



Heavy-Light Susceptibilities in a Strongly Coupled Quark-Gluon Plasma

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Hunan University

In collaboration: Ralf Rapp



Quark Matter, April 4-10, 2022 Kraków, Poland (Online)

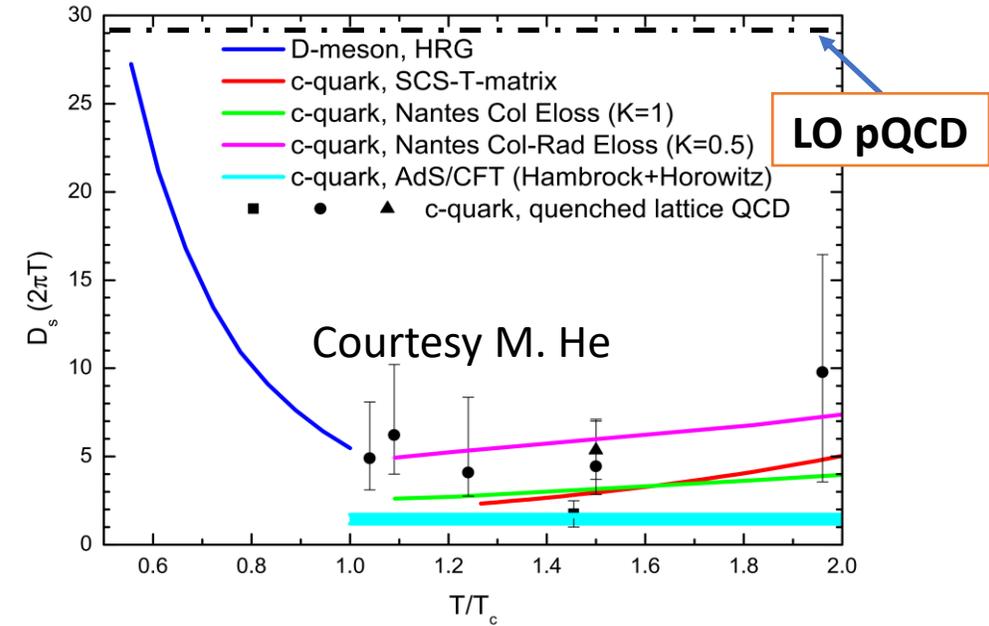
Based on work, Liu and Rapp, arXiv:2111.13620, Phys.Rev.C 97 (2018) 3, 034918

Outline

- 1) Background and motivation
- 2) T-matrix approach
- 3) Extend to finite chemical potential
- 4) Heavy-light susceptibilities
- 5) Summary

A Strongly Coupled Picture of QGP

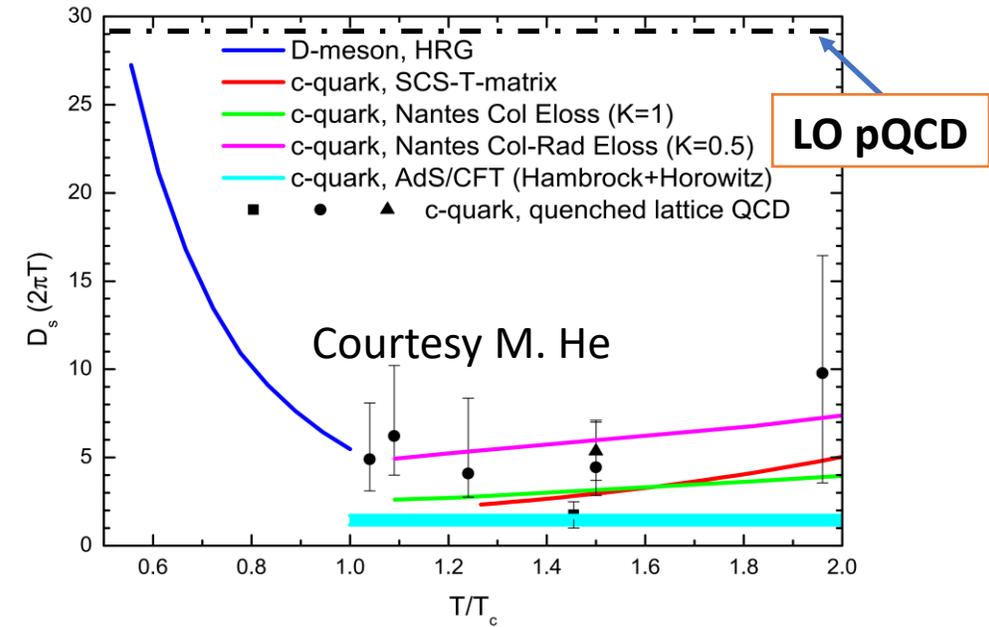
- ❖ $(2\pi T)D_s$, close to strongly coupled limit, an order of magnitude smaller than LO pQCD.
- ❖ QGP is strongly coupled
- ❖ **Microscopically ?**



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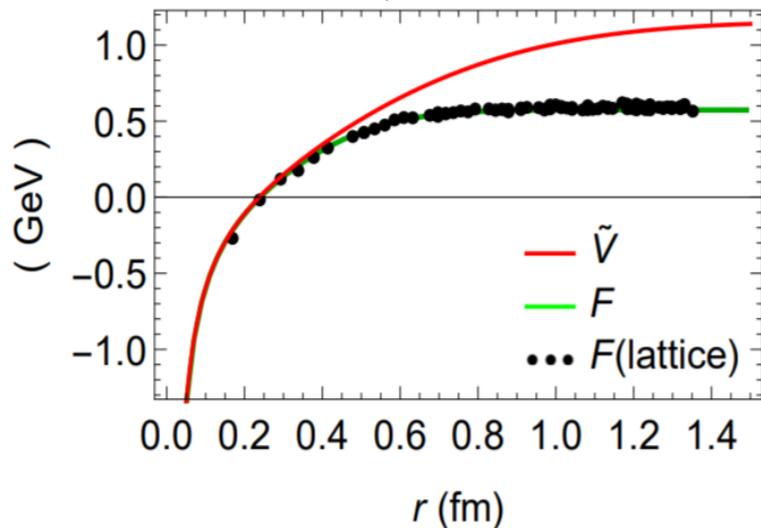
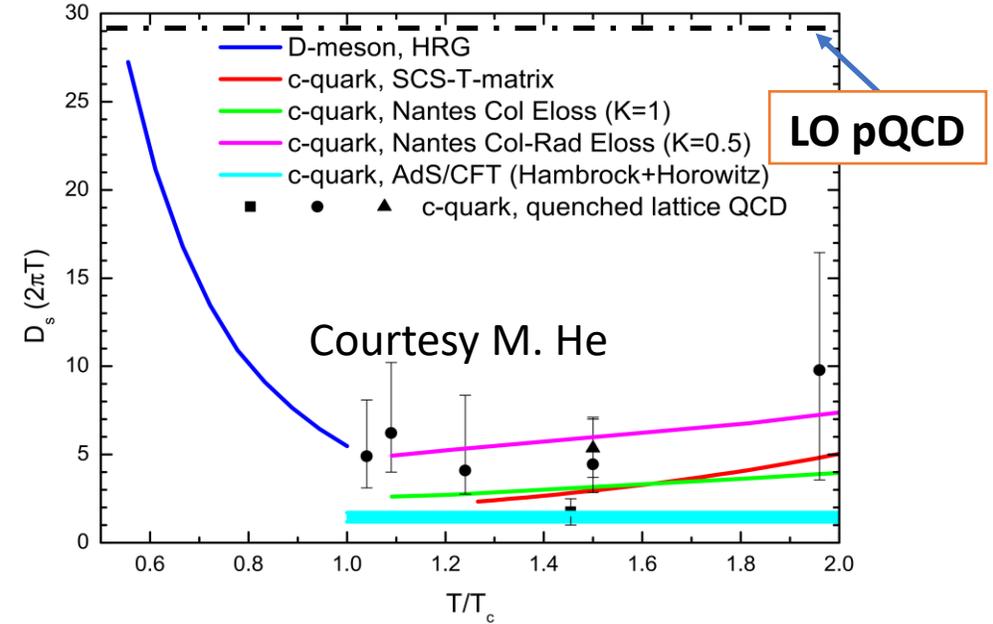
Liu&Rapp, PRC 2018



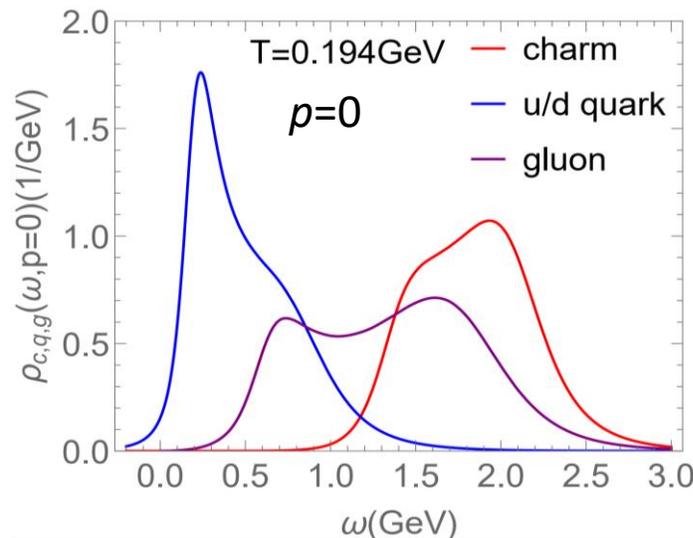
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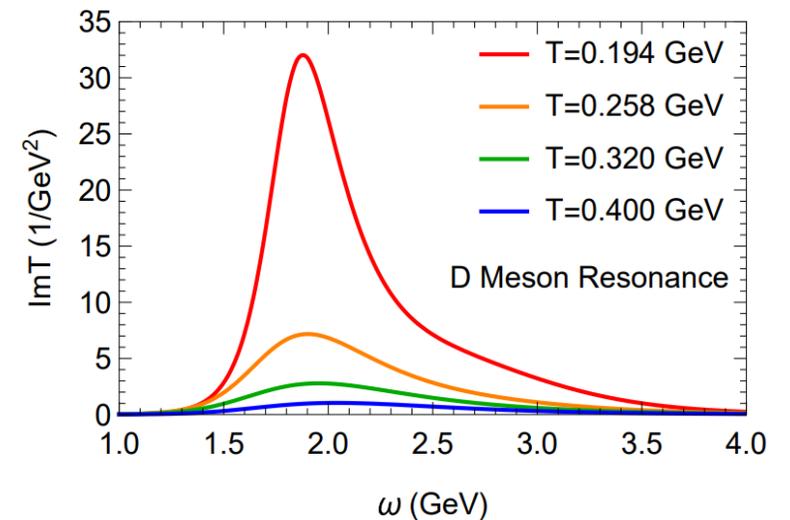
- Large confining interactions Liu&Rapp, PRC 2018



No quasiparticle partons

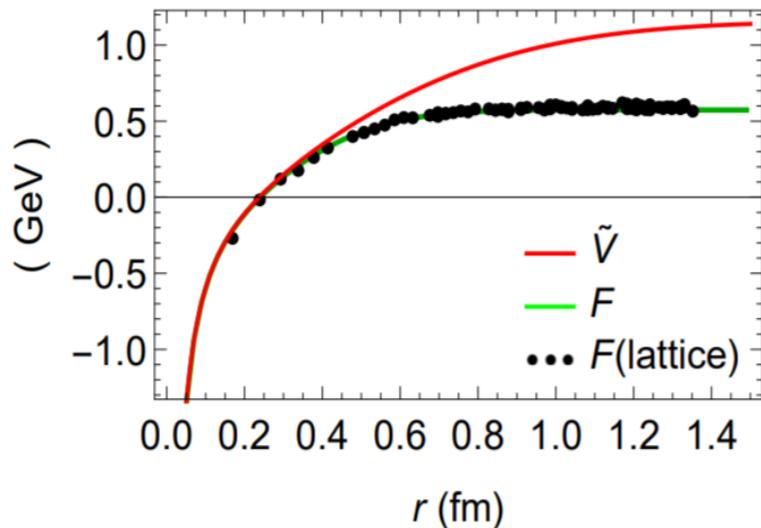
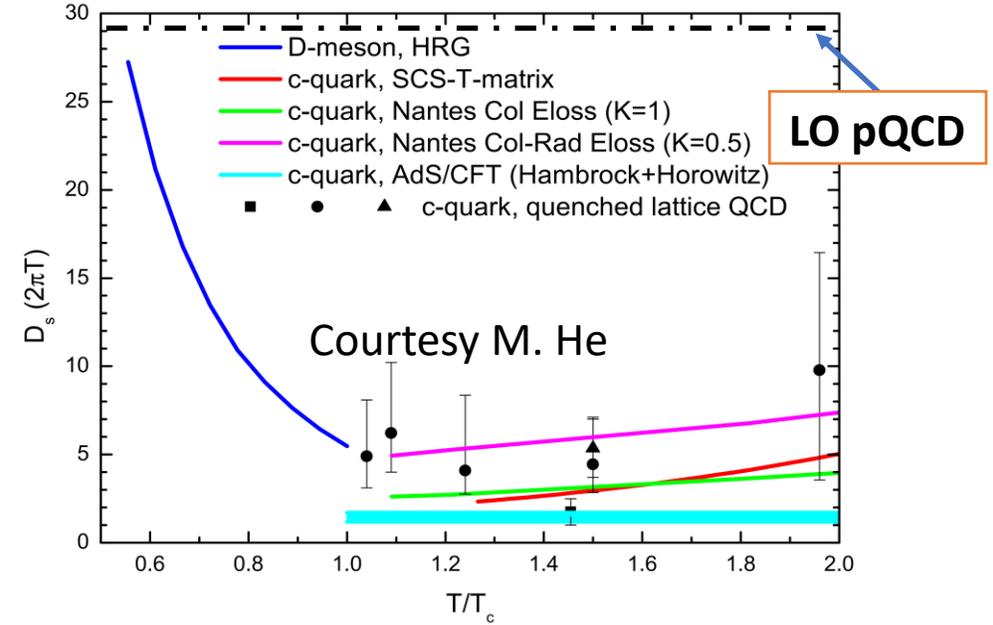


