Charmed hadron interactions and correlation functions

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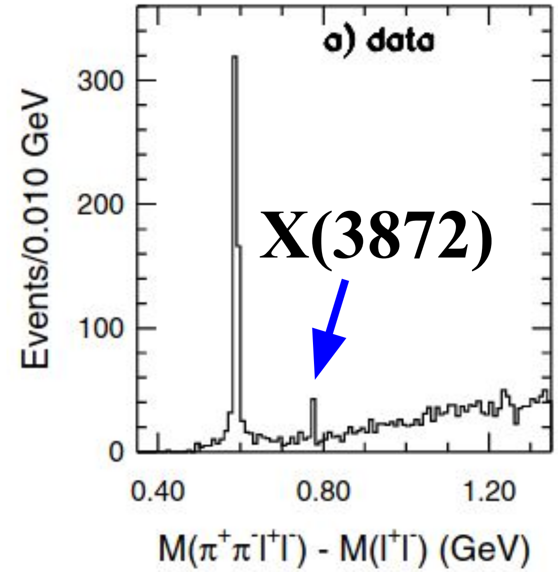
- **Introduction**
- **Charmed hadron interactions –** DD^* and $D\bar{D}^*$
- **Summary**

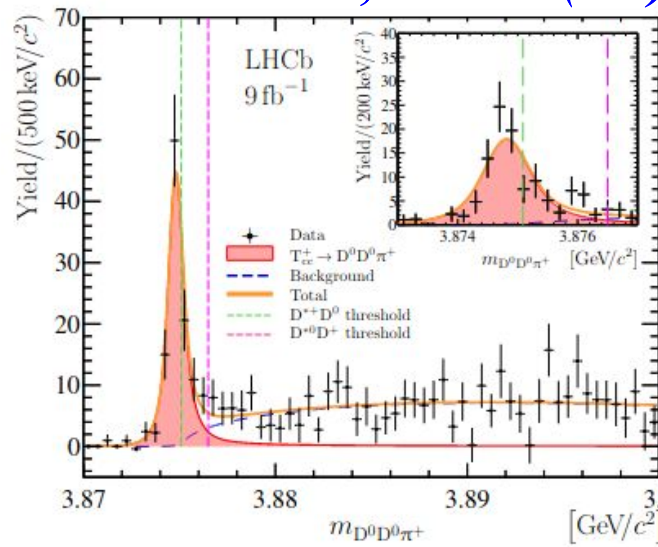
Y. Kamiya, T. Hyodo, A. Ohnishi, arXiv:2203.13814 [hep-ph];

Exotic Hadrons including $c\bar{c}$ / cc / $\bar{c}\bar{c}$

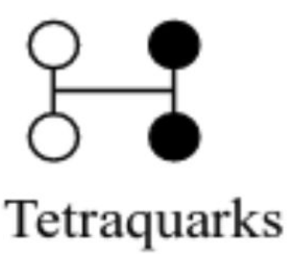
- **Main play ground of exotic hadron physics**
  - X(3872) *Belle ('03)* $c\bar{c}q\bar{q}$
  - Many X,Y,Z states
    Belle, CDF, BaBar, LHCb, CMS, BESIII, ...
  - Charmed pentaquark Pc *LHCb ('15, '19)*
  - Doubly charmed tetraquark state Tcc
    LHCb ('21) $cc\bar{q}\bar{q}$
- **Structure of exotic hadrons**
  - Compact multiquark states
    → "good" [ud] diquark gains energy
  - Hadronic molecules
    → Many exotic states around thresholds
  - Their mixture...

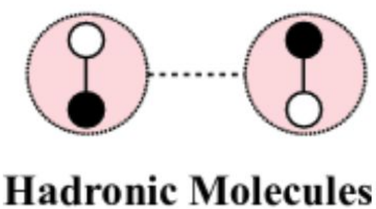


S.K.Choi+[Belle], PRL91, 262001 ('03)



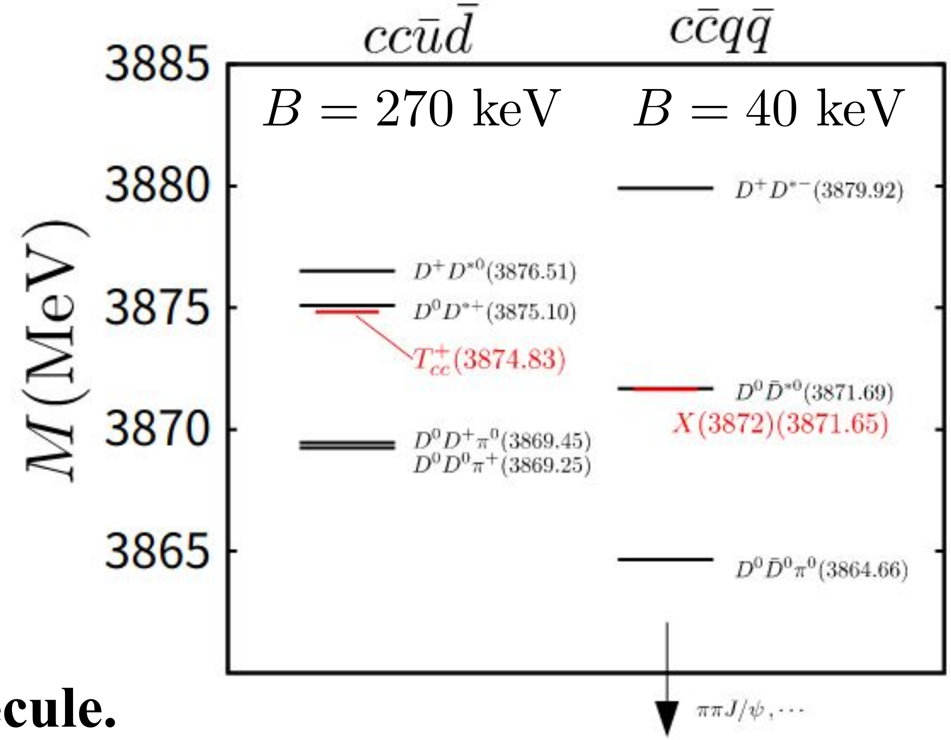
R. Aaji+ [LHCb], 2109.01038, 2109.01056





Compact Tetraquarks or Hadronic Molecules

- Tcc = Compact Tetraquark ?
  Good $[\bar{u}\bar{d}]$ diquark gains energy
  S. Zouzou+('86), ZPC30,457.

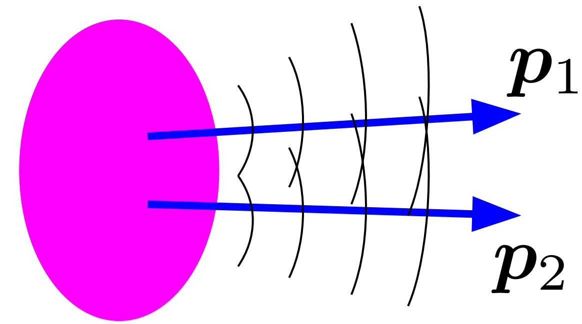


- X(3872)
  - $c\bar{c}$ component ? production cross section *Bignamini+ (0906.0882)*
  - Large yield in Pb+Pb → Molecule ?
    Sirunyan+ [CMS] (2102.13048)
    c.f. Δr/Δp is similar in HIC and molecule.
    ExHIC ('11,'11,'17)

- Hadronic Molecule Conditions
  - Appears around the threshold → OK
  - Have large size $R \simeq 1/\sqrt{2\mu B}$ → Yield
  - Described by the *hh* interaction

How can we access hh int. with charm ?
→ Femtoscopy

Femtoscopic study of hadron-hadron interaction



- **How can we study interactions between short-lived particles ? → Femtoscopy !**
- **Correlation function (CF)**
  - **Koonin-Pratt formula**

    Koonin('77), Pratt+('86), Lednicky+('82)

source fn. **relative w.f.**

$$C(\boldsymbol{p}_1,\boldsymbol{p}_2) = \frac{N_{12}(\boldsymbol{p}_1,\boldsymbol{p}_2)}{N_1(\boldsymbol{p}_1)N_2(\boldsymbol{p}_2)} \simeq \int d\boldsymbol{r}\, \underline{S_{12}(\boldsymbol{r})}\, \underline{|\varphi_{\boldsymbol{q}}(\boldsymbol{r})|^2}$$

- **Source size from quantum stat. + CF (Femtoscopy)**

  Hanbury Brown & Twiss ('56); Goldhaber, Goldhaber, Lee, Pais ('60)
- **Hadron-hadron interaction from source size + CF**
  - **CF of non-identical pair from Gaussian source**

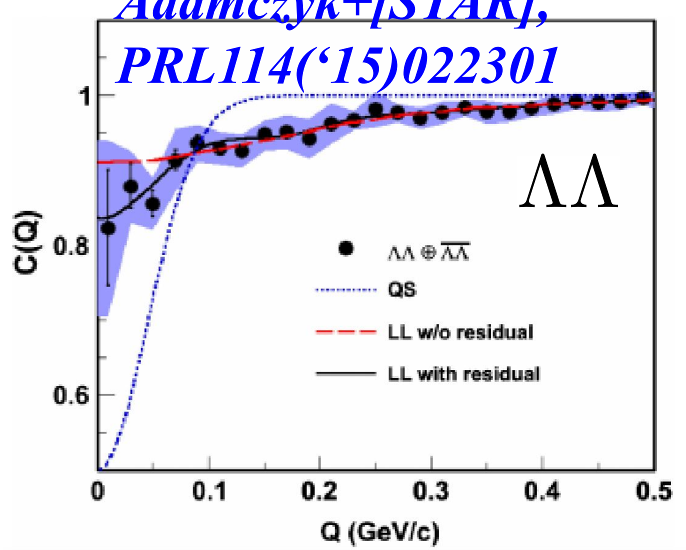
    R. Lednicky, V. L. Lyuboshits ('82); K. Morita, T. Furumoto, AO ('15)

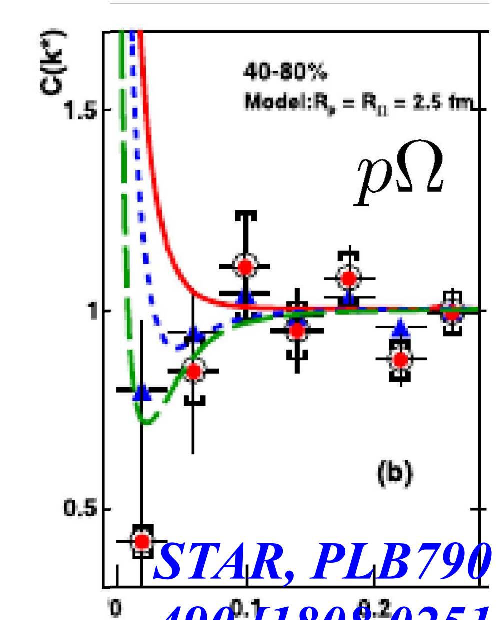
$$C(\boldsymbol{q}) = 1 + \int d\boldsymbol{r}\, S(r)\left\{|\varphi_0(r)|^2 - |j_0(qr)|^2\right\} \quad (\varphi_0 = \text{s-wave w.f.})$$

CF shows how much $|\varphi|^2$ is enhanced → V_{hh} effects !

Measured Correlation Functions (examples)

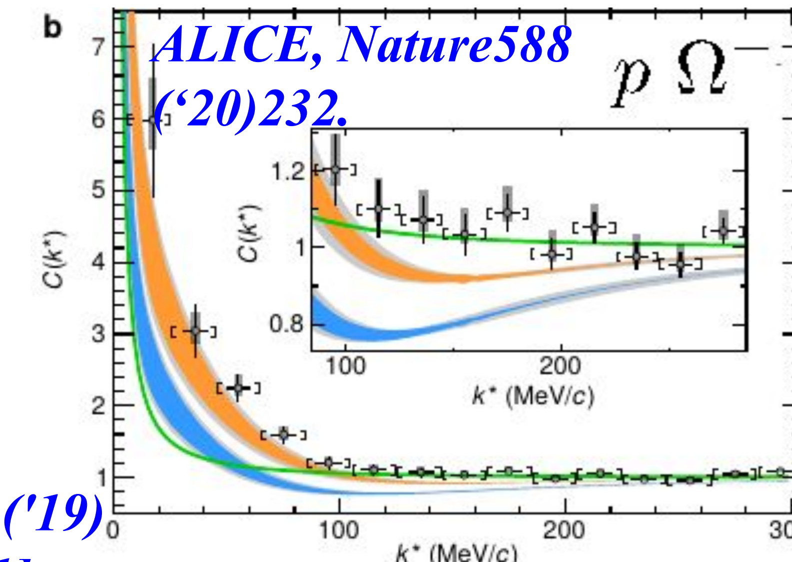
Adamczyk+[STAR], PRL114('15)022301

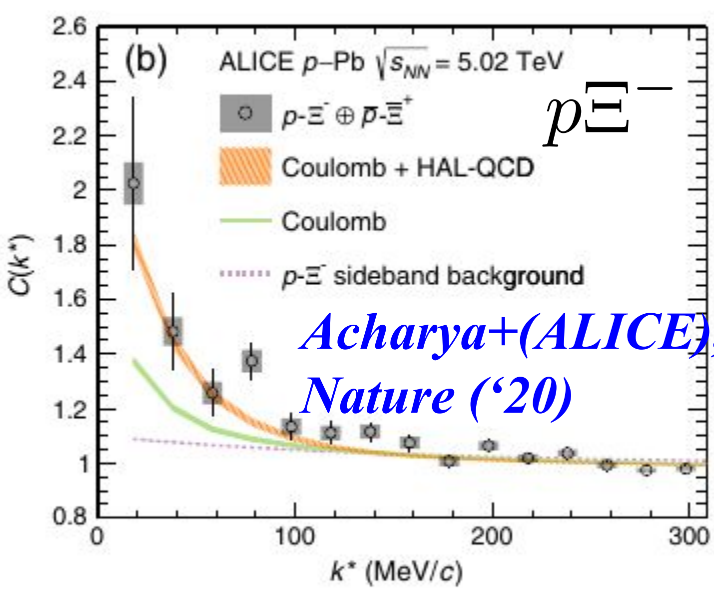




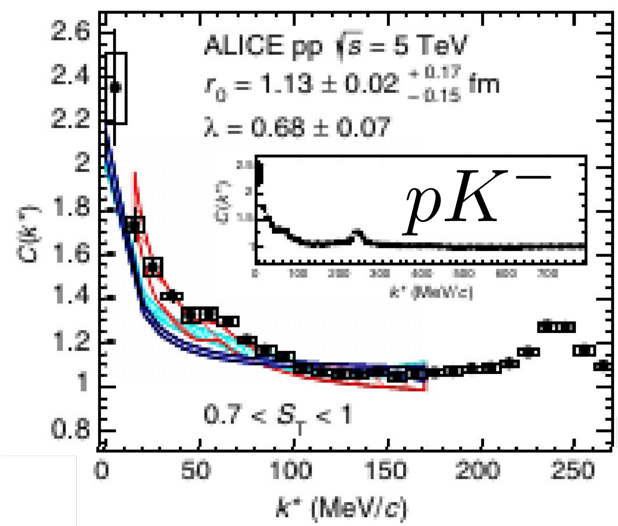
STAR, PLB790 ('19) 490 [1808.02511].

ALICE, Nature588 ('20)232.

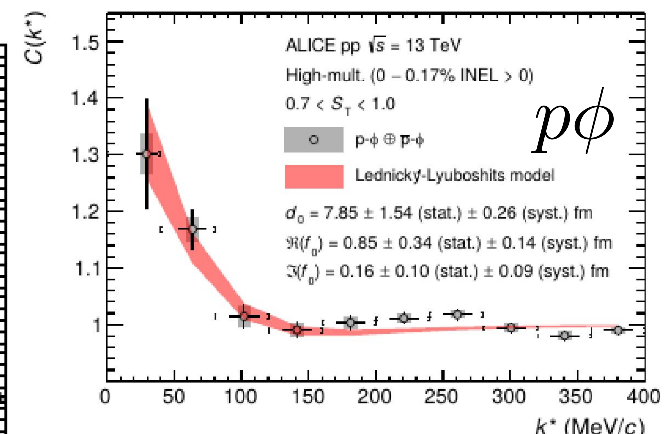




Acharya+(ALICE), Nature ('20)



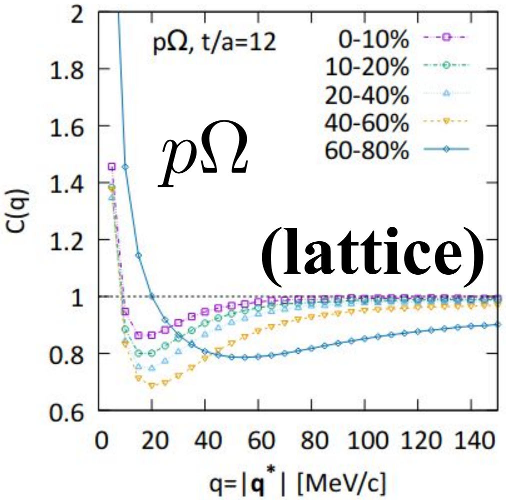
S. Acharya+[ALICE], PRL124('20)092301



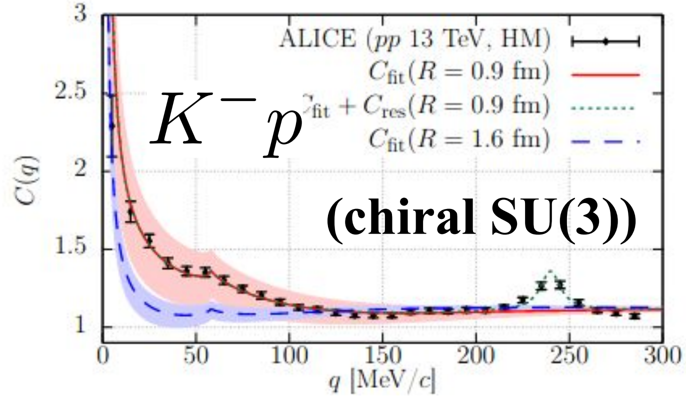
ALICE, 2105.05578

c.f. talk by V. Mantovani Sarti, F. Grosa, N. Agrawal

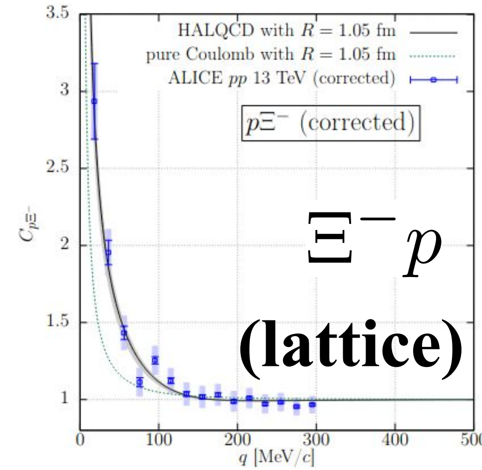
Theoretical femtoscopic study of hh int. (examples)



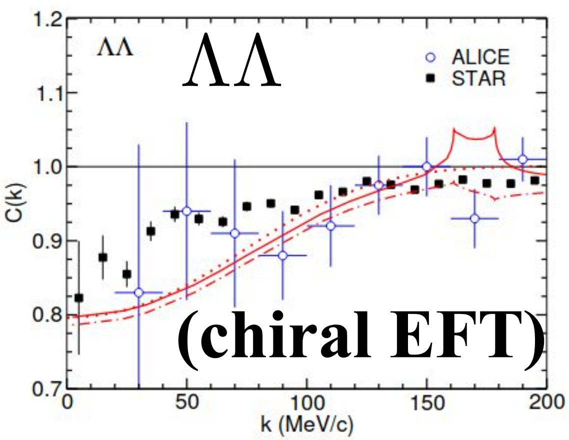
Morita, Gongyo et al.,(1908.05414), Morita, AO, Etminan, Hatsuda (1605.06765)



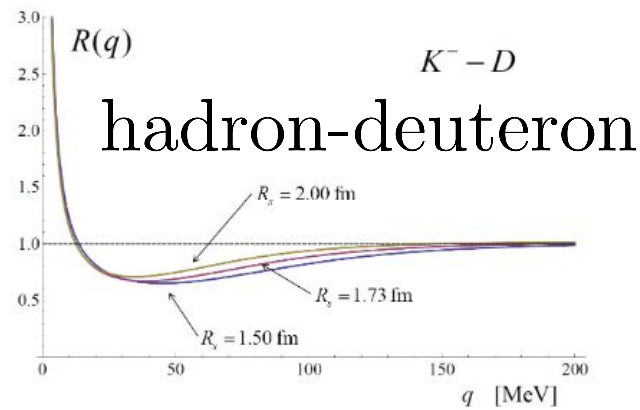
Kamiya+(1911.01041)



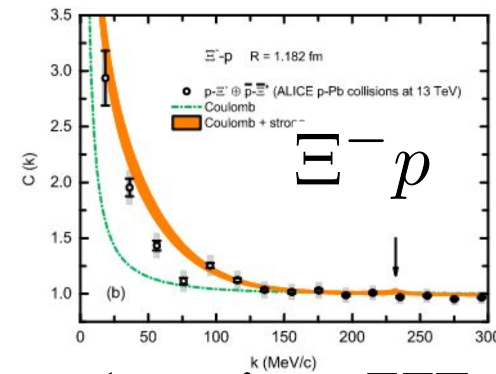
Y.Kamiya, K.Sasaki, et al., (2108.09644)



Haidenbauer(1808.05049), Morita+(1408.6682)



Mrówczyński, Słón (1904.08320, K⁻d), Haidenbauer (2005.05012, Λd), Etminan, Firoozabadi (1908.11484, Ωd), K.Ogata+ (Ξ⁻ d,2103.00100)



(covariant χEFT, S=-2)

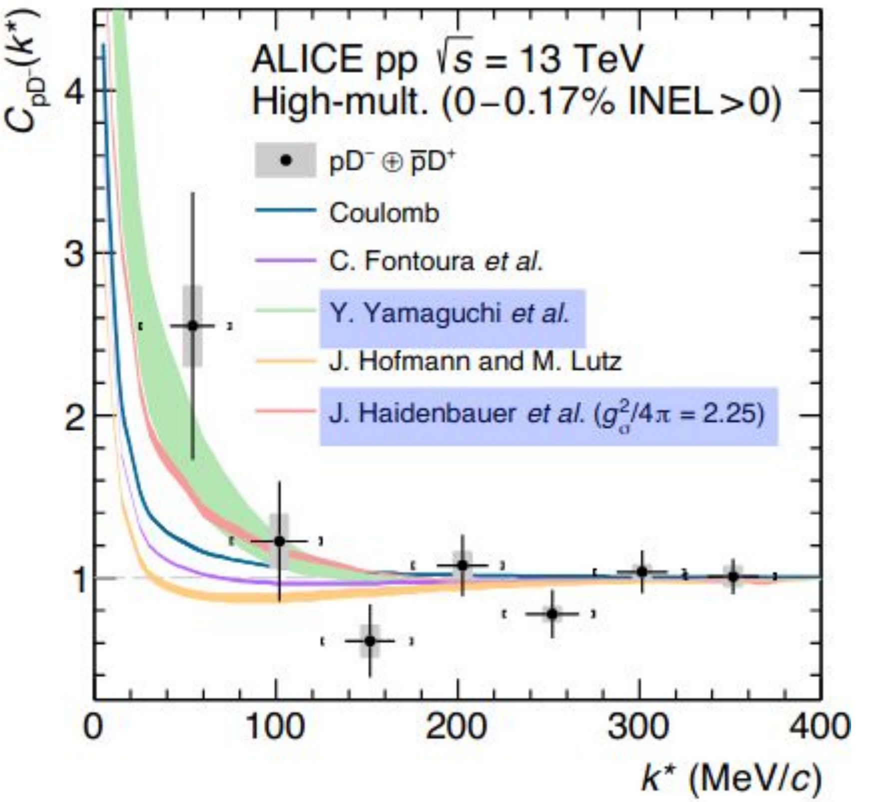
Z.-W. Liu, K.-W. Li, L.-S. Geng (2201.04997)

Femtoscopic study of charmed hadron int.

