# A first step towards quantum simulating jet evolution in a dense medium

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João Barata, BNL

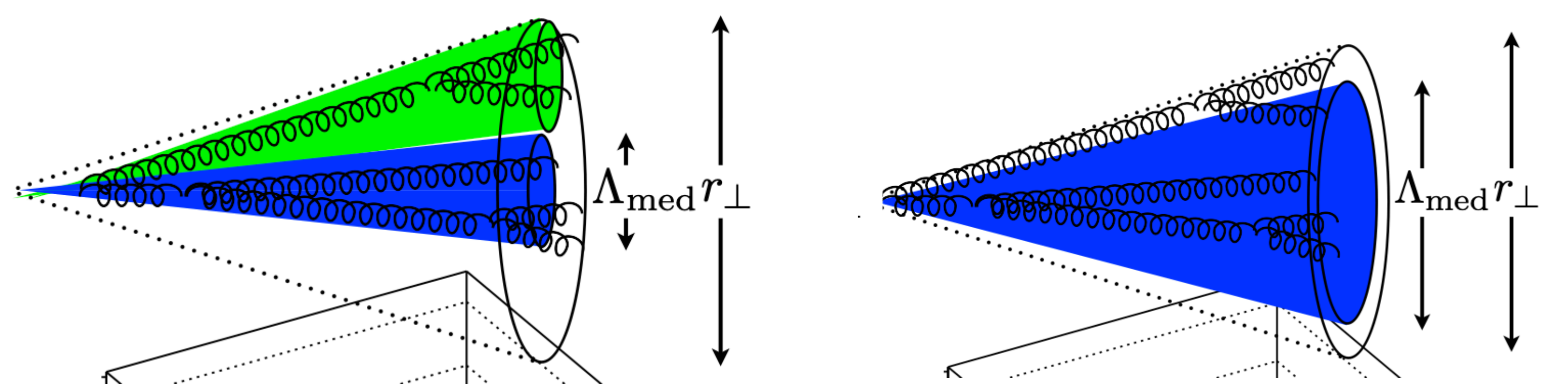
Based on: 2104.04661, with C. Salgado

ongoing, with C. Salgado, M. Li, W. Qian, X. Du

# Why quantum computing for jets?

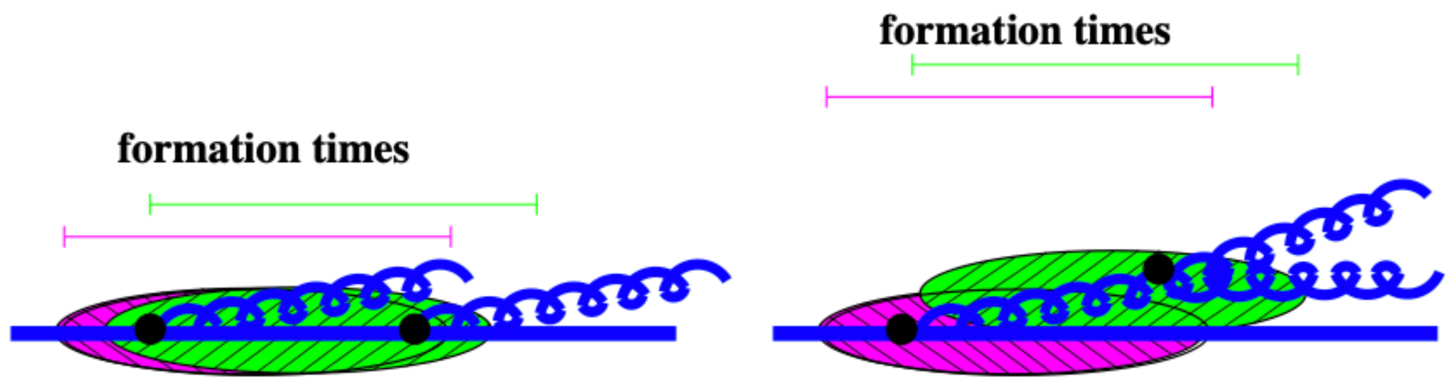
Many of the pheno relevant effects in jet quenching have a quantum origin

e.g.