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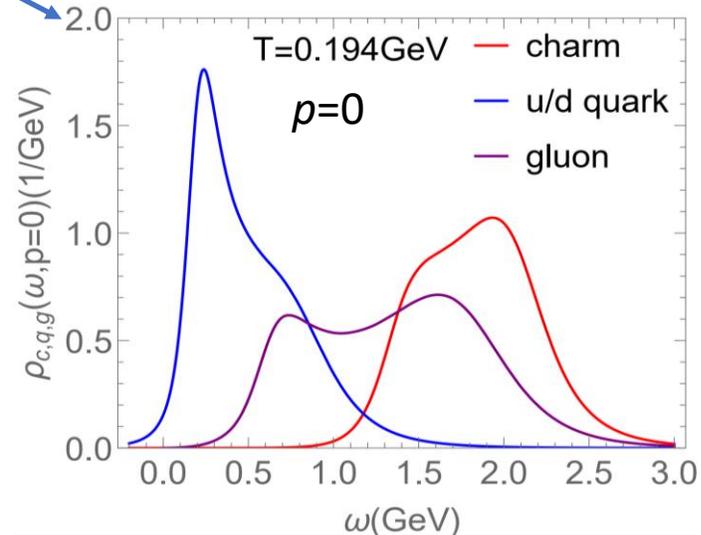


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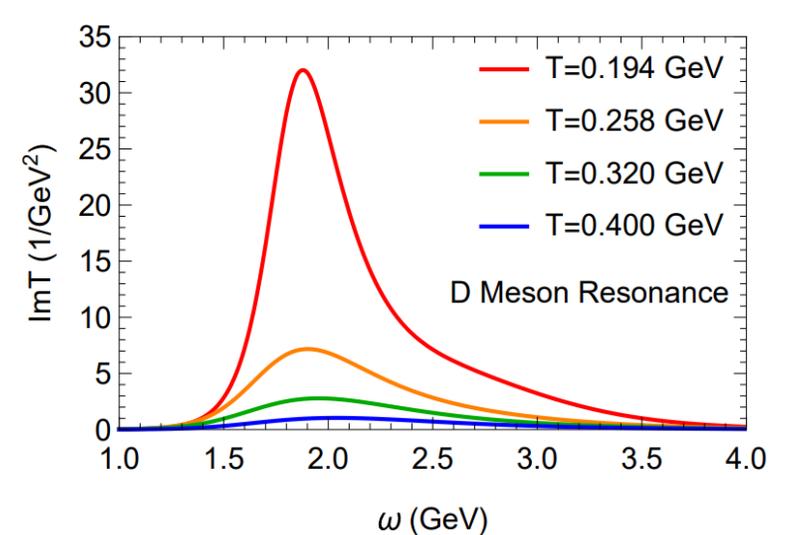
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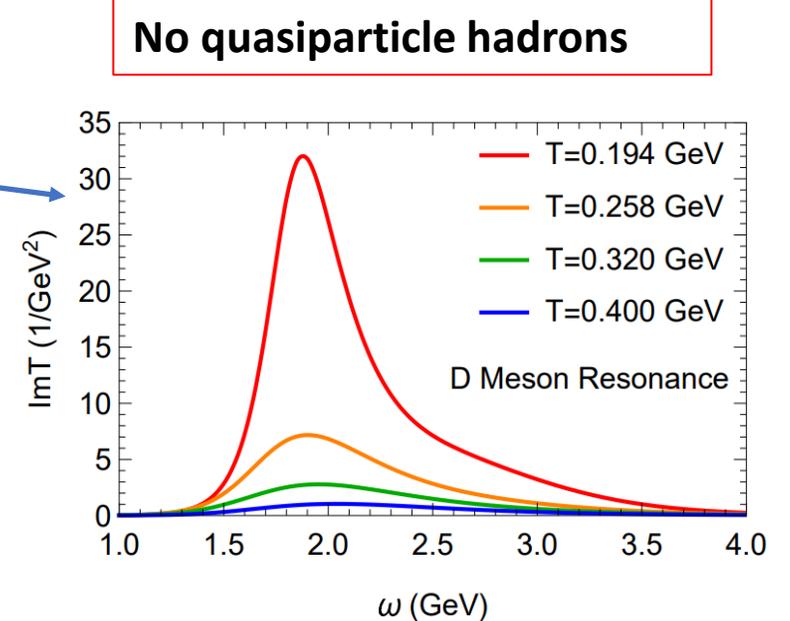
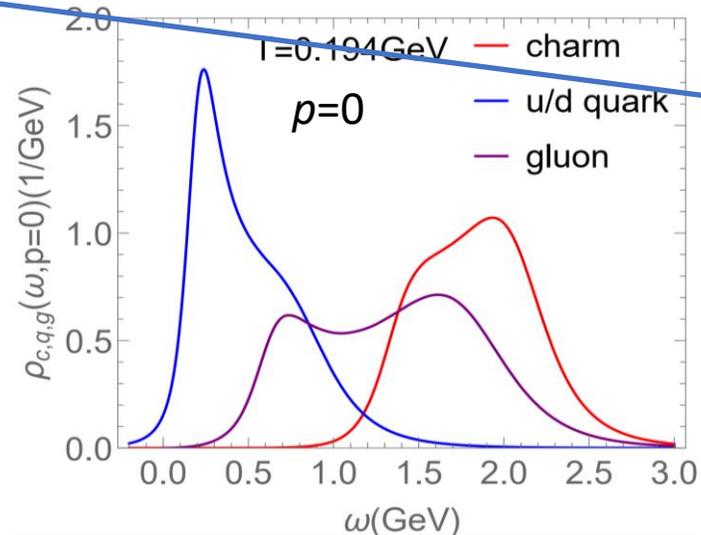
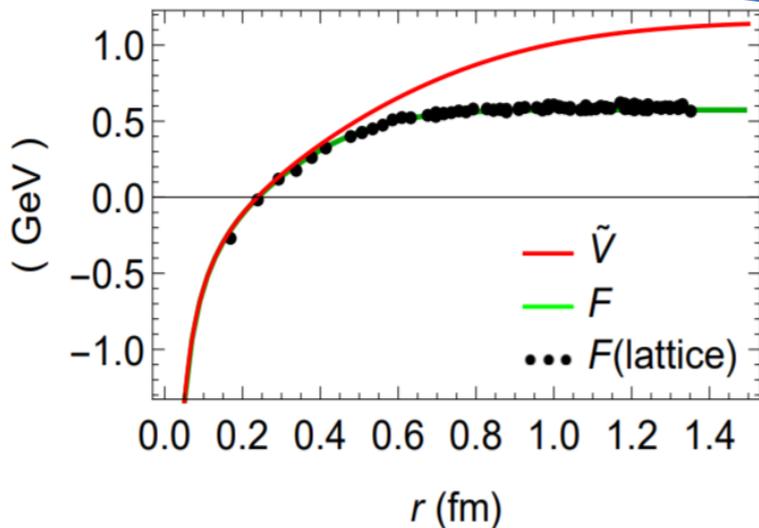
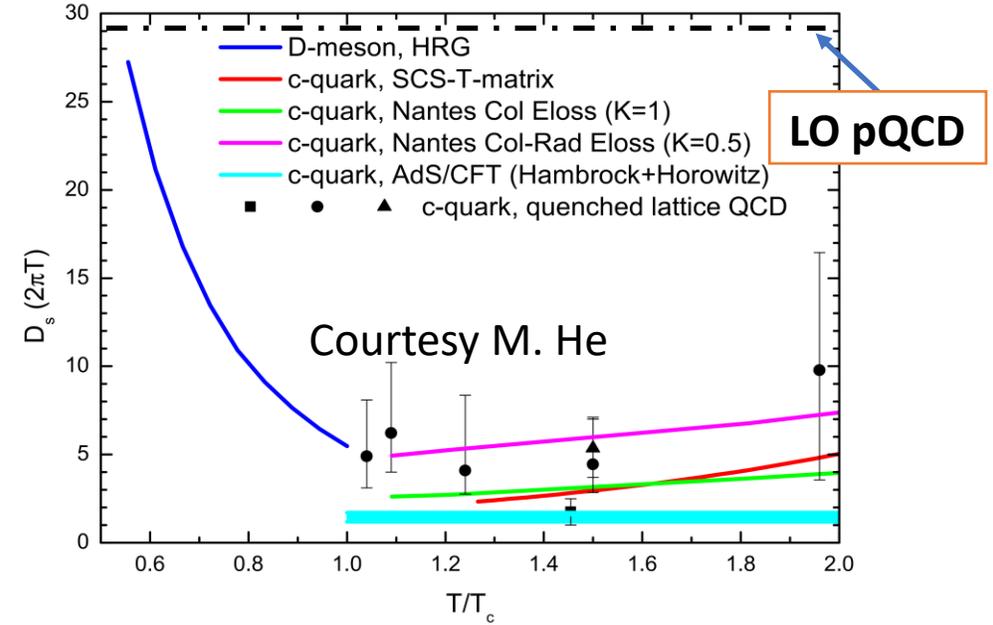


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 - **Broad hadronic resonances**

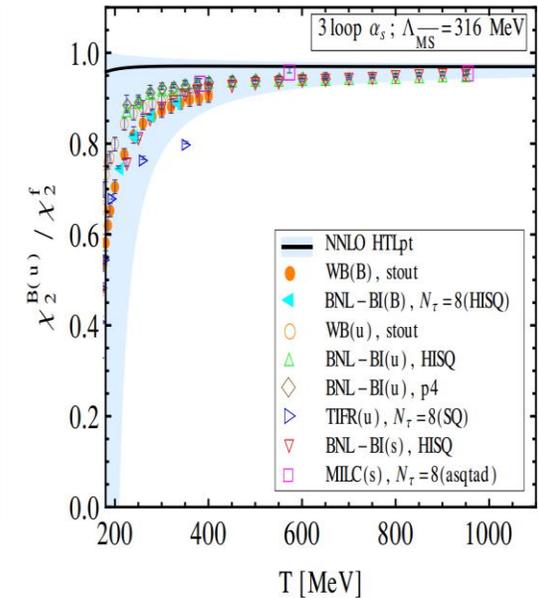
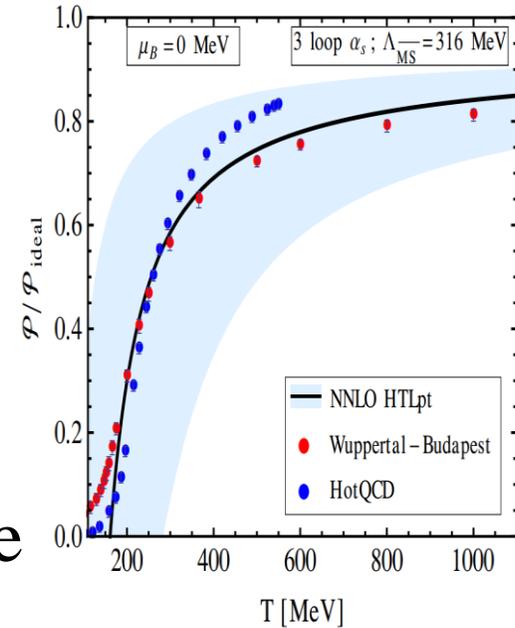


Weakly Coupled Studies on QGP

❖ Weakly coupled calculations, such as Hard Thermal Loop (HTL), are consistent with LQCD data:

- Equation of state (EoS)
- Quark number susceptibility
- Heavy-light susceptibilities

❖ Off-diagonal susceptibilities, expected to be sensitive to the nature of the coupling strength



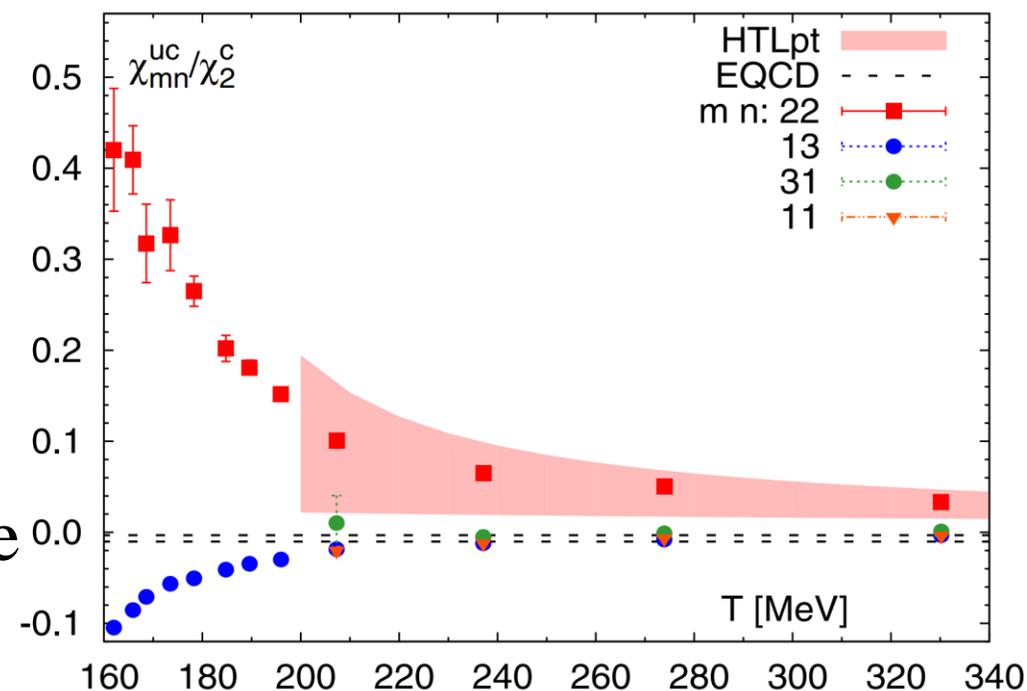
Haque, Bandyopadhyay, Andersen,
Mustafa, Strickland, Su, JHEP 2014

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Mukherjee, Petreczky, Sharma, PRD 2016

Does the agreement between weakly coupled calculations and LQCD results mean that QGP is weakly coupled?