- "First study of the two-body scattering involving charm hadrons"

Acharya+[ALICE] (2201.05352) [F. Grosa]

  - D^- p corr. func. is measured.
  - Enhanced CF from Coulomb.
  - One range gaussian potential with strength fitted to the I=0 scattering length of the model → attractive potentials are favored



Model	f_0 (I = 0)	f_0 (I = 1)	n_σ
Coulomb			(1.1–1.5)
Haidenbauer et al. [21]			
$-\ g_\sigma^2/4\pi = 1$	0.14	−0.28	(1.2–1.5)
$-\ g_\sigma^2/4\pi = 2.25$	0.67	0.04	(0.8–1.3)
Hofmann and Lutz [22]	−0.16	−0.26	(1.3–1.6)
Yamaguchi et al. [24]	−4.38	−0.07	(0.6–1.1)
Fontoura et al. [23]	0.16	−0.25	(1.1–1.5)

[21] Haidenbauer+(0704.3668) (weakly / mildly attractive (I=0))
[22] Hofmann, Lutz (hep-ph/0507071) (repulsive (I=0))
[23] Fontoura+(1208.4058) (weakly attractive (I=0))
[24] Yamaguchi, Ohkoda, Yasui, Hosaka (1105.0734) (att., w/ bound state (I=0))

To be bound or not to be bound

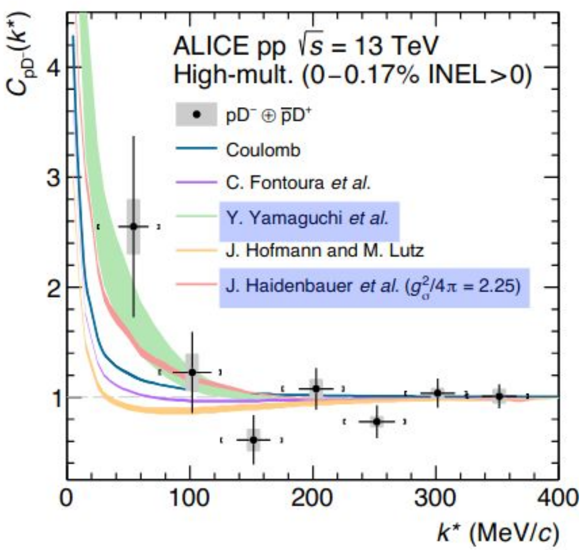
- **When there is a bound state, CF shows interesting dependence on the source size and relative momentum.**
- **D^- p corr. func. shows the behavior with a bound state, and the best fit parameter set (R, a_0) is in the bound region. (If bound, it is the first weakly decaying pentaquark state.)**

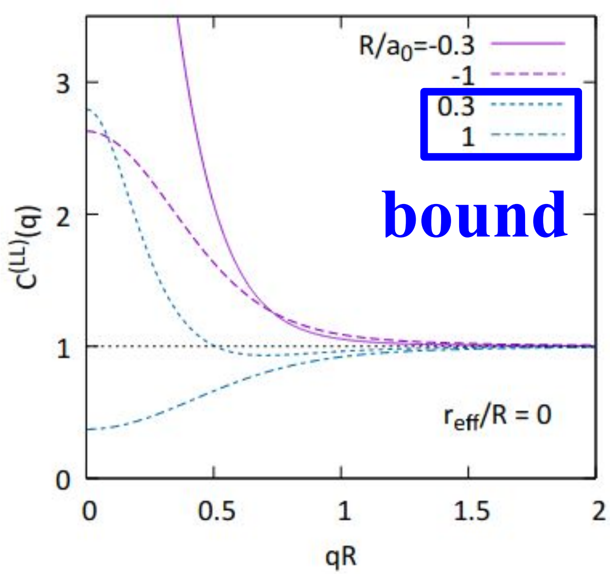
$$k \cot\delta = -\frac{1}{a_0} + \frac{1}{2} r_{\mathrm{eff}} k^2 + \mathcal{O}(k^3)$$

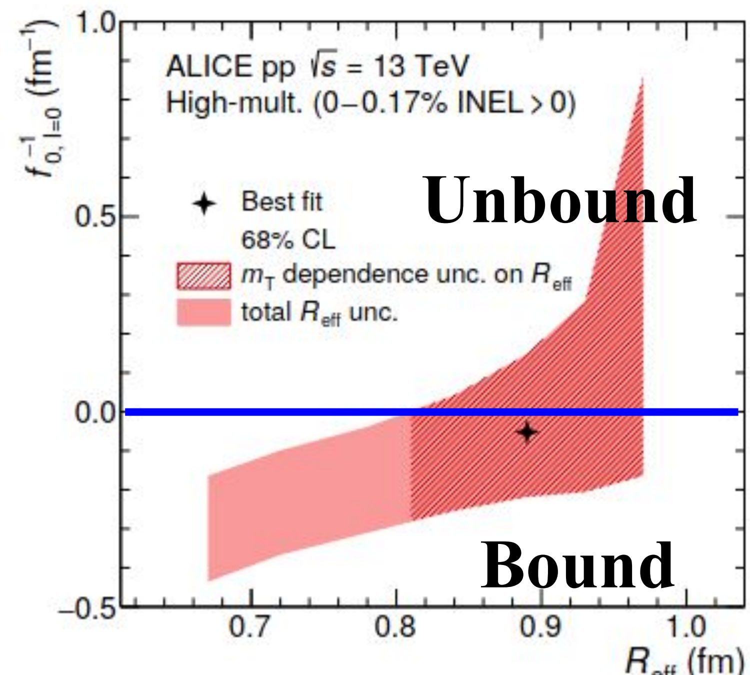
(Nuclear and atomic phys. convention.)

$$k \cot\delta = +\frac{1}{f_0} + \frac{1}{2} r_{\mathrm{eff}} k^2 + \mathcal{O}(k^3)$$

(High-E. phys. convention.)







Morita+(1908.05414) *[ALICE] (2201.05352)*

Femtoscopic study of charm hadron interactions

- DD^* and $D\bar{D}^*$ correlation func. -

Femtoscopic study of charmed hadron int. (2)

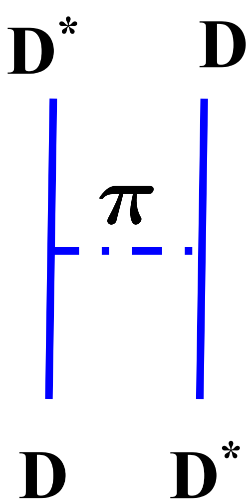
- DD^* and $D\bar{D}^*$ correlation functions. *Kamiya, Hyodo, AO (2203.1381)*
  - Related with Tcc and X(3872)
  - DD^* and $D\bar{D}^*$ interactions

$$V = \frac{1}{2}\begin{pmatrix} V_{I=0} + V_{I=1} & V_{I=0} - V_{I=1} \\ V_{I=0} - V_{I=1} & V_{I=0} + V_{I=1} \end{pmatrix}$$

  - I=0: One range gaussian, strength fitted to the mass
  - I=1: ignored
  - Range = one pion exchange *Yasui, Sudoh (0906.1452)*
  - Strength is fitted to the pole mass.

$$\{D^0\bar{D}^{*0}\} = (D^0\bar{D}^{*0} + \bar{D}^0 D^{*0})/\sqrt{2}\ (C = +1)$$

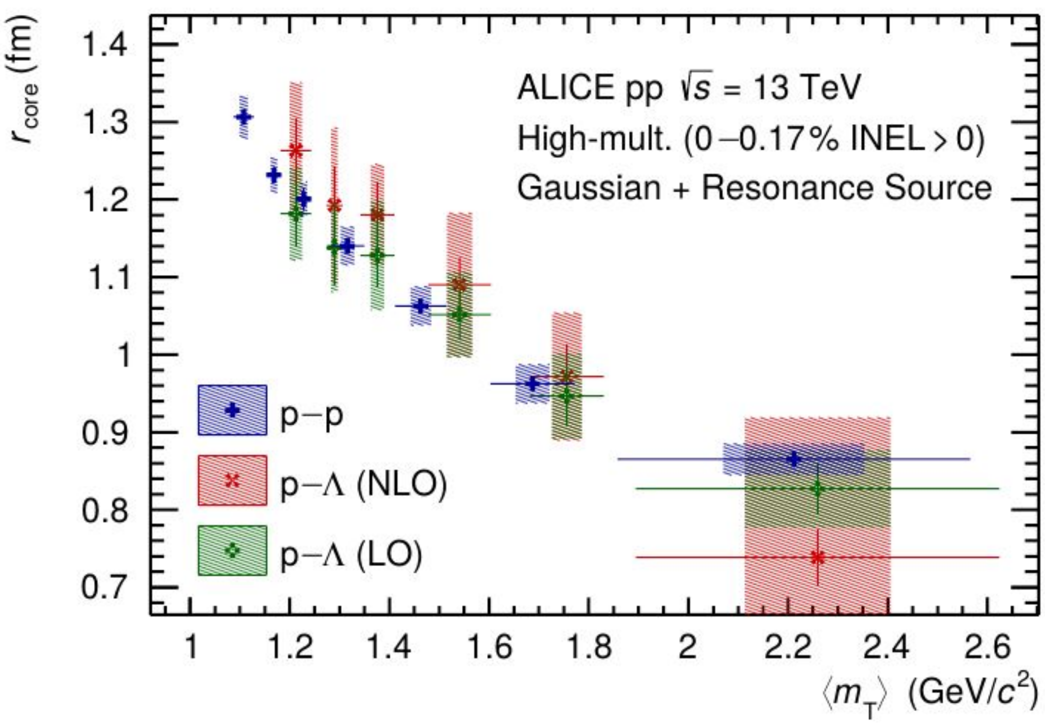
$$\{D^+D^{*-}\} = (D^+D^{*-} + D^-D^{*+})/\sqrt{2}\ (C = +1)$$



DD^*	V_0 [MeV]	$a_0^{D^0D^{*+}}$ [fm]	$a_0^{D^+D^{*0}}$ [fm]
	$-36.569 - i1.243$	$-7.16 + i1.85$	$-1.75 + i1.82$
$\{D\bar{D}^*\}$	V_0 [MeV]	$a_0^{\{D^0\bar{D}^{*0}\}}$ [fm]	$a_0^{\{D^+D^{*-}\}}$ [fm]
	$-43.265 - i6.091$	$-4.23 + i3.95$	$-0.41 + i1.47$

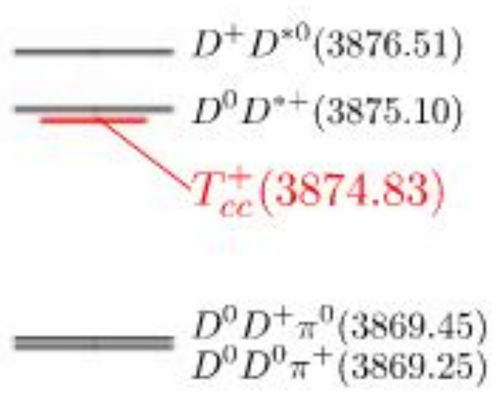
We are sorry, but we use a Gaussian Source !

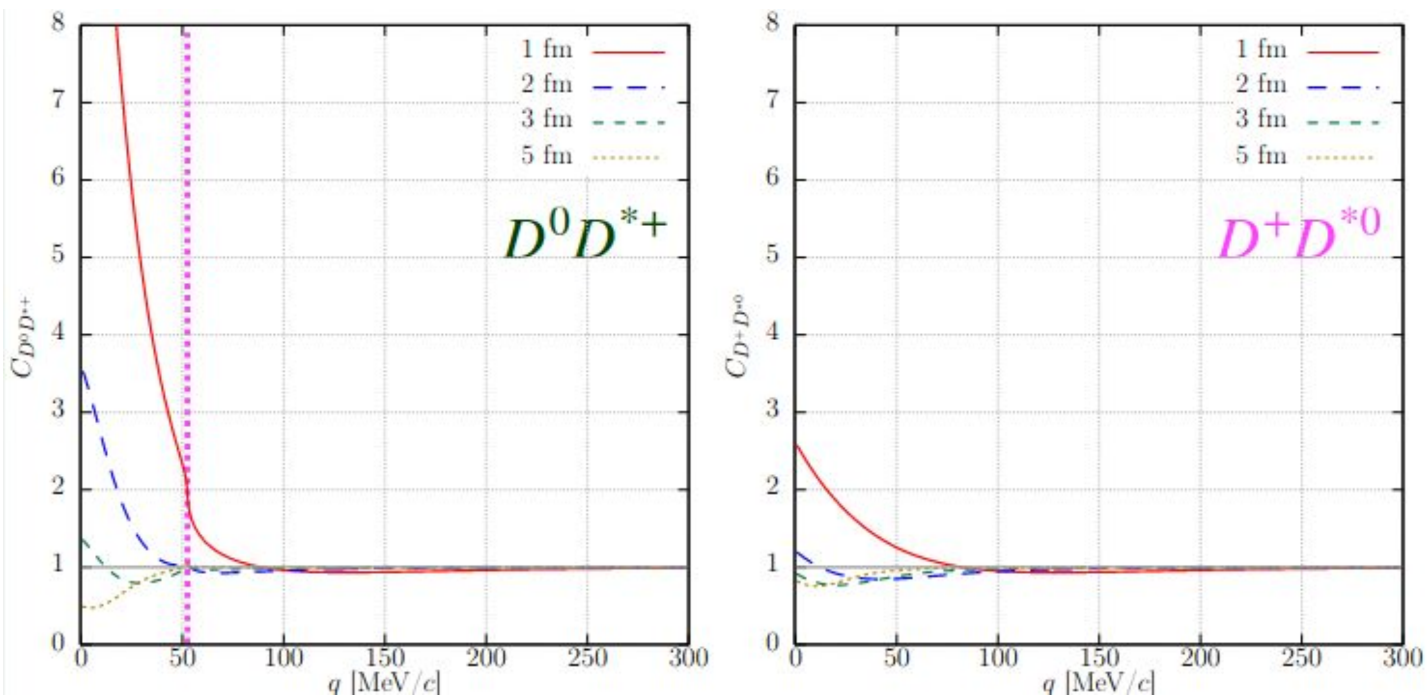
- **Calculating HBT radius in dynamical models is not easy**
  M.A.Lisa, S.Pratt, R.Soltz, U.Wiedemann, Ann.Rev.Nucl.Part.Sci.55('05)357 [nucl-ex/0505014]; S. Pratt, PRL102('09)232301 [0811.3363].
- **and a Gaussian source seems to work at the current precision of hh interaction studies.**
  S. Acharya+[ALICE], PLB811('20)135849.
  - **primary (universal ?)+ decay of short-lived resonances ~ eff. Gaussian**
  - **Flow and source geometry effects are seen in CF, but the uncertainty of hh int. is the largest.**



$D^0 D^{*+}$ and $D^+ D^{*0}$ Correlation Functions

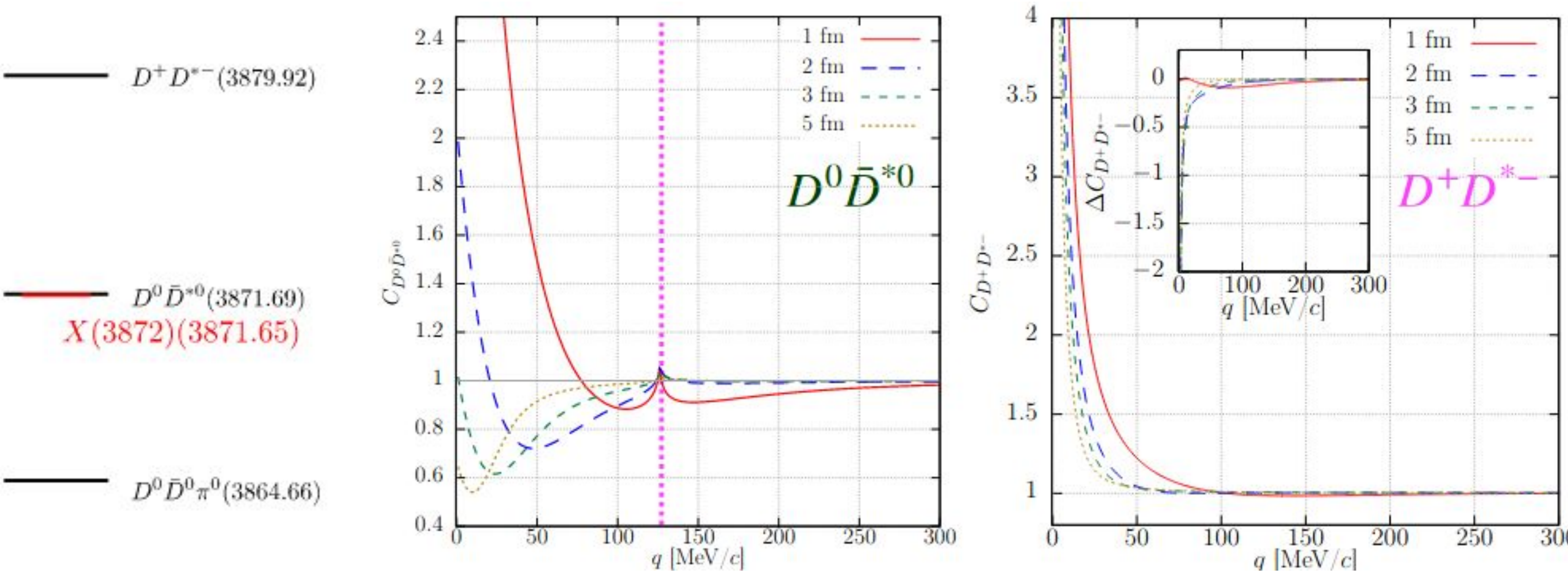
- **For small source (R= 1 fm)**
  C(q) > 8 for the lower channel (D^0D^{*+}) (Very strong)
  C(q) ~ 2.5 for upper channel (D^+D^{*0}) (strong)
- **For large source (R=5 fm), CF show a dip**
- **Strong enhancement for small source, dip for large source**
  → Characteristic dependence with a bound state (Tcc)
- **Cusp is not significant**





$D^0\bar{D}^{*0}$ and D^+D^{*-} Correlation Functions

- $C(D^0\bar{D}^{*0})$: **Strong enh. for small source, dip for large source**
  → Characteristic dependence with a bound state (X(3872))
- $C(D^+D^{*-})$: **Coulomb dominant**
- **Cusp may be observed for small size**



Tcc and X(3872) structure

- **Hadronic molecule structure is assumed**
  → Eigenmomentum $k \simeq -i/a_0\,,\ a_0 \simeq R = 1/\sqrt{2\mu B}$
- **What happens when multiquark state mixes ?**
  → Deviation from weak binding relation (X=compositeness)

Weinberg, Phys. Rev. 137, B672 (1965), Hyodo, Jido, Hosaka (1108.5524), Kunigawa, Hyodo (2112.00249)

$$a_0 = R\left[\frac{2X}{1+X}\right] + \mathcal{O}(R_{\rm typ})$$

$$\left[R_{\rm typ} = \max(m_\pi^{-1}, r_{\rm eff}), R = 1/\sqrt{2\mu B}\right]$$

- **Smaller scattering length in DD* may signal the *genuine* tetraquark nature of Tcc.**

$$f = \frac{1}{k\cot\delta - ik} \simeq \frac{1}{1/a_0 - ik}$$

(high-energy phys. convention)

Summary

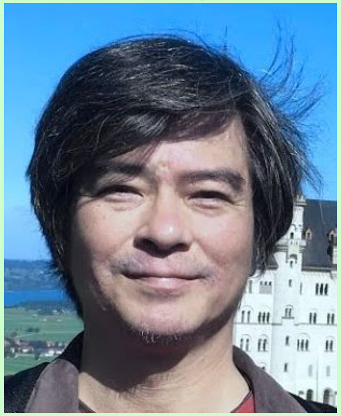
- **Two-particle correlation functions are useful to deduce**
  - **Scattering length**
  - **Existence of a bound state**
  - **and hopefully the compositeness**
- **Charm hadron interactions are within the reach.**
  - **D^- p correlation function has been measured, and the data favor attractive interaction.** ***ALICE (2201.05352)***
  - DD^* and $D\bar{D}^*$ **correlation functions are predicted to reflect the existence of bound states (Tcc and X(3872)) by using simple potentials fitting to the mass and width.** ***Y. Kamiya, T. Hyodo, AO (2203.13814)***
  - **Precise measurement of the correlation function will constrain the scattering length, which may tell us the structure of exotic hadrons.**

Thank you for attention !

Y. Kamiya

T. Hyodo

AO



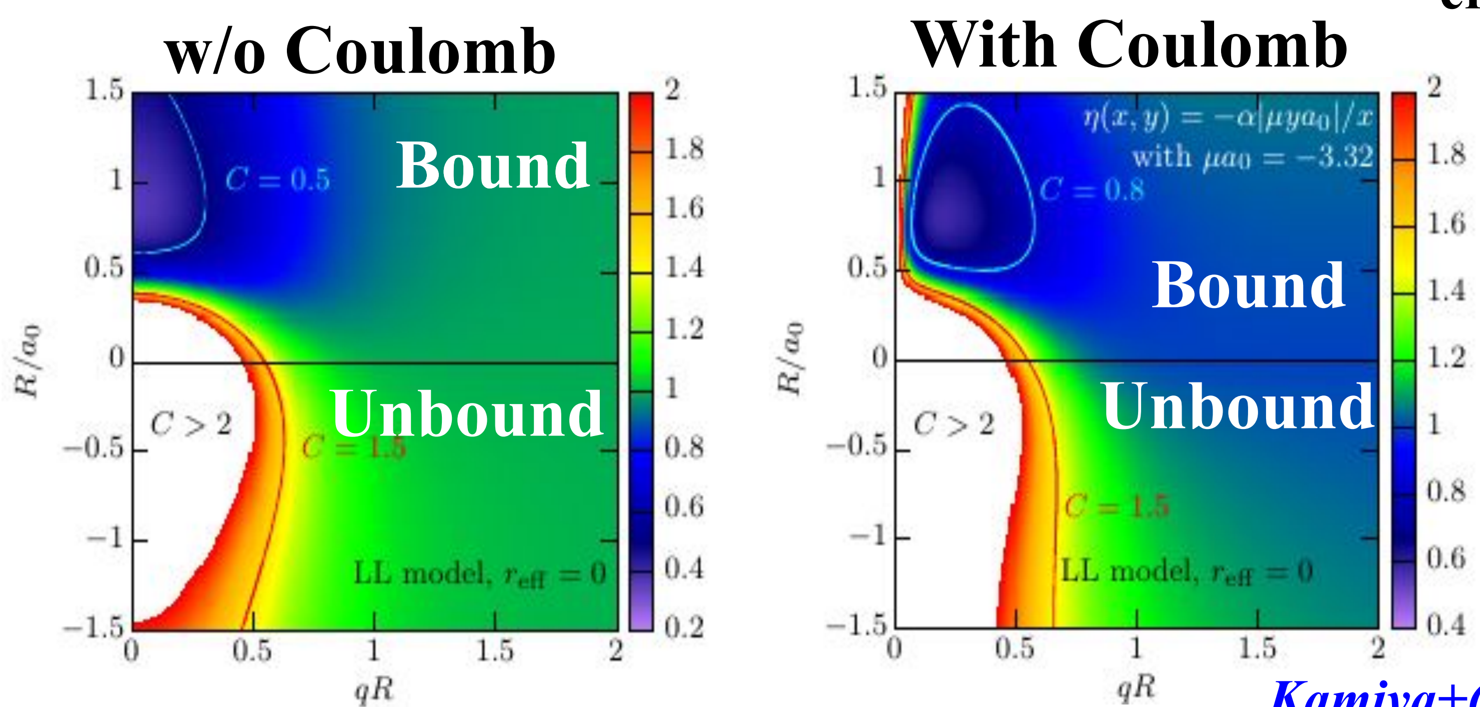
R Dependence of Correlation Function

- **Source size (R) dependence of C(q) is helpful to deduce the existence of a bound state.**

  Morita+('16, '20), Kamiya+('20), Kamiya+(2108.09644)

  - **With a bound state, C(q) is suppressed at small q when $R \sim |a_0|$. (w.f. has a node at $r \sim |a_0|$ with a bound state.)**
  - **Qualitative understanding by the analytic model (LL formula)**

    [Lednickey, Lyuboshits ('82)] **with the zero range approx. (r_{eff}=0)**



Kamiya+(2108.09644)

Coupled-Channel Correlation Function

- **Correlation function with CC effects (KPLLL formula)**
  → sum of j-th channel contributions leading to j=1 with outgoing momentum *q*