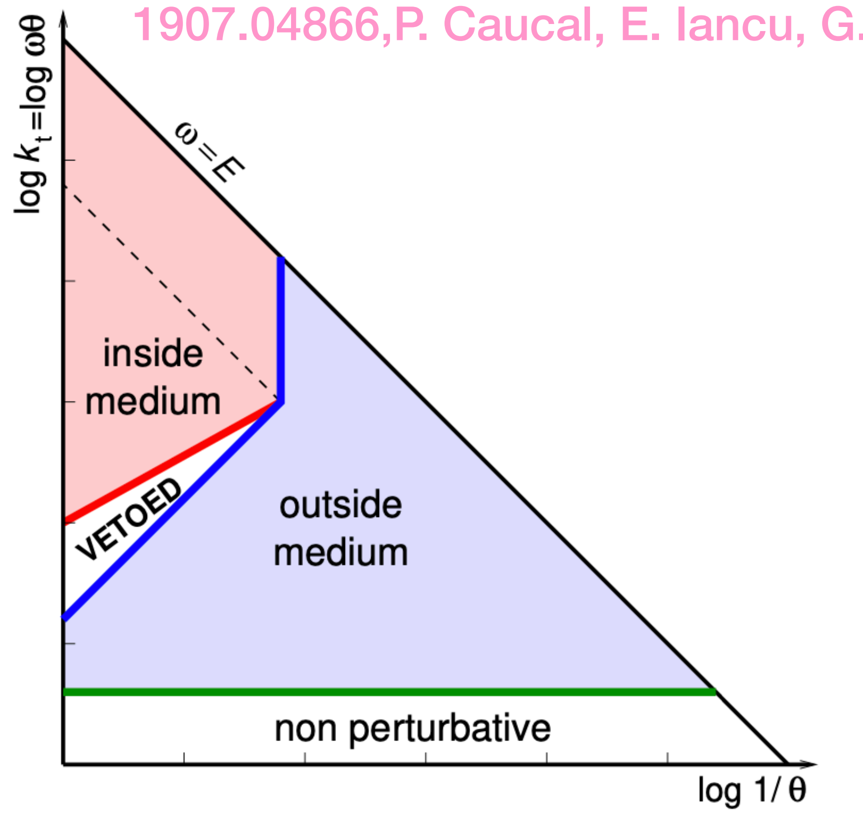



1210.7765, J. Casalderrey-Solana , Y. Mehtar-Tani, C.Salgado, K. Tywoniuk

1501.04964, P. Arnold, S. Iqbal




1907.04866,P. Caucal, E. Iancu, G. Soyez

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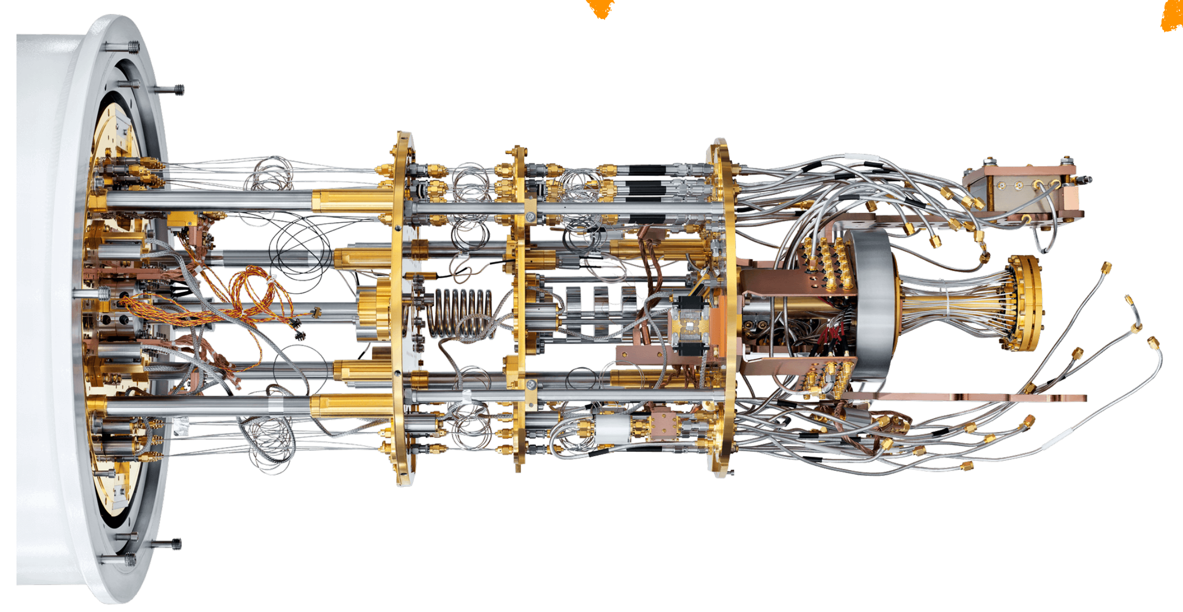

**Qcomputers should be able to handle quantum systems naturally!**

**What can we learn about jets from these machines ?**

# Jet quenching in a QC: a possible approach

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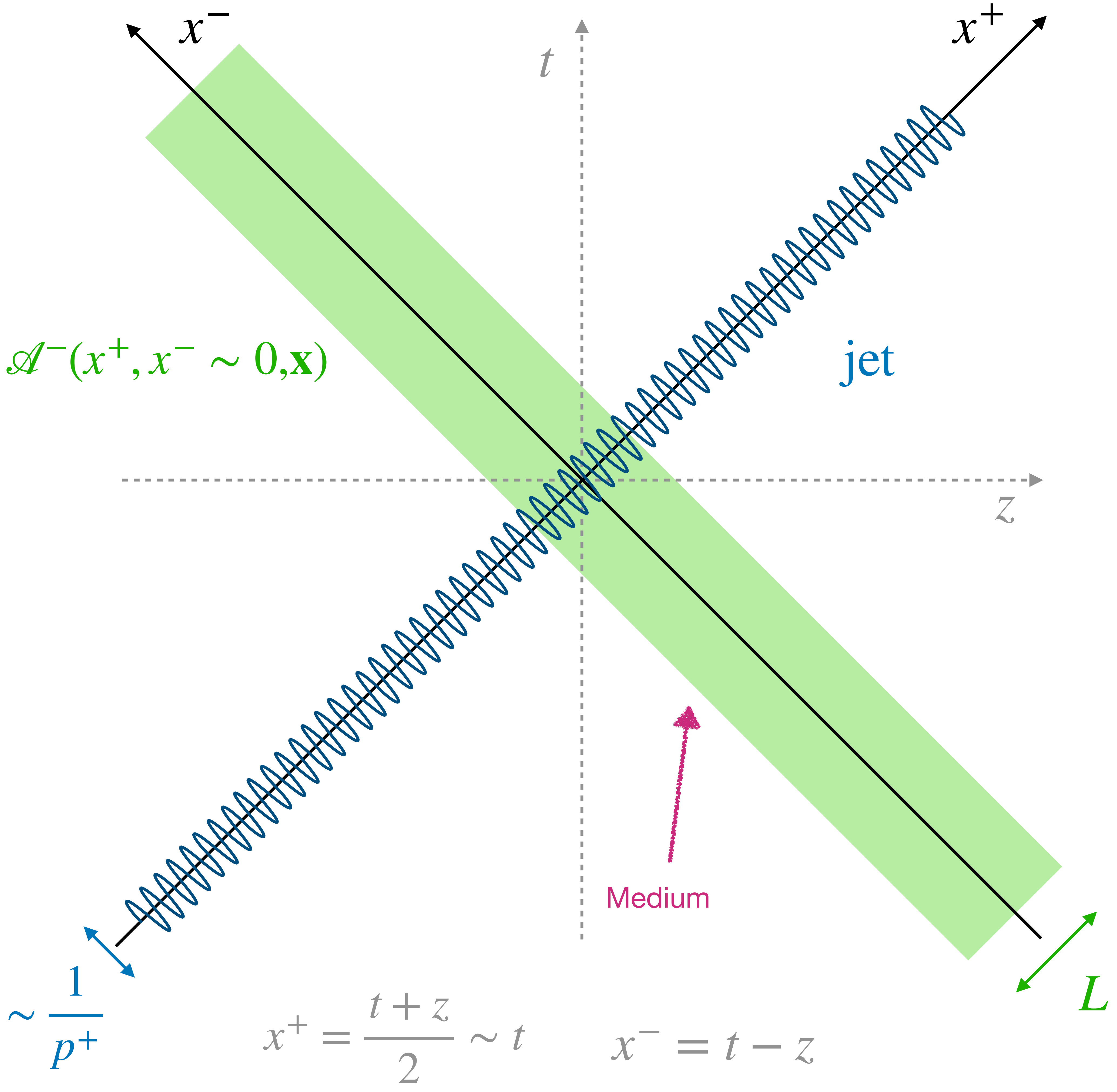