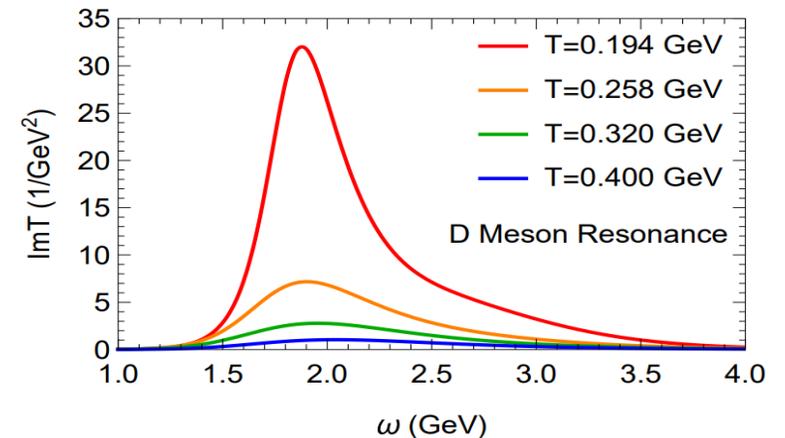
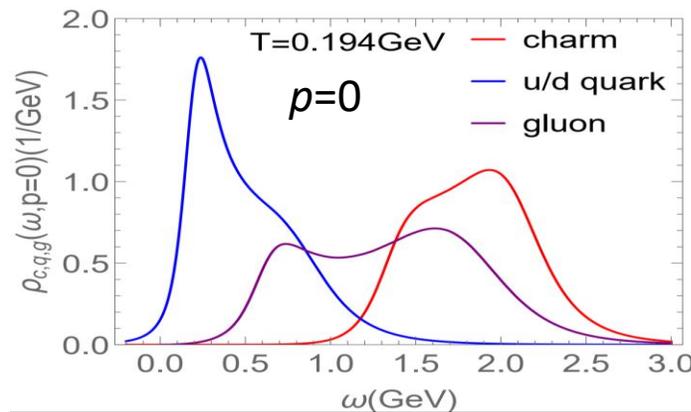
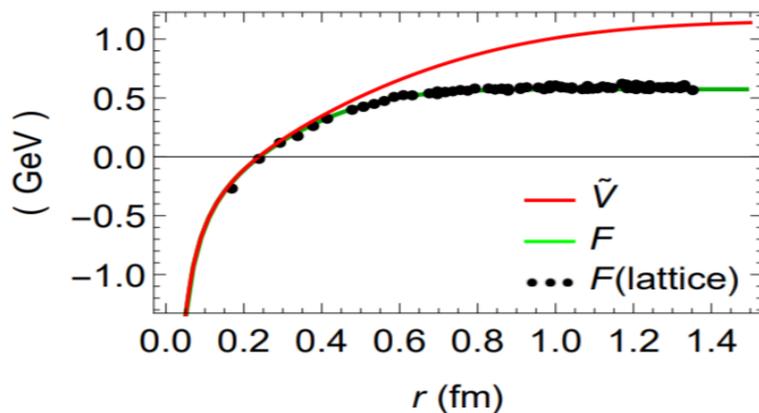
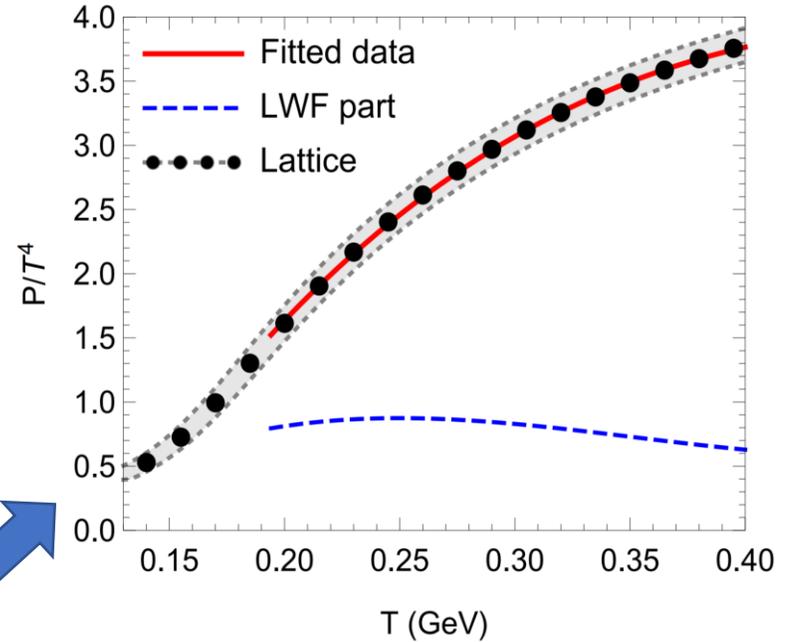
One way to address: do a calculation in a strongly coupled approach for these susceptibilities and compare it to LQCD

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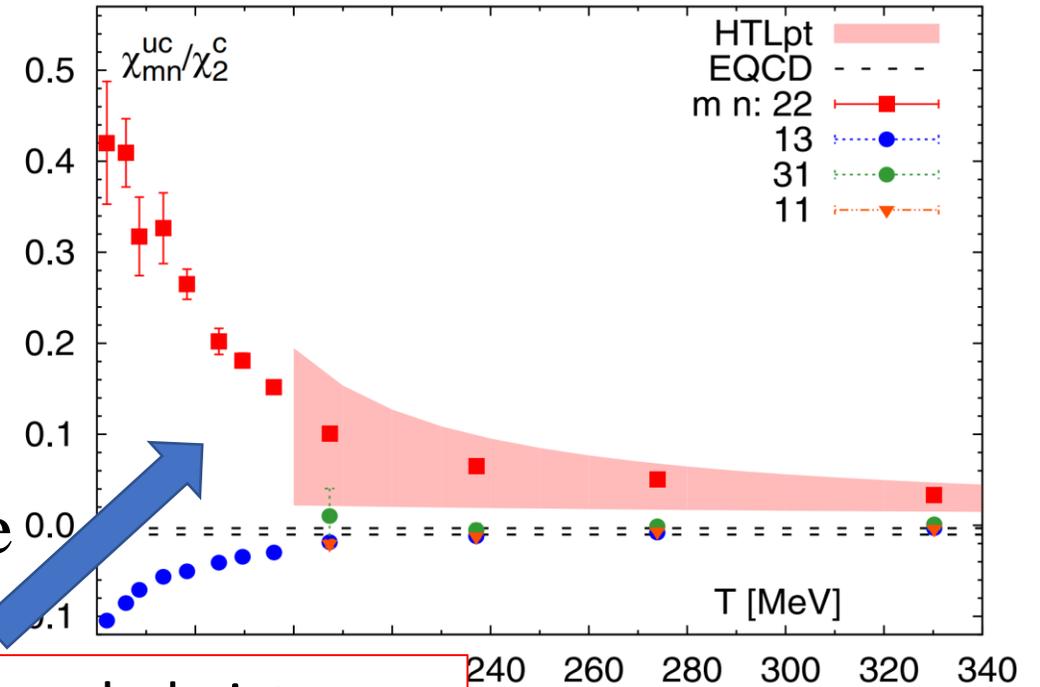


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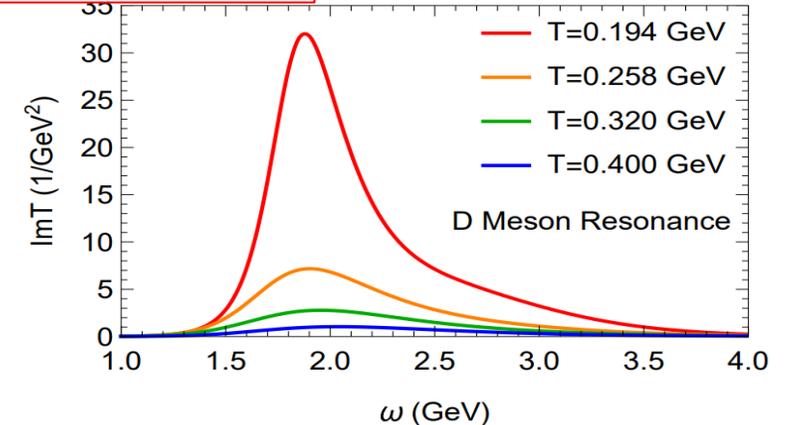
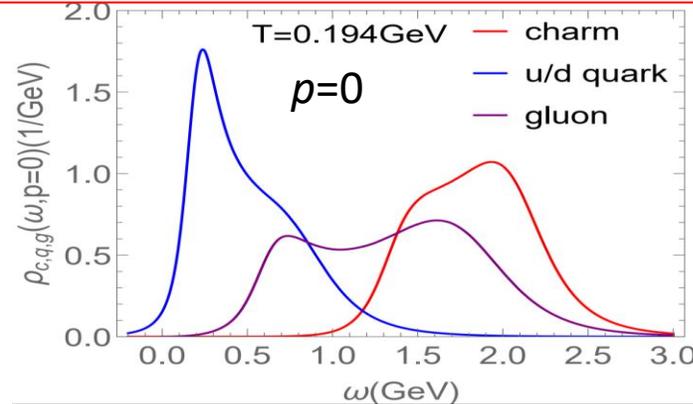
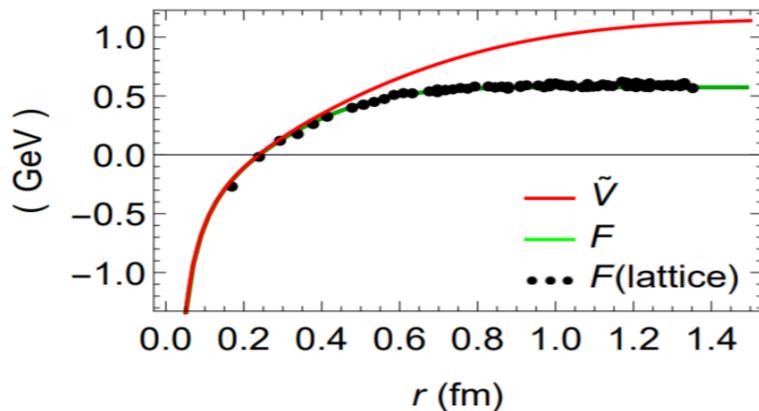
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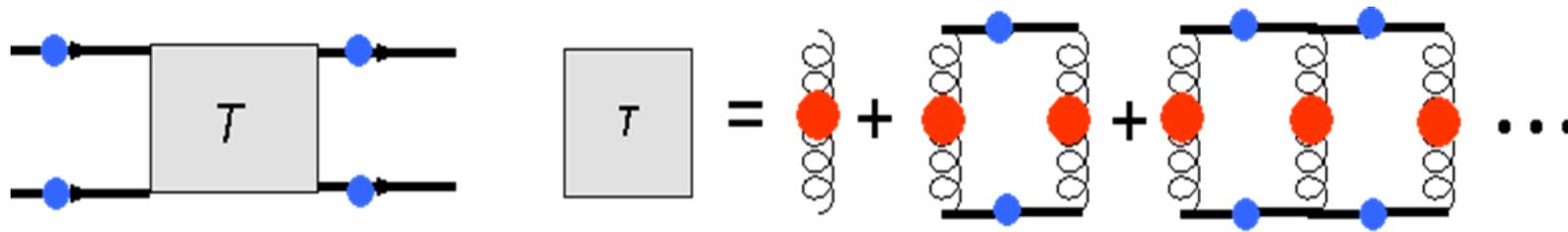
❖ Off-diagonal susceptibilities, expected to be sensitive to the nature of the coupling strength



Could a model with a strongly coupled picture, such as **T-matrix Approach**, explain data?