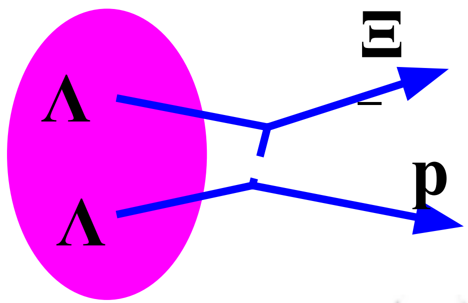
  Lednicky, Lyuboshits, Lyuboshits ('98); Haudenbauer ('19)

$$C(\boldsymbol{q}) = \sum_j \omega_j \int d\boldsymbol{r} S_j(\boldsymbol{r}) |\Psi_j^{(-)}(\boldsymbol{r})|^2$$

$$\Psi_j^{(-)}(\boldsymbol{r}) = [e^{i\boldsymbol{q}\cdot\boldsymbol{r}} - j_0(qr)]\delta_{1j} + \psi_j^{(-)}(r)$$

$$\psi_j^{(-)}(q) \propto e^{-iqr}/r \text{ or } e^{-\kappa r}/r \ (r \to \infty)$$

(No Coulomb case)



Outgoing Mom. q in channel j=1

$$\omega_j S_j(\boldsymbol{r}) |\Psi_j^{(-)}(\boldsymbol{r})|^2$$

Source weight (ω_j) — **Normalized Source fn.** ($S_j(\boldsymbol{r})$)

  - **Effects of coupled-channel, strong & Coulomb pot., and threshold difference are taken into account in the charge base, $p\Xi^-$, $n\Xi^0$, ΛΛ.**
    Y. Kamiya+, PRL('20, K^-p)
  - **Source size (R) and source weight (ω_j) need to be determined.**

A New Insight from CMS: Exotic/Normal Ratio

- **ExHIC index = Coalescence / Statistical Ratio**

$$R_h^{\rm CS} = \frac{\text{Yields in Coalescence}}{\text{Yields in Statistical model}}$$

- **CMS index = Exotic / Normal Ratio**

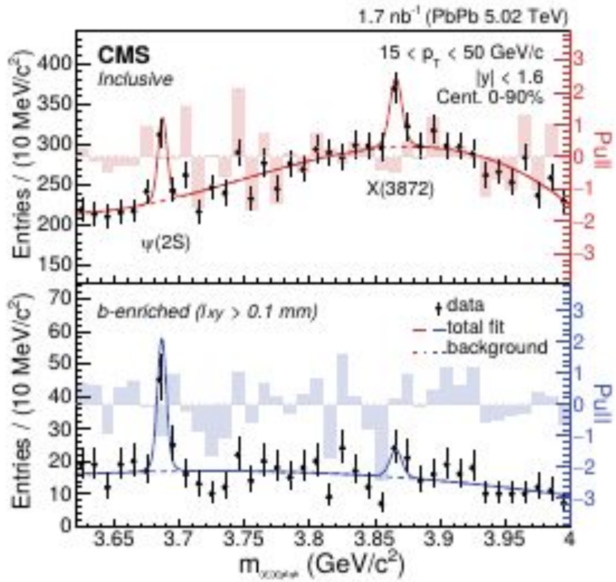
  Sirunyan+ [CMS], arXiv:2102.13048

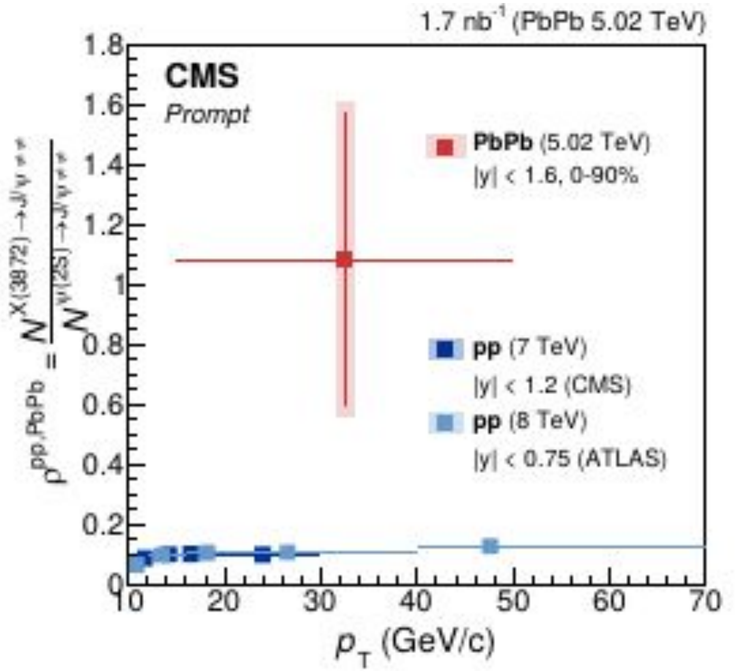
$$\rho_{\rm exo/nor} = \frac{N(\text{Exotic hadron candidate})}{N(\text{Normal hadron})}$$

  - **X(3872) / ψ(2S) ratio in pp and PbPb collisions.**

$$\rho_{X/\psi}(\text{PbPb}) = 1.08 \pm 0.49(\text{stat.}) \pm 0.52(\text{syst.})$$
$$\rho_{X/\psi}(pp) \simeq 0.1$$

ExHIC prediction is found to be (qualitatively) true !

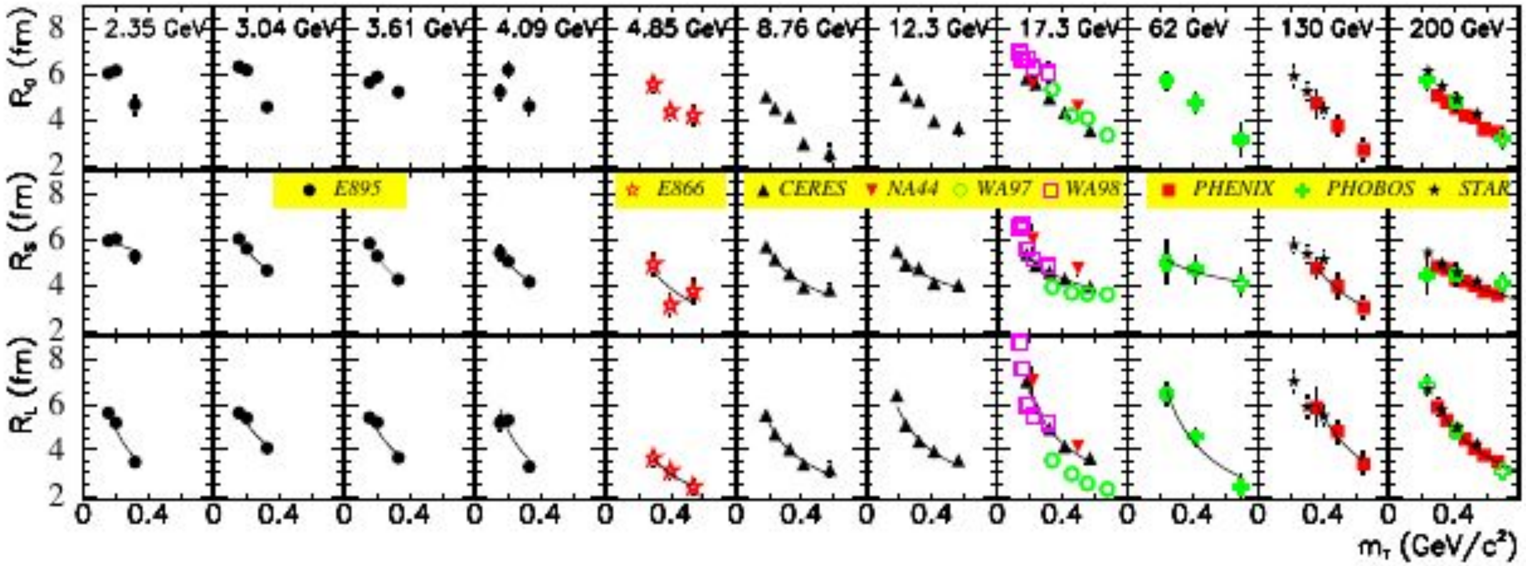




State-of-the-art Femtoscopy of radii

- **Systematic measurement of 3D HBT radii (side, out, long)**

M. A. Lisa, S. Pratt, R. Soltz, U. Wiedemann, Ann.Rev.Nucl.Part.Sci. 55 (2005) 357-402.



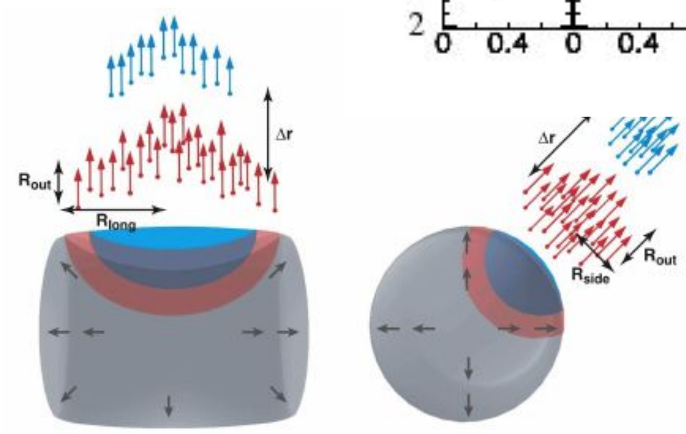
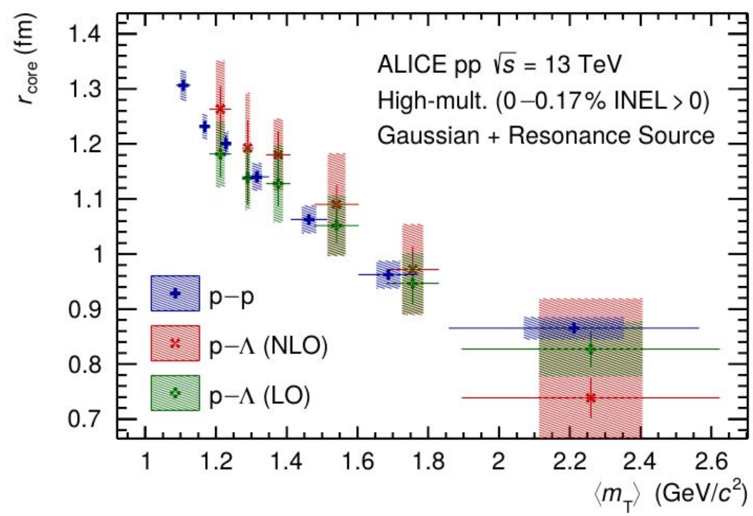


Figure 3: because particles with heavier masses have smaller thermal velocities, their source volumes are more strongly confined by collective flow. For longitudinal flow (*left panel*) this results in smaller values of $R_{\rm long}$ for particles with higher $m_T = \sqrt{m^2 + p_T^2}$. For radial flow (*right panel*) this confines heavier particles toward the surface, which results in both a reduced volume and an offset Δr in the outward direction.



S. Acharya+[ALICE], PLB811('20)135849

We are sorry for using a Gaussian Source !

- **Calculating HBT radius in dynamical models is not easy (HBT puzzle).**
  - *M.A.Lisa, S.Pratt, R.Soltz, U.Wiedemann, Ann.Rev.Nucl.Part.Sci.55('05)357 [nucl-ex/0505014];*

choices then tends to exceed the number of experimental constraints. In fact, all the model results that we review in the current subsection remain unsatisfactory with this respect: They either deviate significantly from femtoscopic data, or they reproduce these data at the price of missing other important experimental information. In particular, there is so far no dynamically consistent model that reproduces quantitatively both the systematic trends discussed in Section 4 and the corresponding single inclusive spectra. In this situation, the scope of this subsection is

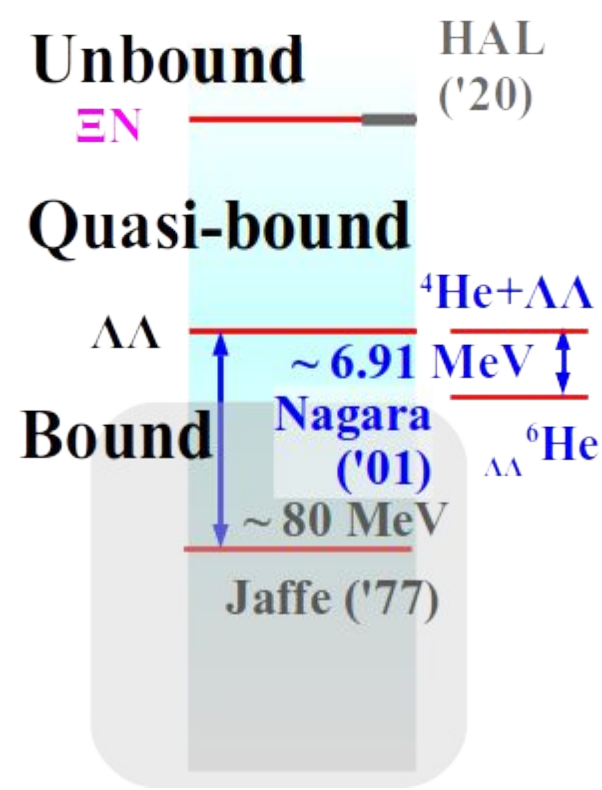
  - *S. Pratt, PRL102('09)232301 [0811.3363].*

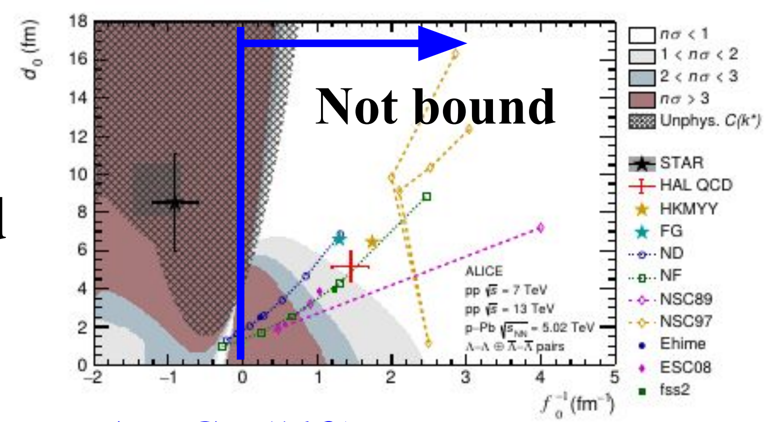
Two particle correlation data from the BNL Relativistic Heavy Ion Collider have provided detailed femtoscopic information describing pion emission. In contrast with the success of hydrodynamics in reproducing other classes of observables, these data had avoided description with hydrodynamic-based approaches. This failure has inspired the term "HBT puzzle," where HBT refers to femtoscopic studies which were originally based on Hanbury Brown–Twiss interferometry. Here, the puzzle is shown to originate not from a single shortcoming of hydrodynamic models, but the combination of several effects: mainly prethermalized acceleration, using a stiffer equation of state, and adding viscosity.

How about afterburner effects ?

Impact of S=-2 Baryon-Baryon Interactions (1)

- Is "H(uuddss)" bound, unbound, or quasi-bound ?
- It is plausible not to be bound below ΛΛ.
  - Bound H in the $SU(3)_f$ limit.
    Bag model: Jaffe, PRL38(1977)195.
    LQCD: HALQCD('11), NPLQCD('11,'13), Mainz ('19).
  - But no discovery of bound H.
    No $M(\Lambda p \pi^-)$ peak; ΛΛ hypernucl.:Takahashi+ ('01);
    Femtoscopy:STAR('15); ALICE('19); Morita+('15).
- Quasi-bound state below NΞ or Unbound ?
  - Resonance "H" from (K^-, K^+) ?
    KEK-E522 ('07)
  - LQCD at almost physical m_q → Unbound
    HAL QCD('20).



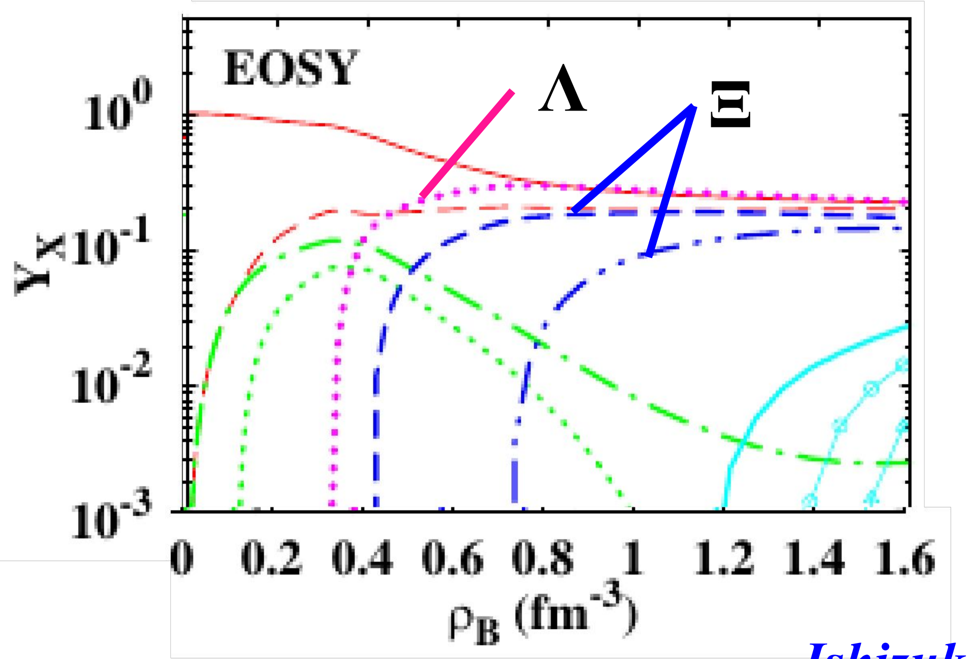


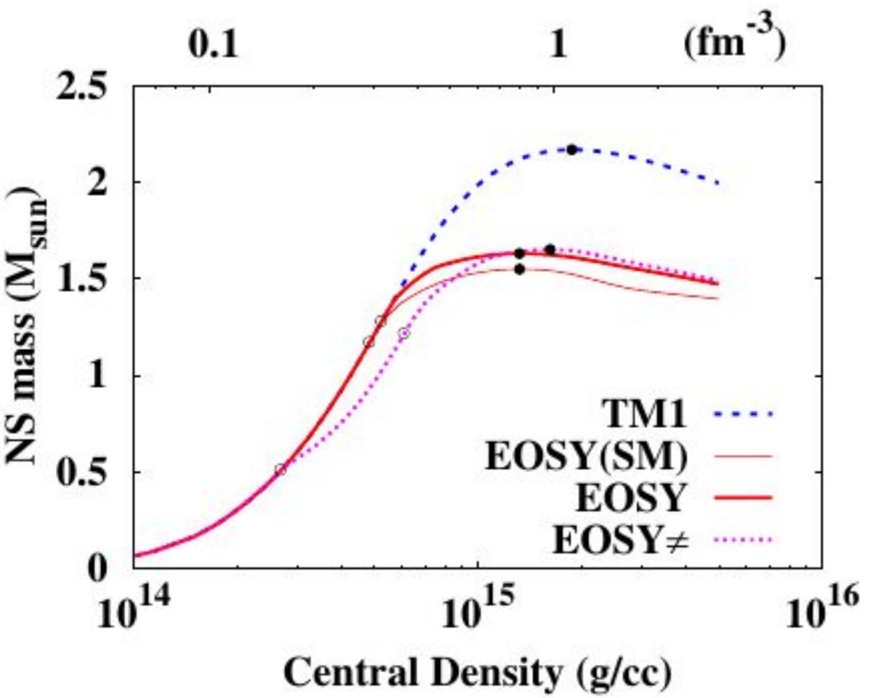
ALICE('19)

Impact of S= -2 Baryon-Baryon Interactions (2)

- **ΛΛ and NΞ interactions are relevant to "Hyperon Puzzle"**
  - **Λ and Ξ are predicted to appear at (2-4)ρ_0, and softened EOS cannot support 2 $M_\odot$ neutron stars.**
    → Repulsive YNN interactions, Quark Matter, Modified Gravity ?
  - **Precise ΛN, ΛΛ, NΞ, and ΛNN interactions need to be known.**
    - **Repulsive ΞN interaction (I=1) may help support 2 $M_\odot$ NS**

Weissborn et al., NPA881 ('12) 62.





Ishizuka, AO, Tsubakihara, Sumiyoshi, Yamada ('08)

S=-2 Baryon-Baryon Interactions

- **Theoretical Approaches**
  - **Phenomenological (Nijmegen, Jülich, Ehime, Quark model, …)**
  - **Chiral EFT** ***[Haidenbauer, Meissner, Petschauer ('16); Li, Hyodo, Geng ('18)]***
  - **Lattice QCD** ***[Sasaki+ [HAL QCD] ('20)]***
- **Experimental Information**
  - **Double Λ and Ξ hypernuclei**

    Takahashi+('01); Nakazawa+('15); Hayakawa+[E07]('21); Yoshimoto+[E07]('21).
  - **Femtoscopic study of hadron-hadron interactions**

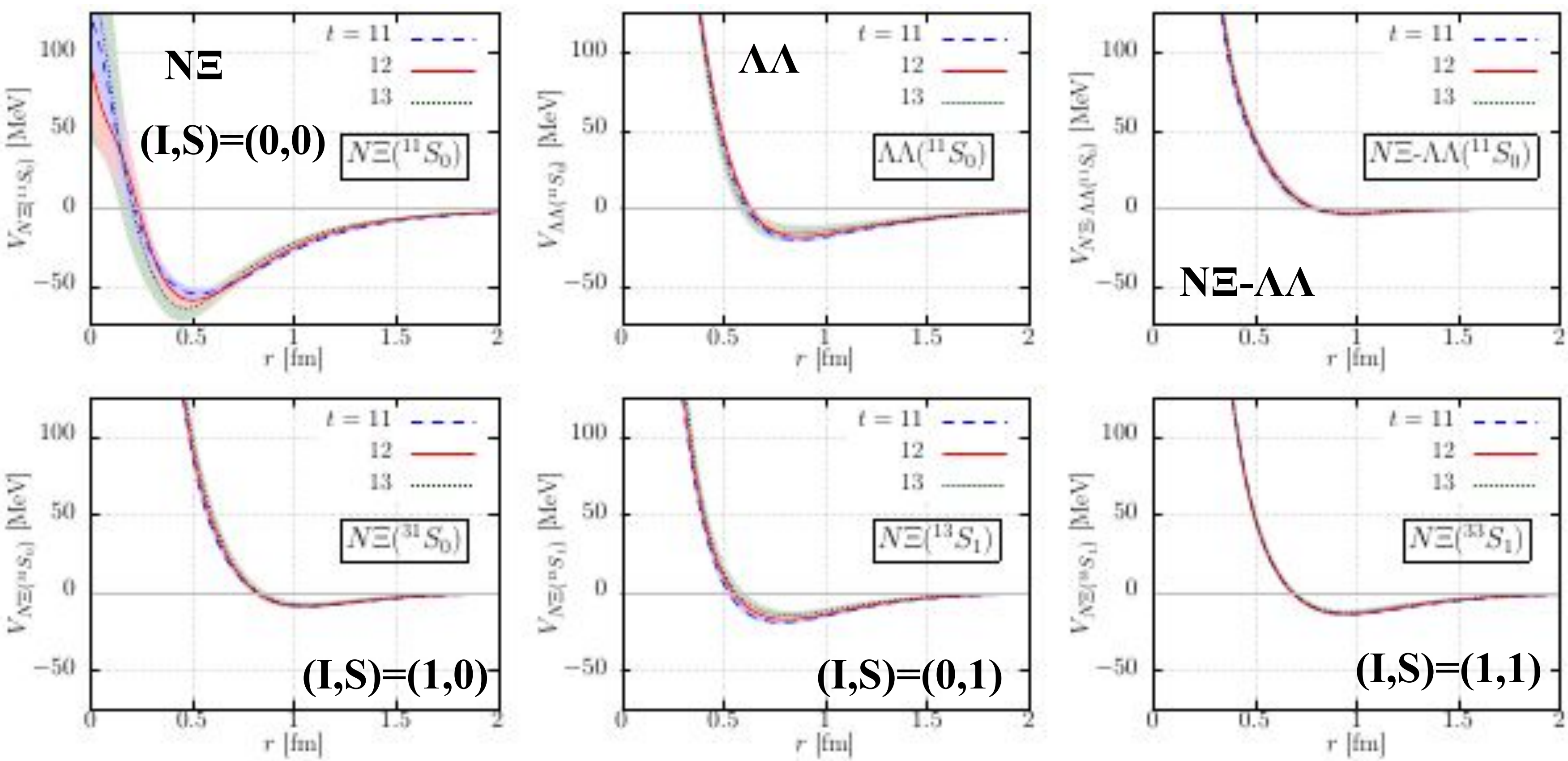
    [See also Valentina Mantovani Sarti (Wed), Laura Šerkšnytė (Sun)]

    Adamczyk+[STAR]('15, ΛΛ); Acharya+[ALICE]('19(ΛΛ), '19(NΞ), '20(NΞ));
    Morita, Furumoto, AO ('15, ΛΛ); Hatsuda, Morita, AO, Sasaki ('17, NΞ);
    Haidenbauer ('19, ΛΛ-NΞ); Haidenbauer+ ('20).