Integrating out $x^-$ the **quark propagator** satisfies

$$\left( i\partial_t + \frac{\partial^2_{\boldsymbol{x}}}{2\omega} + g\mathcal{A}^-(t, \boldsymbol{x}) \cdot T \right) G(t, \boldsymbol{x}; 0, \boldsymbol{y}) = i\delta(t)\delta(\boldsymbol{x} - \boldsymbol{y})$$

Parton evolution is equivalent to **2+1d non-rel. QM**

$$\mathcal{H}(t) = \underbrace{\frac{\boldsymbol{p}^2}{2\omega}}_{\text{p-space}} + \underbrace{g\mathcal{A}^-(t, \boldsymbol{x}) \cdot T}_{\text{x-space}} = \mathcal{H}_K + \mathcal{H}_\mathcal{A}(t)$$

+vertices

Consider the **simplest** case:

1. $|q\rangle$ Fock space only
2. $T = 1$
3. Stochastic background (hybrid approach)

# The quantum simulation algorithm

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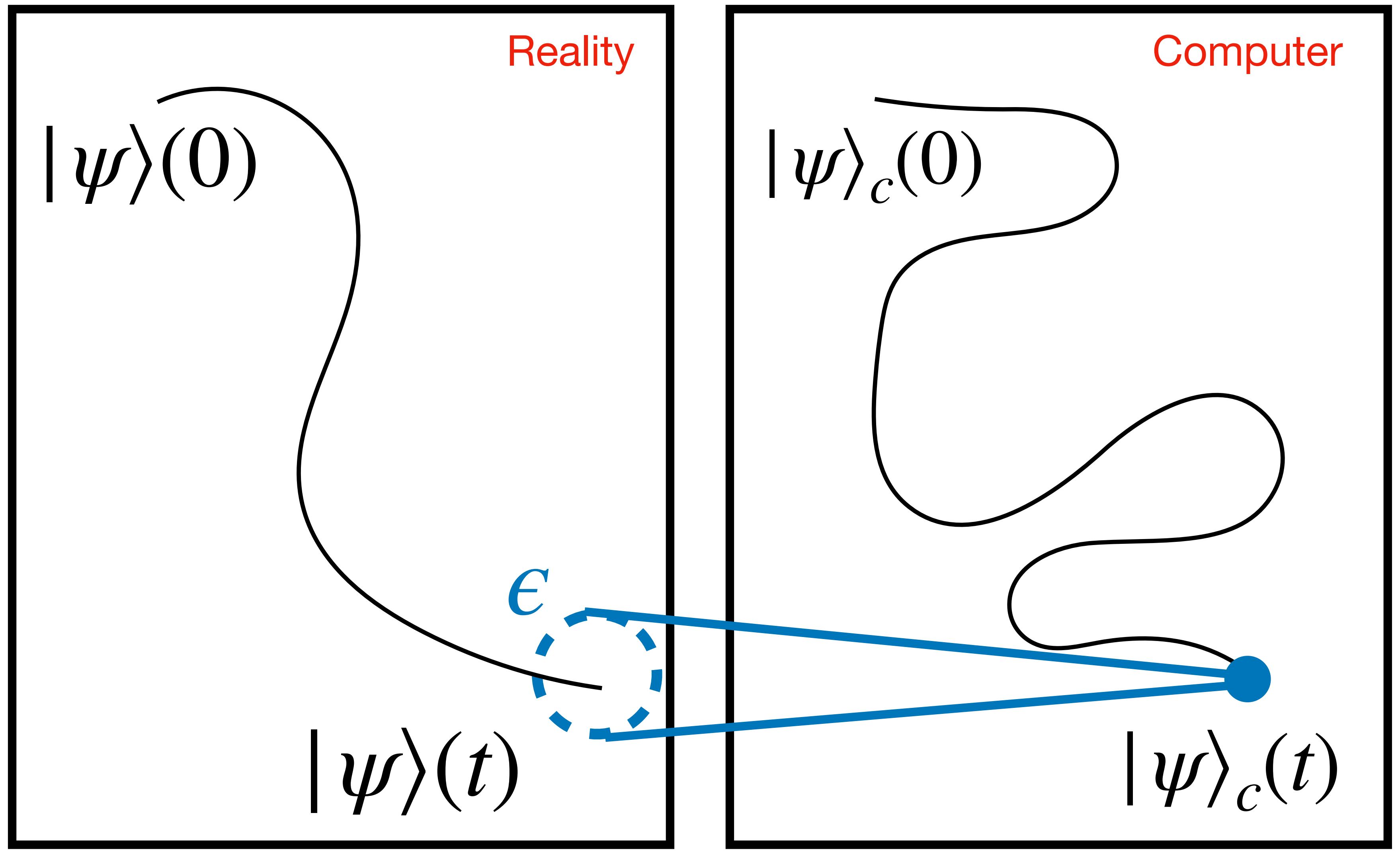
QComputers can **efficiently** simulate real time evolution ruled by:

$$|\psi\rangle(t) = \exp(-iHt)\,|\psi\rangle(0)$$




The **5** main steps of the **Quantum Simulation Algorithm**:

1. Provide $H = \sum_k H_k$ and $\psi(0)$
2. Encode the physical d.o.f's in terms of qubits and decompose $H_k$ in terms of gates
3. Prepare the initial wave function from a fiducial state ($|0\rangle^{\otimes n_{\text{qubits}}}$)
4. Time evolve according to $\exp(-iHt)$
5. Implement a measurement protocol

