also Valentina Mantovani Sarti (Wed), Laura Šerkšnytė (Sun)]
    Adamczyk+[STAR](‘15, ΛΛ); Acharya+[ALICE](‘19(ΛΛ), ‘19(ΝΞ), ‘20(ΝΞ));
    Morita, Furumoto, AO (‘15, ΛΛ); Hatsuda, Morita, AO, Sasaki (‘17, ΝΞ);
    Haidenbauer (‘19, ΛΛ-ΝΞ); Haidenbauer+ (‘20).
Coupled-channel $\Xi-\Lambda\Lambda$ potential and correlation functions
**NΞ-ΛΛ Potential from Lattice QCD**

- NΞ-ΛΛ potential at almost physical quark masses ($m_\pi = 146$ MeV) by HAL QCD Collaboration

  *K. Sasaki et al. [HAL QCD], NPA 998 (‘20) 121737 (1912.08630)*

- Significant attraction in (I,S)=(0,0) of NΞ.
- Weak attraction in ΛΛ (Coupling with NΞ causes ΛΛ attraction).

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![Graphs showing NΞ-ΛΛ potential for different states](attachment:graphs.png)
\textbf{NΞ-ΛΛ Potential from Lattice QCD}

- Low-energy scattering parameters
  - Nuclear physics convention
    \[ k \cot \delta = -\frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} k^2 + \mathcal{O}(k^2) \]

- \( \text{Re} (a_0) < 0 \rightarrow \text{No bound state in ΛΛ-NΞ systems.} \)
  (except for Ξ⁻ atom)

- There is a virtual pole around the NΞ threshold
  (3.93 MeV below nΞ⁰ threshold)
  on the irrelevant Riemann sheet, (+, −, +) [quasi-bound → (−,+,+)]

\[ E_{\text{pole}} = 2250.5 - i0.3 \, \text{MeV} \]

\( \text{sign of Im(eigenn momentum)} \)
Virtual Pole

- Virtual pole (single channel case) = Eigen energy of the pole is below the threshold, but the wave function diverges at $r \to \infty$.
  (Imaginary part of eigen momentum is negative, $\exp(iqr)/r \to \infty$.)

- Lattice BB potential at almost physical quark masses (HAL QCD)
  - With Coulomb potential and threshold mass difference, virtual pole appears on $(+,-,+)$ Riemann sheet (w.f. of $n\Xi^0$ channel diverges).
  - Atomic states are well separated from VP. ($\mu\alpha^2/2n^2=14.6 \text{ keV/n}^2$)
Coupled-Channel Correlation Function

- Correlation function with CC effects (KPLL formula)
  \( \rightarrow \) sum of j-th channel contributions leading to j=1
  with outgoing momentum \( q \)

**Lednicky, Lyuboshits, Lyuboshits (‘98); Haudenbauer (‘19)**

\[
C(q) = \sum_j \omega_j \int d r S_j(r) |\Psi_j^{(-)}(r)|^2
\]

\[
\Psi_j^{(-)}(r) = [e^{i q \cdot r} - j_0(q r)] \delta_{1j} + \psi_j^{(-)}(r)
\]

\[
\psi_j^{(-)}(q) \propto e^{-i q r / r} \text{ or } e^{-\kappa r / r} \text{ (} r \rightarrow \infty \text{)}
\]

(No Coulomb case)

- Effects of coupled-channel, strong & Coulomb pot., and threshold difference are taken into account in the charge base, \( p\Xi^-, n\Xi^0, \Lambda\Lambda \).
  **Y. Kamiya+, PRL(‘20, \( K^- p \))**

- Source size (R) and source weight (\( \omega_j \)) need to be determined.
Theoretical $p\Xi^-$ and $\Lambda\Lambda$ Correlation Function

- $p\Xi^-$ correlation function
  - Strongly enhanced at low $q$ by the strong interaction, and further enhanced by the Coulomb potential at $q < 50$ MeV/c
  - $\Lambda\Lambda$ source effect is small.