T-matrix Approach to the Strongly Coupled Plasma



- Dynamical bound states generation
- Scattering states in equal footing

❖ T-matrix equation: $T(E, \mathbf{p}, \mathbf{p}') = V(\mathbf{p}, \mathbf{p}') + \int \frac{d^3 p}{(2\pi)^3} V(\mathbf{p}, \mathbf{k}) G_{(2)}(E, \mathbf{k}) T(z, \mathbf{p}, \mathbf{p}')$

$$G_{(2)} = G G \quad G = \frac{1}{z - \varepsilon_{\mathbf{p}} - \Sigma} \quad \Sigma = \sum_{s,c,f} \int d^4 k T(G) G$$

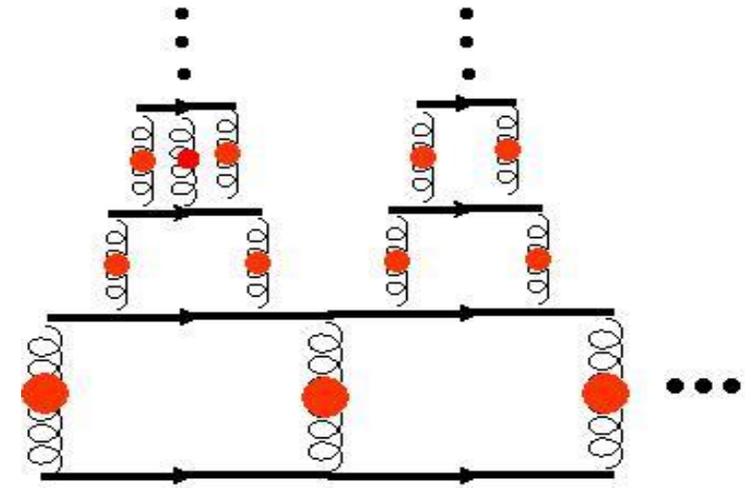
Self consistent self energy and many-body effects

❖ Luttinger–Wald functional (LWF):

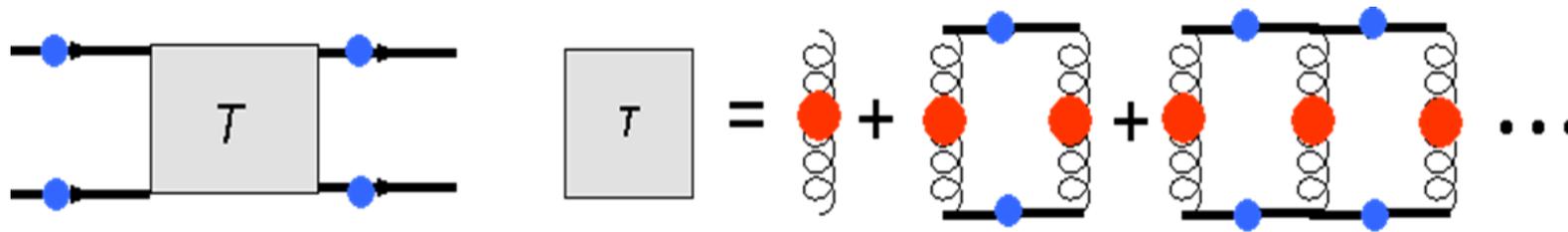
$$\Phi = \sum_{n,v} \text{Tr} \left\{ \frac{1}{2v} \Sigma_v G \right\} = \frac{1}{2} \sum_n \text{Tr} \{ -\ln(1 - GG V) \}$$

❖ Grand potential in Luttinger–Wald Formalism:

$$\Omega = \mp \frac{-1}{\beta} \sum_n \text{Tr} \{ \ln(-G^{-1}) + (G_0^{-1} - G^{-1})G \} \pm \Phi$$



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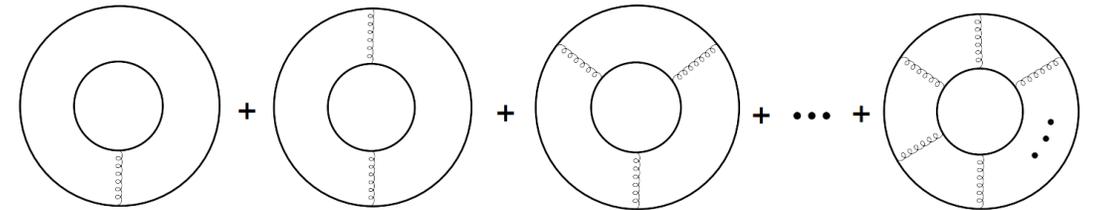
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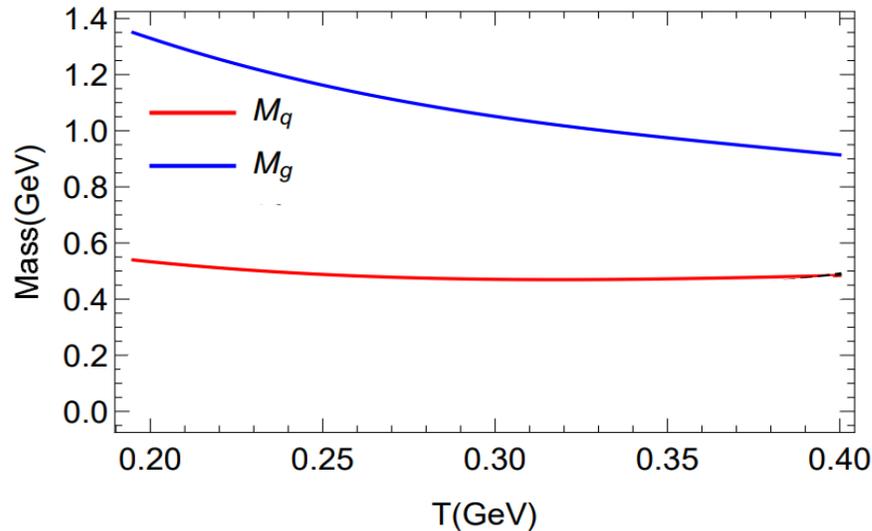
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Inputs of the T-matrix approach

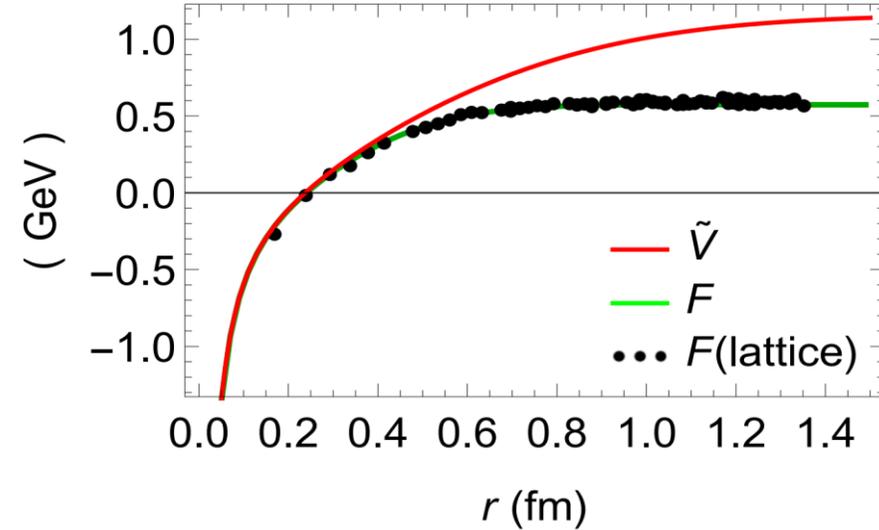
❖ Masses in the free propagators

$$G_0 = \frac{1}{z - \varepsilon_p}, \varepsilon_p = \sqrt{m_i^2 + p^2}$$

Liu&Rapp, PRC 2018



❖ The in-medium potential V

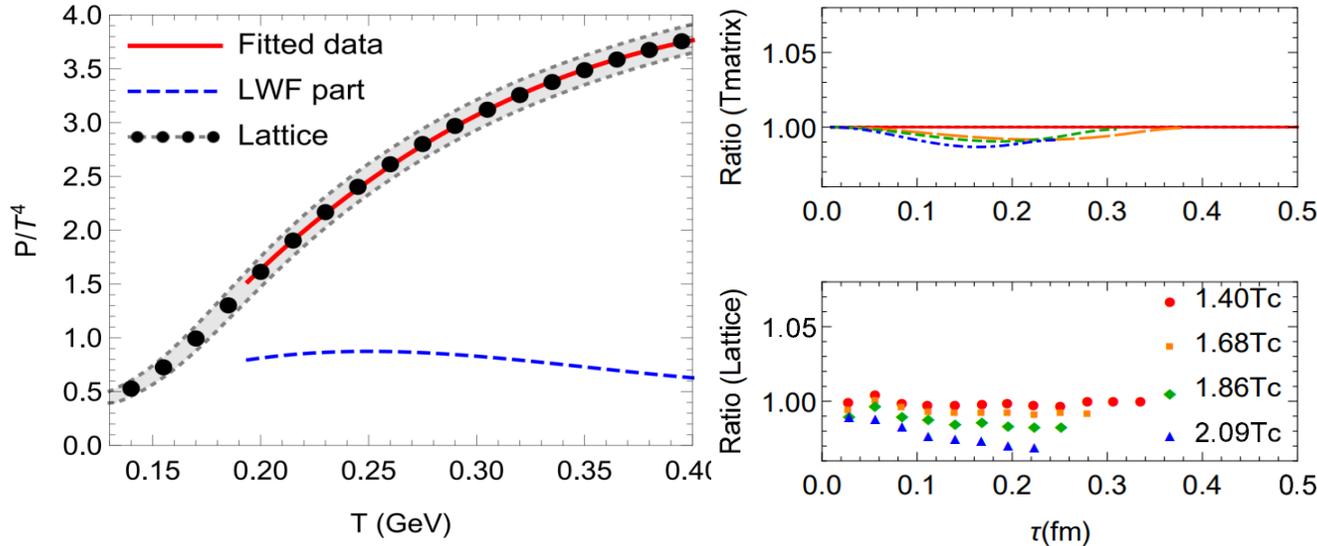


- ❖ These parameters are constrained by LQCD EoS, Euclidean correlators, heavy quark free energy.
- ❖ Resulting transport coefficients, such as D_s and η/s , reasonably agree with those expected from phenomenology and LQCD.