Coupled-channel NΞ-ΛΛ potential and correlation functions

$N\Xi$-$\Lambda\Lambda$ Potential from Lattice QCD

- **NΞ-ΛΛ potential at almost physical quark masses (m_π=146 MeV) by HAL QCD Collaboration**
  K. Sasaki et al. [HAL QCD], NPA 998 ('20) 121737 (1912.08630)
  - **Significant attraction in (I,S)=(0,0) of NΞ.**
  - **Weak attraction in ΛΛ (Coupling with NΞ causes ΛΛ attraction).**



$N\Xi$-$\Lambda\Lambda$ Potential from Lattice QCD

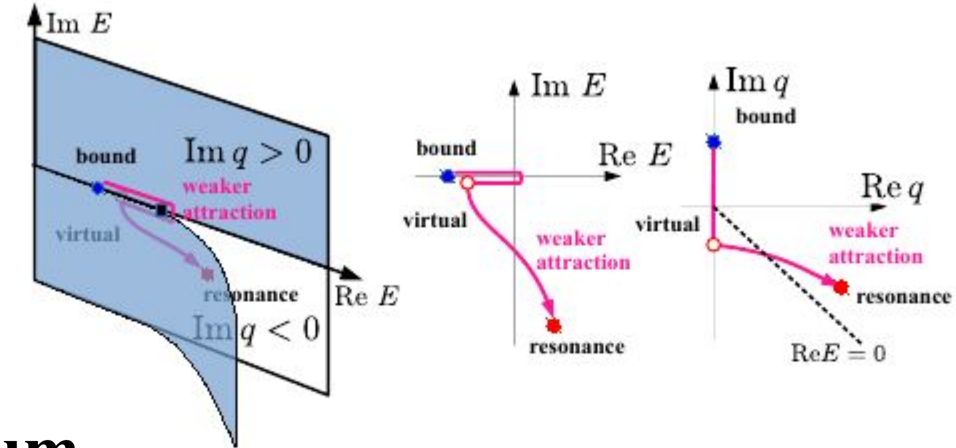
- **Low-energy scattering parameters**
  - **Nuclear physics convention** $$k \cot \delta = -\frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} k^2 + \mathcal{O}(k^2)$$
  - **Re (a_0) < 0 → No bound state in ΛΛ-NΞ systems.**
    (except for Ξ^- atom)
- **There is a virtual pole around the NΞ threshold**
  (3.93 MeV below $n\Xi^0$ threshold)
  on the irrelevant Riemann sheet, (+, –, +) [quasi-bound → (–,+,+)]

$$E_{\text{pole}} = 2250.5 - i0.3 \text{ MeV}$$

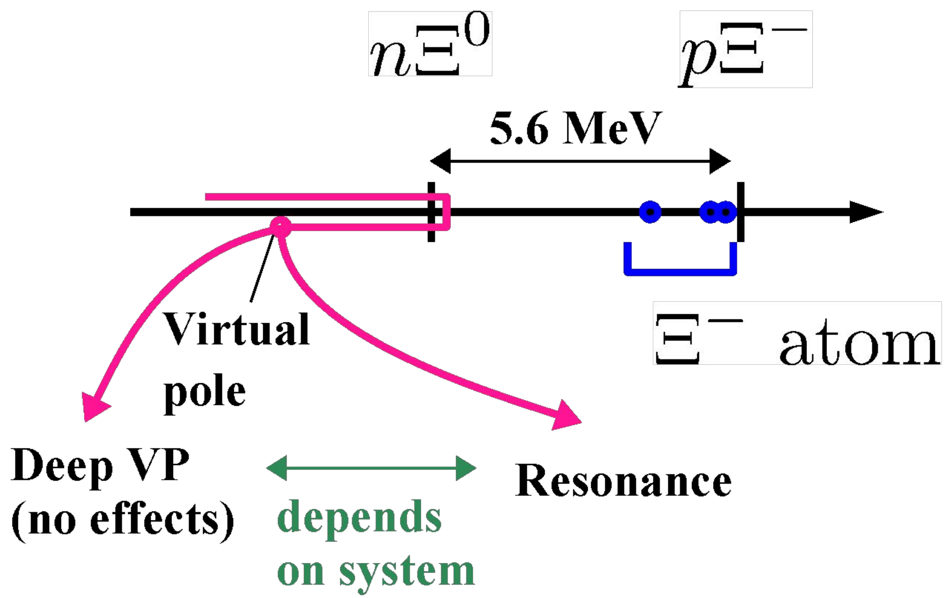
sign of Im(eignen momentum)

Virtual Pole

- Virtual pole (single channel case)
  = Eigen energy of the pole is below the threshold,
  but the wave function diverges at r → ∞.
  (Imaginary part of eigen momentum is negative, exp(iqr)/r → ∞.)



- Lattice BB potential at almost physical quark masses (HAL QCD)
  - With Coulomb potential and threshold mass difference, virtual pole appears on (+,-,+) Riemann sheet (w.f. of $n\Xi^0$ channel diverges).
  - Atomic states are well separated from VP. ($\mu\alpha^2/2n^2$=14.6 keV/n^2)



Coupled-Channel Correlation Function

- **Correlation function with CC effects (KPLLL formula) –> sum of j-th channel contributions leading to j=1 with outgoing momentum *q***

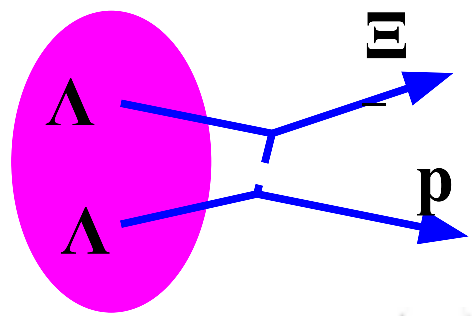
Lednicky, Lyuboshits, Lyuboshits ('98); Haudenbauer ('19)

$$C(\boldsymbol{q}) = \sum_j \omega_j \int d\boldsymbol{r} S_j(\boldsymbol{r}) |\Psi_j^{(-)}(\boldsymbol{r})|^2$$

$$\Psi_j^{(-)}(\boldsymbol{r}) = [e^{i\boldsymbol{q}\cdot\boldsymbol{r}} - j_0(qr)]\delta_{1j} + \psi_j^{(-)}(r)$$

$$\psi_j^{(-)}(q) \propto e^{-iqr}/r \text{ or } e^{-\kappa r}/r \; (r \to \infty)$$

(No Coulomb case)



Outgoing Mom. q in channel j=1

$$\omega_j S_j(\boldsymbol{r}) |\Psi_j^{(-)}(\boldsymbol{r})|^2$$

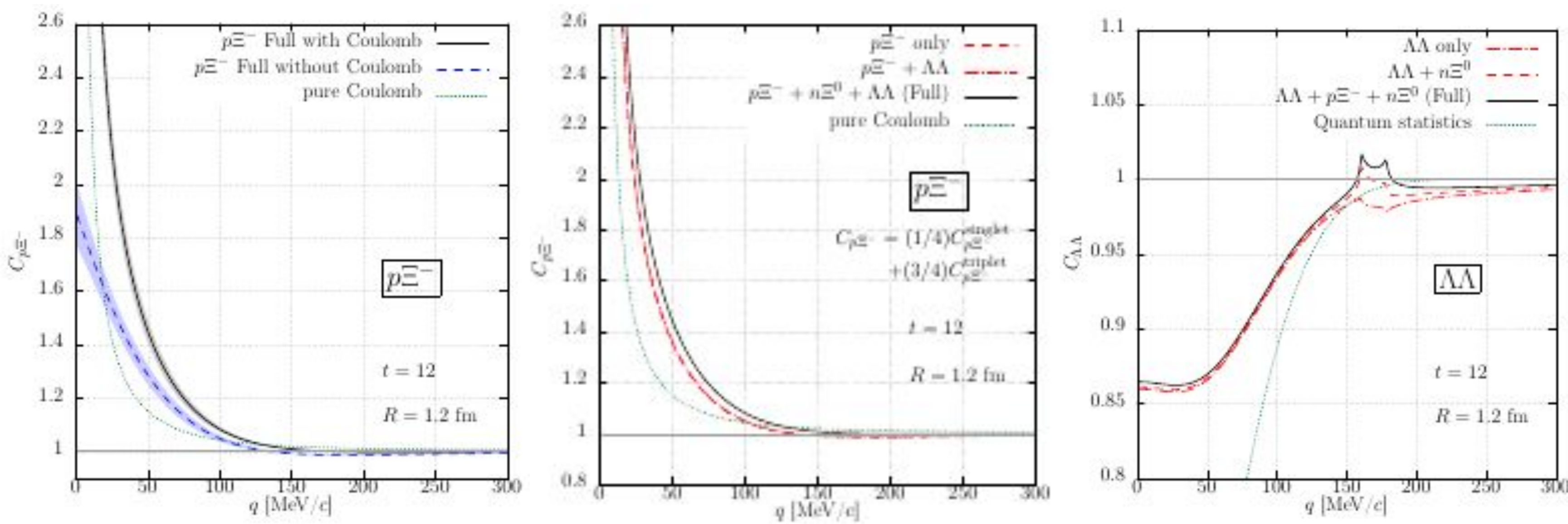
Source weight — **Normalized Source fn.**

- **Effects of coupled-channel, strong & Coulomb pot., and threshold difference are taken into account in the charge base, $p\Xi^-$, $n\Xi^0$, $\Lambda\Lambda$.**
  Y. Kamiya+, PRL('20, K^-p)

- **Source size (R) and source weight (ω_j) need to be determined.**

Theoretical pΞ⁻ and ΛΛ Correlation Function

- **pΞ⁻ correlation function**
  - **Strongly enhanced at low q by the strong interaction, and further enhanced by the Coulomb potential at q < 50 MeV/c**
  - **ΛΛ source effect is small.**
- **ΛΛ correlation function**
  - **Suppressed by quantum statistics, but enhanced by the strong interaction at low q.**
  - **NΞ source effect is visible only around the thresholds.**



Kamiya+ (2108.09644)

Comparison with pΞ^- and $\Lambda\Lambda$ correlation function data

Parameters in Correlation Function Data

- **Actual data contains non-femtoscopic effects → Pair purity < 1. (jets, misidentified particles)**

$$C_{\rm exp}(q; R, \lambda, N, \omega) = N(q)\,[1 + \lambda(C_{\rm theory}(q; R, \omega) - 1)]$$

  - **We adopt Pair purity (λ) from MC analysis results by ALICE.**
  - **Source Weight (ω_j) is given by a simple statistical model. (Sensitivity is small.)**
  - **Normalization with jet effects (N(q)=a+bq) is determined by the fit to the data.**
  - **Source size (R) is determined by the fit to the data for pp 13 TeV collisions,**

$$R_{p\Xi^-}(pp) \simeq 1.05\ \text{fm} \quad [R^{\rm ALICE}_{p\Xi^-}(pp) = 1.02 \pm 0.05\ \text{fm}]$$

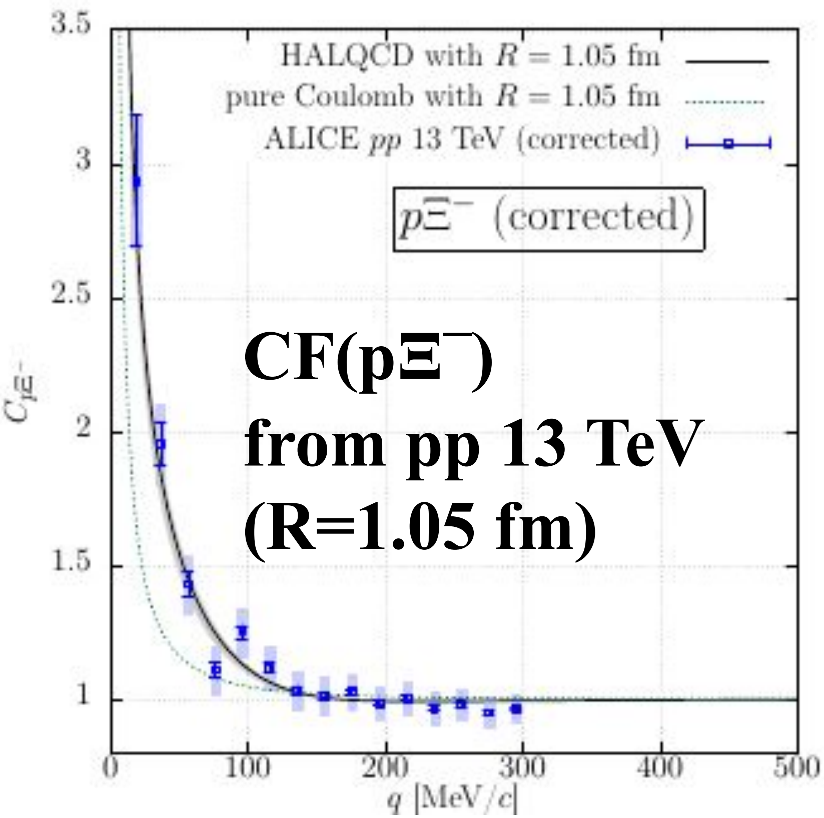
and based on the scaling relation for p Pb 5.02 TeV collisions.

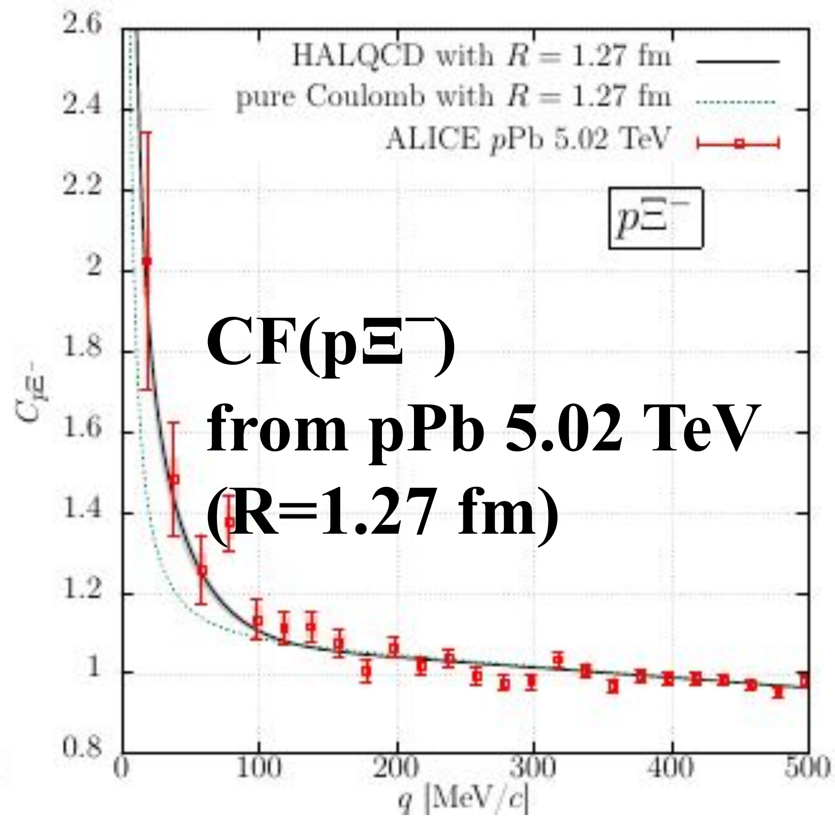
$$R_{p\Xi^-}(p{\rm Pb})/R_{p\Xi^-}(pp) \simeq R^{\rm ALICE}_{pp}(p{\rm Pb})/R^{\rm ALICE}_{pp}(pp)\ [R_{p\Xi^-}(p{\rm Pb}) = 1.27\ \text{fm}]$$

(ΛΛ and pΞ^- source sizes are assumed to be the same.)

pΞ⁻ Correlation Function

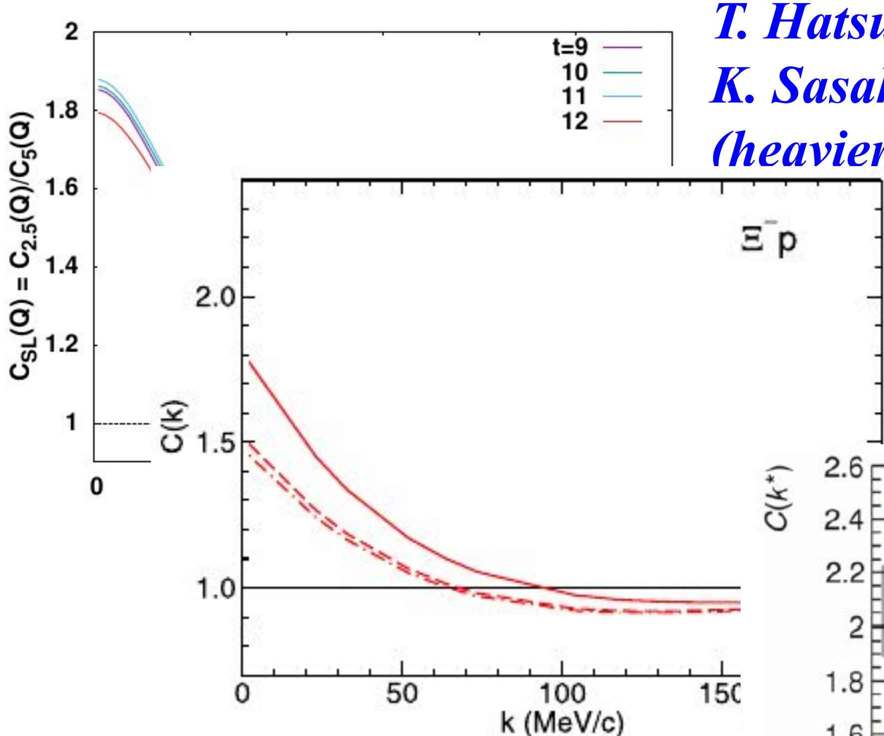
- pΞ⁻ correlation function data implies attractive NΞ interaction.
  - Strong enhancement from pure Coulomb CF
  - ΛΛ source effect is negligible. nΞ⁰ source effect is visible.
  - Calculated CF agrees with ALICE data.





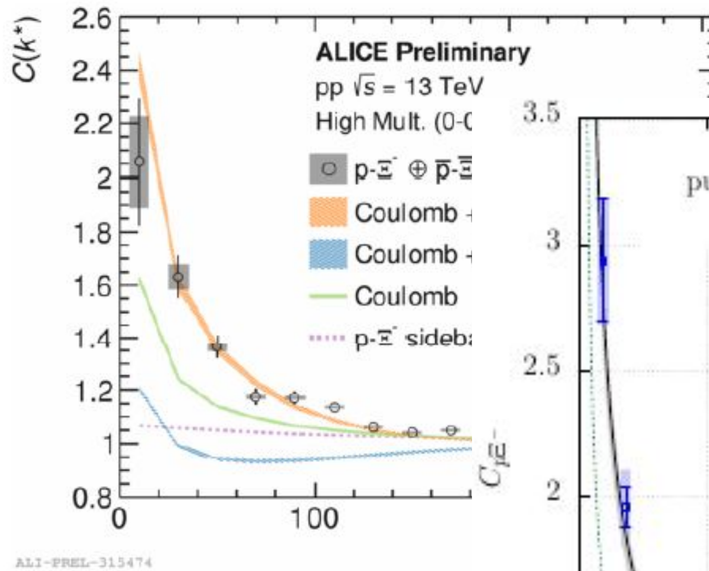
Kamiya+ (2108.09644); Acharya+(ALICE), PRL('19), Nature ('20)

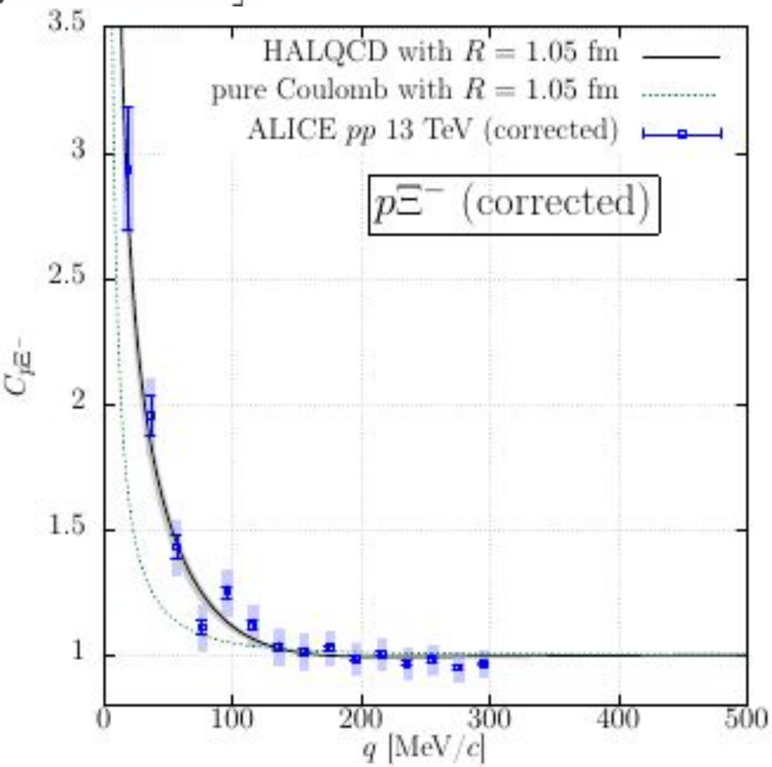
Comparison with other results



T. Hatsuda, K. Morita, AO,
K. Sasaki, NPA967('17)856.
(heavier quark mass)

J. Haidenbauer, NPA981('19)1.
(NLO(600), w/ CC effects, w/o Coulomb)
(w/ Coulomb, it will be comparable with data.)



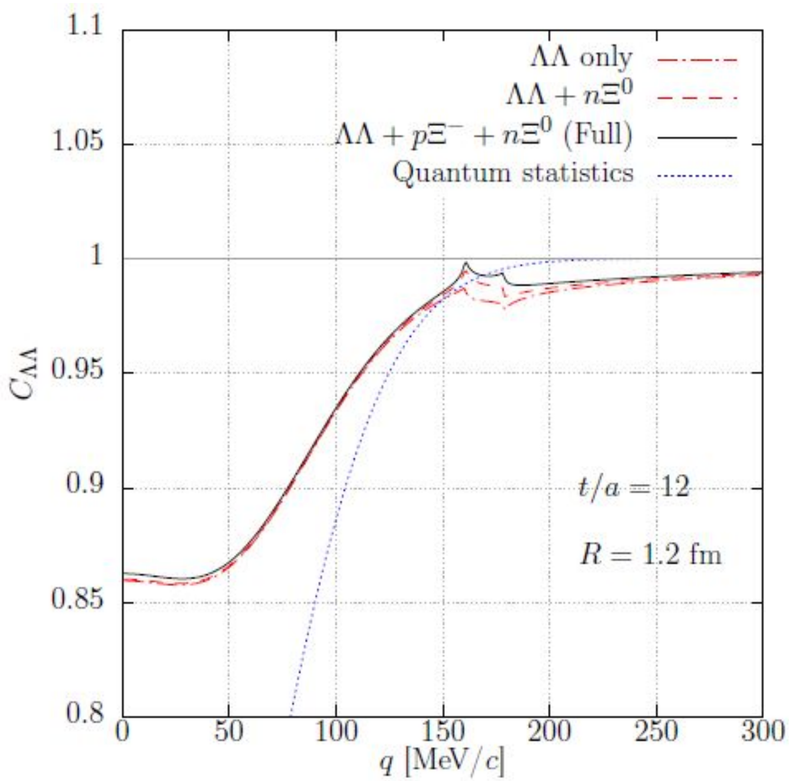


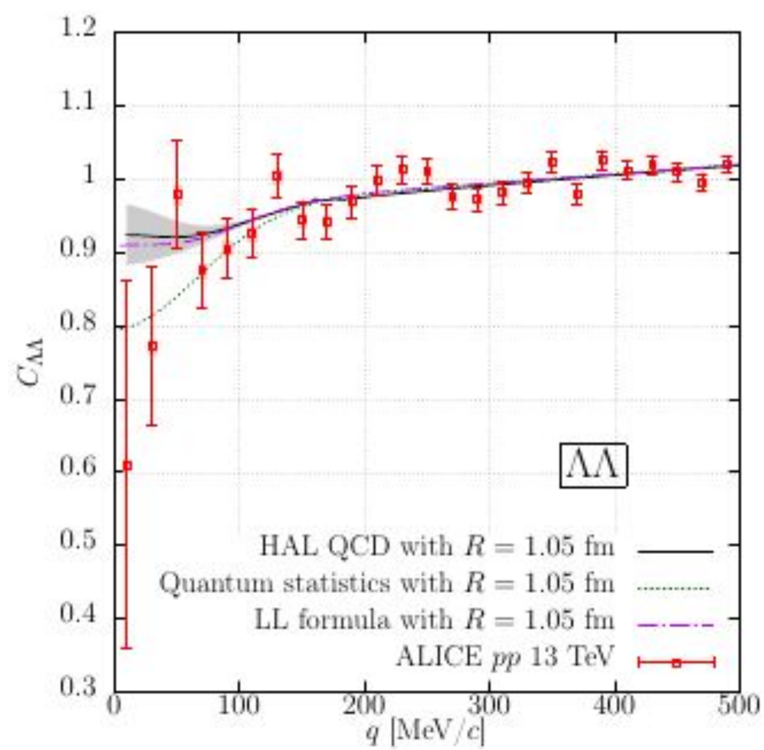
D. L. Mihairov+[ALICE],
NPA1005('21)121760 (QM2019).
(Nijmegen potential does not
explain the data.)