# Set up the algorithm





1. Provide $\mathscr{H} = \mathscr{H}_K + \mathscr{H}_A(t)$ and $\psi(0) = \psi(\mathbf{p}=0)$ + ensemble of $\{\mathscr{A}, p_A\}$

2. Encode the physical d.o.f's in terms of qubits and write $\mathscr{H}$ in terms of gates

   Introduce 2d spatial lattice with $N_s = 2^{n_Q}$ sites per dimension

$$|\mathbf{x}\rangle = |x_1, x_2\rangle = a_\perp |n_1, n_2\rangle$$

   such that

$$H = \frac{\boldsymbol{P}^2}{2E} + gA(t, \boldsymbol{X}) \cdot T = H_K + H_A(t)$$

$a_\perp$
Lattice spacing

$$\hat{P}|p\rangle = p|p\rangle \qquad \hat{X}|x\rangle = x|x\rangle \qquad x, p \in \mathbb{Z}$$

3. Prepare the initial wave function from a fiducial state ($|0\rangle^{\otimes n_{\text{qubits}}}$)

# Set up the algorithm

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**4.** Time evolve according to $\exp(-iHt)$

$$H = \frac{\boldsymbol{P}^2}{2E} + gA(t, \boldsymbol{X}) \cdot T = H_K + H_A(t)$$

Time dependent evolution is not trivial to implement. Simplest product formula

$$U(L', 0) \approx \prod_{k_t=1}^{N_t} \left\{ \exp\left[-iH_K \frac{L'}{N_t}\right] \exp\left[-iH_A\left(k_t \cdot \frac{L'}{N_t}\right) \frac{L'}{N_t}\right] \right\} \equiv \prod_{k_t=1}^{N_t} \{U_K(\varepsilon_t) U_A(k_t \cdot \varepsilon_t, \varepsilon_t)\}$$

valid for very smooth $H_A$

Implement operators with a Fourier Transform in between

$$U_K(\varepsilon_t)\,|\boldsymbol{p}\rangle = \exp\left(-i\frac{\varepsilon_t}{2E}\boldsymbol{p}^2\right)|\boldsymbol{p}\rangle \qquad |\mathbf{p}\rangle \xrightarrow{\text{qFT}} |\mathbf{x}\rangle \qquad U_A(k_t \cdot \varepsilon_t, \varepsilon_t)\,|\boldsymbol{x}\rangle = \exp(-ig\varepsilon_t A(k_t \cdot \varepsilon_t, \boldsymbol{x}))\,|\boldsymbol{x}\rangle$$

# Set up the algorithm

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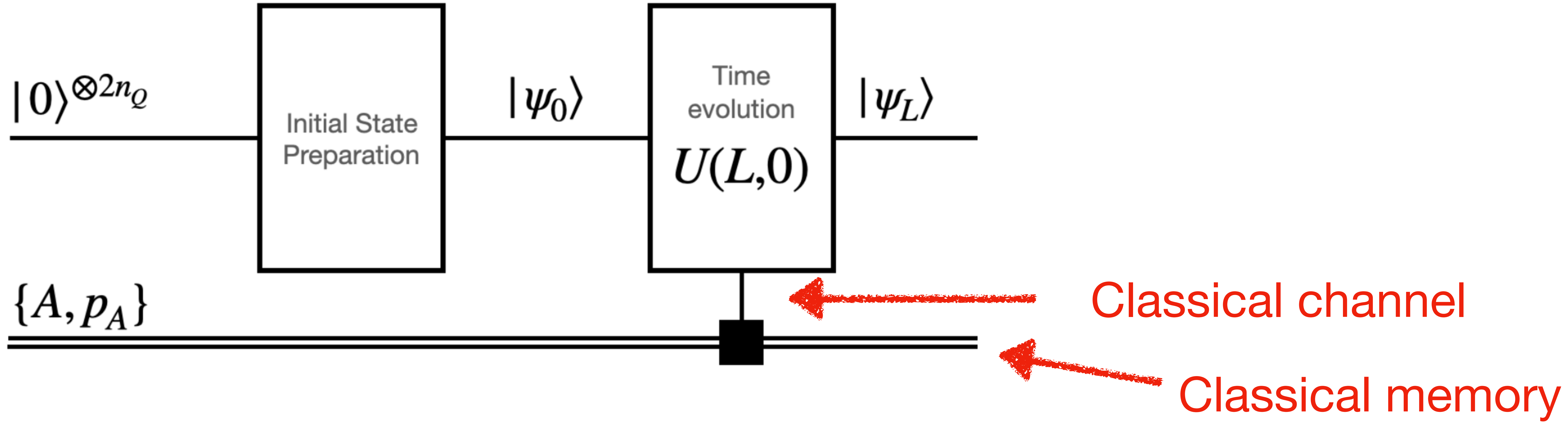
4. Time evolve according to $\exp(-iHt)$ $\quad U_A(k_t \cdot \varepsilon_t, \varepsilon_t)\,|\boldsymbol{x}\rangle = \exp(-ig\varepsilon_t A(k_t \cdot \varepsilon_t, \boldsymbol{x}))\,|\boldsymbol{x}\rangle$

Field insertions require probing the field value. This is done classically




Requires $\mathcal{O}(N_t \times N_s^2)$ field evaluations; **Ok for resolving parton evolution**

