Non-perturbative determination of the collisional broadening kernel and medium-induced radiation in QCD plasmas

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Based on: G.D. Moore, S. Schlichting, N. Schlusser, I.S. DOI: [10.1007/JHEP10(2021)059]
S. Schlichting, I.S. DOI: [10.1103/PhysRevD.105.076002]

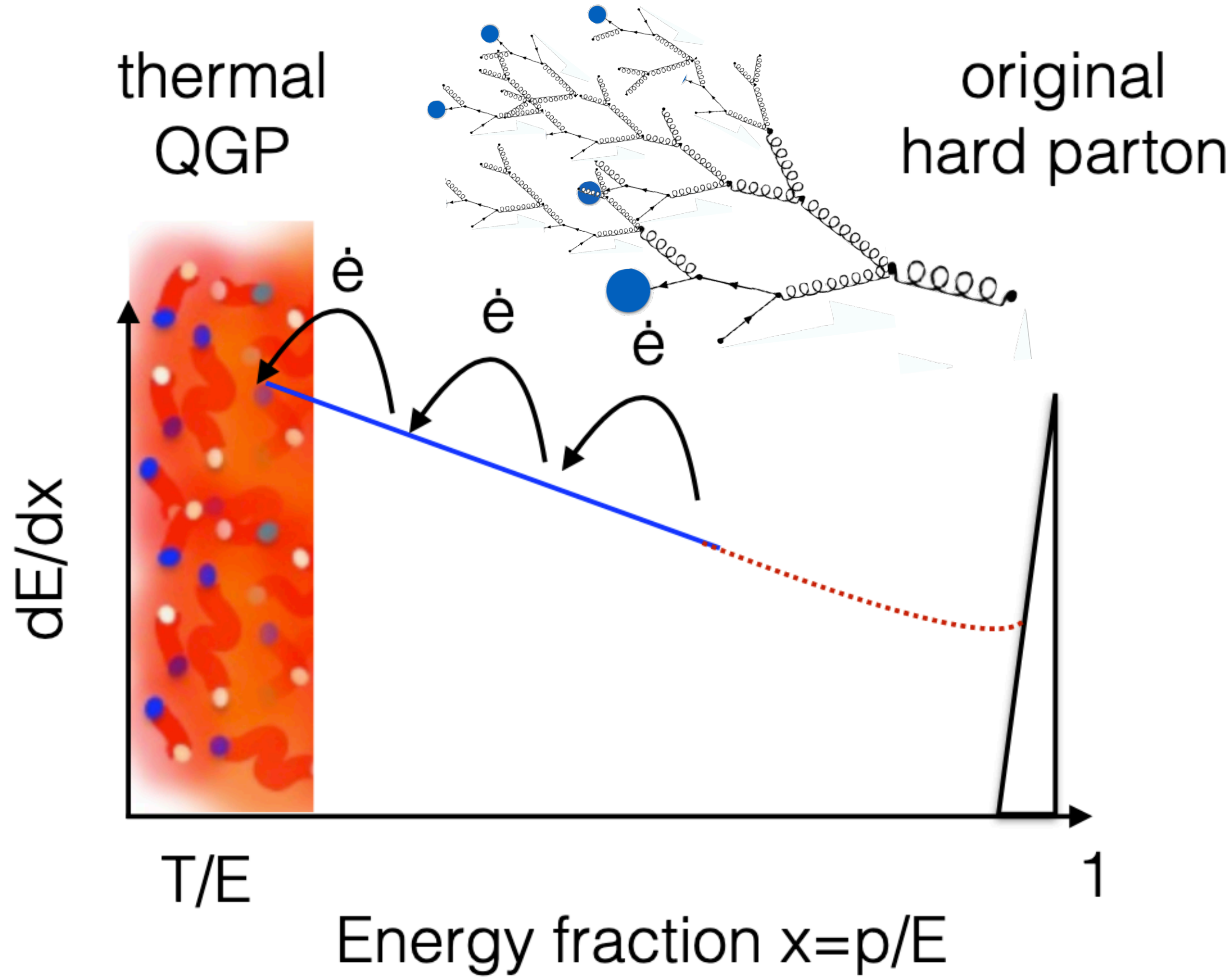
Quark Matter Krakow
April 7th, 2022

Medium energy loss

CRC-TR 211

In-medium energy loss is dominated by an inverse energy cascade, driven by multiple successive splittings.

=>Requires a good grasp on the physics of in-medium splittings



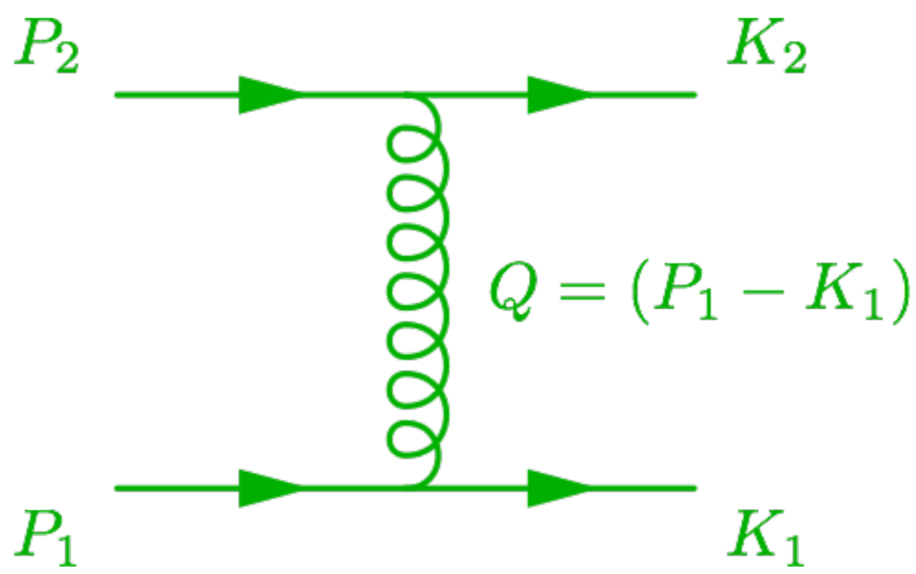
[see poster by S. Schlichting Session 1 T04_1]

[Blaizot et al. arXiv: 1301.6102]
[Mehtar-Tani & Schlichting arXiv: 1807.06181]
[Schlichting & I.S. arXiv: 2008.04928]

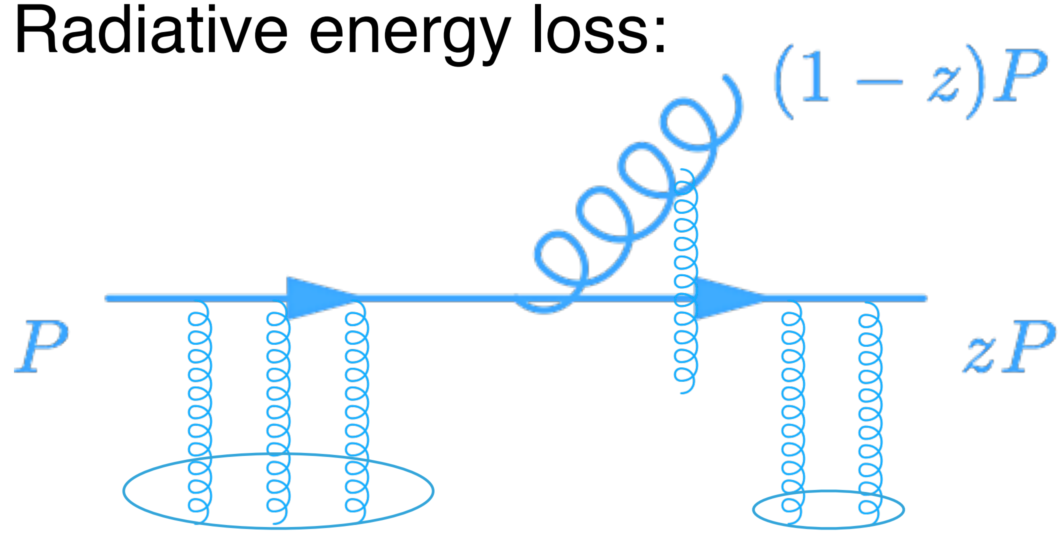
CRC-TR 211

As the high energetic partons traverse the medium they lose energy due to:

- Elastic energy loss:



- Radiative energy loss:



Both require input from the medium transverse scattering rate :

$$\lim_{p\to\infty} \frac{d\Gamma(\boldsymbol{p}, \boldsymbol{p}+\boldsymbol{q}_\perp)}{d^2 q_\perp} = \frac{\mathcal{C}(q_\perp)}{(2\pi)^2}.$$

Transverse momentum broadening rate :

In the literature one employs pQCD broadening kernels:

- Static screened color centers -> $C(q) \propto \frac{1}{(q^2 + m_D^2)^2}$

[X. Wang and M. Gyulassy. *Phys.Rev.Lett.* 68 (1992) 1480-1483]

- Dynamics moving charges -> $C(q) \propto \frac{1}{q^2(q^2 + m_D^2)}$

[P. Aurenche, F. Gelis, and H. Zaraket. JHEP 05 (2002), p. 043.]