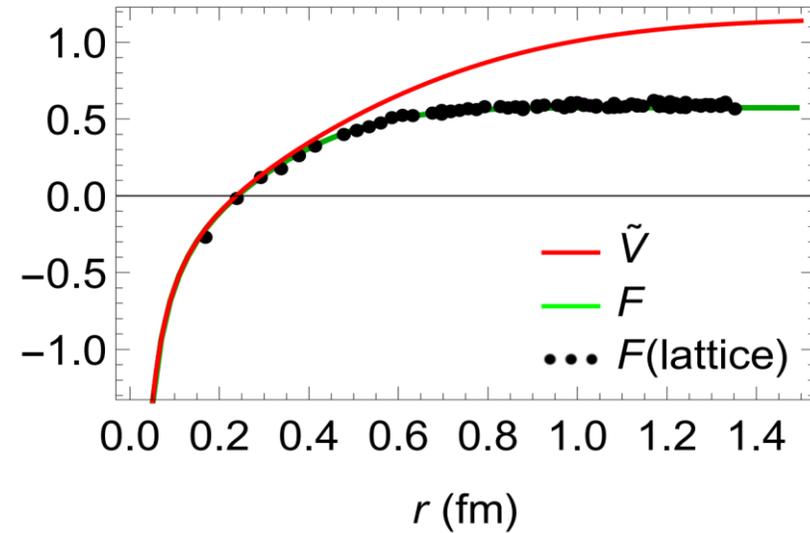
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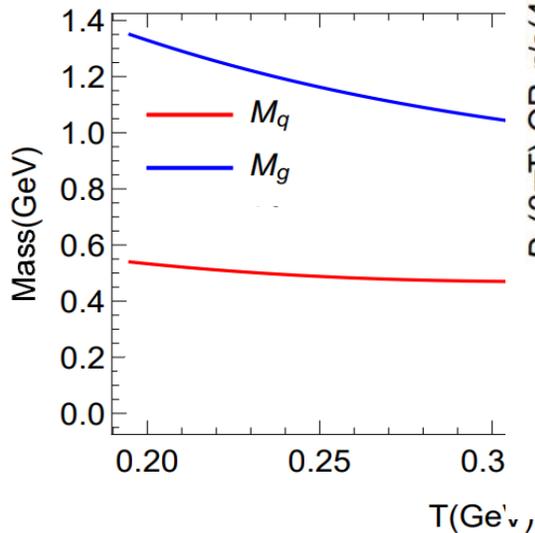
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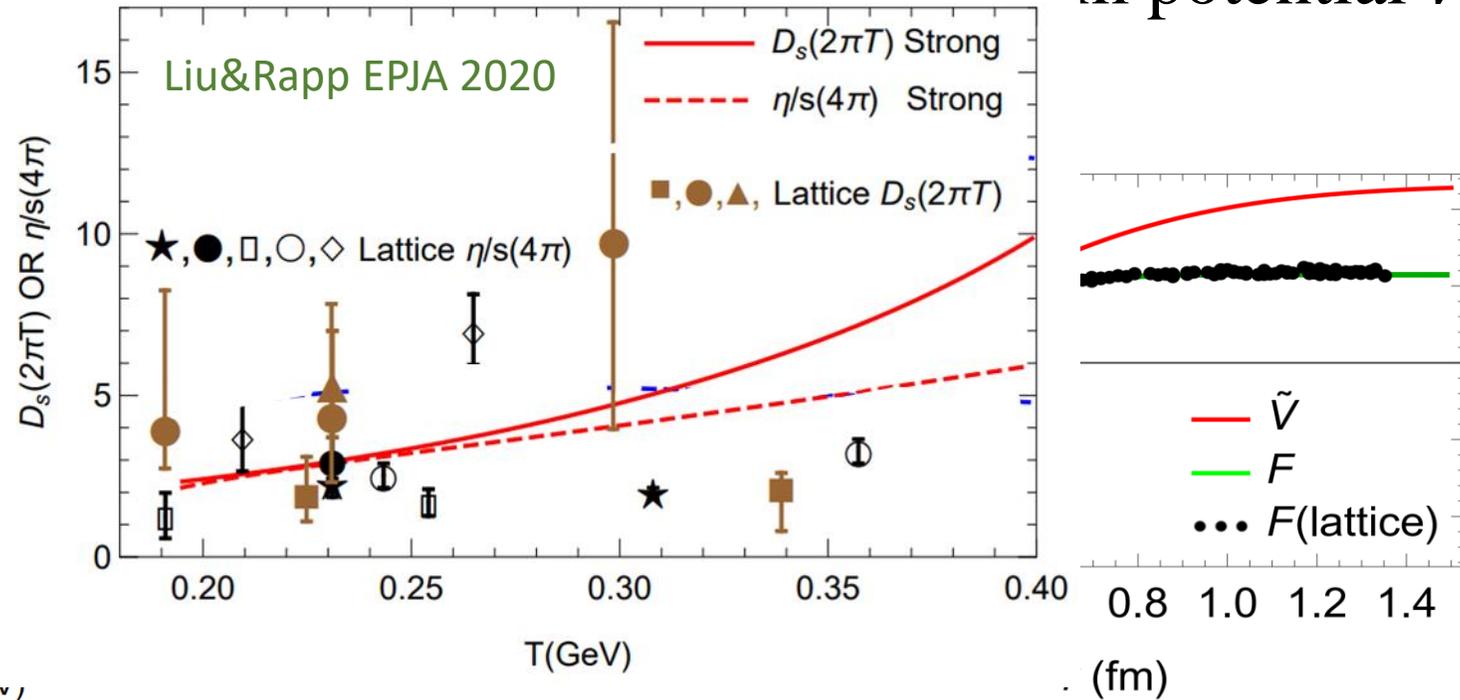
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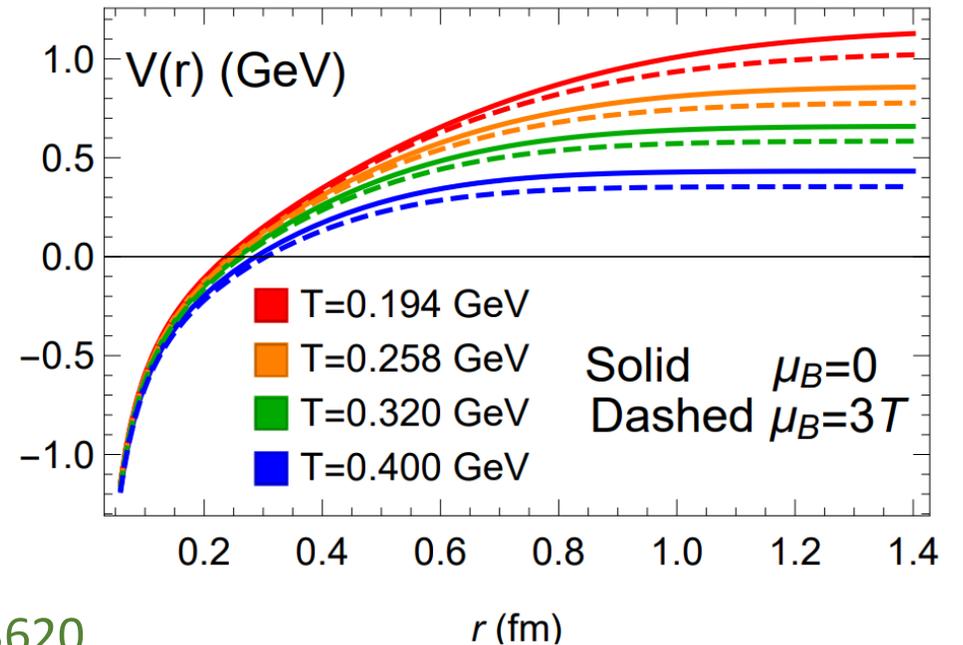
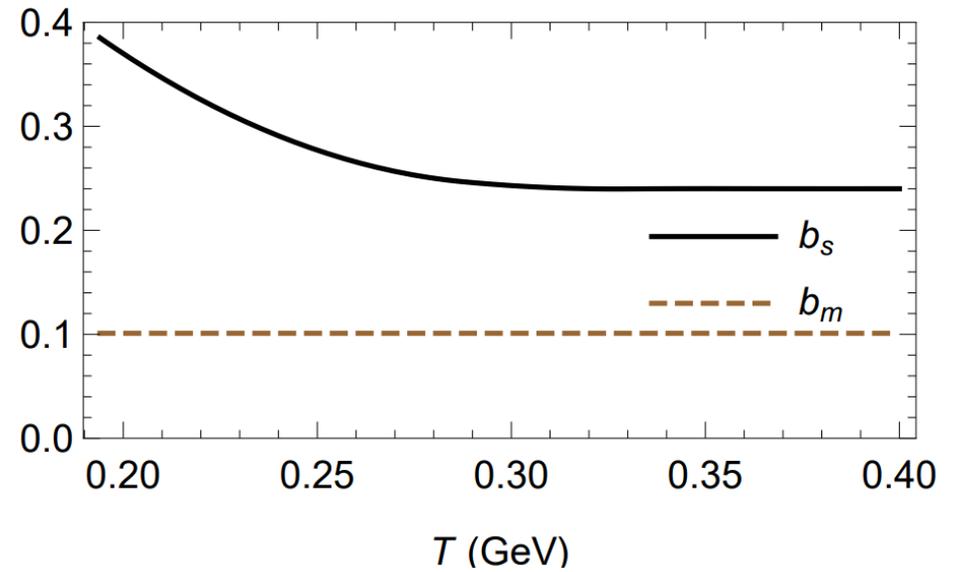
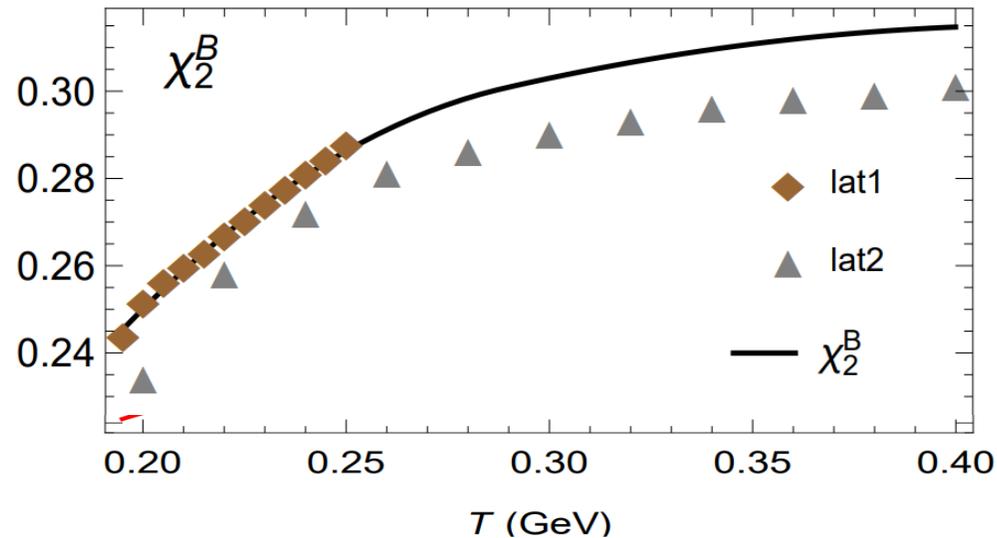
Extend to Finite Chemical Potential

❖ Propagator: $G_i^0(z, \mathbf{p}) = \frac{1}{z - \varepsilon_{\mathbf{p}} \pm \mu_i}, \varepsilon_{\mathbf{p}} = \sqrt{M_i^2 + p^2}$

❖ Masses: $M_i = M_i^0 \sqrt{1 + b_m \left(\frac{\mu_q}{T}\right)^2}$

❖ Screening masses: $m_d = m_d^0 \sqrt{1 + b_s \left(\frac{\mu_q}{T}\right)^2}$

❖ The parameters are fitted to the χ_2^B data



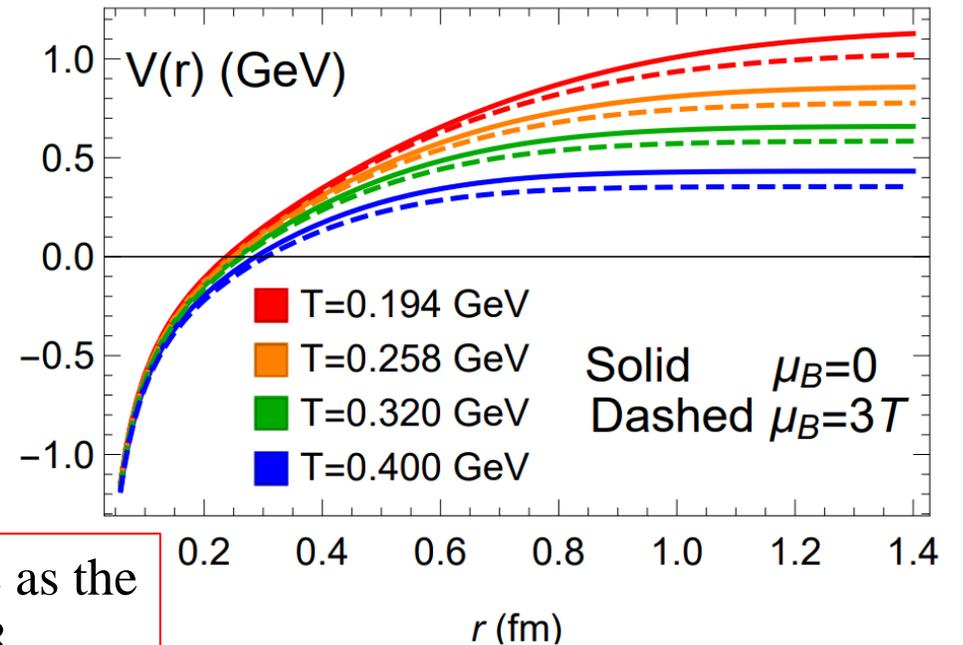
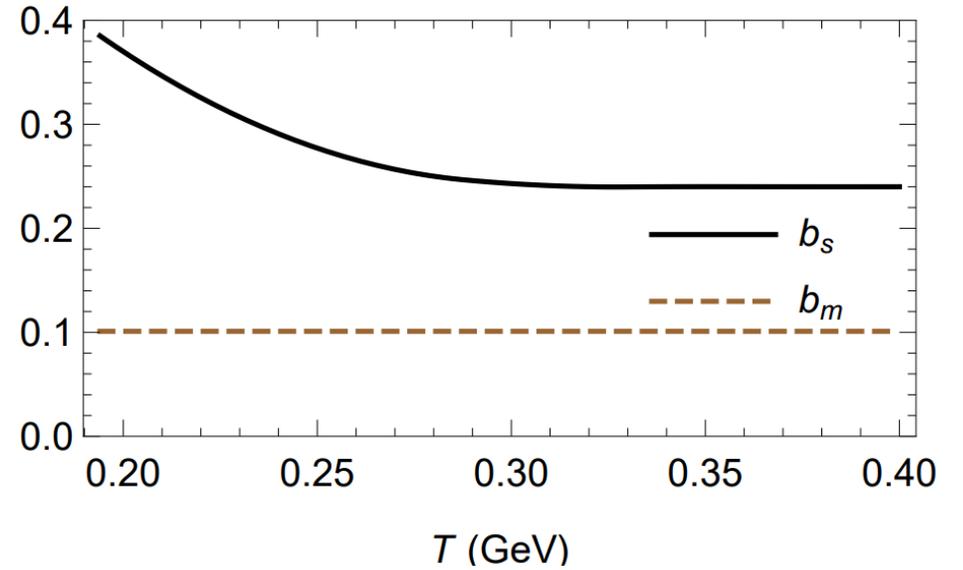
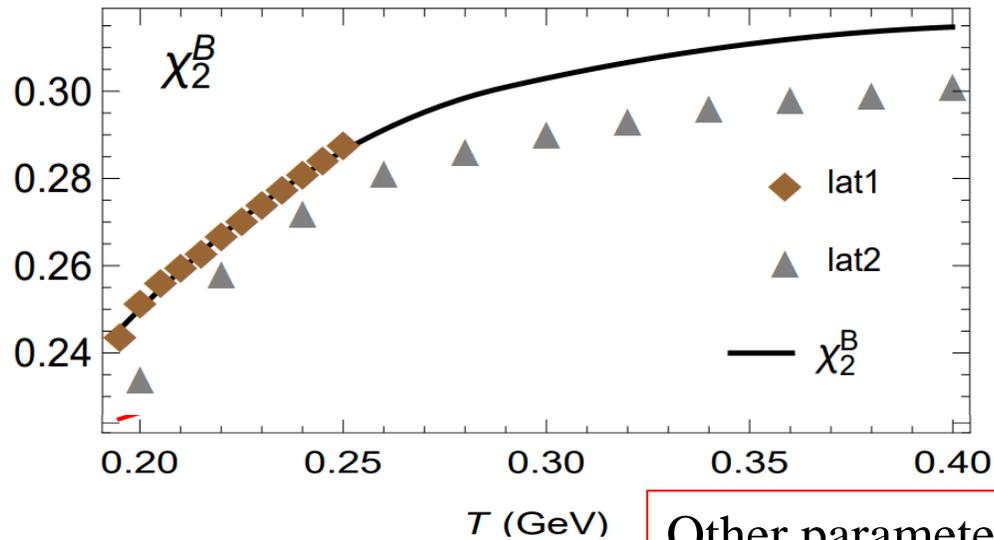
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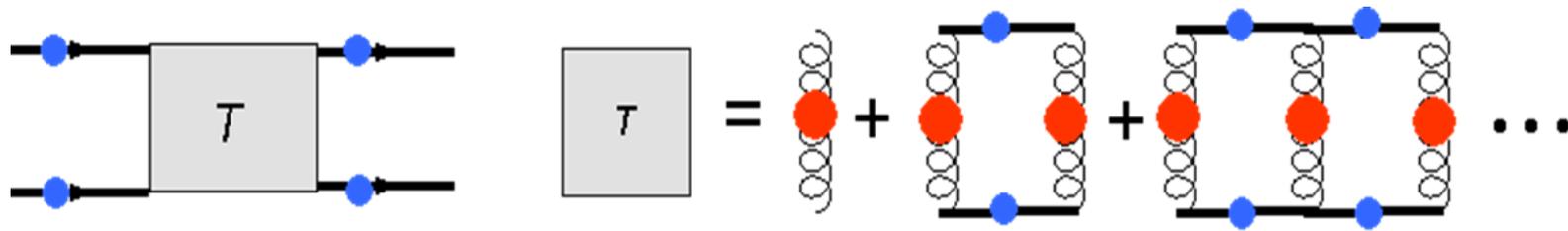
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Other parameters are the same as the SCS in Liu & Rapp, PRC 2018