- $\Lambda\Lambda$ correlation function
  - Suppressed by quantum statistics, but enhanced by the strong interaction at low $q$.
  - $N\Xi$ source effect is visible only around the thresholds.

Kamiya+ (2108.09644)
Comparison with pΞ- and ΛΛ correlation function data
Parameters in Correlation Function Data

- Actual data contains non-femtoscopic effects $\rightarrow$ Pair purity $<1.$ (jets, misidentified particles)

$$C_{\text{exp}}(q; R, \lambda, N, \omega) = N(q) [1 + \lambda (C_{\text{theory}}(q; R, \omega) - 1)]$$

- We adopt Pair purity ($\lambda$) from MC analysis results by ALICE.
- Source Weight ($\omega_j$) is given by a simple statistical model. (Sensitivity is small.)
- Normalization with jet effects ($N(q)=a+bq$) is determined by the fit to the data.
- Source size ($R$) is determined by the fit to the data for pp 13 TeV collisions,

$$R_{p\Xi^-}(pp) \simeq 1.05 \text{ fm} \hspace{1cm} [R_{p\Xi^-}^{\text{ALICE}}(pp) = 1.02 \pm 0.05 \text{ fm}]$$

and based on the scaling relation for p Pb 5.02 TeV collisions.

$$R_{p\Xi^-}(p\text{Pb})/R_{p\Xi^-}(pp) \simeq R_{pp}^{\text{ALICE}}(p\text{Pb})/R_{pp}^{\text{ALICE}}(pp) \ [R_{p\Xi^-}(p\text{Pb}) = 1.27 \text{ fm}]$$

($\Lambda\Lambda$ and $p\Xi^-$ source sizes are assumed to be the same.)
\( p\Xi^- \) Correlation Function

- \( p\Xi^- \) correlation function data implies attractive \( \Lambda\Xi \) interaction.
  - Strong enhancement from pure Coulomb CF
  - \( \Lambda\Lambda \) source effect is negligible. \( n\Xi^0 \) source effect is visible.
  - Calculated CF agrees with ALICE data.

\( p\Xi^- \) (corrected)

- CF(\( p\Xi^- \)) from pp 13 TeV (R=1.05 fm)
- CF(\( p\Xi^- \)) from pPb 5.02 TeV (R=1.27 fm)

Kamiya+ (2108.09644); Acharya+(ALICE), PRL(‘19), Nature (‘20)
Comparison with other results

T. Hatsuda, K. Morita, AO, K. Sasaki, NPA967('17)856. (heavier quark mass)

J. Haidenbauer, NPA981('19)1. (NLO(600), w/ CC effects, w/o Coulomb) (w/ Coulomb, it will be comparable with data.)

D. L. Mihairov+[ALICE], NPA1005('21)121760 (QM2019). (Nijmegen potential does not explain the data.)

Kamiya+(2108.09644). (w/ Lattice BB pot. at phys. mass and CC effects with $\Lambda\Lambda$)
**ΛΛ correlation function**

- **ΛΛ correlation function**
  - Enhancement from pure quantum statistic CF
  - $N\Xi$ source effect is visible only around thresholds.
  - Calculated CF agrees with ALICE data.
  - Analytic model (Lednicky-Lyuboshits formula) works well.

*Kamiya+ (2108.09644); Acharya+[ALICE] (‘19)*
Comparison with other results

C. Greiner, B. Muller, PLB219('89)199.
(Assumed $\Lambda\Lambda$ resonance)

AO, Hirata, Nara, Shinmura, Akaishi,
NPA670('00)297c
(Before NAGARA, interaction was too strong.)

Adamczyk+[STAR], PRL114('15)022301
(Residual source $R \sim 0.5$ fm was assumed.)

Morita, Furumoto, AO, PRC91('15)024916. (Res.Source + flow)

J. Haidenbauer, NPA981('19)1.
(Larger cusp ?)

Kamiya+('21).
Smaller cusp than $\chi$EFT.
CC simulates res. source, but not enough for STAR data
Unbound nature of Ξ confirmed?
**R Dependence of Correlation Function**

- Source size (R) dependence of C(q) is helpful to deduce the existence of a bound state.