- Multiple soft scattering -> $C(b) \propto \frac{\hat{q}}{4} b^2$

[Baier-Dokshitzer-Mueller-Peigne-Schiff]

EQCD broadening kernel

Due to the infamous infrared problem of finite temperature QCD

=> perturbative calculations can receive large non-perturbative contribution even at small coupling.

$$C(\boldsymbol{b}_\perp) :\equiv \int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2} \left(1 - e^{-i\boldsymbol{q}_\perp \cdot \boldsymbol{b}_\perp}\right) C(q_\perp) \,.$$

The collision kernel can be defined in terms of the behavior of certain light-like Wilson loops

$$C(\boldsymbol{b}_\perp) \equiv - \lim_{L\to\infty} \frac{1}{L} \ln \tilde{W}(L, \boldsymbol{b}_\perp) \,,$$

[J. Casalderrey-Solana & D. Teaney, JHEP, vol. 04, p. 039, 2007]

=>For temperatures well above Tc these Wilson loops can be recast in the reduced effective theory of electrostatic QCD (EQCD)

[S. Caron-Huot Phys.Rev.D 79 (2009), 065039]

EQCD broadening kernel

The kernel was computed in a perturbative expansion in the effective theory of EQCD :

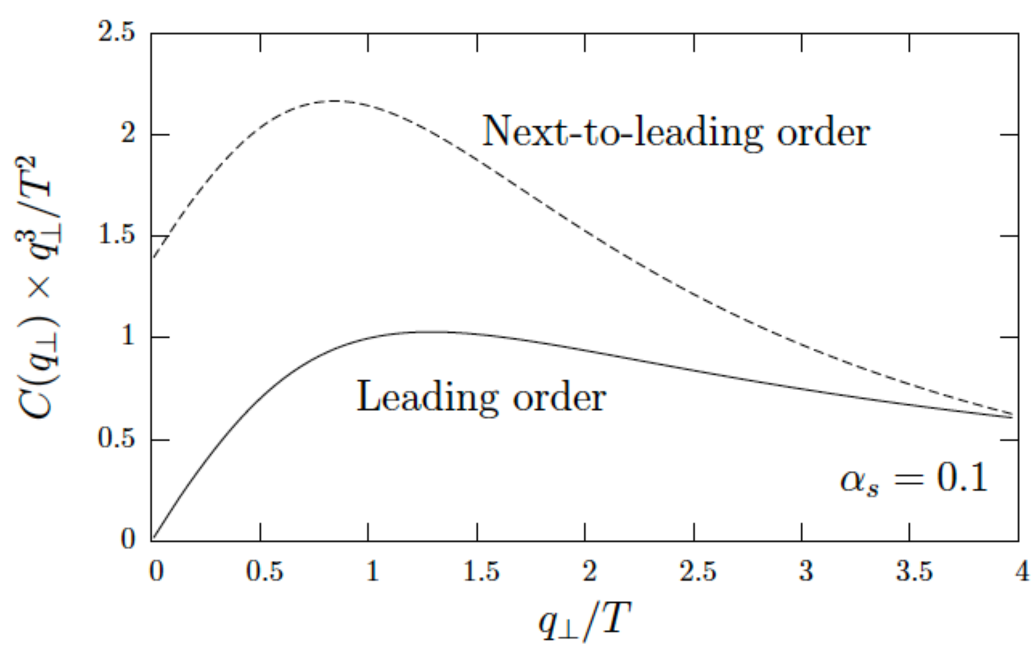
- The LO EQCD kernel

$$C^{\mathrm{LO}}_{\mathrm{QCD}}(q_\perp) = \frac{g_s^4 T^3 C_R}{q_\perp^2 (q_\perp^2 + m_D^2)} \int \frac{d^3p}{(2\pi)^3} \frac{p - p_z}{p} \left[2C_A n_B(p)\left(1 + n_B(p')\right) + 4\, N_f T_f\, n_F(p)\left(1 - n_F(p')\right)\right] ,$$

$$= g_s^2 T C_R \begin{cases} \frac{m_D^2 - g_s^2 T^2 C_A \frac{q_\perp}{16T}}{q_\perp^2 (q_\perp^2 + m_D^2)} , & q_\perp \ll g_s T , \\ \frac{g_s^2 T}{q_\perp^4} \mathcal{N} , & q_\perp \gg g_s T , \end{cases}$$

[P. Arnold & W. Xiao Phys.Rev.D 78 (2008), 125008]

- NLO corrections :