**Major limitation of the approach due to classical treatment of medium**

# Set up the algorithm

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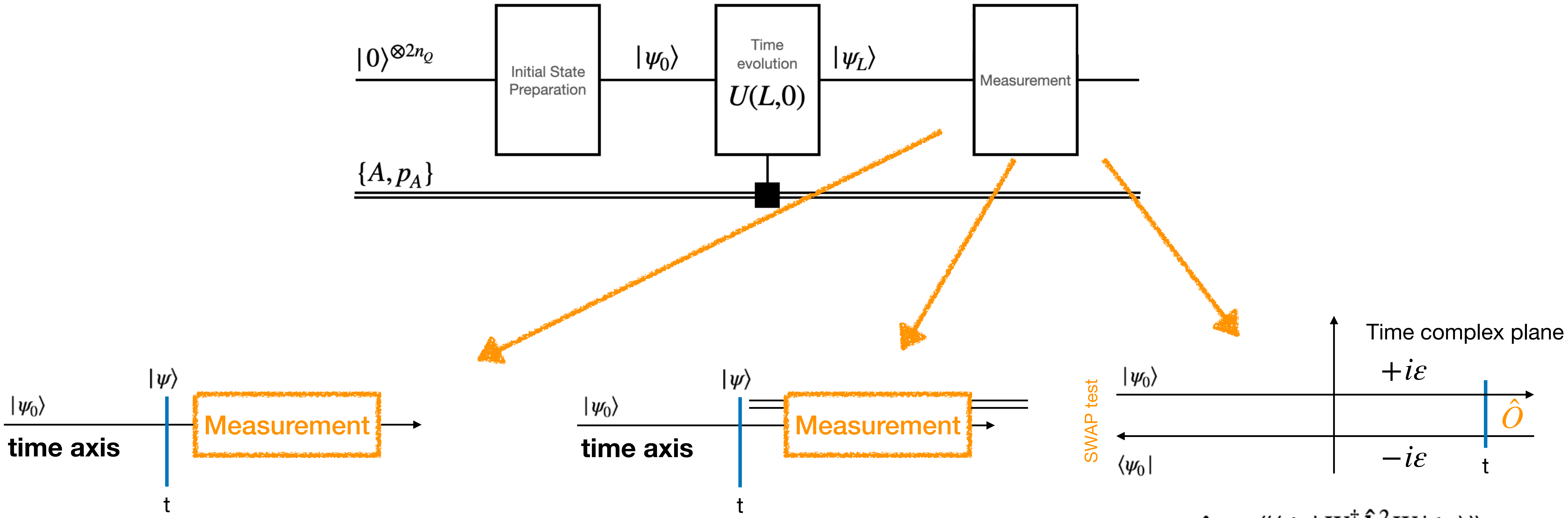
**5.** Implement a measurement protocol

$$|\psi_L\rangle = \sum_{\boldsymbol{q}} \psi_L^{\boldsymbol{q}} \, |\boldsymbol{q}\rangle$$

Generic final state

**Due to quantumness, measurement needs optimization: interference experiment**




Brute force: repeat the experiment many times

Hadamard test: interfere real and imaginary part

$$\hat{q}t = \langle\langle\langle\psi_0|\, W^\dagger \hat{\boldsymbol{k}}^2 W \,|\psi_0\rangle\rangle\rangle$$

SWAP test: find overlap of 2 wave-functions

Rough estimate: for Gaussian errors and an uniform distribution, statistical error of 10% requires $\sim 10^2 \times \#\text{states}$ shots

Rough estimate: $\sim 10^2 \times \#\mathcal{O}(1)$ shots

# Adding color to the algorithm





$$A \cdot T = A^a \frac{\lambda^a}{2}$$

Consider $a = 1$

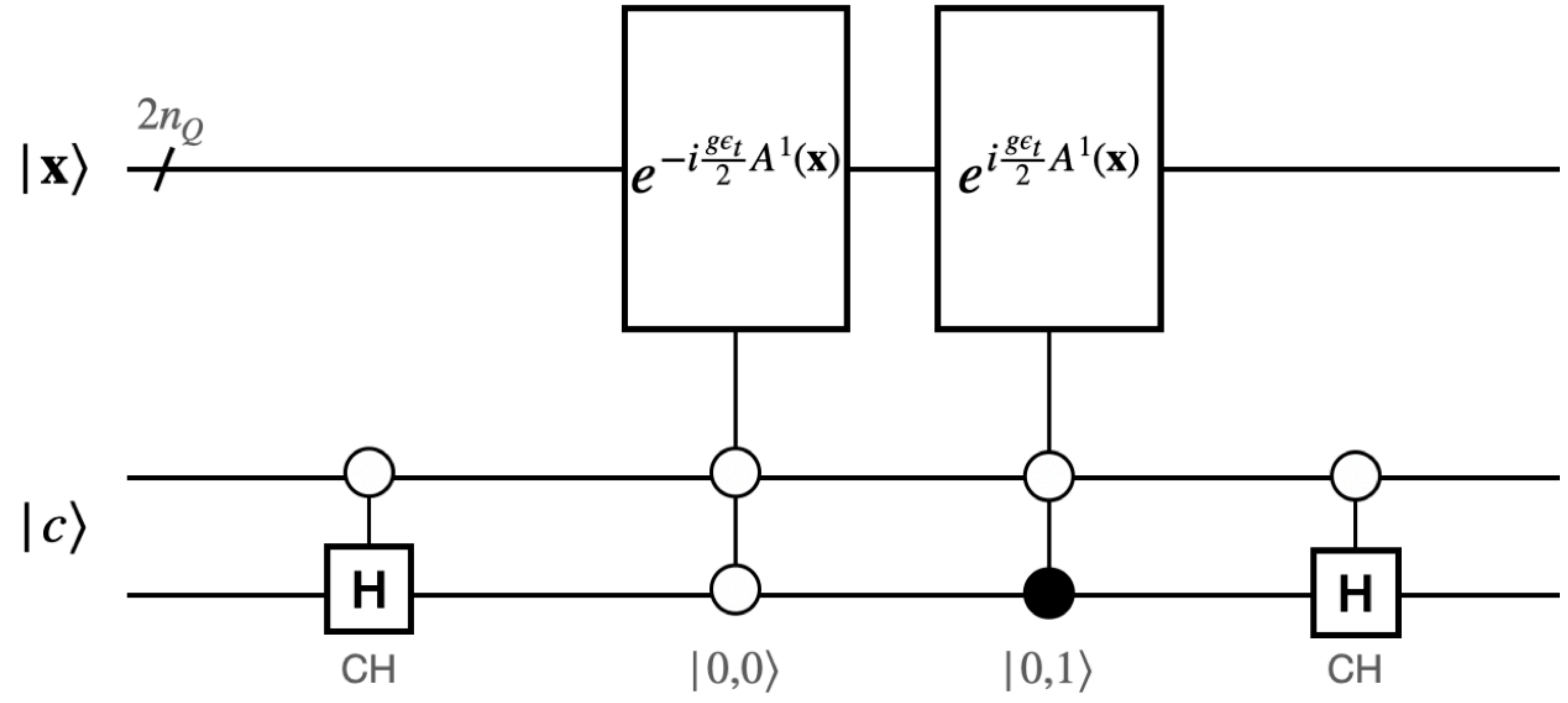
$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \equiv \tilde{\lambda}^1$$