Kamiya+(2108.09644).
(w/ Lattice BB pot. at phys. m_q
and CC effects with ΛΛ)

ΛΛ correlation function

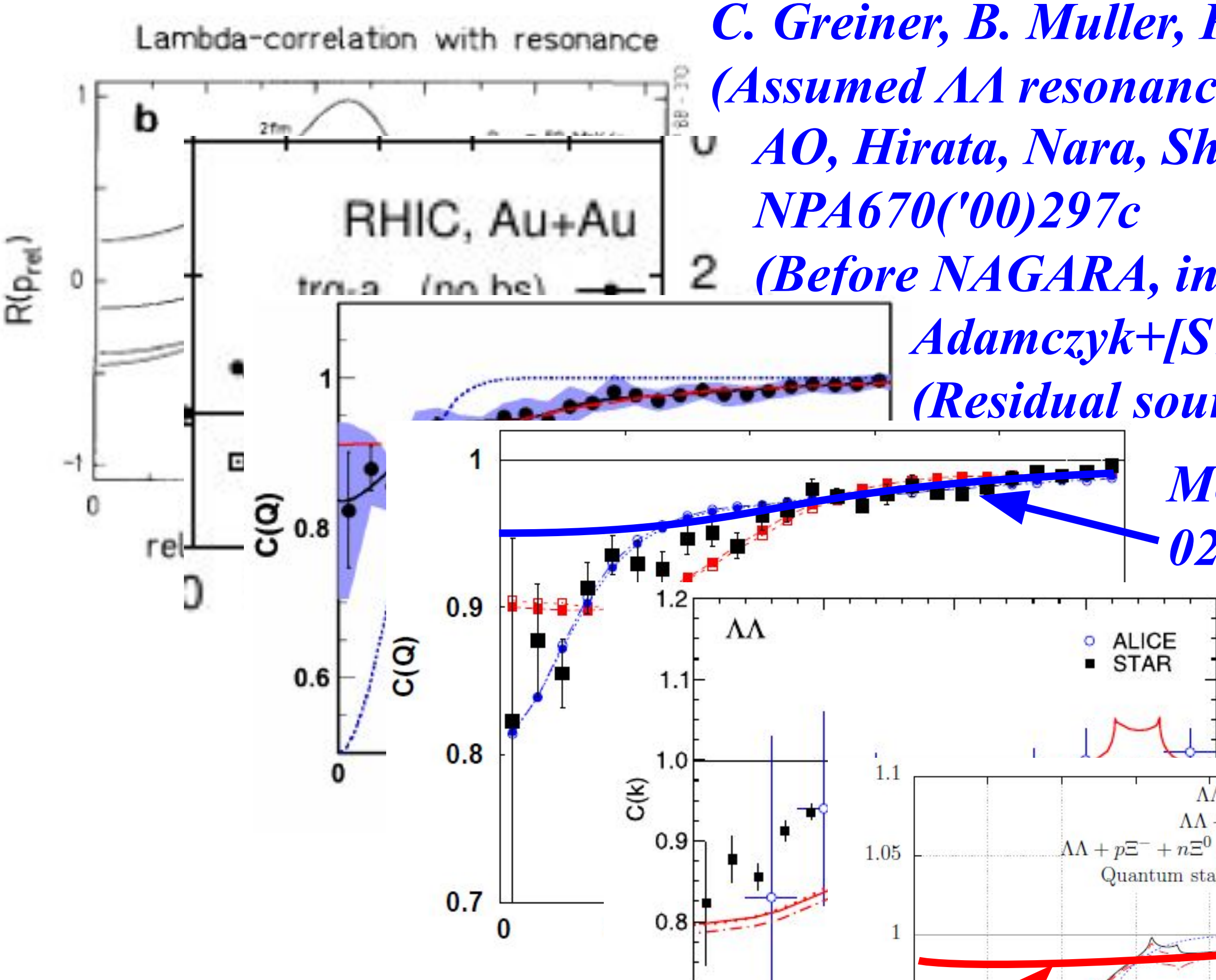
- **ΛΛ correlation function**
  - **Enhancement from pure quantum statistic CF**
  - **NΞ source effect is visible only around thresholds.**
  - **Calculated CF agrees with ALICE data.**
    Analytic model (Lednicky-Lyuboshits formula) works well.





Kamiya+ (2108.09644); Acharya+[ALICE] ('19)

Comparison with other results



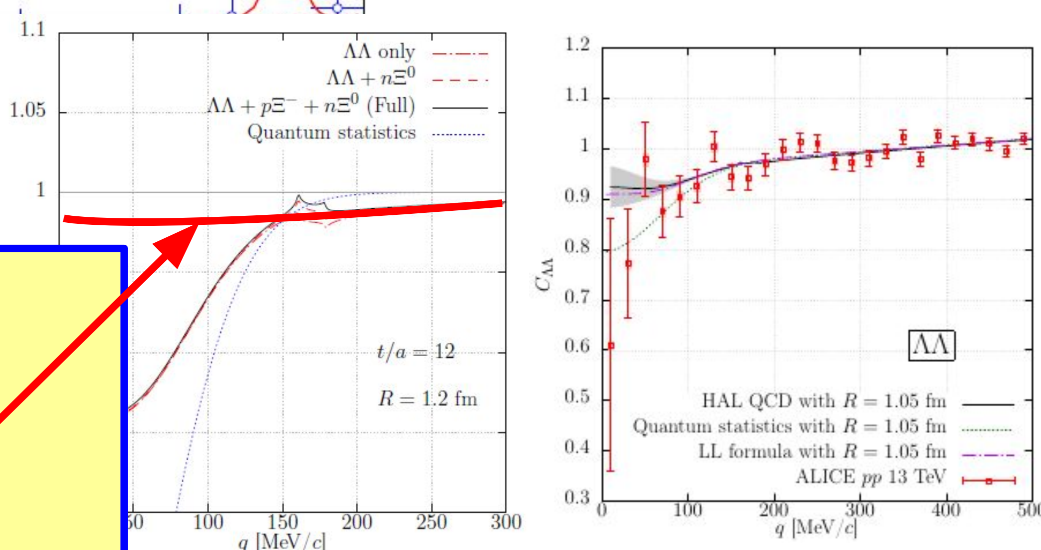
C. Greiner, B. Muller, PLB219('89)199.
(Assumed ΛΛ resonance)

AO, Hirata, Nara, Shinmura, Akaishi, NPA670('00)297c
(Before NAGARA, interaction was too strong.)

Adamczyk+[STAR], PRL114('15)022301
(Residual source R ~ 0.5 fm was assumed.)

Morita, Furumoto, AO, PRC91('15) 024916. (Res.Source + flow)

J. Haidenbauer, NPA981('19)1.
(Larger cusp ?)



Kamiya+('21).
Smaller cusp than χEFT.
CC simulates res. source,
but not enough for STAR data

Unbound nature of NΞ confirmed ?

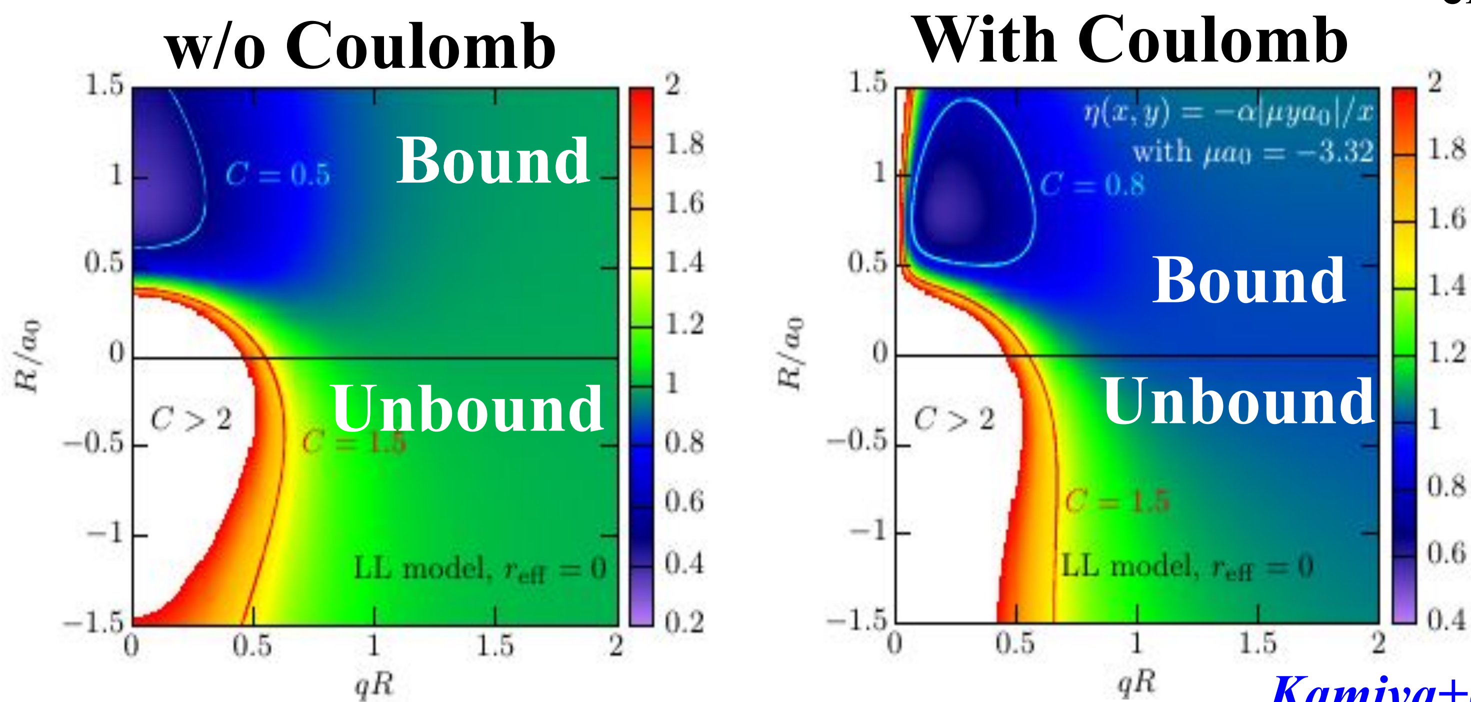
R Dependence of Correlation Function

- **Source size (R) dependence of C(q) is helpful to deduce the existence of a bound state.**

Morita+('16, '20), Kamiya+('20), Kamiya+(2108.09644)

  - **With a bound state, C(q) is suppressed at small q when R ~ $|a_0|$. (w.f. has a node at r ~ $|a_0|$ with a bound state.)**
  - **Qualitative understanding by the analytic model (LL formula)**

[Lednickey, Lyuboshits ('82)] **with the zero range approx. (r_{eff}=0)**

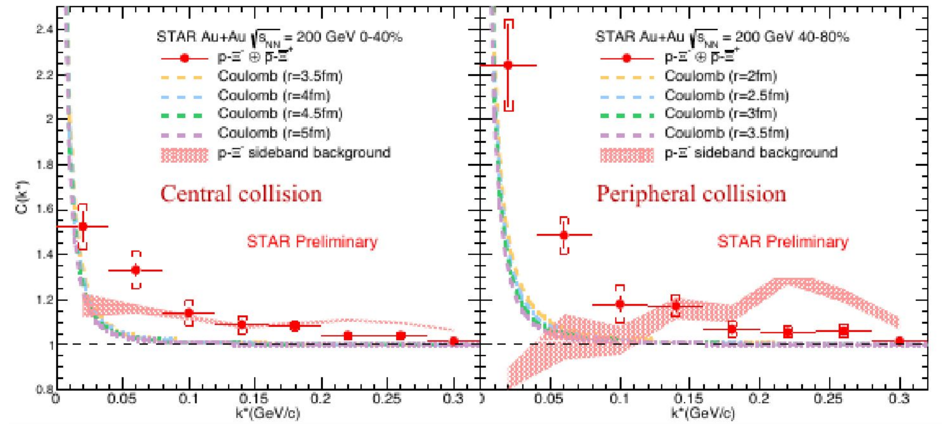


Kamiya+(2108.09644)

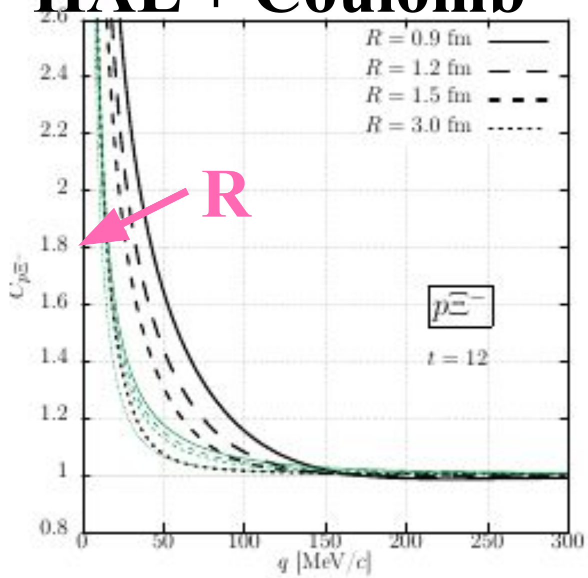
R dependence of $p\Xi^-$ correlation function

- R dep. of calculated results
  → Enhanced region shrinks
  with larger R. No Dip.
- Larger R data from Au+Au
  seem to show similar behavior.

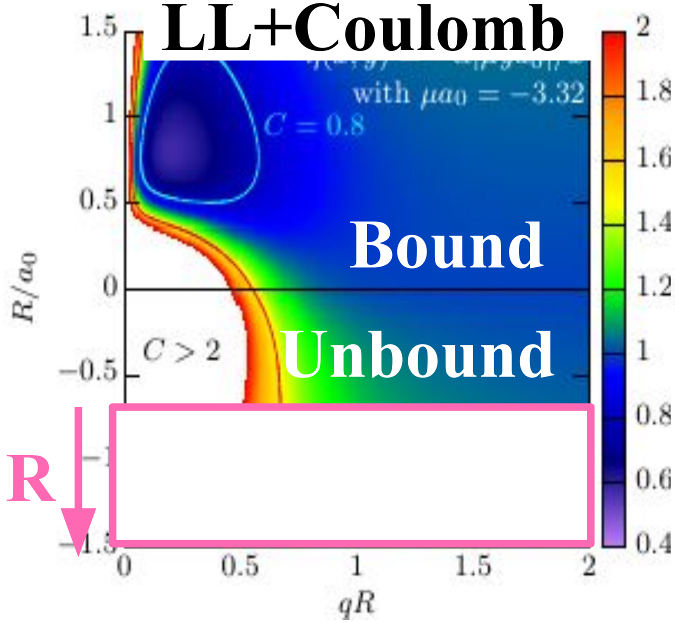
K. Mi+(STAR, preliminary), Au+Au 200 AGeV, APS2021.
(No Dip at larger R)

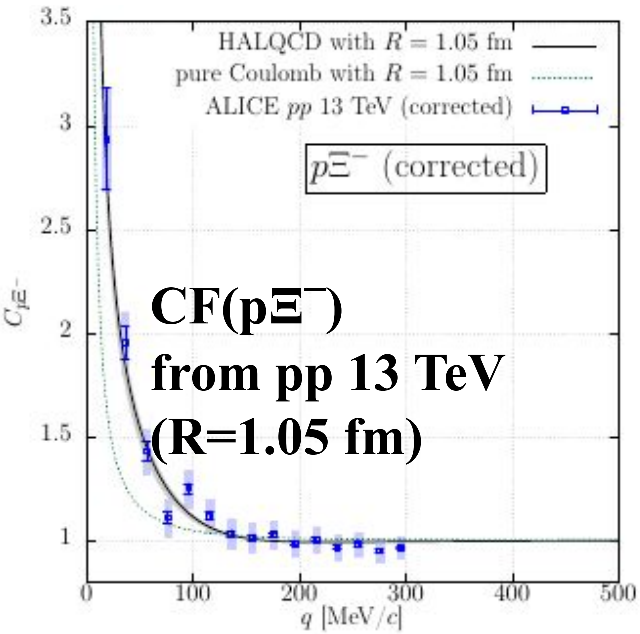


HAL + Coulomb



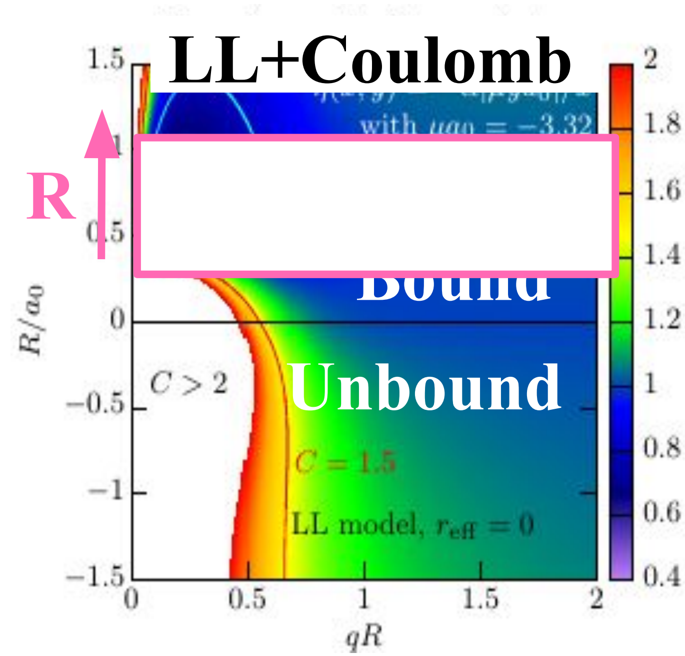
LL+Coulomb



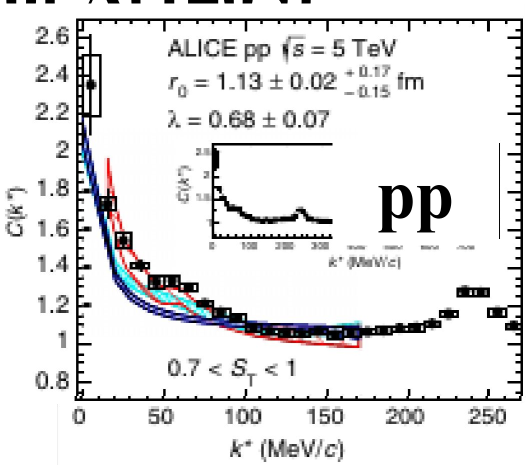


c.f. R dependence of pK^- correlation function

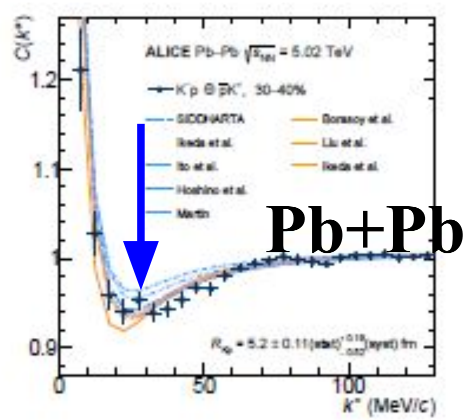
- **Enhanced C(q) from pp collisions, and dip in heavy-ion collisions.**
  = Typical behavior expected from LL formula + Coulomb with a bound state.
  Kamiya+(PRL, '20)



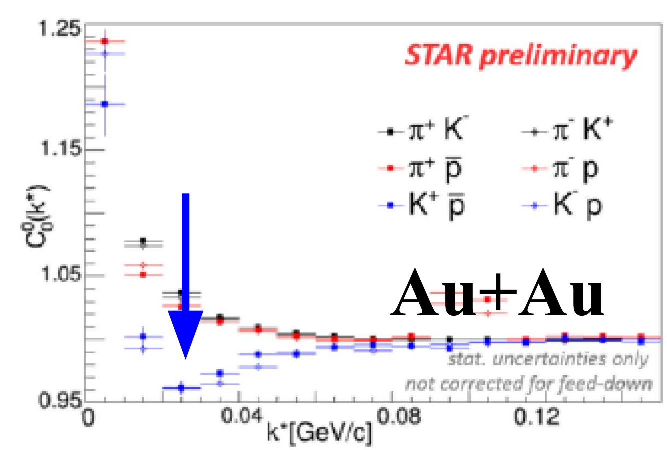
- **These R dependence of C(q) supports again the KN bound state nature of Λ(1405)**



S. Acharya+[ALICE], PRL124('20)092301



S. Acharya+[ALICE], 2105.05683



Siejka+[STAR, preliminary], NPA982 ('19)359.

Summary

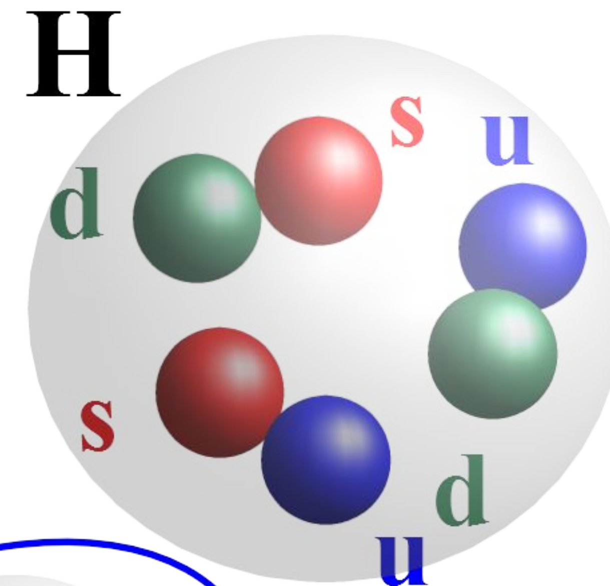
- Correlation functions are helpful to constrain / examine hadron-hadron interactions as well as to deduce the existence of a bound state.
- We have calculated $p\Xi^-$ and $\Lambda\Lambda$ correlation functions by using lattice $N\Xi$-$\Lambda\Lambda$ coupled-channel (CC) potential.
  - w/ effects of CC, Coulomb, threshold difference.
  - ALICE $p\Xi^-$ and $\Lambda\Lambda$ correlation function data are consistent with the HAL QCD potential.
  - Source weight effect from conversion channel is not big, except for the cusps at $N\Xi$ thresholds in $\Lambda\Lambda$ corr. fn. (Solving CC equation is still important.)
- Unbound nature of $N\Xi$ will be supported by studying the source size dependence of the $p\Xi^-$ correlation function. (Any way to confirm the virtual pole nature ?)