*Morita+(‘16, ‘20), Kamiya+(‘20), Kamiya+(2108.09644)*

- With a bound state, C(q) is suppressed at small q when R \( \sim |a_0| \).
  (w.f. has a node at \( r \sim |a_0| \) with a bound state.)

- Qualitative understanding by the analytic model (LL formula)

  [Lednickey, Lyuboshits (‘82)] with the zero range approx. (\( r_{\text{eff}} = 0 \))

---

**Legend**

- **Bound**
- **Unbound**

---

A. Ohnishi @ Quark Matter 2022, Apr.07, 2022, Online/ Krakow, Poland
**R dependence of $p\Xi^-$ correlation function**

- R dep. of calculated results
  - Enhanced region shrinks with larger R. No Dip.
- Larger R data from Au+Au seem to show similar behavior.

---

**HAL + Coulomb**

**LL+Coulomb**

- **Bound**
- **Unbound**

---

(No Dip at larger R)*
c.f. $R$ dependence of $pK^-$ correlation function

- Enhanced $C(q)$ from pp collisions, and dip in heavy-ion collisions.  
  = Typical behavior expected from LL formula + Coulomb with a bound state.  
  
  Kamiya+ (PRL, ’20)

- These $R$ dependence of $C(q)$ supports again the KN bound state nature of $\Lambda(1405)$
  
  S. Acharya+[ALICE], PRL124(‘20)092301
  S. Acharya+[ALICE], 2105.05683
  Siejka+[STAR, preliminary], NPA982 (‘19)359.
Summary

• Correlation functions are helpful to constrain / examine hadron-hadron interactions as well as to deduce the existence of a bound state.

• We have calculated pΞ− and ΛΛ correlation functions by using lattice NΞ-ΛΛ coupled-channel (CC) potential.
  • w/ effects of CC, Coulomb, threshold difference.
  • ALICE pΞ− and ΛΛ correlation function data are consistent with the HAL QCD potential.
  • Source weight effect from conversion channel is not big, except for the cusps at NΞ thresholds in ΛΛ corr. fn. (Solving CC equation is still important.)

• Unbound nature of NΞ will be supported by studying the source size dependence of the pΞ− correlation function. (Any way to confirm the virtual pole nature?)
To be, or not to be, that is the question.

Table 1. Leading $6q \, L = 0$ dibaryon candidates [2], their $BB'$ structure and the CM interaction gain with respect of the lowest $BB'$ threshold calculated by means of Eq. [2]. Asterisks are used for the $10_f$ baryons $\Sigma^* \equiv \Sigma(1385)$ and $\Xi^* \equiv \Xi(1530)$. The symbol $[i,j,k]$ stands for the Young tablau of the $SU(3)_f$ representation, with $i$ arrays in the first row, $j$ arrays in the second row and $k$ arrays in the third row, from which $P_f$ is evaluated. The $10^*$ SU(3)$_f$ representation is denoted here $10^*$.

<table>
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<tr>
<th>$-S$</th>
<th>SU(3)$_f$</th>
<th>$I$</th>
<th>$J^\pi$</th>
<th>$BB'$ structure</th>
<th>$\frac{\Delta(Y_{CM})}{M_0}$</th>
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<tr>
<td>0</td>
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<td>1</td>
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<td>1/2</td>
<td>$2^+$</td>
<td>$\frac{1}{\sqrt{5}}(N\Sigma^* + 2\Delta \Sigma)$</td>
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<td>$\frac{1}{\sqrt{8}}(\Lambda \Lambda + 2N \Xi - \sqrt{3}\Sigma \Sigma)$</td>
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<tr>
<td>3</td>
<td>[3,2,1]</td>
<td>8</td>
<td>1/2</td>
<td>$2^+$</td>
<td>$\frac{1}{\sqrt{5}}(\sqrt{2}N \Omega - \Lambda \Xi^* + \Sigma^* \Xi - \Sigma \Xi^*)$</td>
</tr>
</tbody>
</table>

A. Gal (‘16); M. Oka (‘88)
**Potentially measurable hh pairs**

- Correlation function is useful to access hadron-hadron interactions as well as to deduce the existence of a bound state.