**It should be more efficient to select on color state**

1. $$e^{-\frac{ig\varepsilon_t}{2} A^1 \otimes \tilde{\lambda}^1} = (1 \otimes CH) e^{-\frac{ig\varepsilon_t}{2} A^1 \otimes \tilde{\sigma}^Z} (1 \otimes CH)$$ $\tilde{\sigma}^Z = \mathrm{diag}(1, -1, 0, 0)$

2. $$e^{-i\frac{g\varepsilon_t}{2} A^1 \otimes \tilde{\sigma}^Z} |\boldsymbol{x}\rangle \otimes |c\rangle = \sum_n \frac{(-ig\varepsilon_t)^n}{2^n n!} (A^1(\boldsymbol{X}) \tilde{\sigma}^Z)^n |\boldsymbol{x}\rangle |c\rangle = |\boldsymbol{x}\rangle \sum_n \frac{(-ig\varepsilon_t A^1(\boldsymbol{x}))^n}{2^n n!} (\tilde{\sigma}^Z)^n |c\rangle$$

Circuit form



# Numerical results

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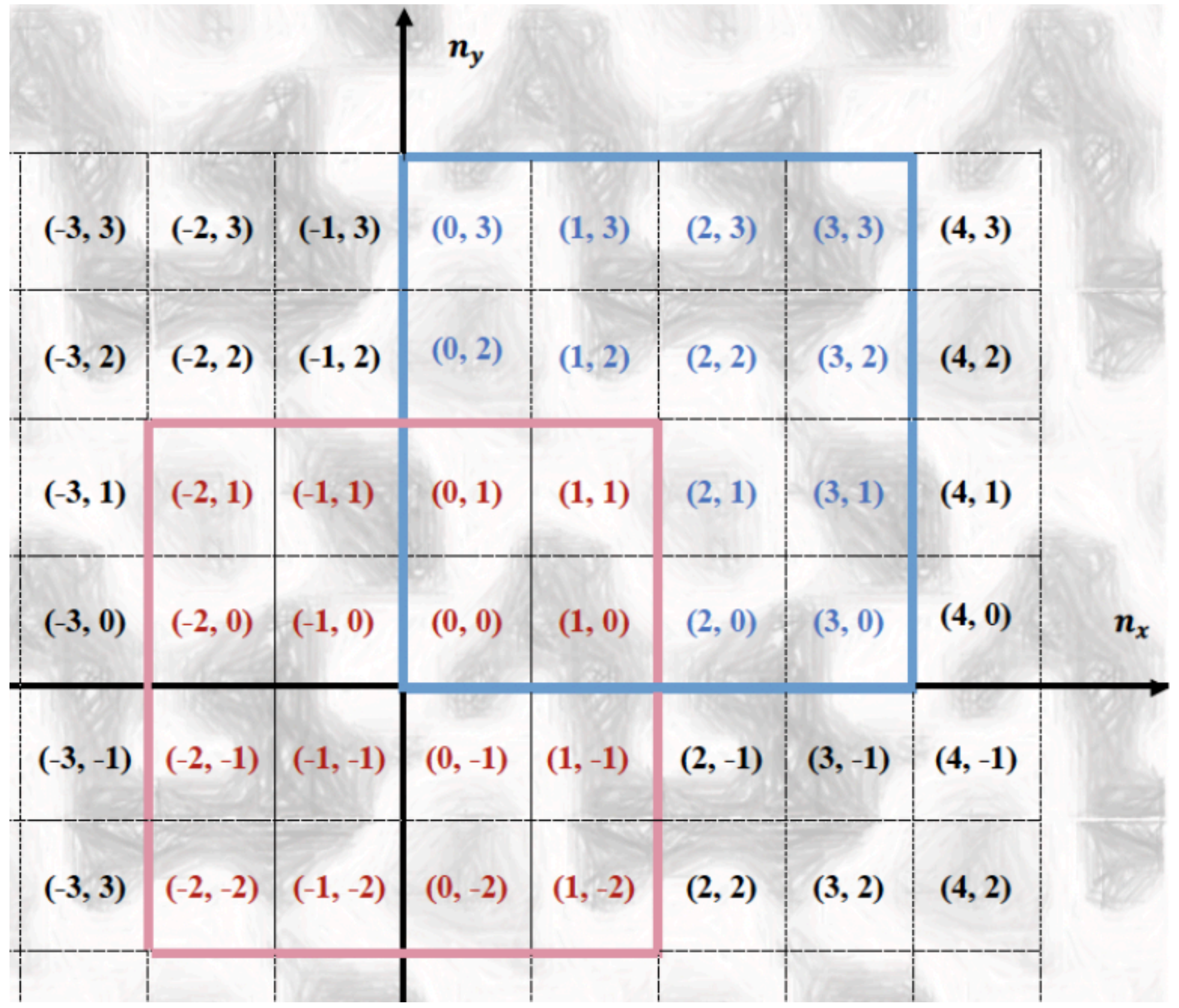
**We consider the simplest possible set up:**