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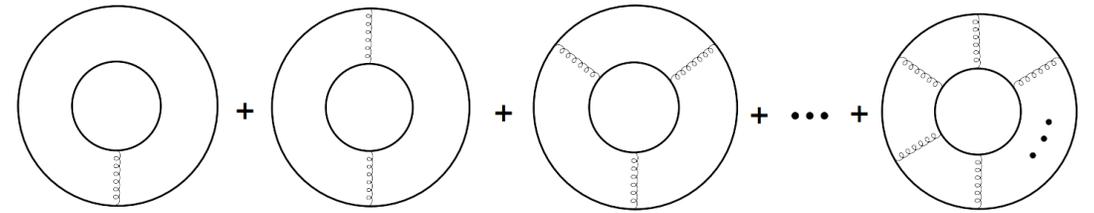
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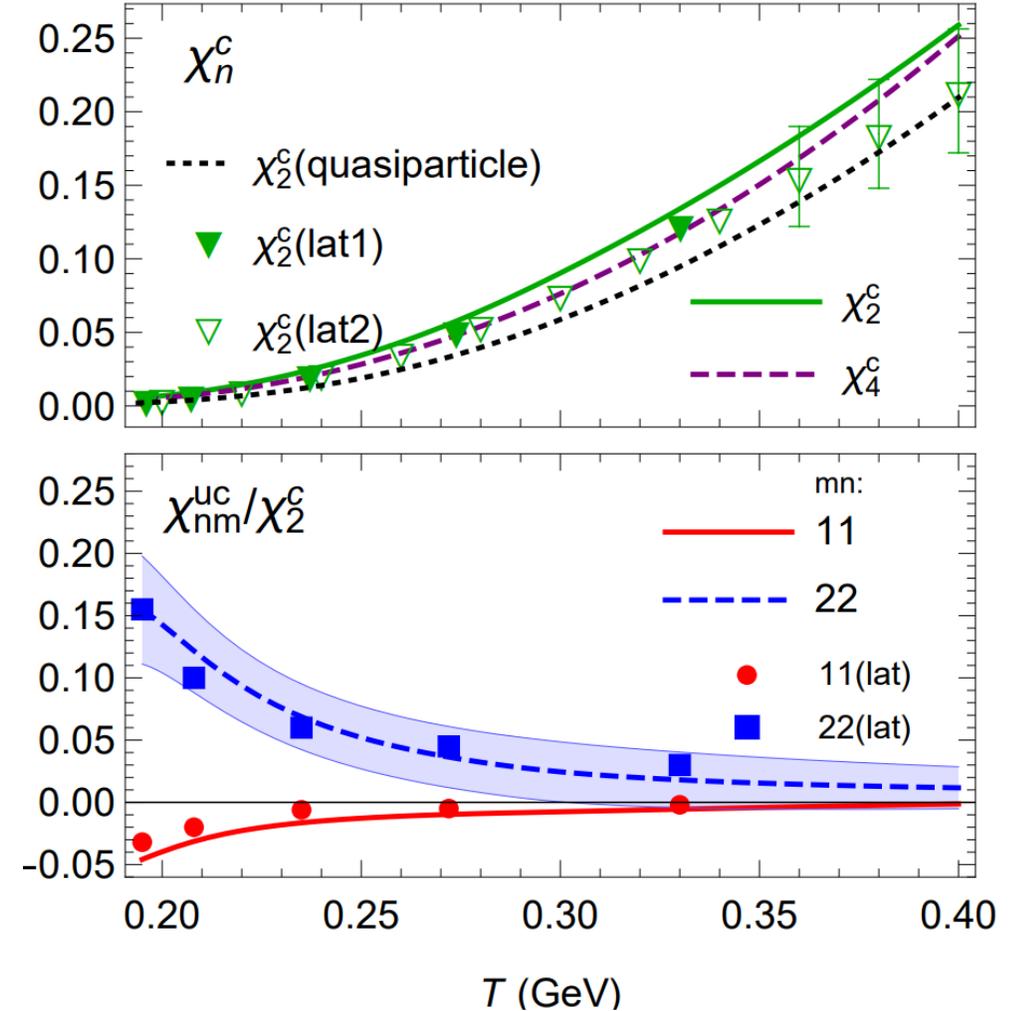
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$$\chi_{mn}^{qc} \sim - \frac{\partial^{m+n} \Omega}{\partial \mu_q^m \partial \mu_c^n}, \text{ with some rescale with } T$$

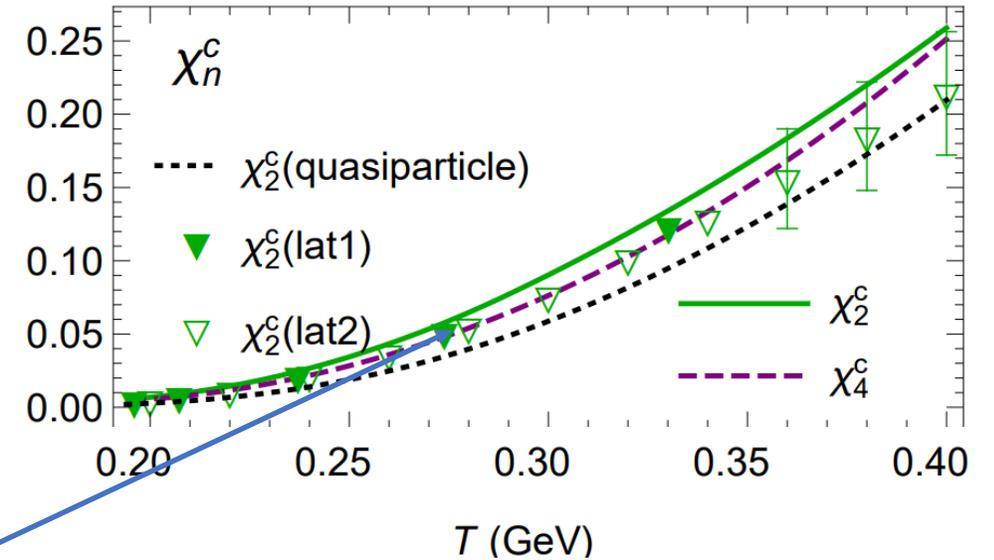
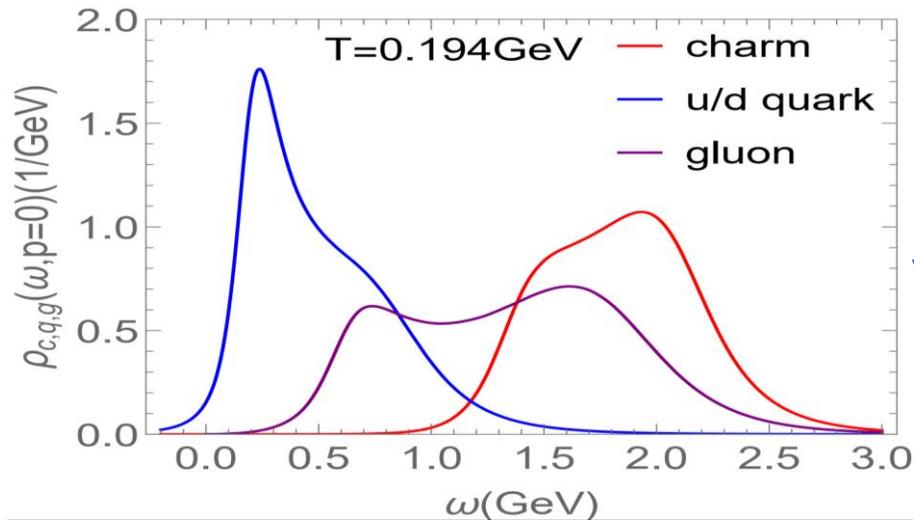
Predictions to Charm & Heavy-Light Susceptibilities

Parameters are all fixed
in previous steps



Predictions to Charm & Heavy-Light Susceptibilities

- ❖ NOT depend on the two new parameters b_m, b_s , since only need $\mu_B = 0$ information
- ❖ $\chi_2^c \approx \chi_4^c$
- ❖ The width of the spectral function enhances the χ_2^c

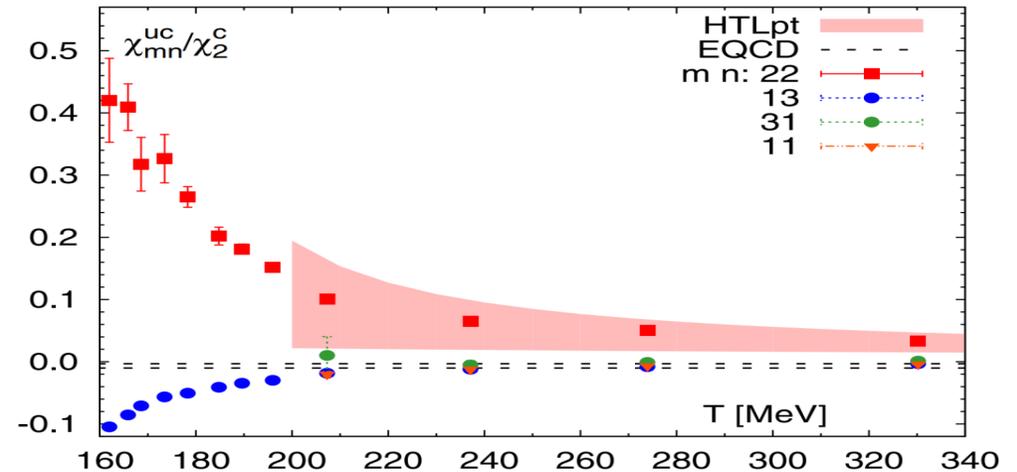
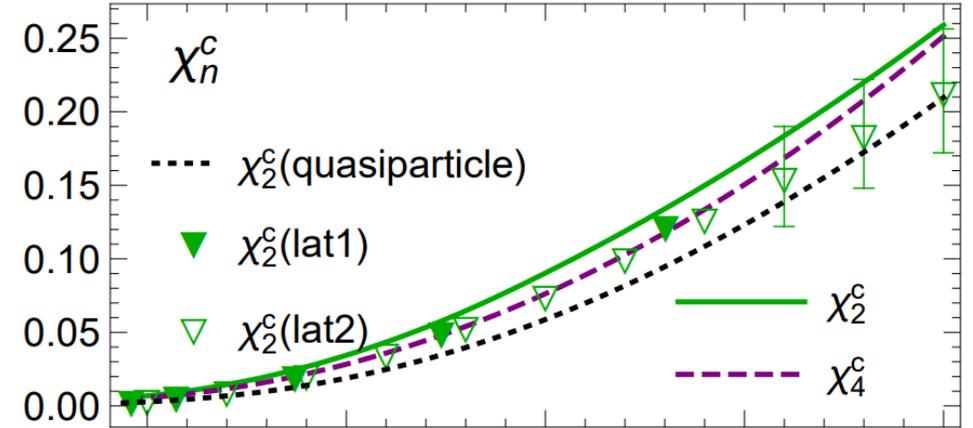


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❖ **Previous interpretation:** Mukherjee, Petreczky, Sharma, PRD 2016

- Quasiparticle parton \rightarrow Quasiparticle hadron
- Hard thermal loop perturbative theory, QGP is weakly coupled?