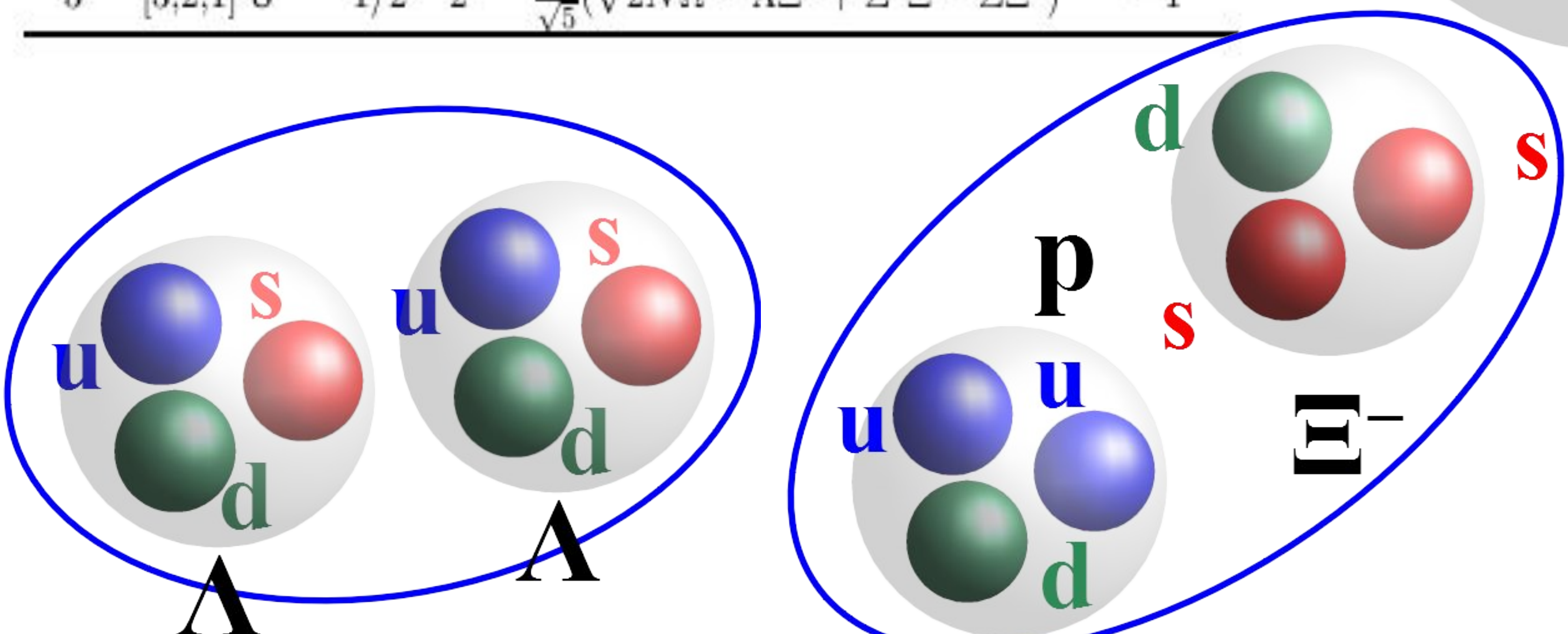
To be, or not to be, that is the question.

Table 1. Leading $6q$ $L = 0$ dibaryon candidates [12], their BB' structure and the CM interaction gain with respect of the lowest BB' threshold calculated by means of Eq. (2). Asterisks are used for the $\mathbf{10}_f$ baryons $\Sigma^* \equiv \Sigma(1385)$ and $\Xi^* \equiv \Xi(1530)$. The symbol [i,j,k] stands for the Young tablaux of the $SU(3)_f$ representation, with i arrays in the first row, j arrays in the second row and k arrays in the third row, from which $\mathcal{P}_f$ is evaluated. The $\overline{\mathbf{10}}$ $SU(3)_f$ representation is denoted here $\mathbf{10}^*$.

$-S$	$SU(3)_f$	I	J^π	BB' structure	$\frac{\Delta\langle V_{CM}\rangle}{M_0}$
0	[3,3,0] $\mathbf{10}^*$	0	3^+	$\Delta\Delta$	0
1	[3,2,1] $\mathbf{8}$	1/2	2^+	$\frac{1}{\sqrt{5}}(N\Sigma^* + 2\Delta\Sigma)$	-1
2	[2,2,2] $\mathbf{1}$	0	0^+	$\frac{1}{\sqrt{8}}(\Lambda\Lambda + 2N\Xi - \sqrt{3}\Sigma\Sigma)$	-2
3	[3,2,1] $\mathbf{8}$	1/2	2^+	$\frac{1}{\sqrt{5}}(\sqrt{2}N\Omega - \Lambda\Xi^* + \Sigma^*\Xi - \Sigma\Xi^*)$	-1

A. Gal ('16); M. Oka ('88)





Potentially measurable hh pairs

- **Correlation function is useful to access hadron-hadron interactions as well as to deduce the existence of a bound state.**

Scatt.+Nuclei

Scatt.+Mesic atom

Scatt. +Hyper Nuclei

	n	p	K^-	K^+	π^-	π^+	Λ	Σ	Ξ^-	Ω^-	D^-	D^+	Ks	d	pp	φ	$+\alpha$
n																	
p		O	O	O	△	△	O	O	O	O	O	O		O	O	O	
K^-		O	O	O	O	O							O				
K^+		O	O	O	O	O							O				
π^-		△	O	O	O	O											
π^+		△	O	O	O	O											
Λ		O					O		O						O		
Σ		O						O									
Ξ^-		O															
Ω^-		O															
D^-		O															
D^+		O															
Ks			O	O													
d		O															
pp		O					O										
φ		O															
$+\alpha$																	

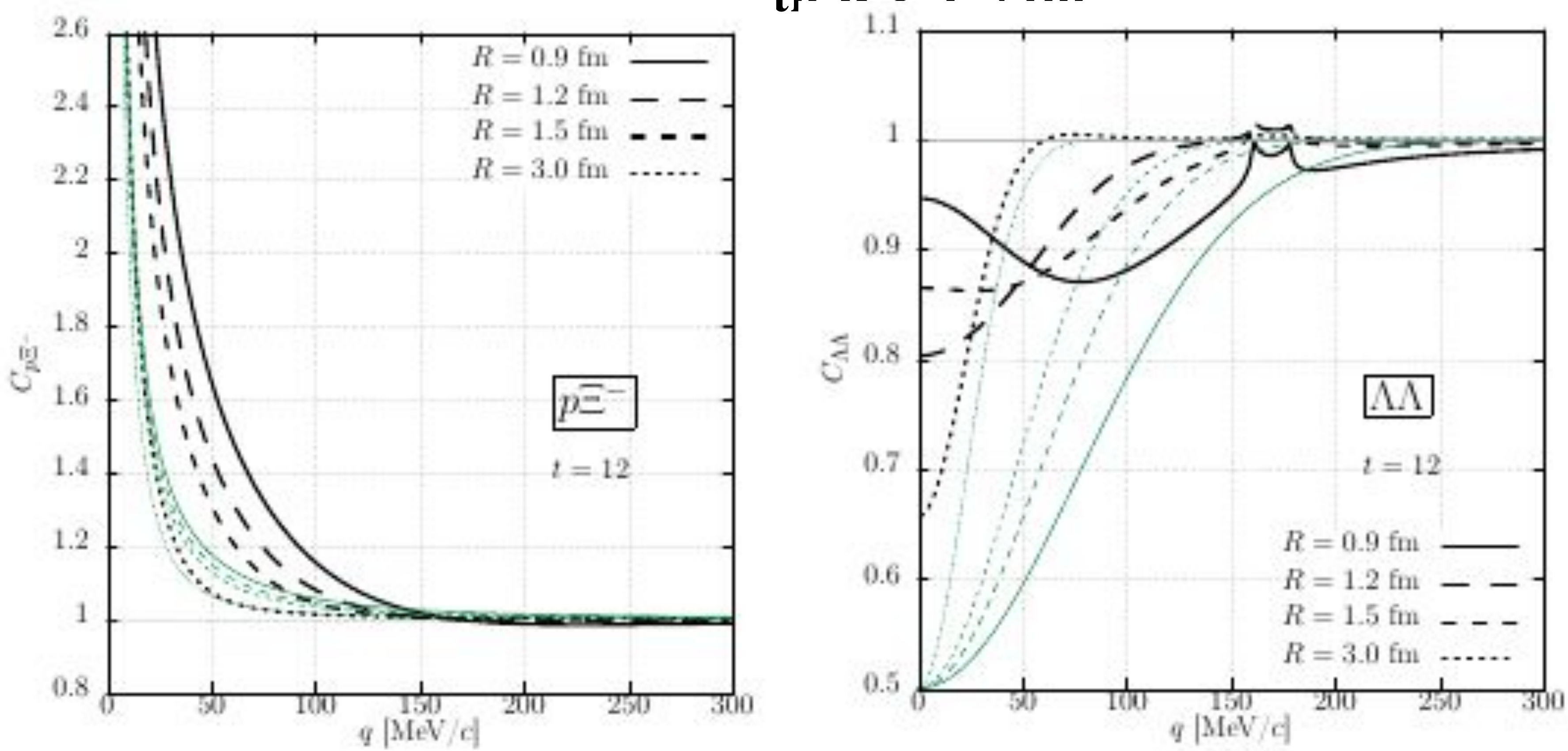
ΛΛ hypernuclei

Femtoscopy

Blue: Pairs we have studied, O: Experimentally measured

Source size dependence of correlation functions

- **pΞ–**
  - **Smooth dependence on R. (No bound state, Non-identical particles)**
- **ΛΛ**
  - **Complicated R dependence (Quantum statistics)**
  - **No long tail ($q > 200$ MeV/c) with $R > 1.5$ fm**



Kamiya+(2108.09644)

Non-Femtoscopic Parameters

- **Relevant parameters=R, λ, N=a+bq**
  (ω's are almost irrelevant for pΞ⁻ and ΛΛ correlation functions.)

$$C_{\rm exp}(q;R,\lambda,N,\omega) = N(q)\left[1+\lambda(C_{\rm theory}(q;R,\omega)-1)\right]$$

collision	pair	λ	a	b [(MeV/c)$^{-1}$]	R [fm]
pp (13 TeV)	$p\Xi^-$	1 [15]	1 [15]	0 [15]	1.05
	$\Lambda\Lambda$	0.338 [9]	0.95	1.28×10^{-4}	
pPb (5.02 TeV)	$p\Xi^-$	0.513 [14]	1.09	-2.56×10^{-4}	$1.27^{(*)}$
	$\Lambda\Lambda$	0.239 [9]	0.99	0.29×10^{-4}	

TABLE II. The pair purity λ, non-femtoscopic parameters a and b, and the effective source size R in the fitting function $C_{\rm th}(q)$. The parameters a and b in pp ($\Lambda\Lambda$ pairs) and pPb ($p\Xi^-$ and $\Lambda\Lambda$ pairs) collisions and R in pp collisions are the actual fitting parameters. Numbers with references are taken from Refs. [9] [14] [15], and the number with $(*)$ is estimated from other other parameters. See the text for details.

Kamiya+(2108.09644)

Correlation function from T-matrix

- **s-wave w.f. using the half-off-shell T-matrix (T_0)**

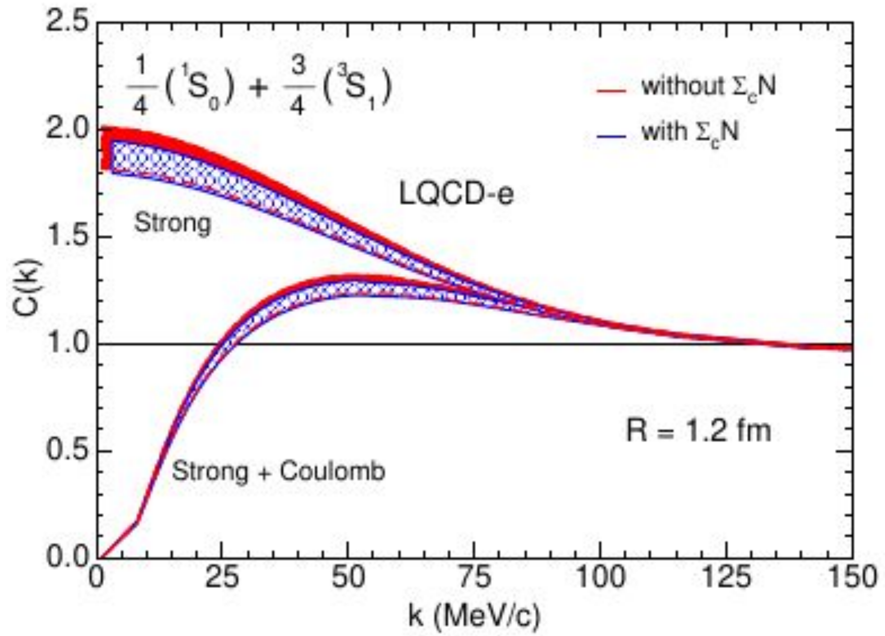
  J. Haidenbauer, NPA 981('19)1.

$$\widetilde{\psi}_0(k,r) = j_0(kr) + \frac{1}{\pi}\int dq\, q^2 j_0(qr)\,\frac{1}{E - E_1(q) - E_2(q) + i\varepsilon} T_0(q,k;E)$$

$$\psi_0^{(-)}(k,r) = e^{-2i\delta_0}\widetilde{\psi}_0(k,r) \to \frac{e^{-i\delta_0}}{kr}\sin(kr+\delta_0) = \frac{1}{2ikr}\left(e^{ikr} - e^{-2i\delta_0}e^{-ikr}\right)$$

- **Strong T-matrix + Coulomb potential**

  J. Haidenbauer, G. Krein, and T. C. Peixoto, EPJA 56 ('20)184;
  using the Vincent-Phatak method
  [C.M. Vincent and S.C. Phatak, PRC10('74)391;
  B. Holzenkamp, K. Holinde and J. Speth,
  NPA 500('89)485 (1989)]



Analytic model of correlation function

- **Asymptotic w.f. is described by the scattering amplitude *f(q)***
  (non-identical particle pair, short range int. (only s-wave is modified),
  single channel, no Coulomb pot.)

$$\Phi^{(+)}(\boldsymbol{r}) = e^{i\boldsymbol{q}\cdot\boldsymbol{r}} - j_0(qr) + \varphi_0^{(+)}(r;q)$$

$$\varphi_0^{(+)}(r;q) \to \frac{e^{i\delta}\sin(qr+\delta)}{qr} = \frac{1}{2iqr}(Se^{iqr} - e^{-iqr}) = \frac{\sin qr}{qr} + f(q)\frac{e^{iqr}}{r}$$

$$\varphi_0^{(-)}(r;q) = S^{-1}\varphi^{(+)}(r;q) \quad \left[S = \exp(2i\delta),\, f = (S-1)/2iq = [q\cot\delta - iq]^{-1}\right]$$

- **Correlation function in Lednicky-Lyuboshits (LL) formula**

  (with static Gaussian source, real δ) (Lednickey, Lyuboshits ('82))

$$C(q) = \int d\boldsymbol{r} S(r) \left|\Phi^{(-)}(\boldsymbol{r})\right|^2 = 1 + \int d\boldsymbol{r} S(r) \left[\left|\varphi_0^{(-)}(\boldsymbol{r})\right|^2 - (j_0(qr))^2\right]$$

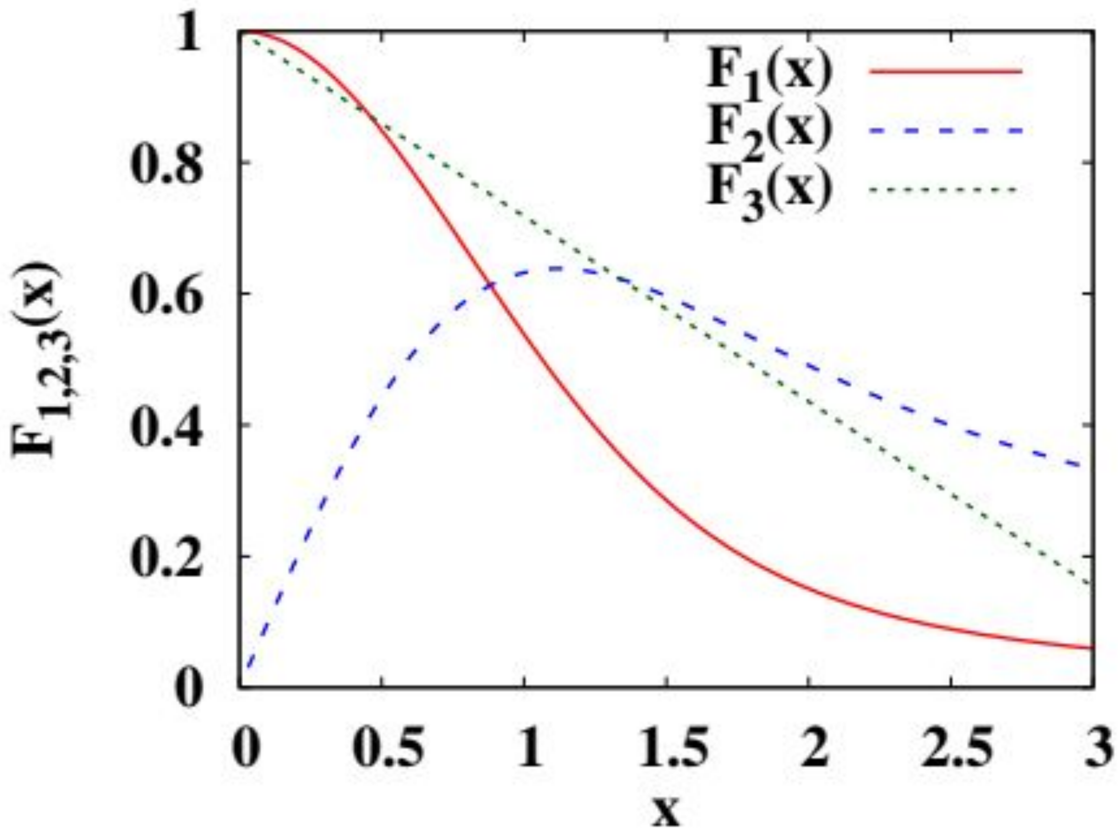
$$\simeq 1 + \int 4\pi dr\, S(r) \left[|f(q)|^2 + \frac{\sin qr}{q}\left\{f(q)e^{iqr} + f^*(q)e^{-iqr}\right\}\right]$$

$$C_{\mathrm{LL}}(q) = 1 + \frac{|f(q)|^2}{2R^2} F_3\left(\frac{r_{\mathrm{eff}}}{R}\right) + \frac{2\mathrm{Re}\, f(q)}{\sqrt{\pi}R} F_1(2qR) - \frac{\mathrm{Im}\, f(q)}{R} F_2(2qR)$$

$$\left[f(q) = (q\cot\delta - iq)^{-1},\ F_1(x) = \frac{1}{x}\int_0^x dt e^{t^2-x^2},\ F_2(x) = (1-e^{-x^2})/x,\ F_3(x) = 1 - \frac{x}{2\sqrt{\pi}}\right]$$

Lednicky-Lyuboshits functions

$$F_1(x) = \frac{1}{x}\int_0^x dt e^{t^2-x^2},\ \ F_2(x) = (1-e^{-x^2})/x,\ \ F_3(x) = 1 - \frac{x}{2\sqrt{\pi}}$$



$$F_1(x) \simeq \frac{1 + c_1x^2 + c_2x^4 + c_3x^6}{1 + (c_1 + 2/3)x^2 + c_4x^4 + c_5x^6 + c_3x^8}\ (0 \le x < 20)$$

$$(c_1, c_2, c_3, c_4, c_5) = (0.123, 0.0376, 0.0107, 0.304, 0.0617)$$

AO,Morita,Mihayara,Hyodo, NPA 954 ('16)294.

Bird's-eye view of C(q)

- **Zero eff. range pot. → C(q)=F(R/a_0, qR)**

$$r_{\rm eff}=0 \to q\cot\delta = -1/a_0 \to f(q) = (q\cot\delta - iq)^{-1} = -\frac{R}{R/a_0 + iqR}$$

$$C(x,y) = 1 + \frac{1}{x^2+y^2}\left[\frac{1}{2} - \frac{2y}{\sqrt{\pi}}F_1(2x) - xF_2(2x)\right] \quad (x=qR, y=R/a_0)$$

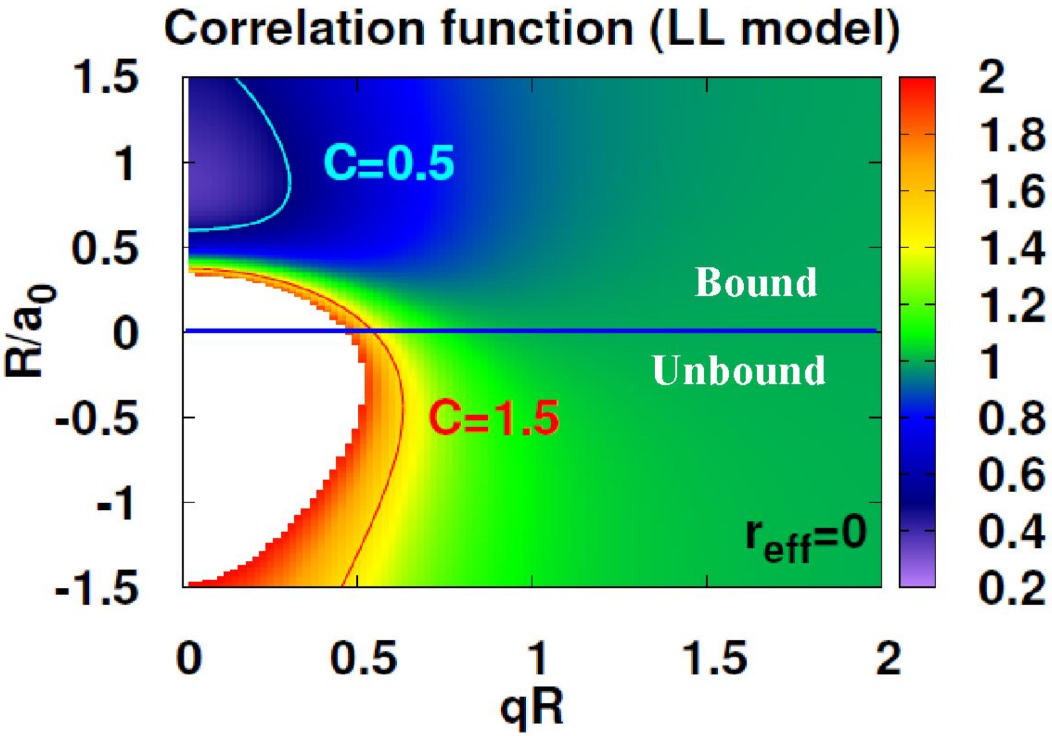
- **Low momentum limit**

$$C(x,y) \to \frac{1}{2}\left(\frac{1}{y}-\frac{2}{\sqrt{\pi}}\right)^2 + 1 - \frac{2}{\pi} \quad (F_1\to 1, F_2 \to 0 \text{ at } x\to 0)$$

  - **Enhanced C(q) at small q with a_0<0**

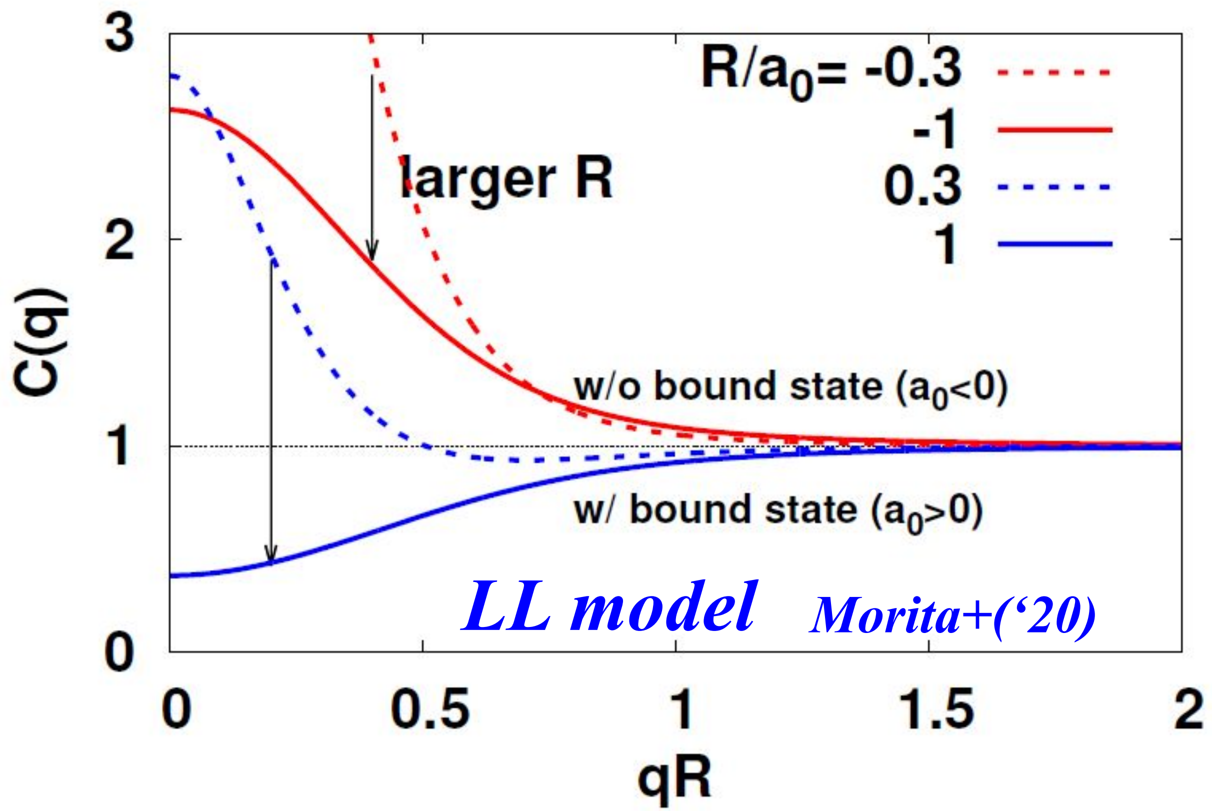
$$C_{\rm LL}(0) = 1 - \frac{2}{\sqrt{\pi}}\left(\frac{a_0}{R}\right) + \frac{1}{2}\left(\frac{a_0}{R}\right)^2$$