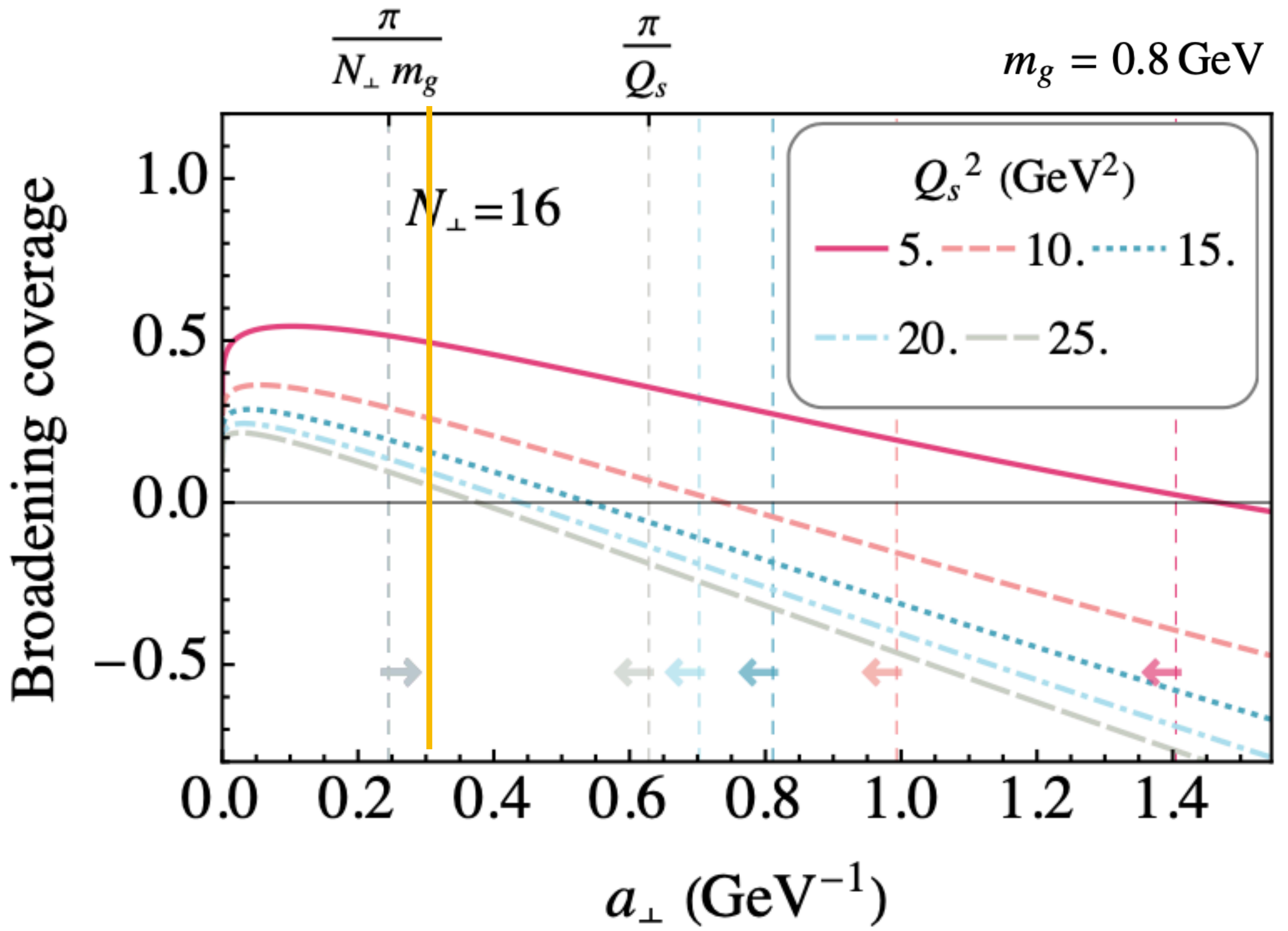

1. $T = 1$ (no colors)
2. Static brick of length 10 fm
3. We use 5/6 qubits per spatial dimension (1024/4096 states in total).
4. We use 5 field configurations. These are determined by lattice spacing and the field strength

Determined by saturation scale: $g^2\tilde{\mu} = \sqrt{\dfrac{2\pi Q_s^2}{C_F L_\eta}}$

Determined by lattice saturation conditions:

$$\frac{\pi}{N_\perp m_g} \ll a_\perp \ll \frac{\pi}{Q_s}$$ (relevant physical region is covered)

$$a_\perp^2 Q_s^2 < \frac{4\pi^2}{3}\left[\log\left(\frac{1}{a_\perp^2 m_g^2/\pi^2}+1\right) - \frac{1}{1+a_\perp^2 m_g^2/\pi^2}\right]^{-1}$$ (edge effects are absent)