No model with a strongly coupled picture calculate heavy-Light Susceptibilities before

T (GeV)

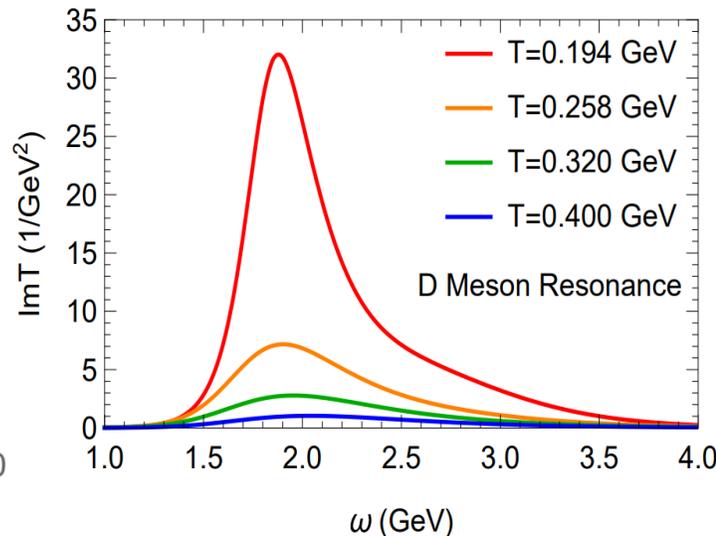
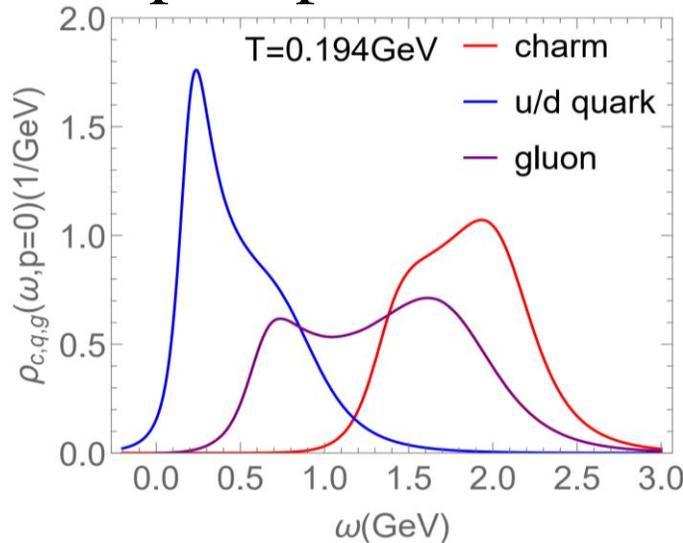
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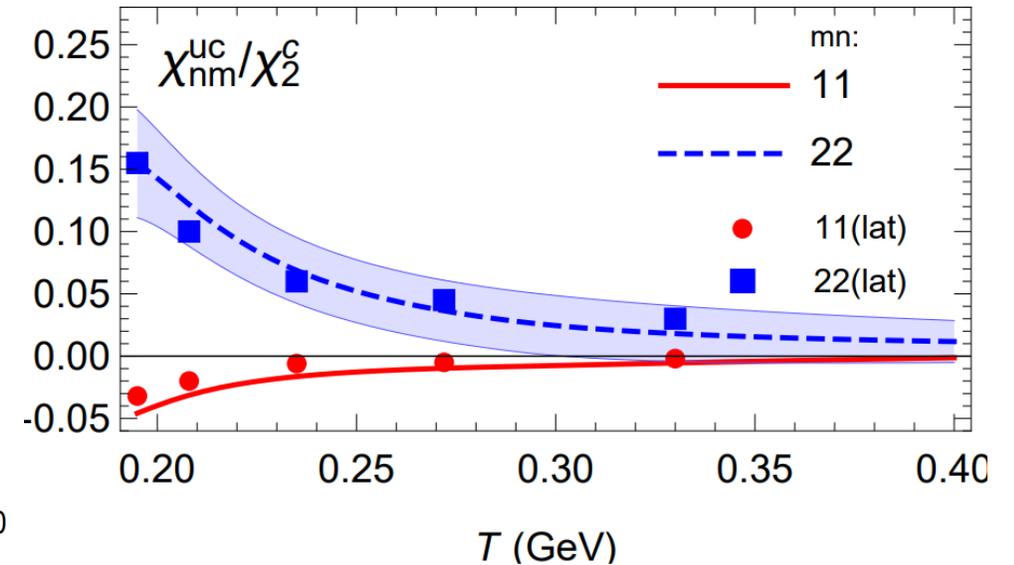
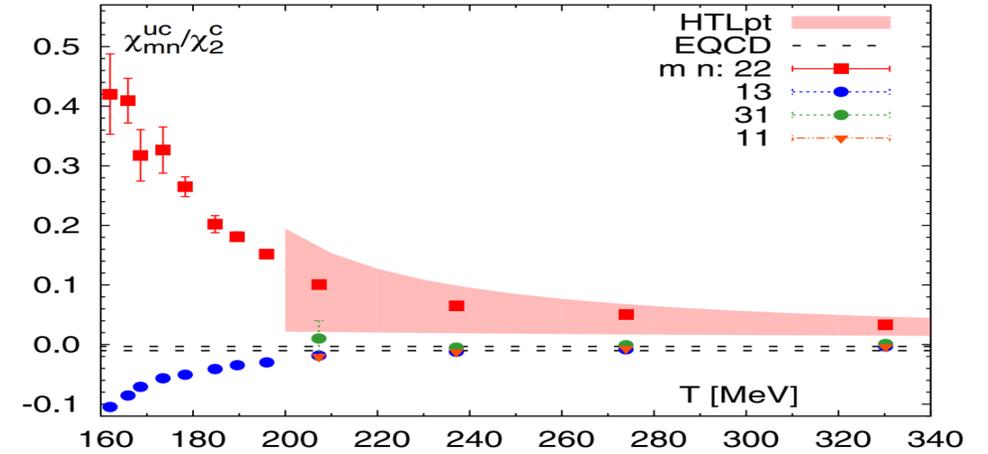
❖ **First calculation with a strongly coupled picture:**



Broad non-quasi particle parton



Broad hadronic resonance



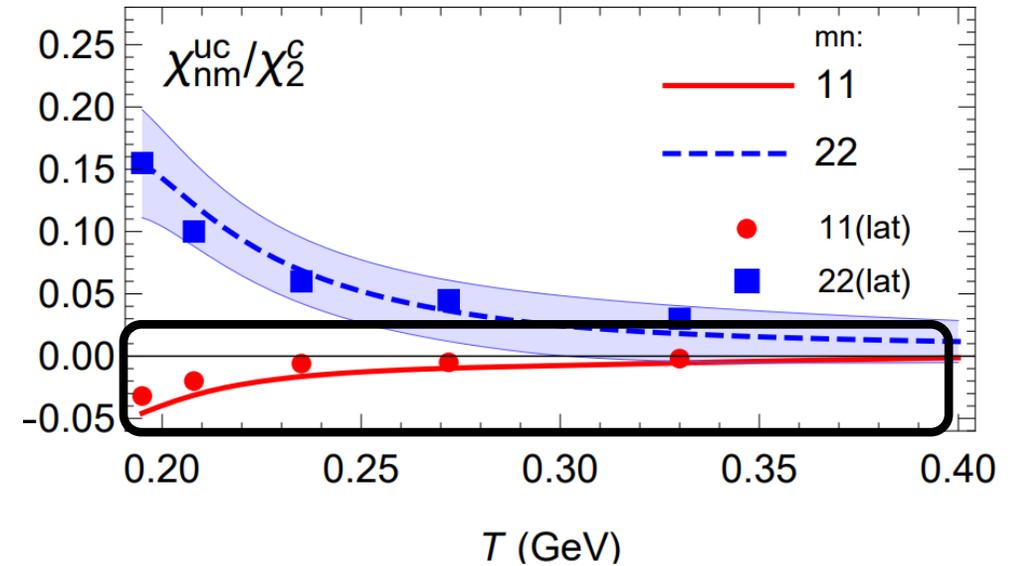
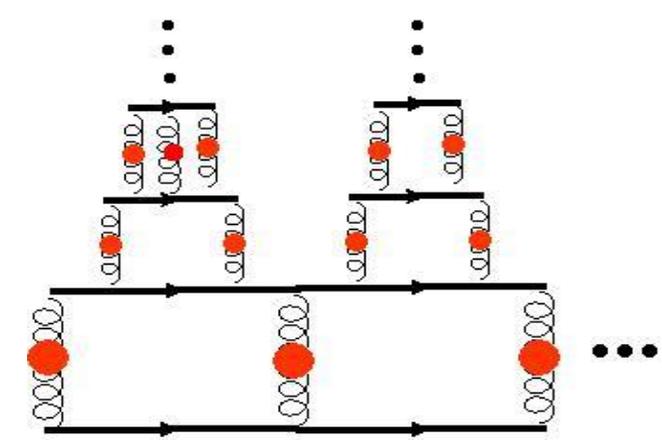
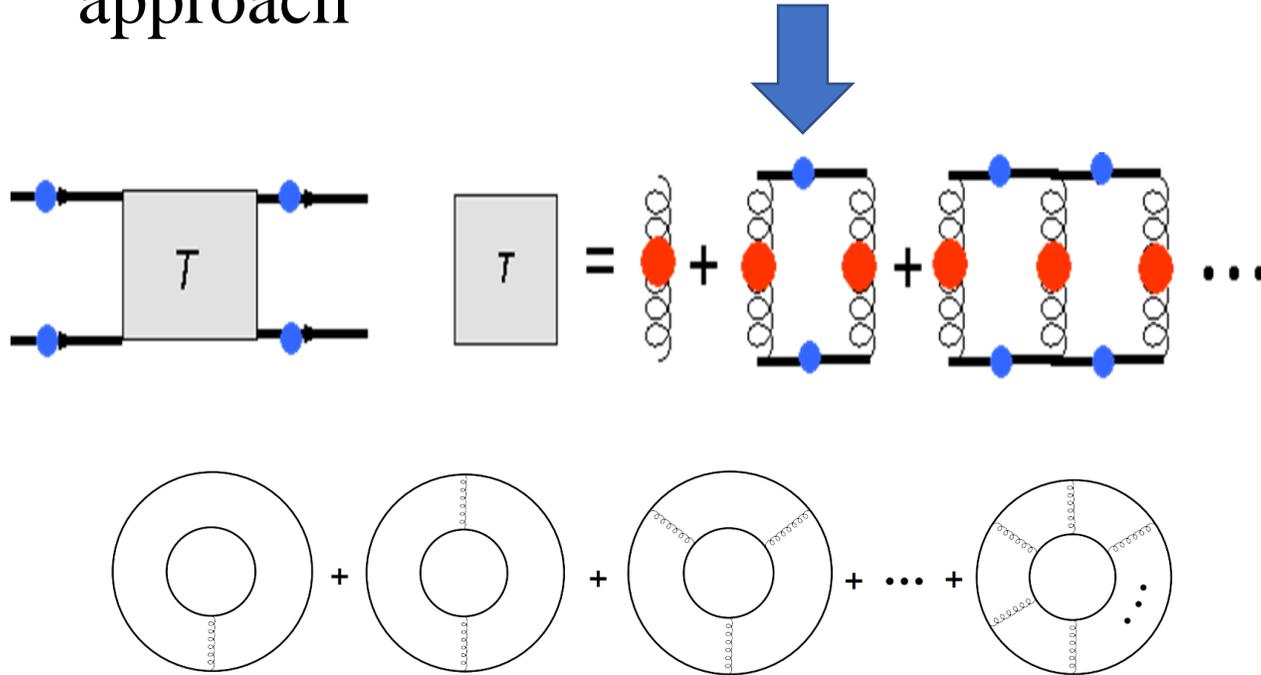
❖ **Strongly coupled picture, also consistent with data!**

Predictions to Charm & Heavy-Light Susceptibilities

❖ $\chi_{11}^{uc} = 0$ for HTL in Ref Haque, Bandyopadhyay, Andersen, Mustafa, Strickland, Su, JHEP 2014

❖ $\chi_{11}^{uc} \neq 0$ for T-matrix approach

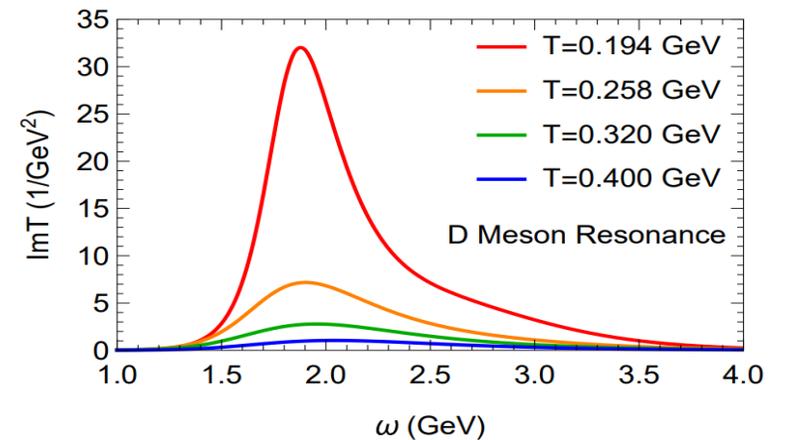
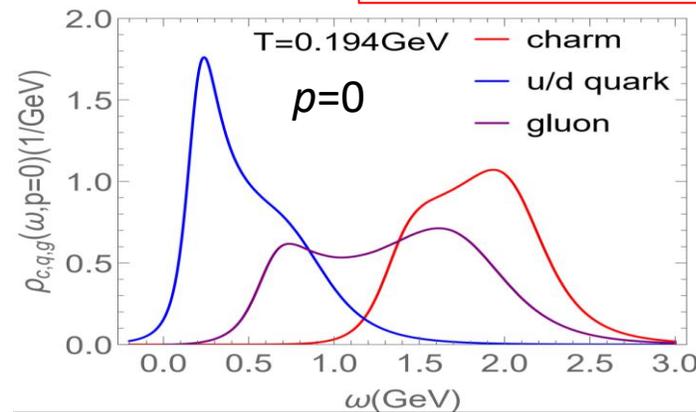
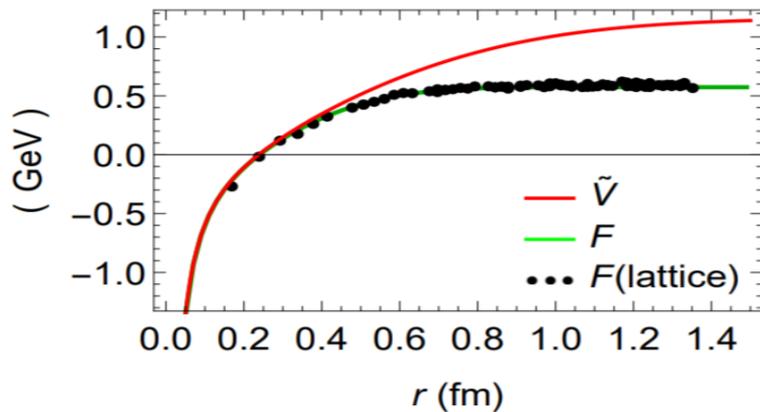
❖ Example of diagrams in T-matrix approach