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**Blue:** Pairs we have studied, **O:** Experimentally measured

---

**Femtoscopy**
Source size dependence of correlation functions

- $p\Xi^-$
  - Smooth dependence on R. (No bound state, Non-identical particles)

- $\Lambda\Lambda$
  - Complicated R dependence (Quantum statistics)
  - No long-tail ($q > 200$ MeV/c) with $R > 1.5$ fm

Kamiya+(2108.09644)
Non-Femtoscopic Parameters

- Relevant parameters = $R$, $\lambda$, $N = a + bq$
  
  ($\omega$’s are almost irrelevant for $p\Xi^-$ and $\Lambda\Lambda$ correlation functions.)

\[
C_{\text{exp}}(q; R, \lambda, N, \omega) = N(q) \left[ 1 + \lambda (C_{\text{theory}}(q; R, \omega) - 1) \right]
\]

<table>
<thead>
<tr>
<th>collision</th>
<th>pair</th>
<th>$\lambda$</th>
<th>$a$</th>
<th>$b$ [(MeV/c)$^{-1}$]</th>
<th>$R$ [fm]</th>
</tr>
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<tr>
<td>$pp$</td>
<td>$p\Xi^-$</td>
<td>1</td>
<td>0.15</td>
<td>0.15</td>
<td>1.05</td>
</tr>
<tr>
<td>$(13 \text{ TeV})$</td>
<td>$\Lambda\Lambda$</td>
<td>0.338 [9]</td>
<td>0.95</td>
<td>$1.28 \times 10^{-4}$</td>
<td></td>
</tr>
<tr>
<td>$p\text{Pb}$</td>
<td>$p\Xi^-$</td>
<td>0.513 [14]</td>
<td>0.109</td>
<td>$-2.56 \times 10^{-4}$</td>
<td>1.27(*)</td>
</tr>
<tr>
<td>$(5.02 \text{ TeV})$</td>
<td>$\Lambda\Lambda$</td>
<td>0.239 [9]</td>
<td>0.99</td>
<td>$0.29 \times 10^{-4}$</td>
<td></td>
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</table>

TABLE II. The pair purity $\lambda$, non-femtoscopic parameters $a$ and $b$, and the effective source size $R$ in the fitting function $C_{13}(q)$. The parameters $a$ and $b$ in $pp$ ($\Lambda\Lambda$ pairs) and $p\text{Pb}$ ($p\Xi^-$ and $\Lambda\Lambda$ pairs) collisions and $R$ in $pp$ collisions are the actual fitting parameters. Numbers with references are taken from Refs. [9] [14] [15], and the number with (*) is estimated from other other parameters. See the text for details.
Correlation function from T-matrix

- s-wave w.f. using the half-off-shell T-matrix ($T_0$)

  **J. Haidenbauer, NPA 981(‘19)1.**

  \[
  \tilde{\psi}_0(k, r) = j_0(kr) + \frac{1}{\pi} \int dq \frac{q^2 j_0(qr)}{E - E_1(q) - E_2(q) + i\varepsilon} T_0(q, k; E)
  \]

  \[
  \psi_0^{-}(k, r) = e^{-2i\delta_0}\tilde{\psi}_0(k, r) \to \frac{e^{-i\delta_0}}{kr} \sin(kr + \delta_0) = \frac{1}{2ikr} (e^{ikr} - e^{-2i\delta_0}e^{-ikr})
  \]

- Strong T-matrix + Coulomb potential

  **J. Haidenbauer, G. Krein, and T. C. Peixoto, EPJA 56 (‘20)184; using the Vincent-Phatak method**

  [C.M. Vincent and S.C. Phatak, PRC10(‘74)391; B. Holzenkamp, K. Holinde and J. Speth, NPA 500(‘89)485 (1989)]
Analytic model of correlation function

- Asymptotic w.f. is described by the scattering amplitude $f(q)$
  (non-identical particle pair, short range int. (only s-wave is modified),
  single channel, no Coulomb pot.)