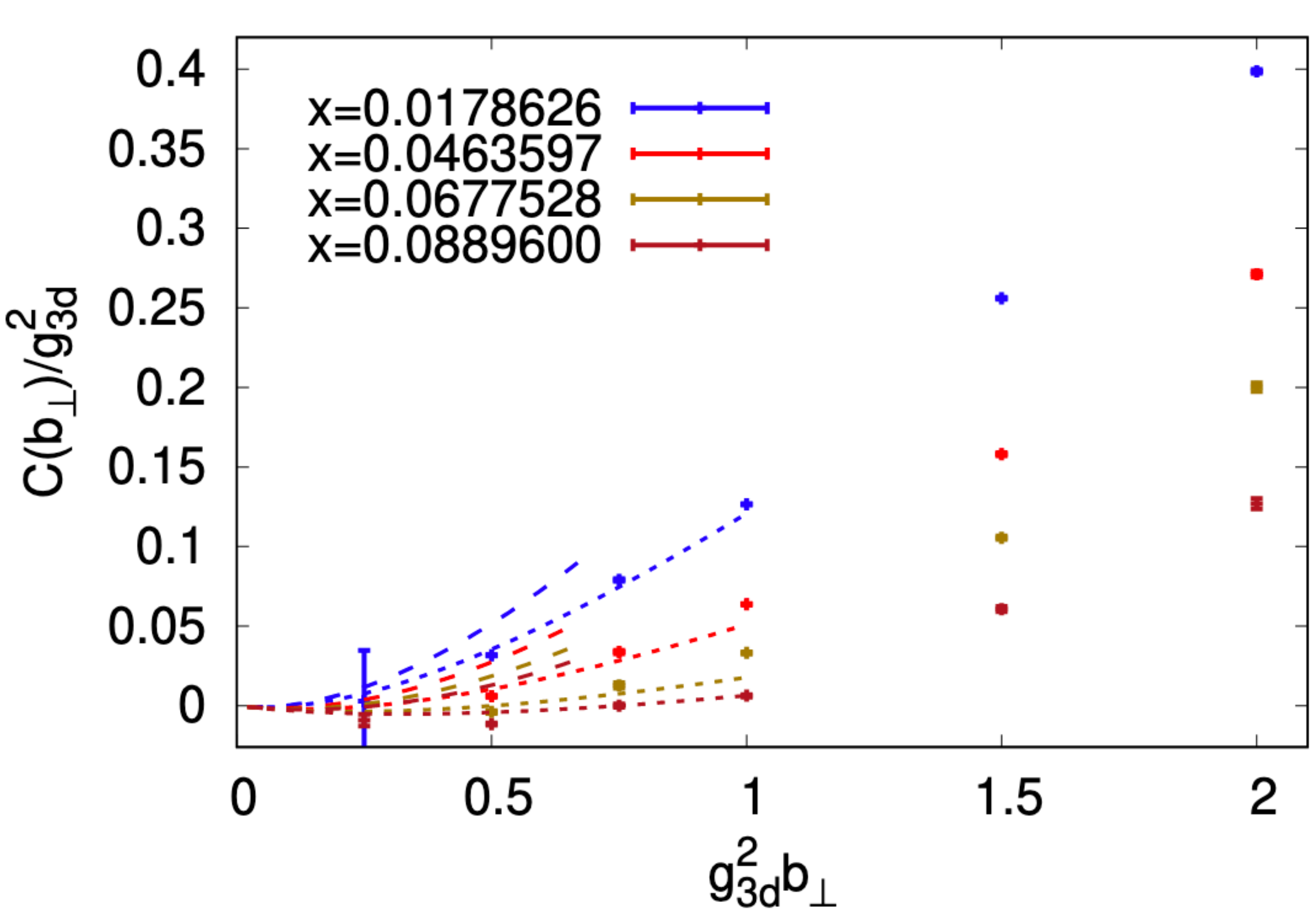


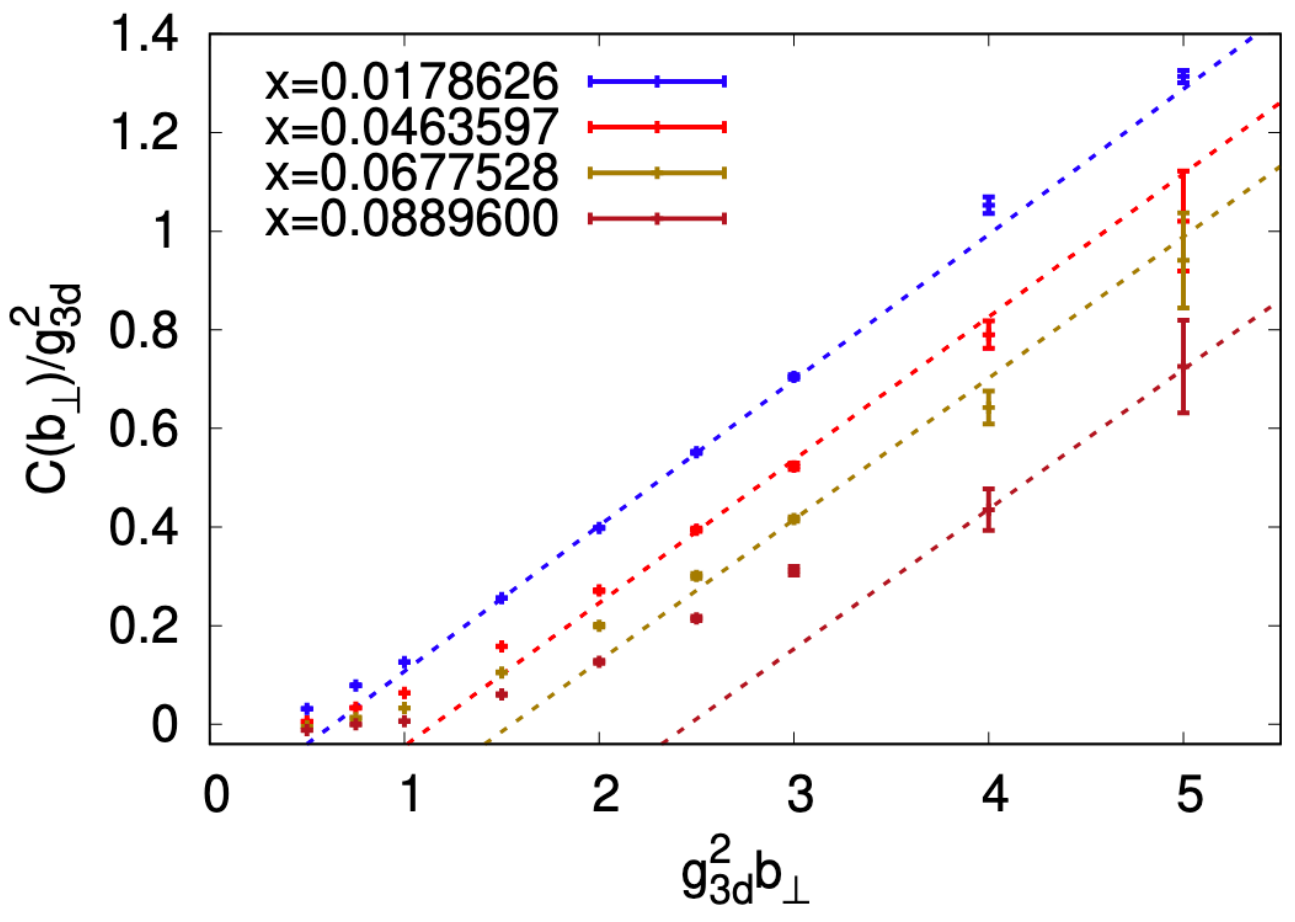
[S. Caron-Huot Phys.Rev.D 79 (2009), 065039]

EQCD broadening kernel

Beyond the perturbative result, lattice extracted non-perturbative contribution were computed

[M. Panero, K. Rummukainen, & A. Schäfer. In: Phys. Rev. Lett. 112.16 (2014), p. 162001]
[G.D. Moore & N. Schlusser Phys.Rev.D 101 (2020) 1, 014505]





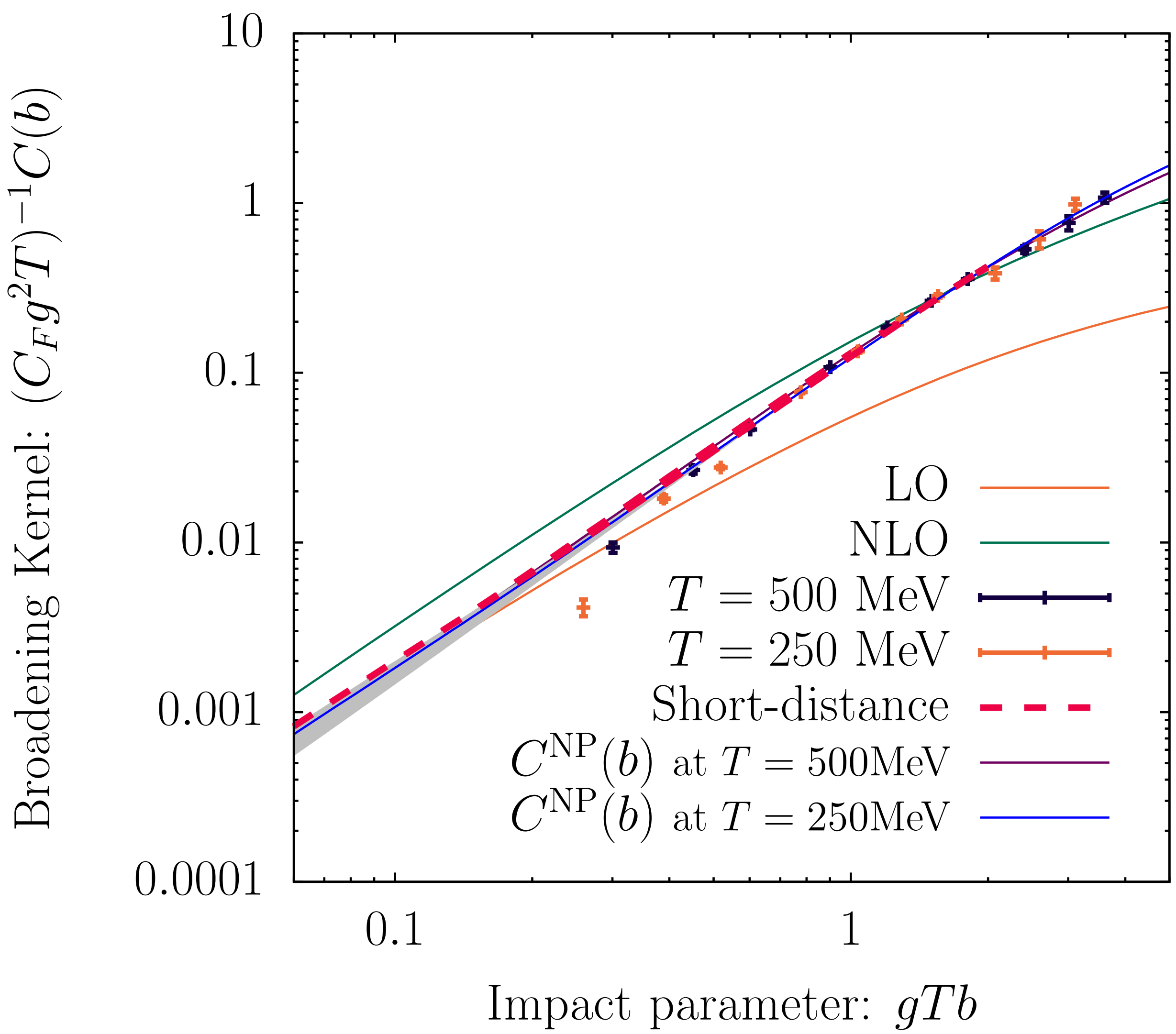
This result here is for the broadening kernel in EQCD which need to be matched to QCD