  - **a_0>0 → Size dependent C(q)**
    - **C(q) > 1 at small R**
    - **C(q) < 1 at R ~ a_0 (w.f. node at r ~ a_0)**

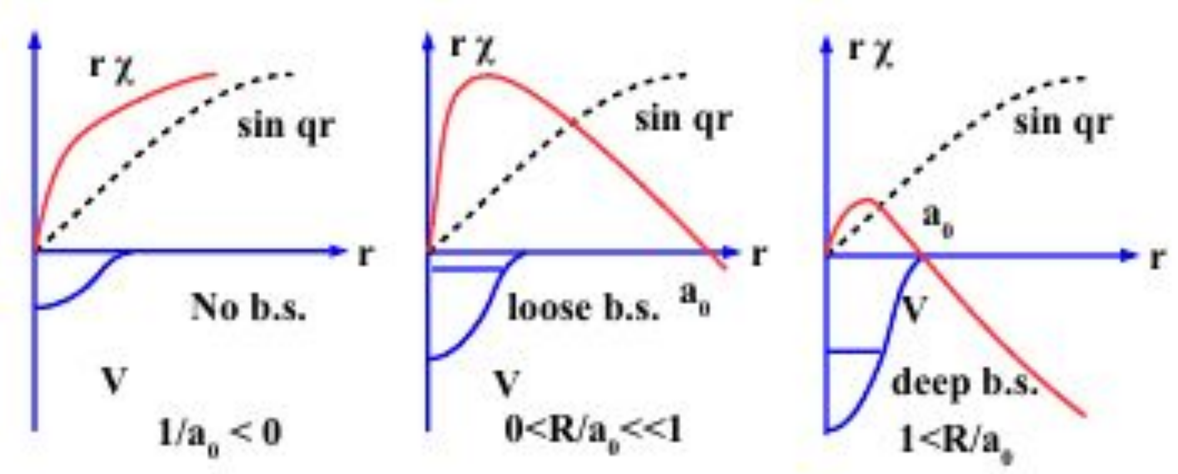


Bound state diagnosis by femtoscopy

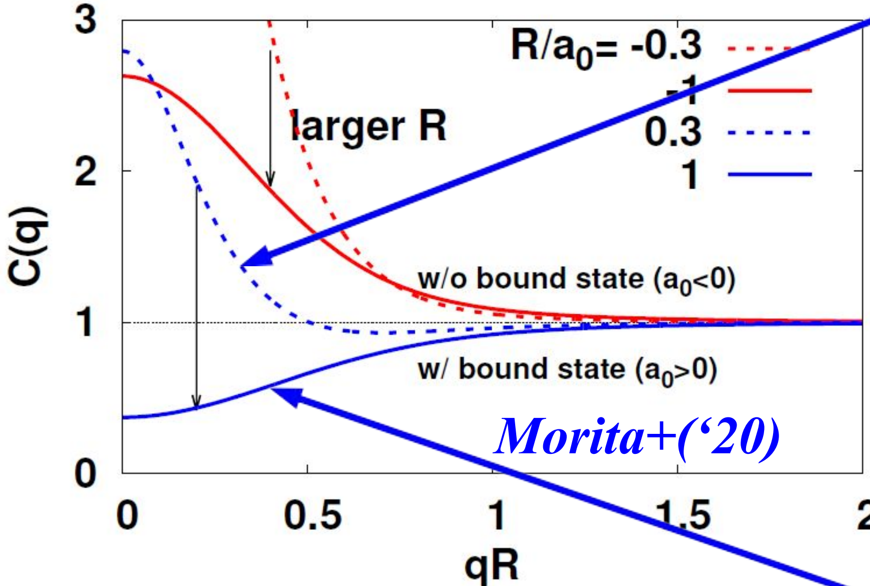
- Source size dep. of CF tells the sign of the scattering length (a_0).
  - With attraction, Large CF at small R.
  - With a bound state ($a_0>0$), CF is suppressed at $R \sim a_0$



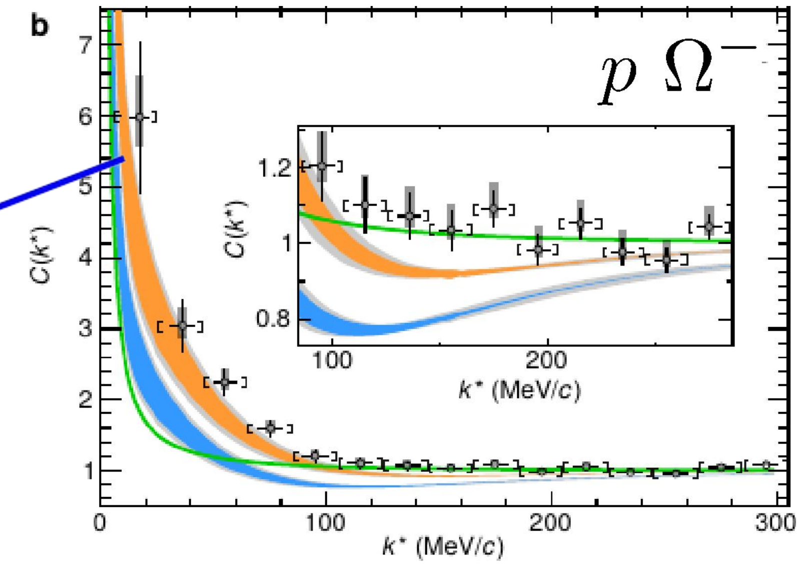
Source size dep. of CF
→ To be bound,
or not to be bound.



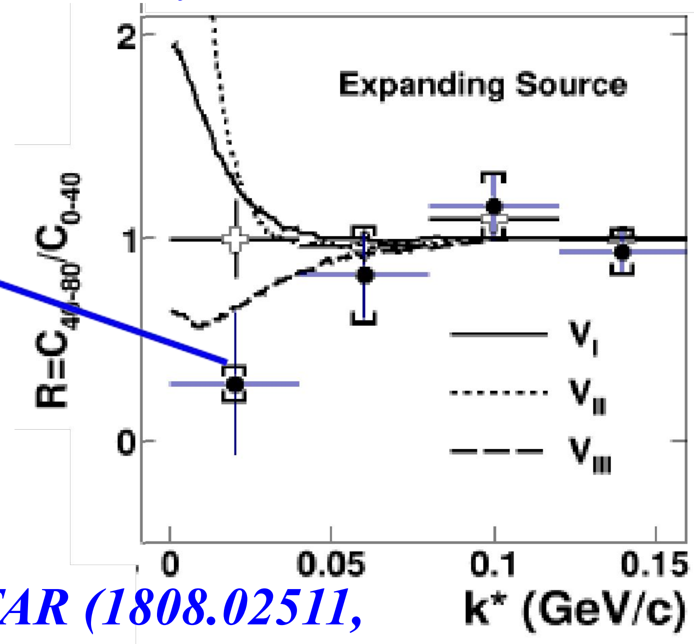
ALICE+STAR = NΩ Dibaryon



Morita+('20)

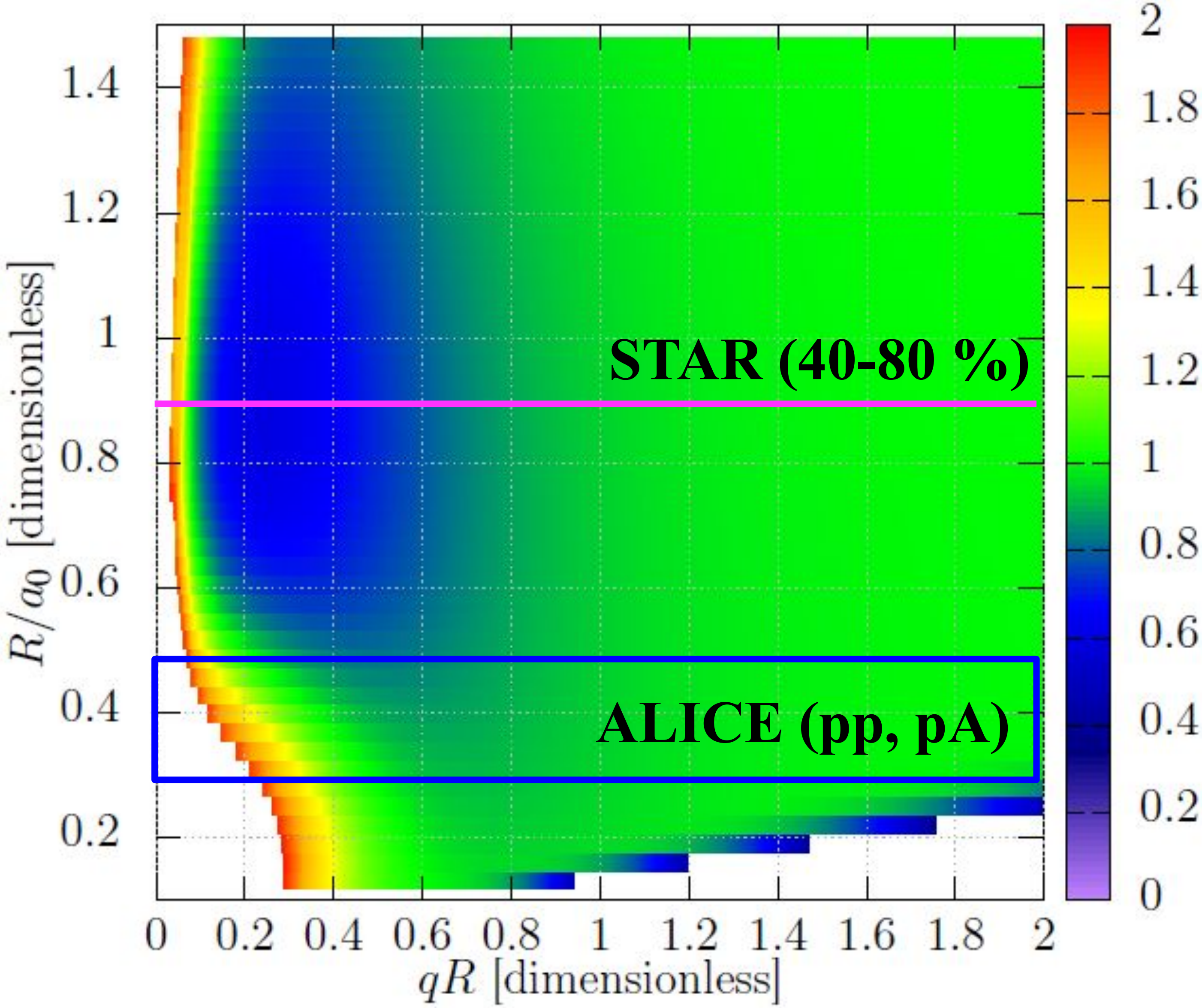


Acharya+(ALICE), Nature ('20)



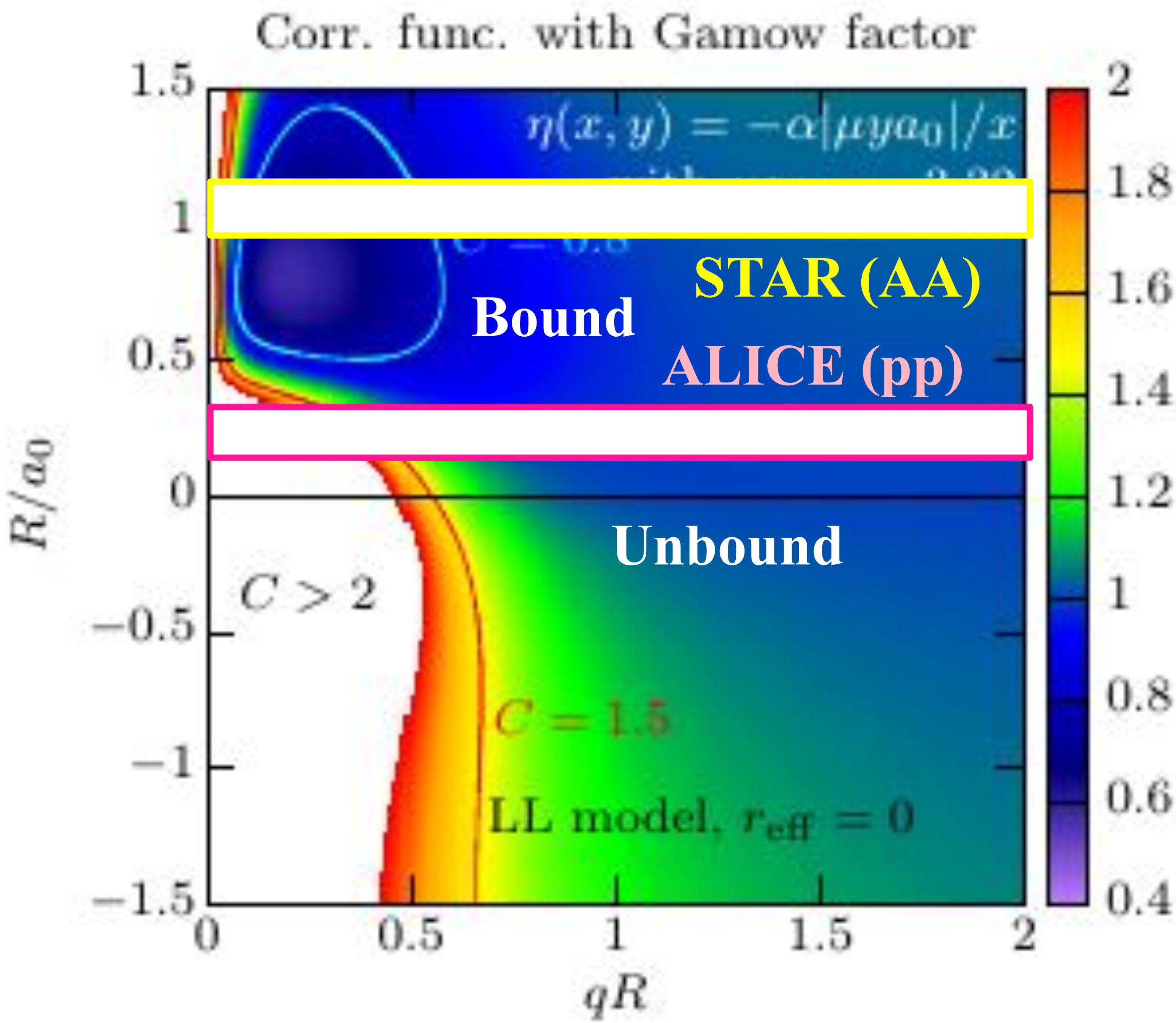
STAR (1808.02511, PLB790 ('19) 490)

Correlation Function with Gaussian source



NΩ potential (J=2, HAL QCD, a_0=3.4 fm) + Coulomb

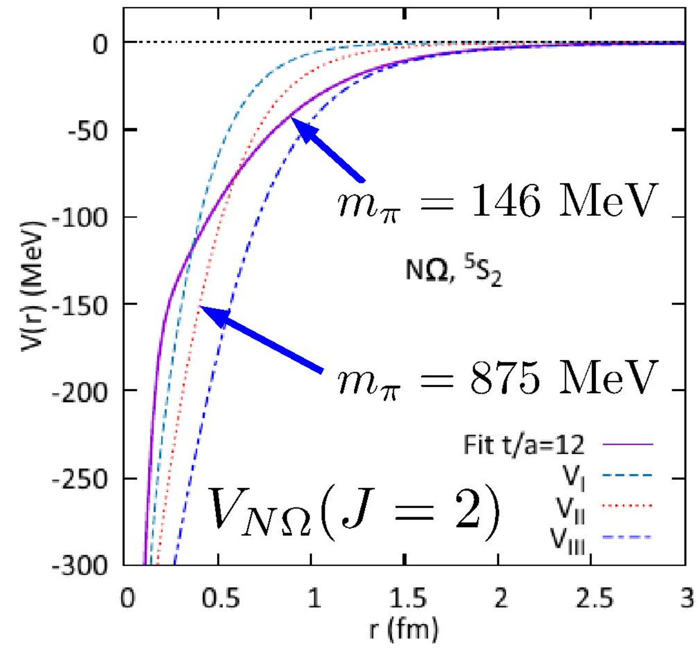
Source Size Dep. of CF w/ Coulomb potential



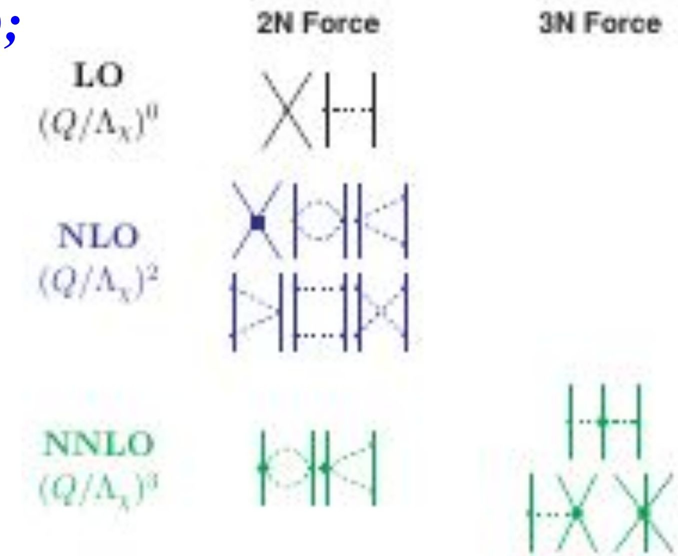
Y. Kamiya+(2108.09644) (LL × Gamow factor)

Modern Hadron-Hadron Interactions

- **Lattice QCD *hh* potential**
  - V_{hh} is obtained from the Schrödinger eq. for the Nambu-Bethe-Salpeter (NBS) amplitude.
    N. Ishii, S. Aoki, T. Hatsuda, PRL99('07)022001.
    → ΩΩ, NΩ, ΛΛ-NΞ potentials at phys. quark mass are published



- **Chiral EFT / Chiral SU(3) dynamics**
  - V_{hh} at low E. can be expanded systematically in powers of Q/Λ.
    S. Weinberg ('79);R. Machleidt, F. Sammarruca ('16); Y. Ikeda, T. Hyodo, W. Weise ('12).
    → NN, NY, YY, KN-πΣ-πΛ, ...

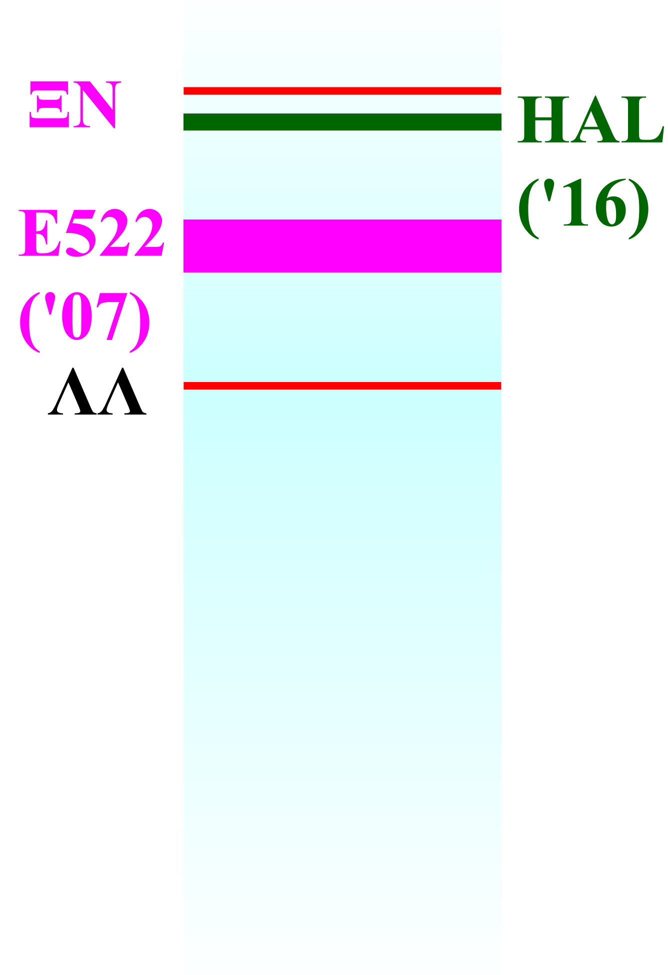


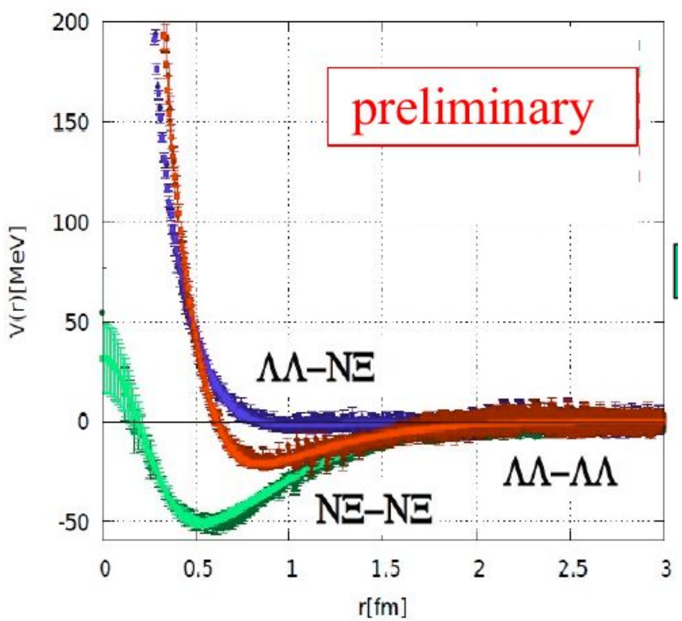
- **Quark cluster models, Meson exchange models, More phenomenological models, …**

Let us examine modern hh interactions !

Relevance of ΞN interaction to physics

- **H-particle: 6-quark state (uuddss) may be realized as a loosely bound state of ΞN (I=0)**
  K. Sasaki et al. (HAL QCD, ‘16,’17)
- **Repulsive ΞN interaction (I=1) may help to support 2 $M_\odot$ Neutron Star**
  Weissborn et al., NPA881 (‘12) 62.

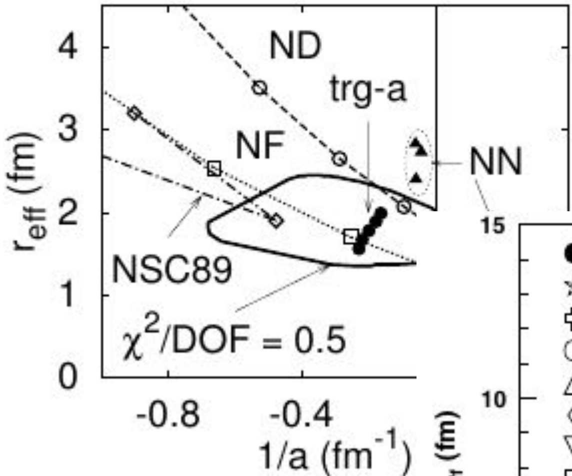




K. Sasaki et al. (HAL QCD Collab.), EPJ Web Conf. 175 (‘18) 05010.

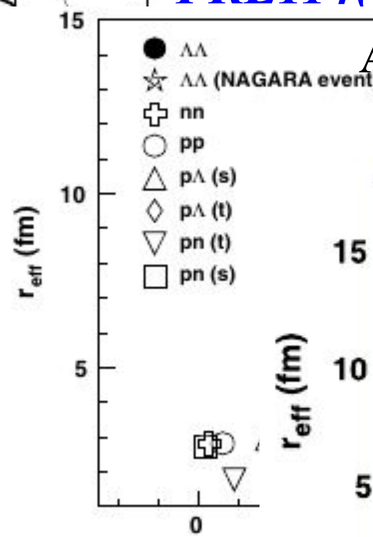
ΛΛ correlation and ΛΛ interaction

It is unlikely that ΛΛ bound state exists.