  $\Phi^{(+)}(r) = e^{i q \cdot r} - j_0(q r) + \varphi^{(+)}_0(r; q)$

  $\varphi^{(+)}_0(r; q) \rightarrow \frac{e^{i \delta} \sin(q r + \delta)}{q r} = \frac{1}{2 i q r} (S e^{i q r} - e^{-i q r}) = \frac{\sin q r}{q r} + f(q) \frac{e^{i q r}}{r}$

  $\varphi^{(-)}_0(r; q) = S^{-1} \varphi^{(+)}(r; q)$ \[ S = \exp(2i \delta), f = (S - 1)/2iq = [q \cot \delta - iq]^{-1} \]

- Correlation function in Lednicky-Lyuboshits (LL) formula
  (with static Gaussian source, real $\delta$) (Lednickey, Lyuboshits (‘82))

  $C(q) = \int dr S(r) \left| \Phi^{(-)}(r) \right|^2 = 1 + \int dr S(r) \left[ \left| \varphi^{(-)}_0(r) \right|^2 - (j_0(q r))^2 \right]$

  $\approx 1 + \int 4\pi dr S(r) \left[ |f(q)|^2 + \frac{\sin q r}{q} \{ f(q)e^{i q r} + f^*(q)e^{-i q r} \} \right]$
Lednicky–Lyuboshits functions

\[
F_1(x) = \frac{1}{x} \int_0^x e^{t^2-x^2} \, dt, \quad F_2(x) = \frac{1 - e^{-x^2}}{x}, \quad F_3(x) = 1 - \frac{x}{2\sqrt{\pi}}
\]

\[
F_1(x) \approx \frac{1 + c_1 x^2 + c_2 x^4 + c_3 x^6}{1 + (c_1 + 2/3)x^2 + c_4 x^4 + c_5 x^6 + c_3 x^8} \quad (0 \leq x < 20)
\]

\[
(c_1, c_2, c_3, c_4, c_5) = (0.123, 0.0376, 0.0107, 0.304, 0.0617)
\]

AO, Morita, Mihayara, Hyodo, NPA 954 (‘16)294.
Bird’s-eye view of $C(q)$

- **Zero eff. range pot. → $C(q)=F(R/a_0, qR)$**
  \[
  r_{\text{eff}} = 0 \rightarrow q \cot \delta = -1/a_0 \rightarrow f(q) = (q \cot \delta - i q)^{-1} = -\frac{R}{R/a_0 + i q R}
  \]
  \[
  C(x, y) = 1 + \frac{1}{x^2 + y^2} \left[ \frac{1}{2} - \frac{2y}{\sqrt{\pi}} F_1(2x) - x F_2(2x) \right] \quad (x = q R, y = R/a_0)
  \]
  - **Low momentum limit**
  \[
  C(x, y) \rightarrow \frac{1}{2} \left( \frac{1}{y} - \frac{2}{\sqrt{\pi}} \right)^2 + 1 - \frac{2}{\pi} \quad (F_1 \rightarrow 1, F_2 \rightarrow 0 \text{ at } x \rightarrow 0)
  \]

- **Enhanced $C(q)$ at small $q$**
  with $a_0 < 0$

  \[
  C_{\text{LL}}(0) = 1 - \frac{2}{\sqrt{\pi}} \left( \frac{a_0}{R} \right) + \frac{1}{2} \left( \frac{a_0}{R} \right)^2
  \]

- **$a_0 > 0$ → Size dependent $C(q)$**
  - $C(q) > 1$ at small $R$
  - $C(q) < 1$ at $R \sim a_0$
    (w.f. node at $r \sim a_0$)
Bound state diagnosis by femtoscopy

- Source size dep. of CF tells the sign of the scattering length ($a_0$).
  - With attraction, Large CF at small R.
  - With a bound state ($a_0 > 0$), CF is suppressed at $R \sim a_0$

**Source size dep. of CF → To be bound, or not to be bound.**
Do you know any mechanism to suppress $C(q)$ other than the existence of bound state, when the interaction is attractive?