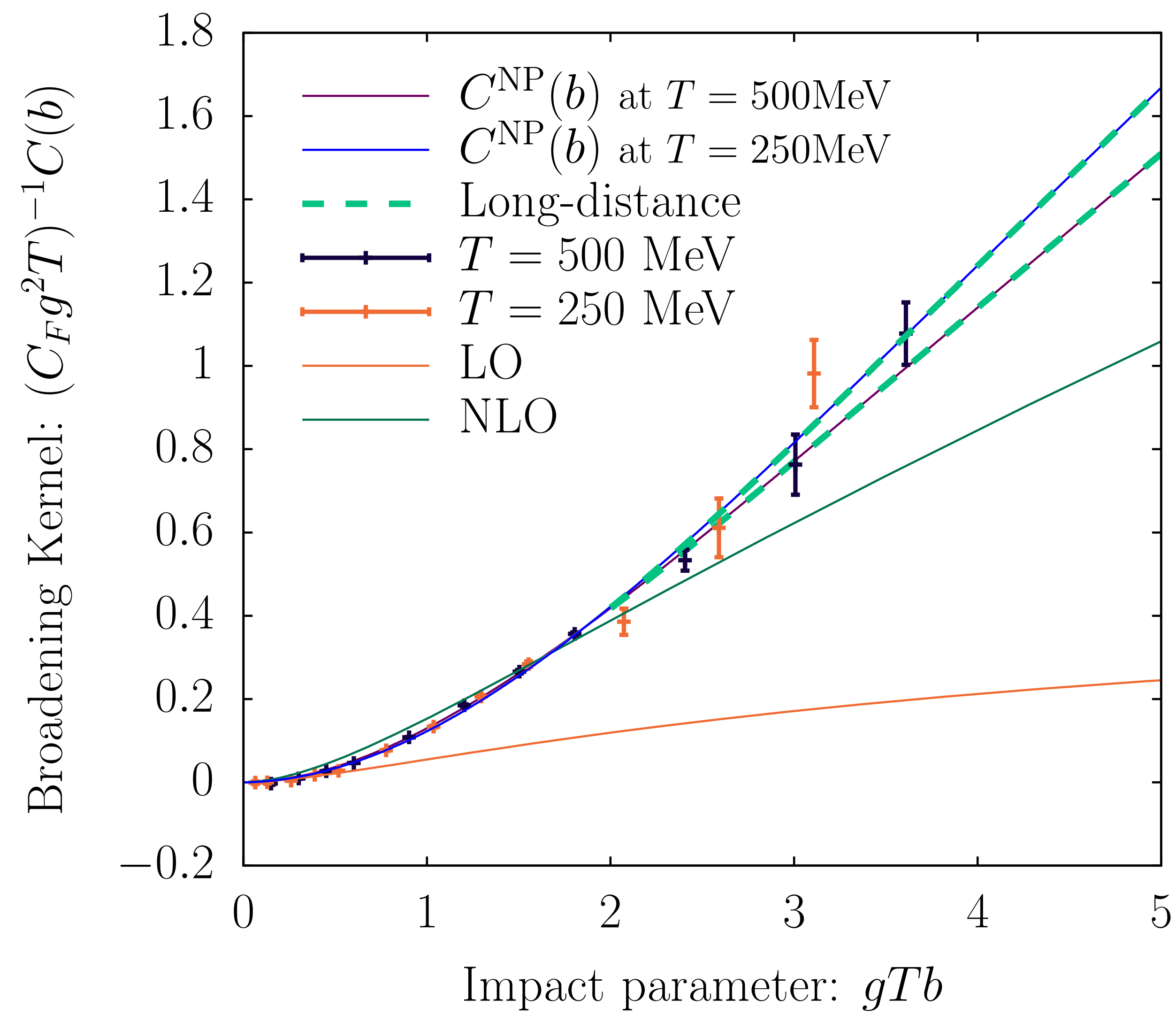
Non-perturbative broadening kernel

Since EQCD is a low-energy effective theory for QCD they should both agree in the IR regime but in the UV they can be different.

In order to ensure the right UV behavior while keeping the IR behavior from the lattice result we write the full kernel:

$$C_{\mathrm{QCD}}(b_\perp) \approx \left(C^{\mathrm{pert}}_{\mathrm{QCD}}(b_\perp) - C^{\mathrm{pert}}_{\mathrm{EQCD}}(b_\perp)\right) + C^{\mathrm{latt}}_{\mathrm{EQCD}}(b_\perp)\,.$$





[G.D. Moore, S. Schlichting, N. Schlusser, I.S arXiv:2105.01679]

Non-perturbative broadening kernel

Long-distance behavior :

The kernel follows an area-law with sub-leading logarithm corrections

[M. Laine, Eur. Phys. J. C, vol. 72]

$$\frac{C_{\rm QCD}}{g_{\rm 3d}^2}(b_\perp) \xrightarrow{b_\perp \gg 1/g_{\rm 3d}^2} \boxed{A + \frac{\sigma_{\rm EQCD}}{g_{\rm 3d}^4} g_{\rm 3d}^2 b_\perp} + \frac{g_{\rm s}^4 C_{\rm R}}{\pi}\left[\frac{y}{4}\left(\frac{1}{6} - \frac{1}{\pi^2}\right) + \frac{C_{\rm A}}{8\pi^2 g_{\rm s}^2}\right]\log(g_{\rm 3d}^2 b_\perp)\,,$$

Short-distance behavior :

The kernel follows the same behavior as the LO one, where we determine $\hat{q}_0$ from the data :

$$\frac{C_{\rm QCD}}{g_{\rm 3d}^2}(b_\perp) \xrightarrow{b_\perp \ll 1/m_{\rm D}} -\frac{C_{\rm R}}{8\pi}\frac{\zeta(3)}{\zeta(2)}\left(-\frac{1}{2g_{\rm s}^2} + \frac{3y}{2}\right)(g_{\rm 3d}^2 b_\perp)^2 \log(g_{\rm 3d}^2 b_\perp) + \frac{1}{4}\frac{\hat{q}_0}{g_{\rm 3d}^6}(g_{\rm 3d}^2 b_\perp)^2\,,$$

Broadening kernel in momentum-space

Results in finite medium

To compute the rate in finite medium it is best to work in momentum space

The rate follows the limits :

- $q \gg 1$:

$$C^{\mathrm{UV}}(q_\perp) = \frac{C_{\mathrm{R}}}{8\pi}\frac{\zeta(3)}{\zeta(2)}\left(-\frac{1}{2g_s^2}+\frac{3y}{2}\right)\frac{8\pi}{q_\perp^4}\,.$$

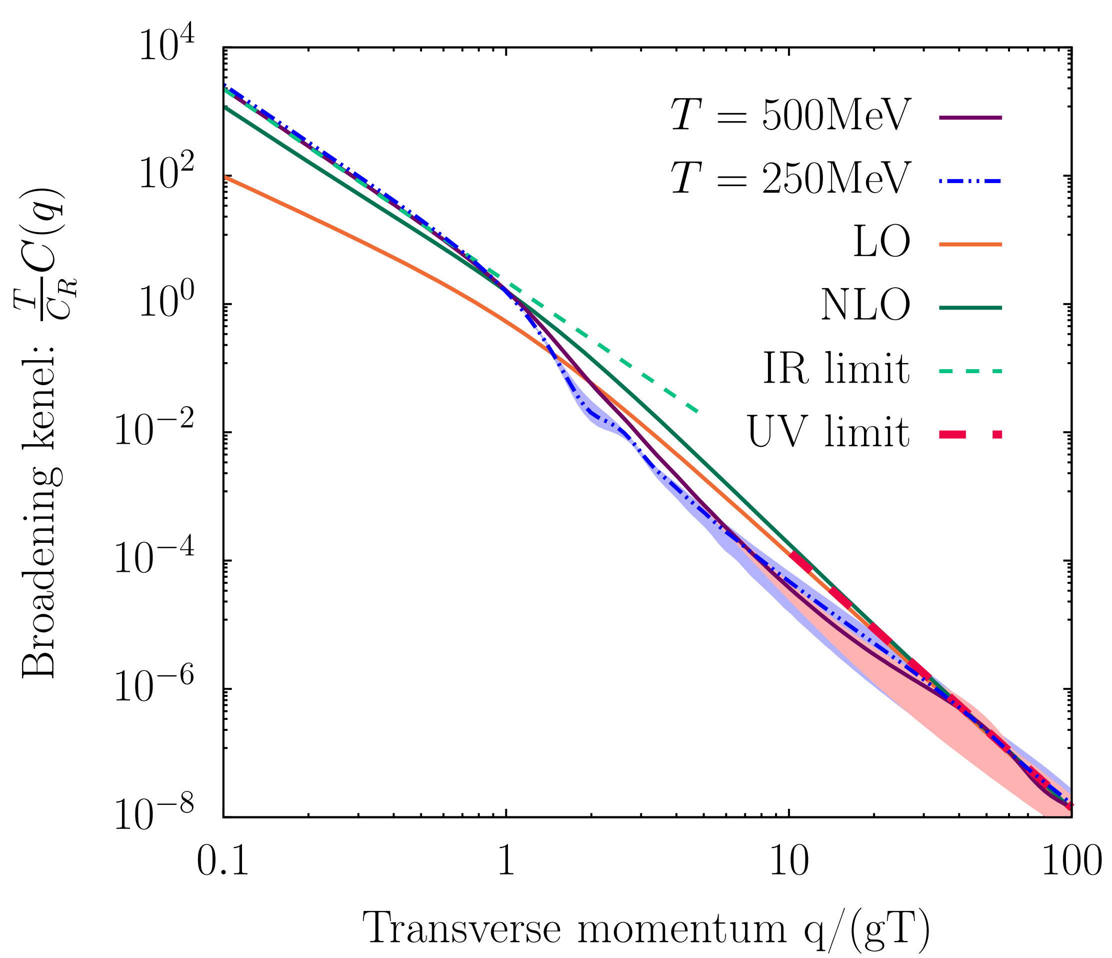
Similar to the LO rate

- $q \ll 1$:

$$C^{\mathrm{IR}}(q_\perp) = \frac{2\pi}{q_\perp^3}\frac{\sigma_{\mathrm{EQCD}}}{g_{3d}^2} + \frac{g^4 C_{\mathrm{R}}}{\pi}\left[\frac{y}{4}\left(\frac{1}{6}-\frac{1}{\pi^2}\right)+\frac{C_{\mathrm{A}}}{8\pi^2 g_s^2}\right]\frac{2\pi}{q_\perp^2}\,. \tag{6}$$

stems from the area law with string tension

Transverse Momentum Broadening:

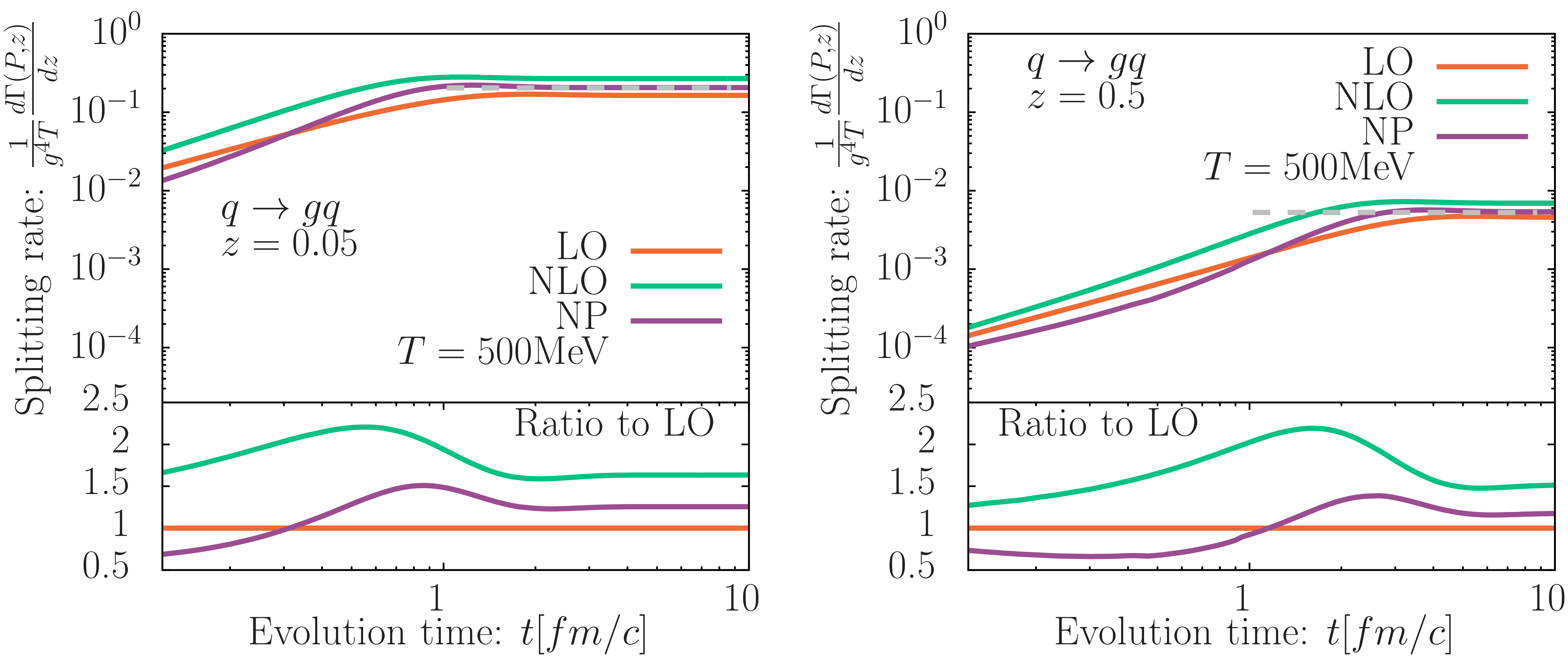


Comparison of different Kernels

- We compute the rate in finite-medium following the approach of S. Caron-Huot and C. Gale

[S. Caron-Huot, C. Gale Phys.Rev.C 82 (2010), 064902]

Medium-induced splitting rates: $\frac{d\Gamma^a_{bc}}{dz}(P,z,t)$ $\quad P = 300T$



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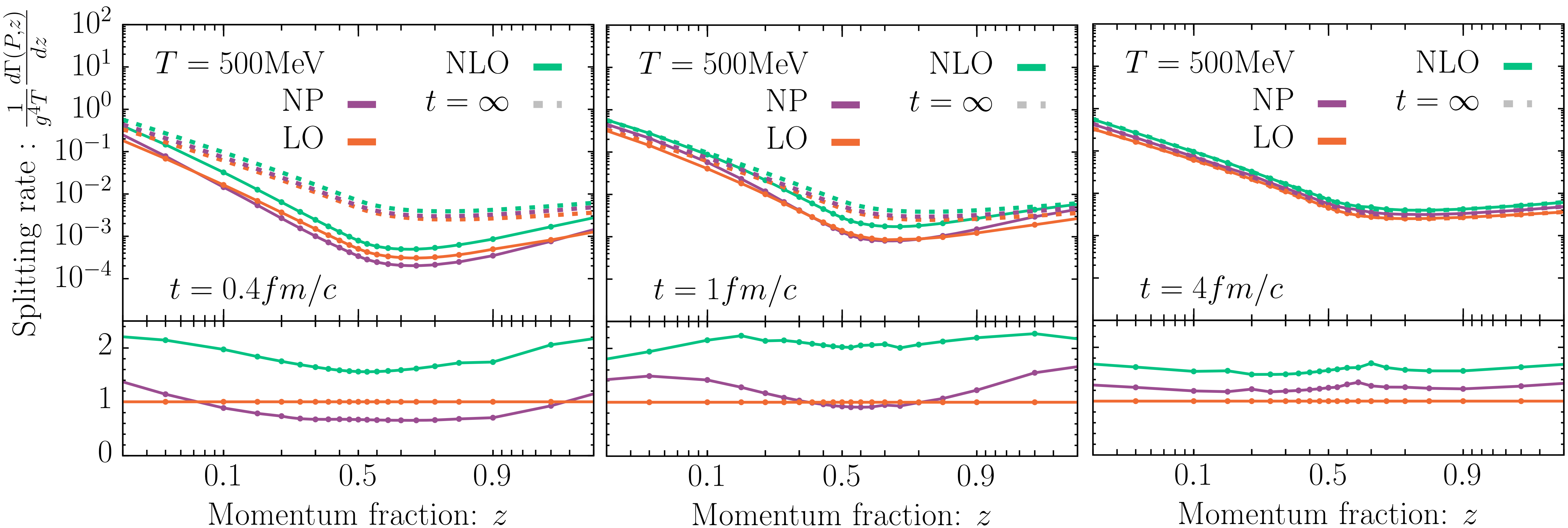
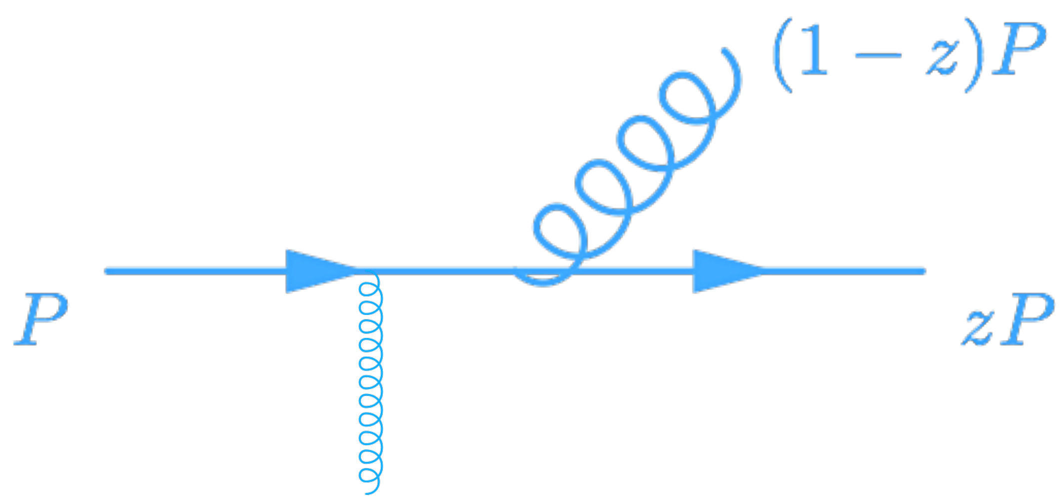