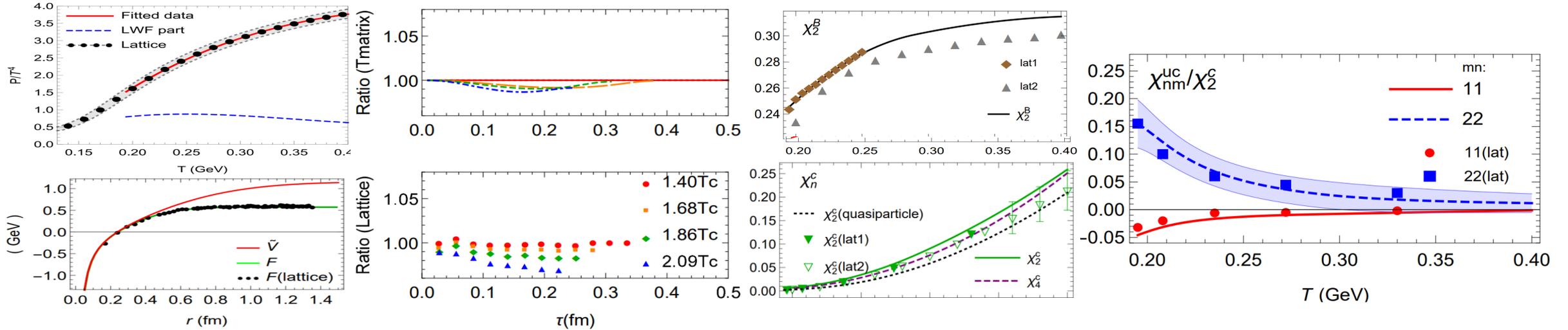
Summary

- ❖ Extend a strongly coupled approach—T-matrix approach to finite chemical potential
- ❖ Tune to explain the baryon number susceptibility
- ❖ **Predict** the charm susceptibilities χ_2^c and χ_4^c , consistent with LQCD
- ❖ **Predict** the heavy-light susceptibilities χ_{11}^{uc} , χ_{22}^{uc} , at least qualitatively consistent with LQCD [arXiv:2111.13620](https://arxiv.org/abs/2111.13620)
- ❖ **Strongly coupled picture obtained in T-matrix approach is consistent with various LQCD data.** (EoS, HQ free energy, correlator ratio, baryon&charm number susceptibility, heavy-light susceptibility)

[Link to the series of work on T-matrix approach](#)

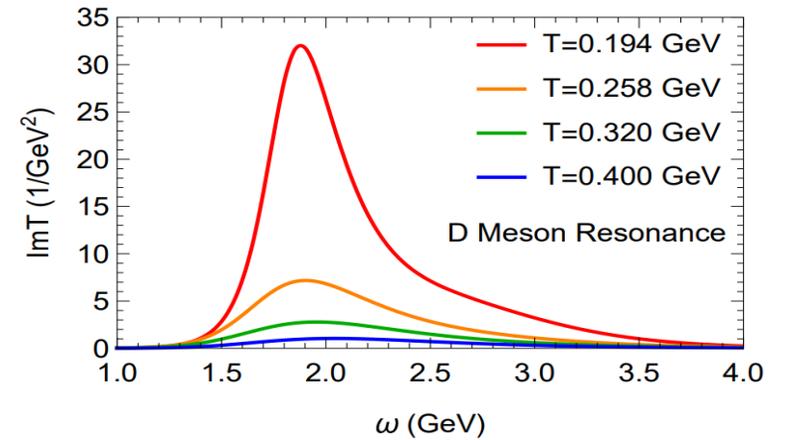
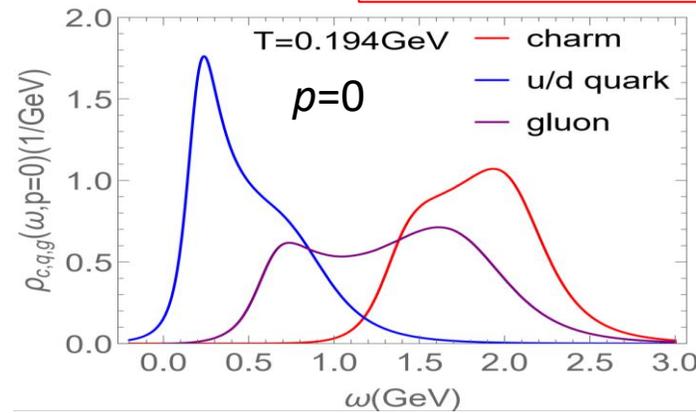
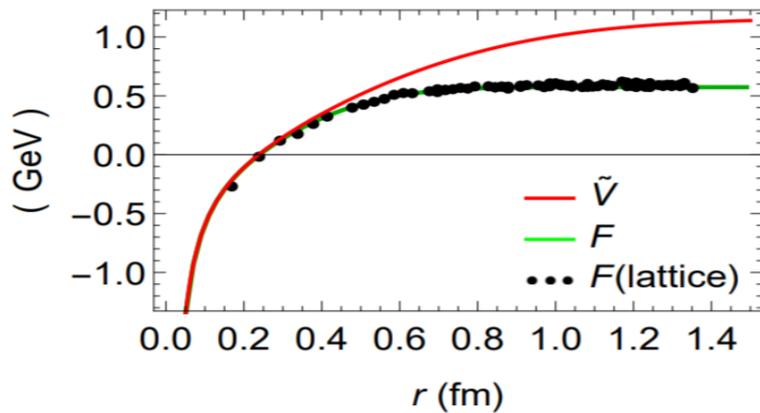


Summary

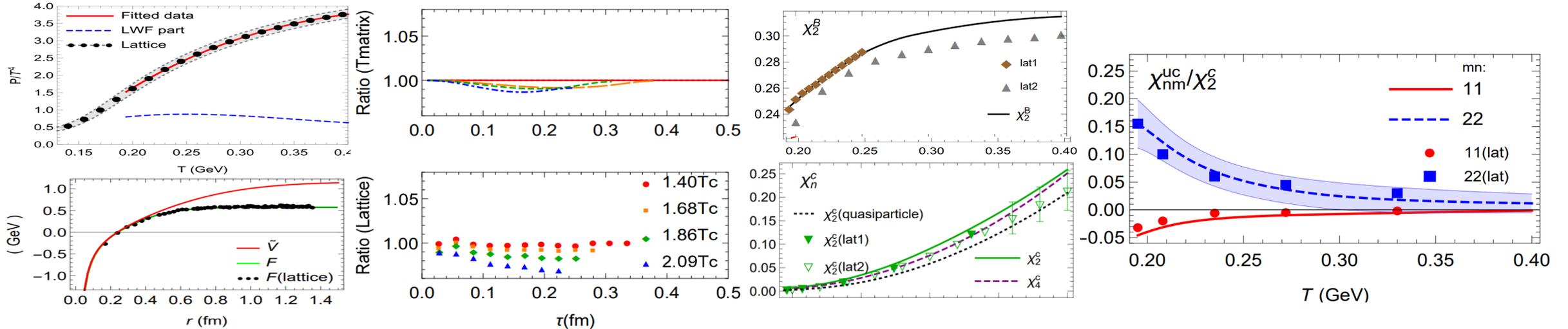


❖ Strongly coupled picture obtained in T-matrix approach is consistent with various LQCD data. (EoS, HQ free energy, correlator ratio, baryon&charm number susceptibility, heavy-light susceptibility)

[Link to the series of work on T-matrix approach](#)



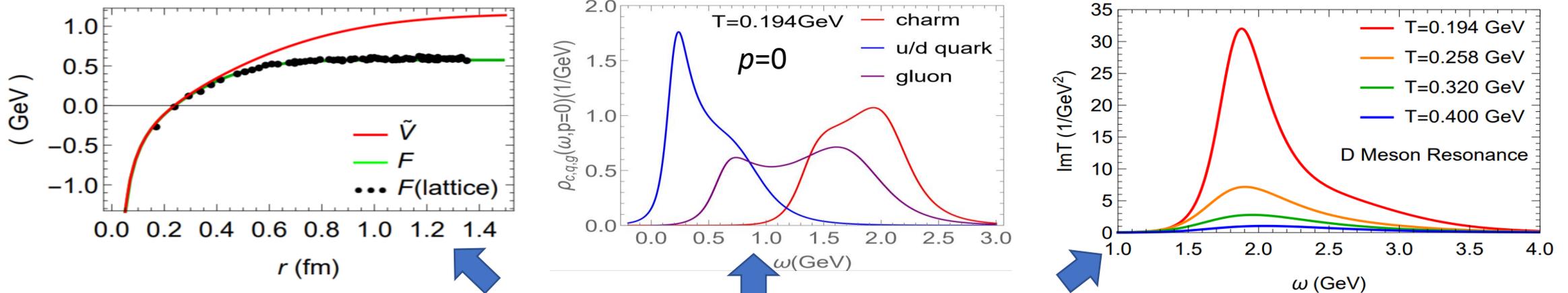
Summary



❖ Strongly coupled picture obtained in T-matrix approach is consistent with various LQCD data. (EoS, HQ free energy, correlator ratio, baryon&charm number susceptibility, heavy-light susceptibility)

❖ Also, reasonable for transport properties.

[Link to the series of work on T-matrix approach](#)

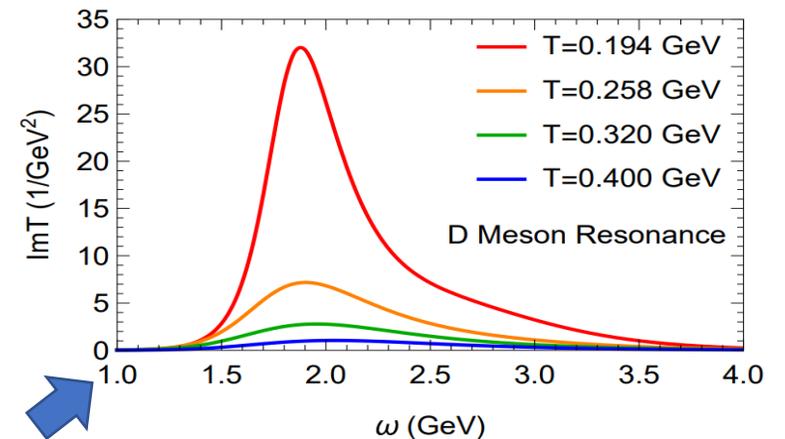
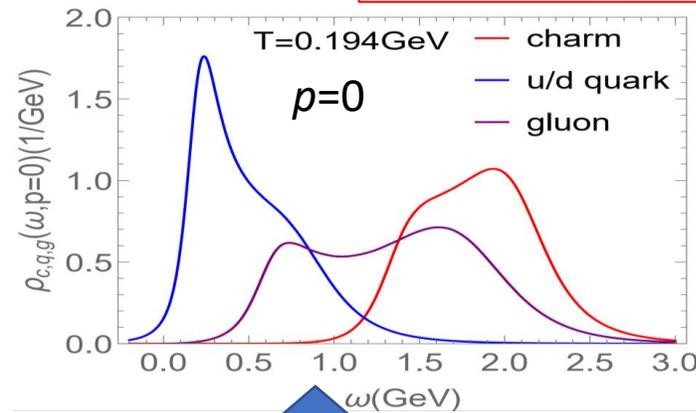
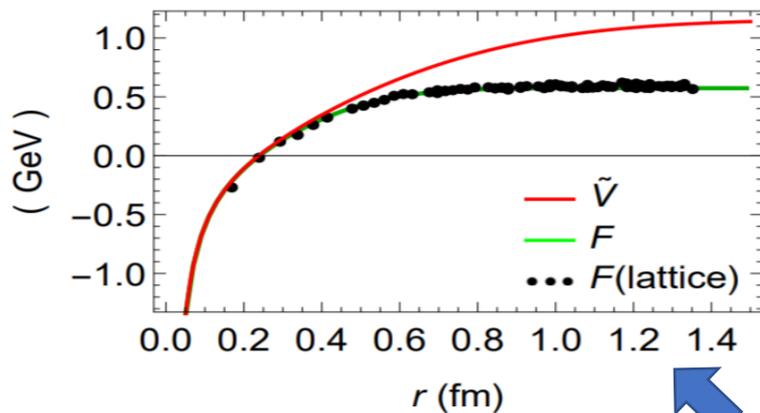


This should be the microscopic picture of QGP!

Backups

Summary

- ❖ Extend a strongly coupled approach—T-matrix approach to finite chemical potential
- ❖ Tune to explain the baryon number susceptibility
- ❖ **Predict** the charm susceptibilities χ_2^c and χ_4^c , consistent with LQCD
- ❖ **Predict** the heavy-light susceptibilities χ_{11}^{uc} , χ_{22}^{uc} , at least qualitatively consistent with LQCD [arXiv:2111.13620](https://arxiv.org/abs/2111.13620)
- ❖ Strongly coupled picture obtained in T-matrix approach is consistent with various LQCD data. (EoS, HQ free energy, correlator ratio, baryon&charm number susceptibility, heavy-light susceptibility)
- ❖ Also, reasonable for transport properties. [Link to the series of work on T-matrix approach](#)



This should be the microscopic picture of QGP!

Summary and Perspective

❖ Summary

- Extend a strongly coupled approach—T-matrix approach to finite chemical potential
- Tune to explain the baryon number susceptibility
- **Predict** the charm susceptibilities χ_2^c and χ_4^c , consistent with LQCD
- **Predict** the heavy-light susceptibilities χ_{11}^{uc} , χ_{22}^{uc} , at least qualitatively consistent with LQCD [arXiv:2111.13620](https://arxiv.org/abs/2111.13620)
- **Strongly coupled picture obtained in T-matrix approach is consistent with various LQCD data.** (EoS, HQ free energy, correlator ratio, quark&charm number susceptibility, heavy-light susceptibility)

[Link to the series of work on T-matrix approach](#)

❖ Perspective

- Include the full baryonic degree of freedom (currently only have the di-quarks)
- Improve the spin structure of the theory
- Could we use the approach to study the phase structure of QCD Matter?