(Strong flow, ...)

Morita+(‘20)
Correlation Function with Gaussian source

$N\Omega$ potential (J=2, HAL QCD, $a_0 = 3.4$ fm) + Coulomb

STAR (40-80 %)

ALICE (pp, pA)
Source Size Dep. of CF w/ Coulomb potential

Y. Kamiya+(2108.09644) (LL × Gamow factor)
Modern Hadron-Hadron Interactions

- Lattice QCD $hh$ potential
  - $V_{hh}$ is obtained from the Schrödinger eq. for the Nambu-Bethe-Salpeter (NBS) amplitude.
  - \cite{Ishii2007}

  \[ V_{\Omega \Omega}, N\Omega, \Lambda\Lambda-N\Xi \text{ potentials at phys. quark mass are published} \]

- Chiral EFT / Chiral SU(3) dynamics
  - $V_{hh}$ at low $E$. can be expanded systematically in powers of $Q/\Lambda$.
    - \cite{Weinberg1979, Machleidt2016, Ikeda2012}

  \[ NN, NY, YY, KN-\pi\Sigma-\pi\Lambda, ... \]

- Quark cluster models,
  Meson exchange models,
  More phenomenological models, ...

Let us examine modern $hh$ interactions!
Relevance of $\Xi N$ interaction to physics

- H-particle: 6-quark state (uuddss) may be realized as a loosely bound state of $\Xi N$ (I=0)
  
  *K. Sasaki et al. (HAL QCD, ‘16,’17)*

- Repulsive $\Xi N$ interaction (I=1) may help to support 2 $M_\odot$ Neutron Star
  
  *Weissborn et al., NPA881 (‘12) 62.*

---

K. Sasaki et al. (HAL QCD Collab.), EPJ Web Conf. 175 (‘18) 05010.

A. Ohnishi @ Quark Matter 2022, Apr.07, 2022, Online/ Krakow, Poland
ΛΛ correlation and ΛΛ interaction

It is unlikely that ΛΛ bound state exists.

L. Adamczyk+[STAR], PRL114(‘15)022301

Au+Au → ΛΛ

δ ~ − a₀q

δ ~ + a₀q

S. Acharya+[ALICE], PLB797(‘19)134822

ΛΛ inv. mass spectrum from

\(^{12}\text{C}(K^-, K^+\Lambda\Lambda)\)

\(P_{K^-} > 0.95\ \text{GeV/c}\)
pΩ⁻ correlation

K. Morita, AO, F. Etminan, T. Hatsuda, PRC94(‘16)031901(R) (w/ Lattice potential with heavier quark mass)

J. Adam+[STAR], PLB790(‘19)490.

K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, Y. Kamiya, AO, PRC 101(‘20)015201. (w/ Lattice potential at physical quark mass)

S. Acharya+[ALICE], 2005.11495 [nucl-ex] (pp 13 TeV)

Bound state ?

A. Ohnishi @ Quark Matter 2022, Apr.07, 2022, Online/ Krakow, Poland
pK− correlation

S. Cho+ [ExHIC], PPNP95(‘17)279.
(Insufficient coupled-channel effects)

J. Haidenbauer, NPA981(‘19)1.
(w/ CC effects, w/o Coulomb)

S. Acharya+[ALICE], PRL124(‘20)092301

Source size dep. shows interesting feature.

Y. Kamiya, T. Hyodo, K. Morita, AO,
W. Weise, PRL124(‘20)132501.
(Chiral SU(3) dynamics)
Other bound states?

- **ΛΛ-ΝΞ**
  - \( C_{ΛΛ}(q) \) in AA(RHIC) and pp(LHC) are similar (No b.s. below ΛΛ).
  - LQCD predicts a virtual pole near NΞ threshold, which can be detected as the cusp in \( C_{ΛΛ}(q) \).
  - NLO(600) potential predicts the same.
  - (The fate of H particle)
    - K. Sasaki+ [HAL QCD], NPA998(‘20)121737;
    - Y. Kamiya+, in prep.; Haidenbauer(‘19).