Approximation to in-medium splitting rates

Opacity expansion



- We compare the finite medium with an opacity expansion at N=1, where we consider a single scattering with the medium

$$\left.\frac{d\Gamma^a_{bc}}{dz}\right|_{N=1}(P,z,\tilde{t})$$
$$=\frac{g^4 T P^a_{bc}(z)}{\pi}\int_{\tilde{\boldsymbol{p}}}\frac{1-\cos\left(\delta\tilde{E}(\tilde{p})\tilde{t}\right)}{\delta\tilde{E}(\tilde{p})}\tilde{\boldsymbol{p}}\cdot\tilde{\Gamma}_3\circ\frac{i\tilde{\boldsymbol{p}}}{\delta\tilde{E}(\tilde{\boldsymbol{p}})}\,.$$

Resummed opacity expansion

- We compare the finite medium with an improved opacity expansion where after we cut low momentum interactions, we exponentiate higher order of the expansion

$$\frac{d\Gamma^a_{bc}}{dz}\bigg|_{N=X}(P,z,\tilde{t}) = \frac{g^4 T P^a_{bc}(z)}{\pi}\mathrm{Re}\int_0^{\tilde{t}} d\Delta\tilde{t}\int_{\tilde{\boldsymbol{p}}} e^{-(i\delta\tilde{E}(\tilde{p})+\Lambda\Sigma_3(\tilde{p}^2))\Delta\tilde{t}}\tilde{\psi}_I^{(1)}(\tilde{p})\ ,$$

[C. Andres et Al. JHEP 03 (2021), 102]

where the first order wave function is the collision integral of the initial condition

$$\psi^{(1)}(\boldsymbol{p}) = \int_{\boldsymbol{q}} n(t)C(\boldsymbol{q})\bigg\{C_1\Big[\frac{p^2}{\epsilon(\boldsymbol{p})} - \frac{p^2-\boldsymbol{p}\cdot\boldsymbol{q}}{\epsilon(\boldsymbol{p}-\boldsymbol{q})}\Big] + C_z\Big[\frac{p^2}{\epsilon(\boldsymbol{p})} - \frac{p^2+z\boldsymbol{p}\cdot\boldsymbol{q}}{\epsilon(\boldsymbol{p}+z\boldsymbol{q})}\Big] + C_{1-z}\Big[\frac{p^2}{\epsilon(\boldsymbol{p})} - \frac{(p^2+(1-z)\boldsymbol{p}\cdot\boldsymbol{q})}{\epsilon((\boldsymbol{p}+(1-z)\boldsymbol{q}))}\Big]\bigg\}$$

and the subsequent interactions are encoded in the exponential of

$$\Sigma(M^2) \equiv \int_{\vec{k}^2>M^2} n(t)C(\vec{k})(C_1 + C_z + C_{1-z})$$

Improved opacity expansion

- Improved opacity expansion around the Harmonic Oscillator

[Mehtar-Tani, Barata, Soto-Ontoso, Tywoniuk]

$$C(b_\perp) = \frac{g_s^4 T^3}{16\pi}\mathcal{N} b_\perp^2 \ln\left(\frac{4Q^2}{\xi m_D^2}\right) + \frac{g_s^4 T^3}{16\pi}\mathcal{N} b_\perp^2 \ln\left(\frac{1}{Q^2 b_\perp^2}\right) = C^{HO}(b_\perp) + C^{\text{pert}}(b_\perp) ,$$

$$\frac{dI^{NLO}}{dz}(P,z,t) = \frac{dI^{HO}}{dz}(P,z,t) + \frac{dI^{(1)}}{dz}(P,z,t) .$$

- By self-consistently solving for the scale

$$Q^2(P,z) = \sqrt{Pz(1-z)\hat{q}_3(Q^2)} ,$$

$$\hat{q}_{\text{eff}}(Q^2) = \frac{g_s^4 T^3}{4\pi}\mathcal{N}\left[C_1 + C_z z^2 + C_{1-z}(1-z)^2\right]\ln\left(\frac{4Q^2}{\xi m_D^2}\right) ,$$