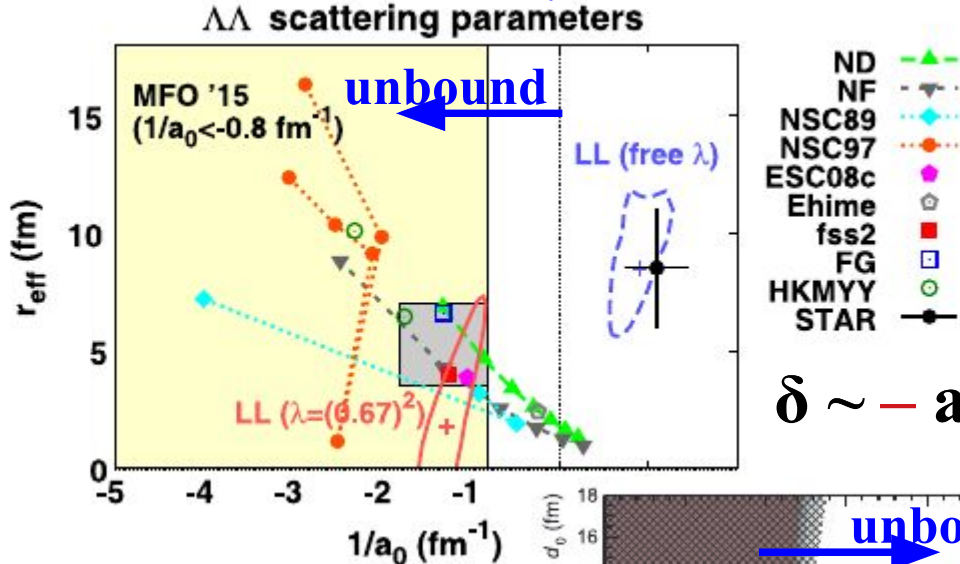


AO+('00)

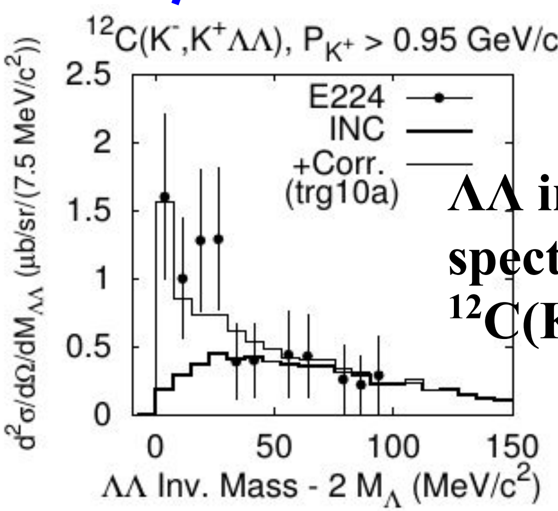
L. Adamczyk+[STAR], PRL114('15)022301



AO, K. Morita, K. Miyahara, T. Hyodo, NPA954('16)294

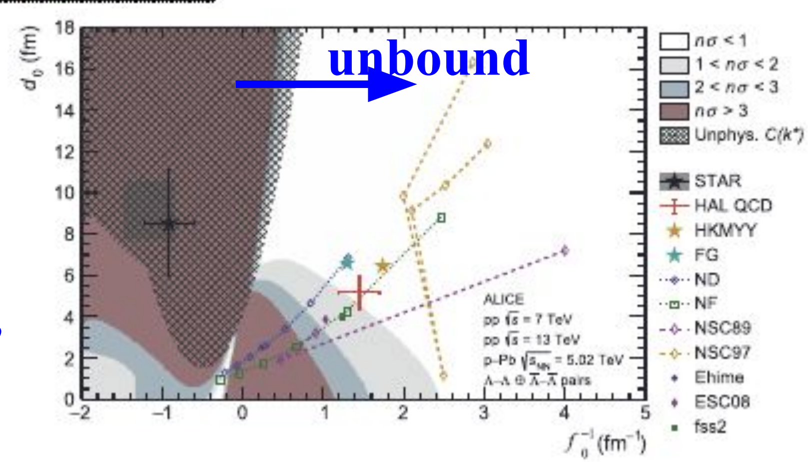


$\delta \sim - a_0 q$



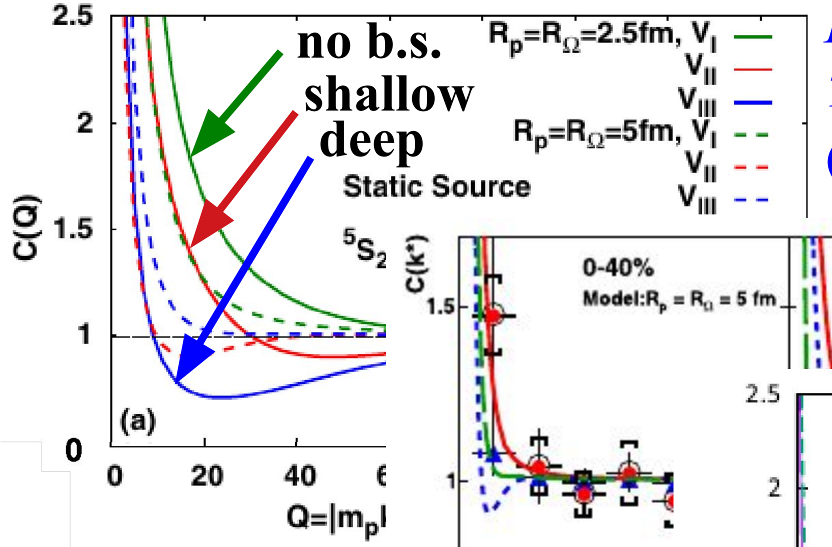
ΛΛ inv. mass spectrum from $^{12}C(K^-, K^+\Lambda\Lambda)$

S. Acharya+[ALICE], PLB797('19)134822

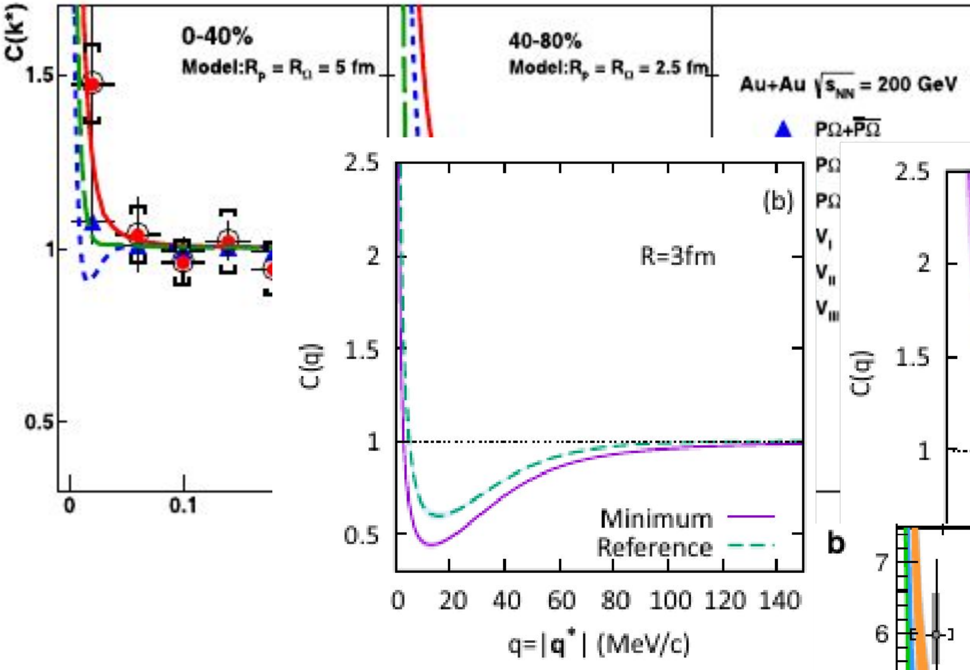


$\delta \sim + a_0 q$

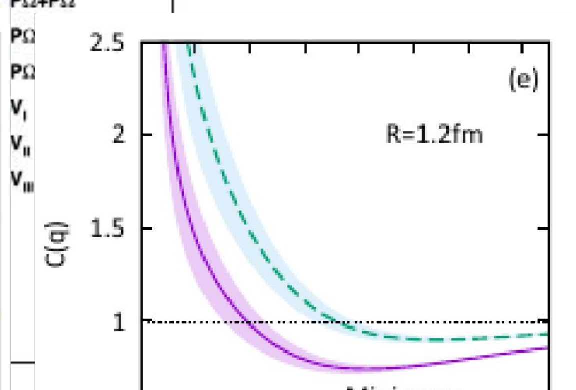
$p\Omega^-$ correlation

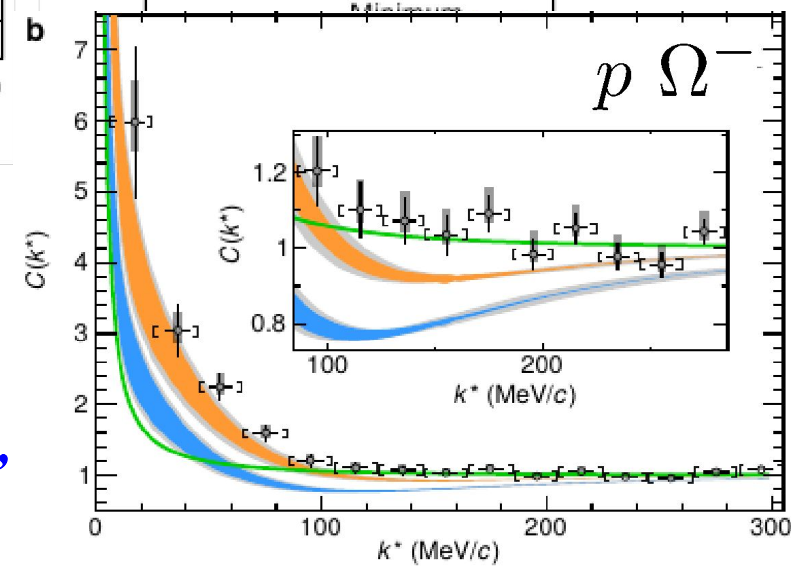


K. Morita, AO, F. Etminan, T. Hatsuda, PRC94('16)031901(R) (w/ Lattice potential with heavier quark mass)



J. Adam+[STAR], PLB790('19)490.





K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, Y. Kamiya, AO, PRC 101('20)015201. (w/ Lattice potential at physical quark mass)

S. Acharya+[ALICE], 2005.11495 [nucl-ex] (pp 13 TeV)

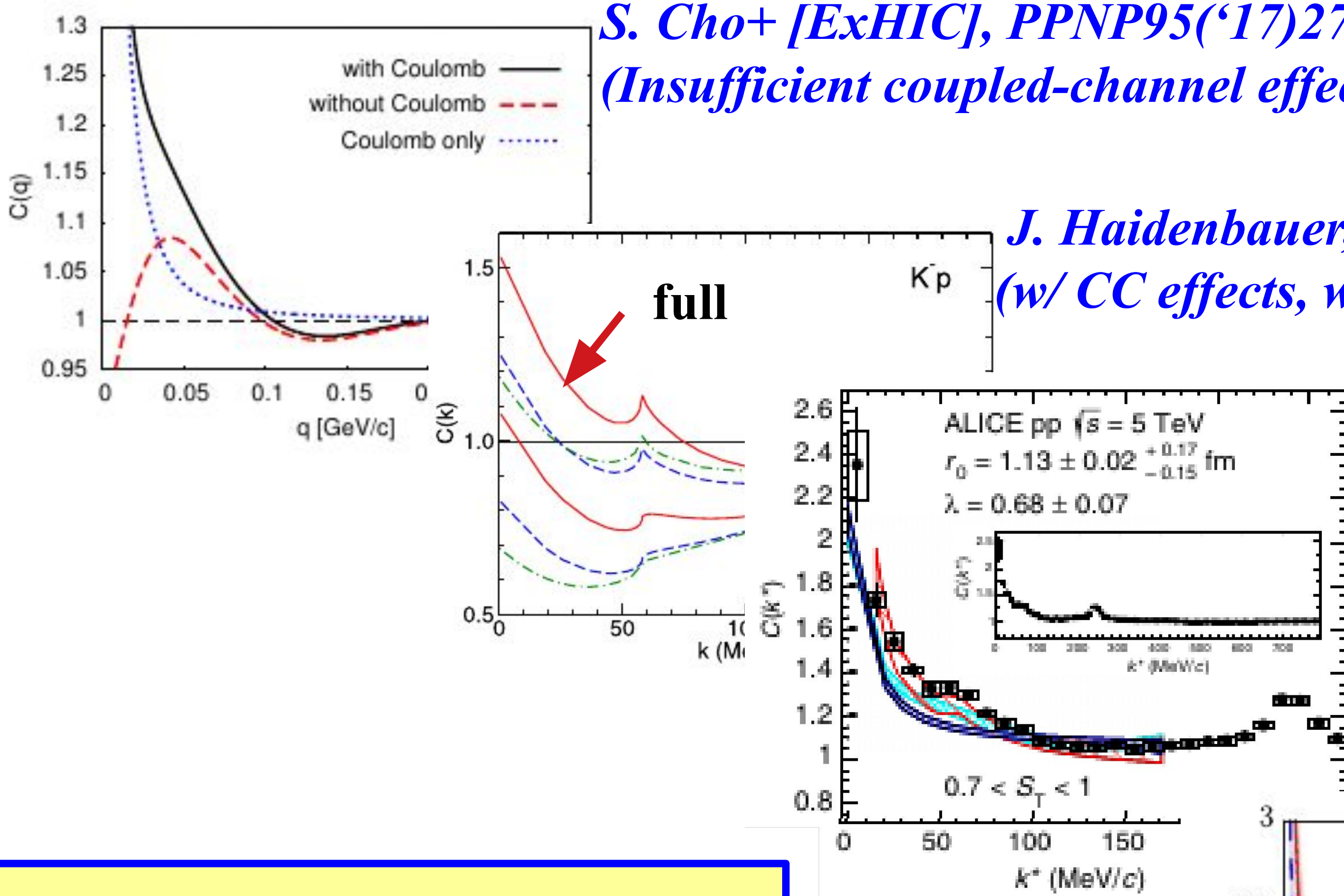
Bound state ?

pK⁻ correlation

S. Cho+ [ExHIC], PPNP95('17)279.
(Insufficient coupled-channel effects)

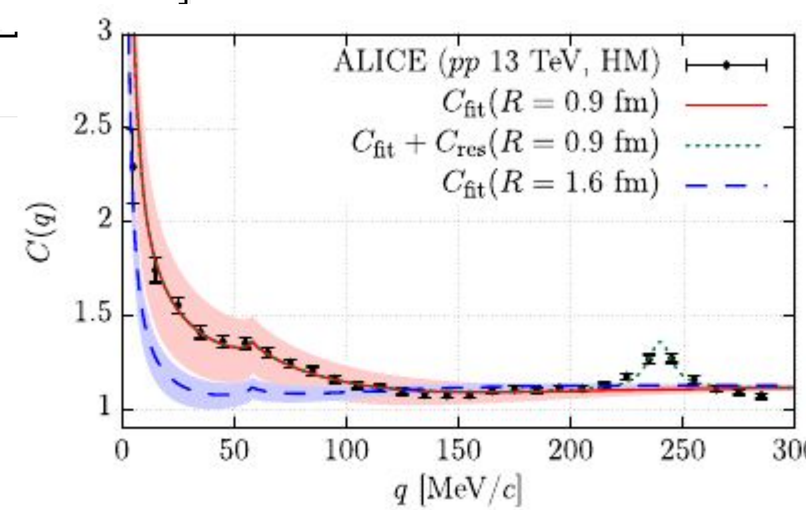
J. Haidenbauer, NPA981('19)1.
(w/ CC effects, w/o Coulomb)

S. Acharya+[ALICE], PRL124('20)092301



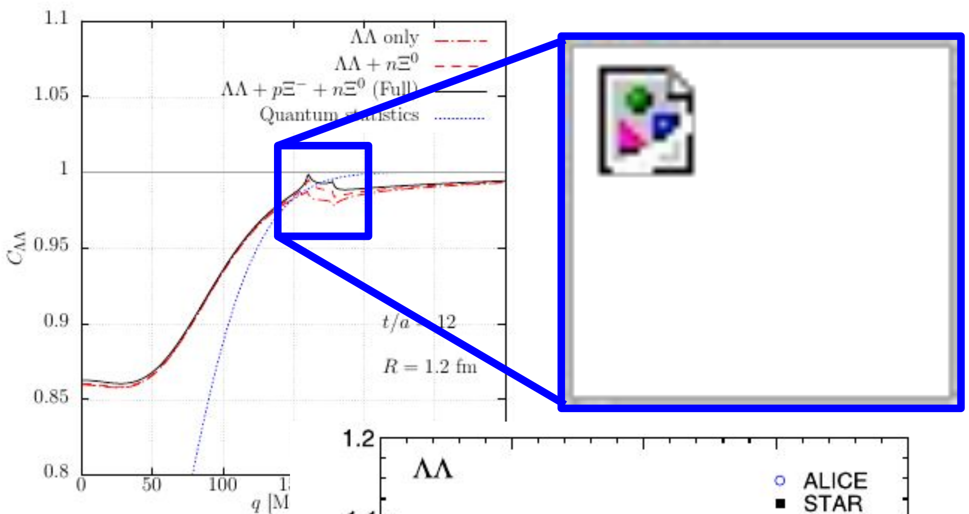
Source size dep. shows interesting feature.

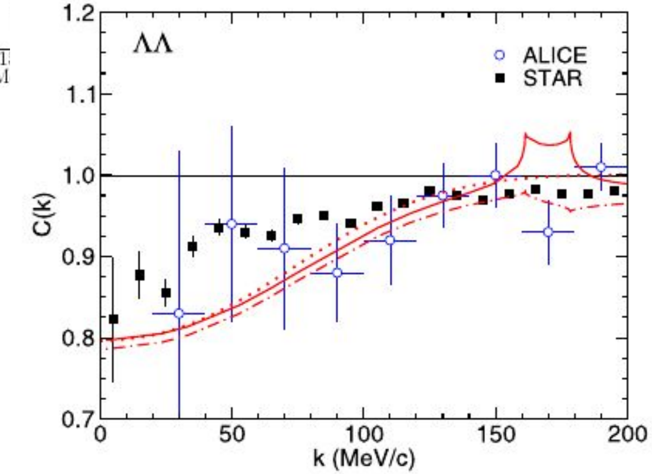
Y. Kamiya, T. Hyodo, K. Morita, AO, W. Weise, PRL124('20)132501.
(Chiral SU(3) dynamics)



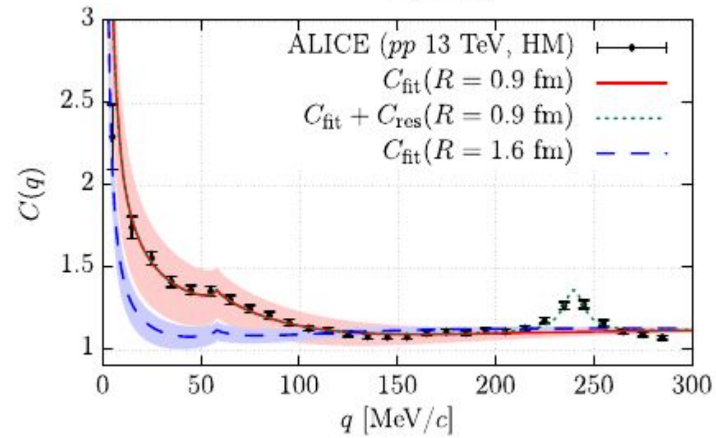
Other bound states ?

- **ΛΛ-NΞ**
  - **$C_{\Lambda\Lambda}(q)$ in AA(RHIC) and pp(LHC) are similar (No b.s. below ΛΛ).**
  - **LQCD predicts a virtural pole near NΞ threshold, which can be detected as the cusp in $C_{\Lambda\Lambda}(q)$. NLO(600) potential predicts the same. (The fate of H particle)**
    K. Sasaki+[HAL QCD], NPA998('20)121737;
    Y. Kamiya+, in prep.; Haidenbauer('19).





- **KN**
  - **Λ(1405) is believed to be the bound state of KN, and "dip" is expected at $R \sim a_0$.**
  - **However, Coulomb and coupled-channel effects modify the dip-like behavior.**
    Kamiya+ ('20).



Correlation Function with Coupled-Channels Effects

J. Haidenbauer, NPA 981('19)1; R. Lednicky, V. V. Lyuboshits, V. L. Lyuboshits, Phys. At. Nucl. 61('98)2950.

- **Single channel, w/o Coulomb (non-identical pair)**

$$C(\boldsymbol{q}) = 1 + \int d\boldsymbol{r} S(\boldsymbol{r}) \left[|\chi^{(-)}(r,q)|^2 - |j_0(qr)|^2 \right]$$

- **Single channel, w/ Coulomb**

$$C(\boldsymbol{q}) = \int d\boldsymbol{r} S(\boldsymbol{r}) \left[|\varphi^{C,\mathrm{full}}(\boldsymbol{q},\boldsymbol{r})|^2 + |\chi^{C,(-)}(r,q)|^2 - |j_0^C(qr)|^2 \right]$$

Full free Coulomb w.f. — **s-wave w.f. with Coul.** — **s-wave Coul. w.f.**

- **Coupled channel, w/ Coulomb**

$$C_i(\boldsymbol{q}) = \int d\boldsymbol{r} S_i(\boldsymbol{r}) \left[|\varphi^{C,\mathrm{full}}(\boldsymbol{q},\boldsymbol{r})|^2 + |\chi_i^{C,(-)}(r,q)|^2 - |j_0^C(qr)|^2 \right]$$

$$+ \sum_{j \neq i} \omega_j \int d\boldsymbol{r} S_j(\boldsymbol{r}) |\chi_j^{C,(-)}(r,q)|^2$$

s-wave w.f. in j-th channel

Outgoing B.C. in the i-th channel, ω_j = Source weight (ω_j=1)

To be, or not to be, that is the question.

A. Gal ('16); M. Oka ('88)

Table 1. Leading 6q $L = 0$ dibaryon candidates [12], their BB' structure and the CM interaction gain with respect of the lowest BB' threshold calculated by means of Eq. (2). Asterisks are used for the $\mathbf{10}_\mathrm{f}$ baryons $\Sigma^* \equiv \Sigma(1385)$ and $\Xi^* \equiv \Xi(1530)$. The symbol [i,j,k] stands for the Young tablaux of the SU(3)$_\mathrm{f}$ representation, with i arrays in the first row, j arrays in the second row and k arrays in the third row, from which $\mathcal{P}_\mathrm{f}$ is evaluated. The $\overline{\mathbf{10}}$ SU(3)$_\mathrm{f}$ representation is denoted here $\mathbf{10}^*$.

$-S$	SU(3)$_\mathrm{f}$	I	J^π	BB' structure	$\frac{\Delta\langle V_{\rm CM}\rangle}{M_0}$
0	[3,3,0] $\mathbf{10}^*$	0	3^+	$\Delta\Delta$	0
1	[3,2,1] $\mathbf{8}$	1/2	2^+	$\frac{1}{\sqrt{5}}(N\Sigma^* + 2\Delta\Sigma)$	-1
2	[2,2,2] $\mathbf{1}$	0	0^+	$\frac{1}{\sqrt{8}}(\Lambda\Lambda + 2N\Xi - \sqrt{3}\Sigma\Sigma)$	-2
3	[3,2,1] $\mathbf{8}$	1/2	2^+	$\frac{1}{\sqrt{5}}(\sqrt{2}N\Omega - \Lambda\Xi^* + \Sigma^*\Xi - \Sigma\Xi^*)$	-1