- **KN**
  - Λ(1405) is believed to be the bound state of KN, and “dip” is expected at \( R \sim a_0 \).
  - However, Coulomb and coupled-channel effects modify the dip-like behavior.
    - Kamiya+ (‘20).
Correlation Function with Coupled-Channels Effects

- Single channel, w/o Coulomb (non-identical pair)
  \[ C(q) = 1 + \int dr S(r) \left| \chi^{(-)}(r, q) \right|^2 - \left| j_0(qr) \right|^2 \]

- Single channel, w/ Coulomb
  \[ C(q) = \int dr S(r) \left[ \varphi^{C,\text{full}}(q, r) \right|^2 + \chi^{C,(-)}(r, q) \right|^2 - \left| j_0^C(qr) \right|^2 \]

- Coupled channel, w/ Coulomb
  \[ C_i(q) = \int dr S_i(r) \left[ \varphi^{C,\text{full}}(q, r) \right|^2 + \chi_i^{C,(-)}(r, q) \right|^2 - \left| j_0^C(qr) \right|^2 \]
  \[ + \sum_{j \neq i} \omega_j \int dr S_j(r) \chi_j^{C,(-)}(r, q) \right|^2 \]
  Outgoing B.C. in the i-th channel, \( \omega_j = \) Source weight (\( \omega_j = 1 \))

J. Haidenbauer, NPA 981(‘19)1; R. Lednicky, V. V. Lyuboshits, V. L. Lyuboshits, Phys. At. Nucl. 61(‘98)2950.
To be, or not to be, that is the question.

A. Gal (‘16); M. Oka (‘88)

Table 1. Leading 6q \(L = 0\) dibaryon candidates \([12]\), their \(BB'\) structure and the CM interaction gain with respect of the lowest \(BB'\) threshold calculated by means of Eq. \([2]\). Asterisks are used for the 10\(_f\) baryons \(\Sigma^* \equiv \Sigma(1385)\) and \(\Xi^* \equiv \Xi(1530)\). The symbol \([i,j,k]\) stands for the Young tablauex of the \(\text{SU}(3)_f\) representation, with \(i\) arrays in the first row, \(j\) arrays in the second row and \(k\) arrays in the third row, from which \(\mathcal{P}_f\) is evaluated. The \(\overline{10}\) \(\text{SU}(3)_f\) representation is denoted here \(10^*\).

<table>
<thead>
<tr>
<th>(\Delta(S))</th>
<th>(\text{SU}(3)_f)</th>
<th>(I)</th>
<th>(J^\pi)</th>
<th>(BB') structure</th>
<th>(\Delta(Y_{CM})_{M_0})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>([3,3,0])</td>
<td>10*</td>
<td>0</td>
<td>3+</td>
<td>(\Delta\Delta)</td>
</tr>
<tr>
<td>1</td>
<td>([3,2,1])</td>
<td>8</td>
<td>1/2</td>
<td>2+</td>
<td>(\frac{1}{\sqrt{5}}(N\Sigma^* + 2\Delta\Sigma))</td>
</tr>
<tr>
<td>2</td>
<td>([2,2,2])</td>
<td>1</td>
<td>0</td>
<td>0+</td>
<td>(\frac{1}{\sqrt{8}}(\Lambda\Lambda + 2N\Xi - \sqrt{3}\Sigma\Sigma))</td>
</tr>
<tr>
<td>3</td>
<td>([3,2,1])</td>
<td>8</td>
<td>1/2</td>
<td>2+</td>
<td>(\frac{1}{\sqrt{5}}(\sqrt{2}N\Omega - \Lambda\Xi^* + \Sigma^<em>\Xi - \Sigma\Xi^</em>))</td>
</tr>
</tbody>
</table>