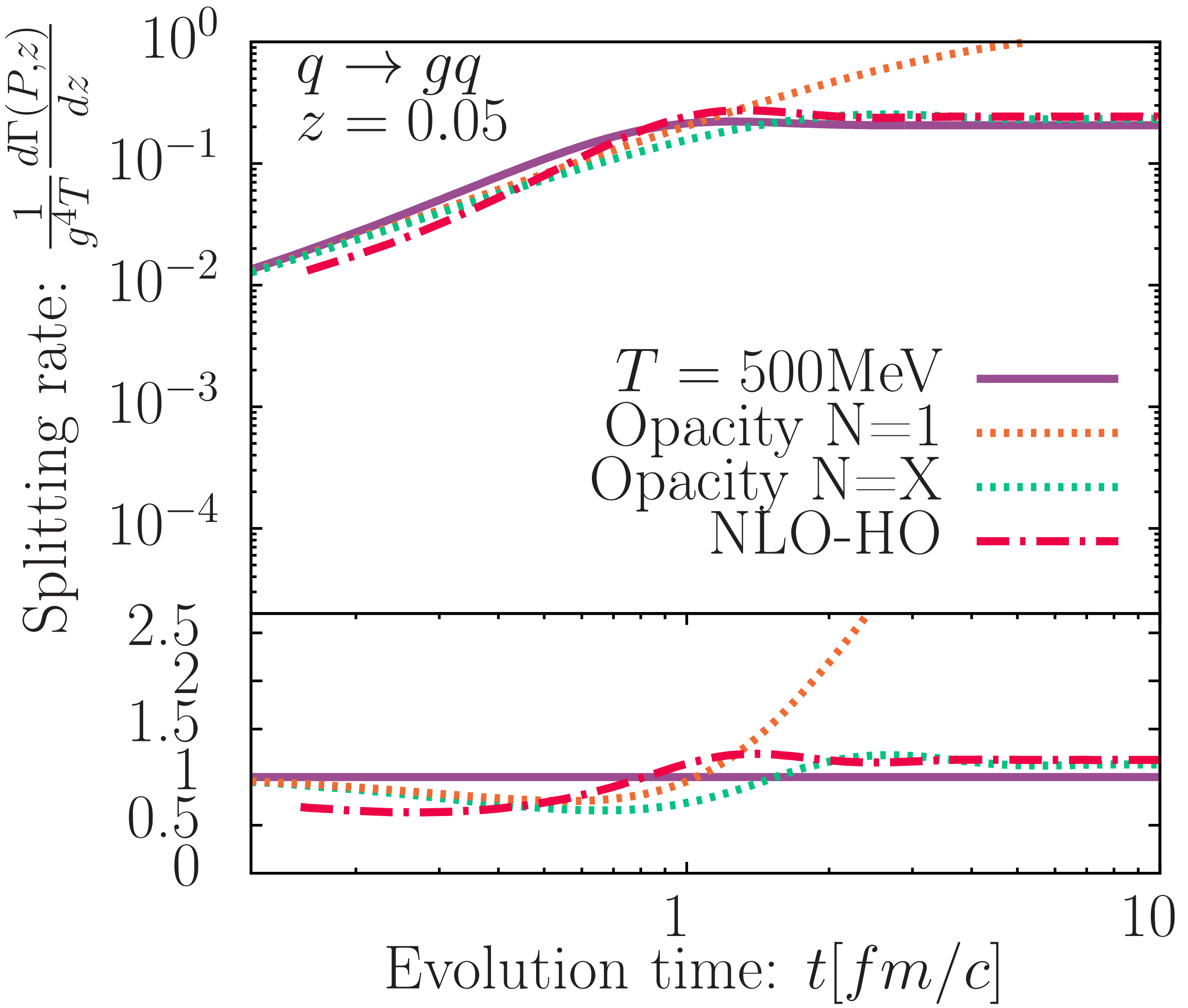
- One can find correction to the HO

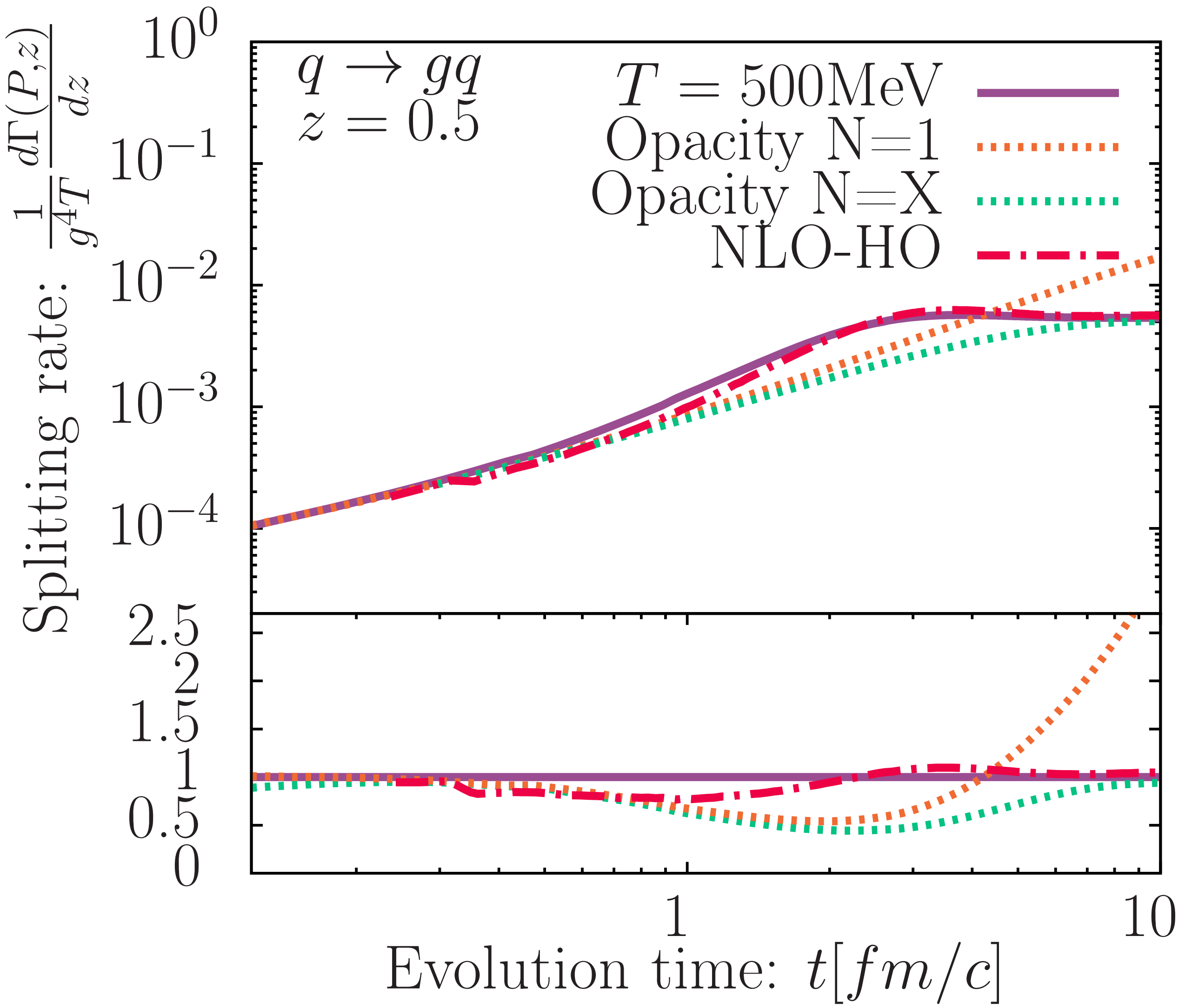
$$\frac{dI^{HO}}{dz}(P,z,t) = \frac{g^2}{4\pi^2}\ln|\cos\Omega t| ,$$

$$\frac{dI^{(1)}}{dz}(P,z,t) = \frac{g^2}{4\pi^2}\text{Re}\int_0^t ds \int_0^\infty \frac{2du}{u}\left[C_1 C^{(1)}(u) + C_z C^{(1)}(zu) + C_{1-z}C^{(1)}((1-z)u)\right] e^{k^2(s)u^2} ,$$

Comparison to the approximations

Medium-induced splitting rates: $P = 300T$





Beyond the broadening kernel

[G.D. Moore & N. Schlusser Phys.Rev.D 102 (2020) 9, 094512]
[J. Ghiglieri et al. JHEP 02 (2022), 058, JHEP 02 (2022), 058]

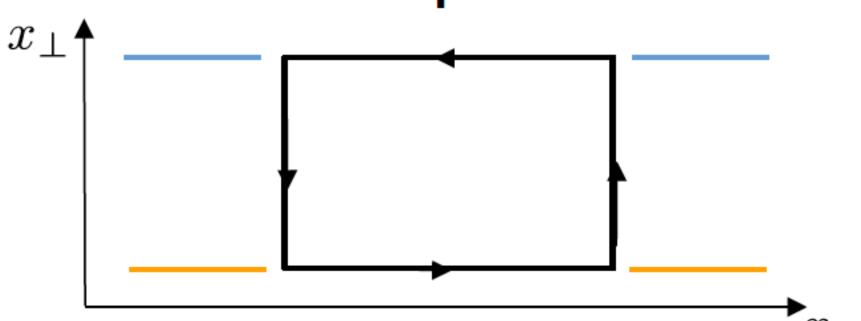
- Collision kernel

$$C(q_\perp) = \frac{\mathrm{d}\Gamma}{\mathrm{d}^2 q_\perp \, \mathrm{d}L}$$

- Asymptotic mass

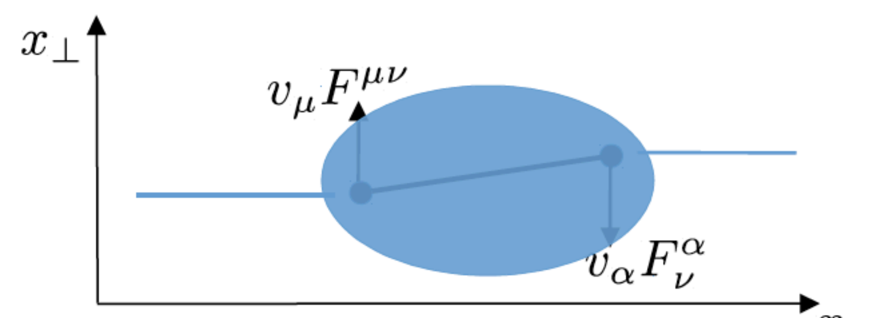
$$m_\infty^2 = C_\mathrm{R}\,(Z_\mathrm{g} + Z_\mathrm{f})$$

- Wilson loop



Casalderrey-Solana, Teaney: 0701123

- Force-force-correlator



Braaten, Pisarski (1992)

> Nonperturbative gluon-zero-mode contributions:
>
> → calculate in lattice EQCD

Caron-Huot: 0811.1603

[See talk by P. Schicho Session T03]

Slide courtesy of N. Schlusser

Conclusion & Outlook

- Approximation to the splitting are can be effective at reproducing the rate within their respective range of validity

[See also Poster of A. Takacs Session 2 T04_2]

- However, differences between the LO kernel which used in phenomenological studies of jet quenching, and the non-perturbative kernel can easily be on the order of 30%.

[G.D. Moore & N. Schlusser Phys.Rev.D 102 (2020) 9, 094512]
[J. Ghiglieri et al. JHEP 02 (2022), 058, JHEP 02 (2022), 058]

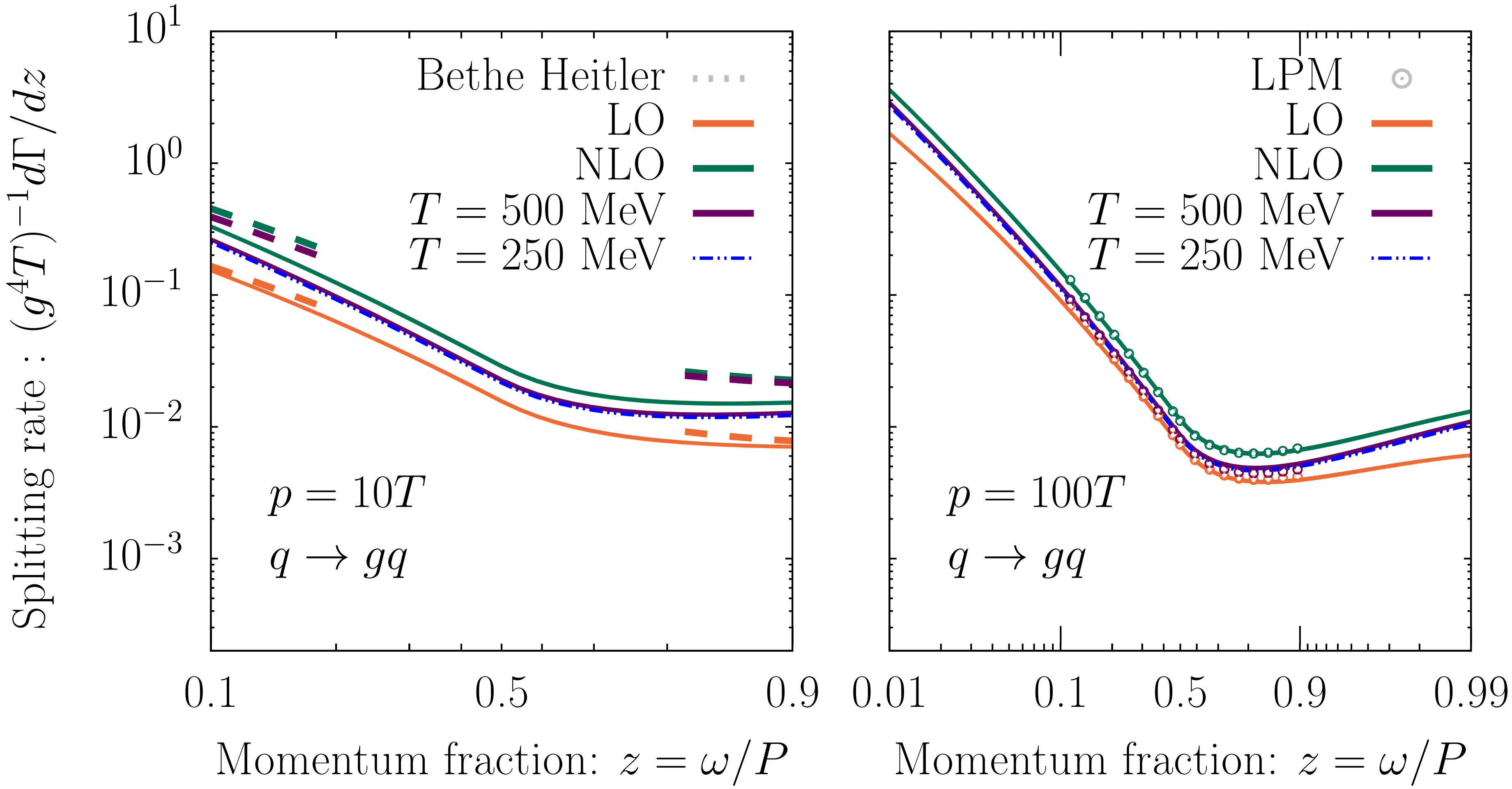
- There have been effort to extract asymptotic masses using the same procedure which still needs to be matched to QCD.
- Would be interesting to include the non-perturbative results to jet studies (elastic and radiative interactions) in kinetic studies or MC

[See also Talk of C. Andres Session T04]

Thank you!

Backup

Results for non-perturbative splitting rates

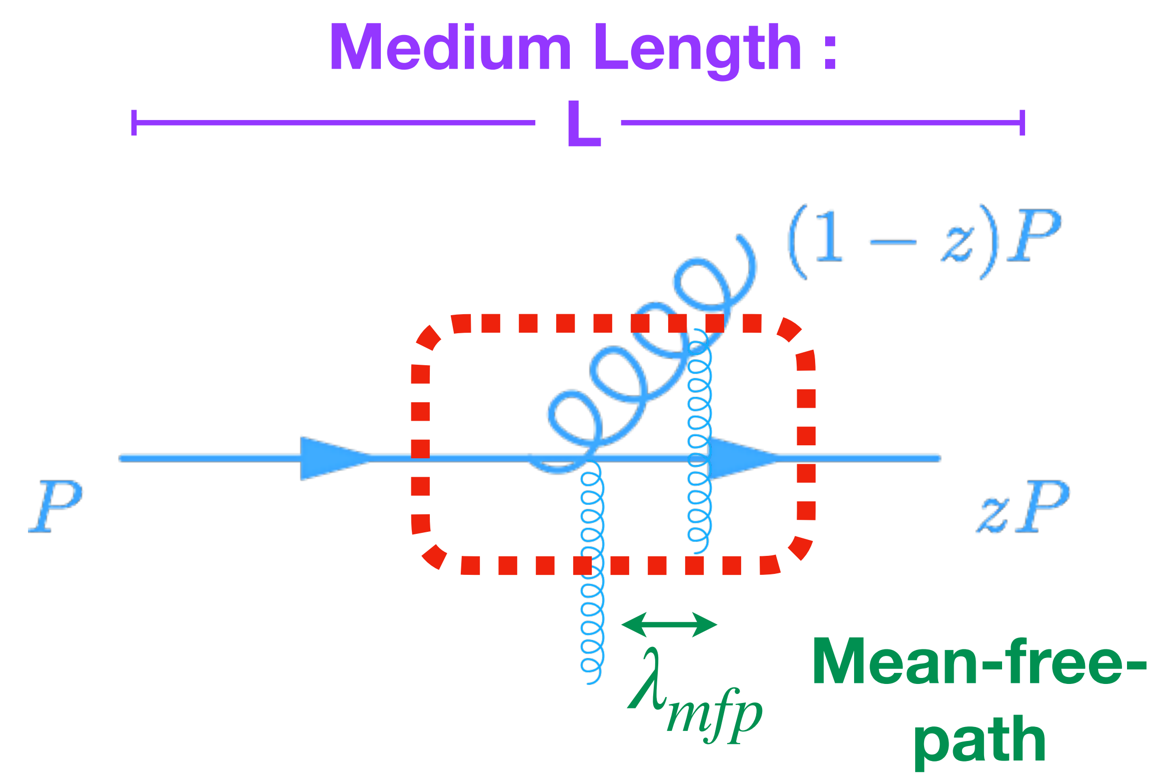


- The two temperatures (250 MeV and 500 MeV) do not display a remarkable difference
- Recover LPM suppression at large momentum.
- In the large energy region the rate is closer to the LO one as they both follow the same behavior in broadening kernel at short distances

Regimes

Interplay between different scales

- $z(1-z) \ll 1 \Rightarrow t_f \ll \lambda_{mfp}$: Few scattering occur during the formation => radiation can be described using opacity expansion
- $z(1-z) \sim 0.25$:
  - $L \ll \dfrac{(zP)^2}{\hat{q}}$: rare hard scattering must lead to formation of the radiation described by an opacity expansion
  - $L \gg \dfrac{(zP)^2}{\hat{q}}$: Multiple scatterings are important => resummed interferences between scattering leads to the LPM effect



$$t_f \sim \sqrt{\frac{2Pz(1-z)}{\hat{q}}}$$

Formation time

$$\hat{q} \sim m_D^2/\lambda_{mfp} \Rightarrow k_\perp^2 \sim \hat{q} t_f \sim m_D^2 t_f/\lambda_{mfp}$$

[C. Andres et Al. JHEP 03 (2021), 102]

In-medium splitting rates

Using the broadening kernel at hand, one can compute in-medium splitting rates

- In the AMY approach:

[Arnold-Moore-Yaffe]

$$\frac{d\Gamma_{ij}}{dz}(P,z) = \frac{\alpha_s P_{ij}(z)}{[2Pz(1-z)]^2} \int \frac{\mathrm{d}^2\mathbf{p}_\perp}{(2\pi)^2} \, \mathrm{Re}\left[2\mathbf{p}_\perp \cdot \mathbf{g}_{(z,P)}(\mathbf{p}_\perp)\right]$$

where $g_{(z,P)}(p_\perp)$ is solution to the integral equation :

$$\begin{aligned} 2\mathbf{p}_\perp = i\delta E(z,P,\mathbf{p}_\perp)\mathbf{g}_{(z,P)}(\mathbf{p}_\perp) + \int \frac{\mathrm{d}^2\mathbf{q}_\perp}{(2\pi)^2} \, \bar{C}(q_\perp) \\ \times \Big\{ C_1 \left[\mathbf{g}_{(z,P)}(\mathbf{p}_\perp) - \mathbf{g}_{(z,P)}(\mathbf{p}_\perp - \mathbf{q}_\perp)\right] \\ + C_z \left[\mathbf{g}_{(z,P)}(\mathbf{p}_\perp) - \mathbf{g}_{(z,P)}(\mathbf{p}_\perp - z\mathbf{q}_\perp)\right] \\ + C_{1-z} \left[\mathbf{g}_{(z,P)}(\mathbf{p}_\perp) - \mathbf{g}_{(z,P)}(\mathbf{p}_\perp - (1-z)\mathbf{q}_\perp)\right] \Big\} \, . \end{aligned}$$

In-medium splitting rates

BH regime :

$$Pz(1-z) \ll \omega_{\mathrm{BH}} \sim T$$

Formation time is small and interference between scatterings can be neglected.

One solves the rate equations in opacity expansion in the number of elastic scatterings with medium.

Amounts to calculating a numerical integral.

[Gyulassy-Levai-Vitev]

LPM regime :

$$Pz(1-z) \gg \omega_{\mathrm{BH}} \sim T$$

Typical number of rescatterings within the formation time of bremsstrahlung can be large.

Interference of many soft scatterings need to be considered.

The broadening kernel follows a $b_\perp^2$ behavior at small distances

$$C(b_\perp) = -\frac{g_s^2 T^2}{16\pi} \mathcal{N} b_\perp^2 \log(\xi m_D^2 b_\perp^2/4)$$

The rate equations become a Harmonic oscillator problem and solved analytically

[P. Arnold and C. Dogan. *Phys. Rev. D* 78 (2008), p. 065008]

Results for non-perturbative splitting rates