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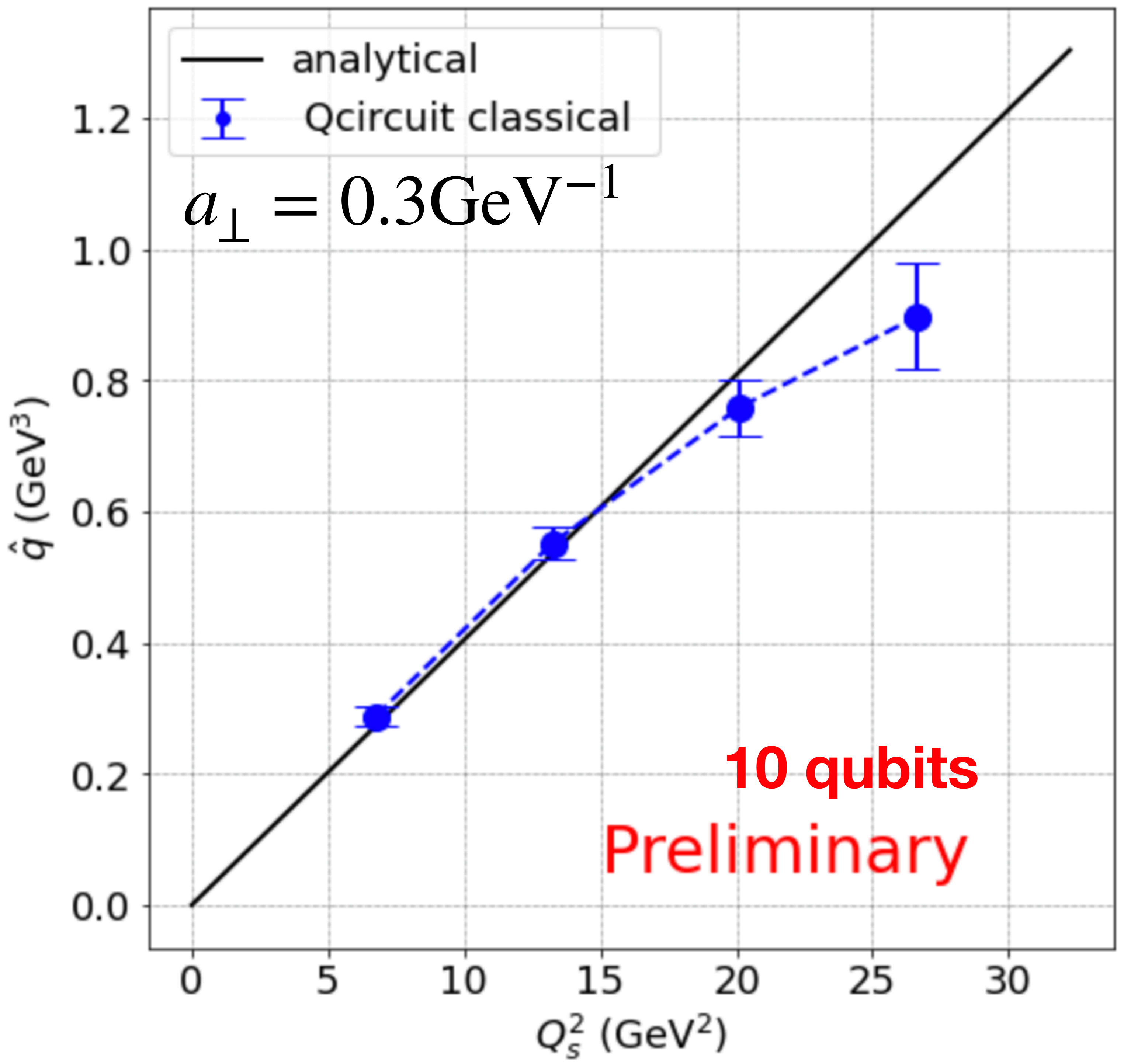
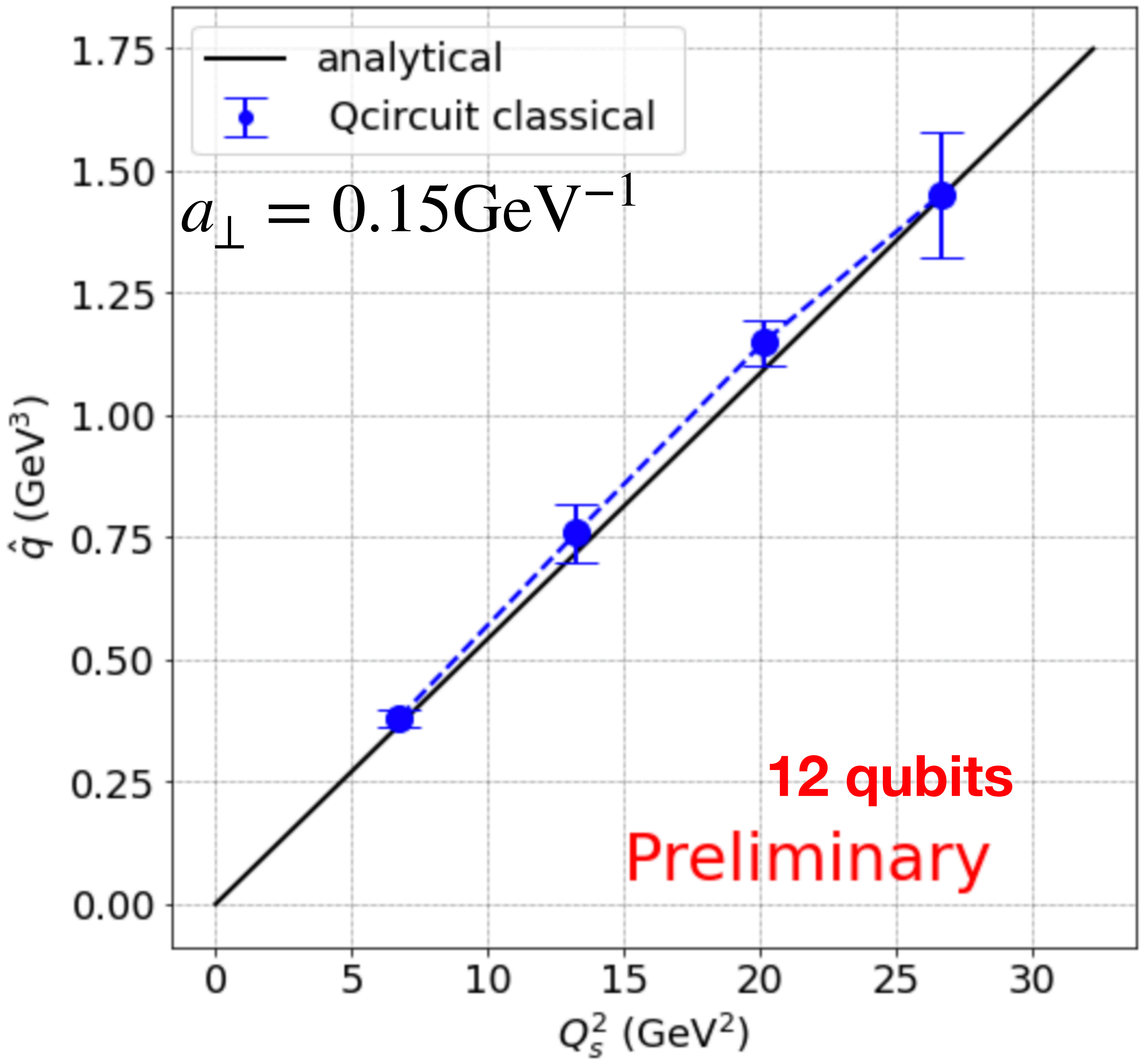
**The jet quenching parameter on the lattice is easily obtained analytically:**

$$\hat{q} = \frac{1}{t}\int_{p,x,y} p^2 e^{-ip\cdot(y-x)}\langle\langle \mathcal{W}^\dagger(y)\mathcal{W}(x)\rangle\rangle = g^2 t\langle\langle \nabla_x \mathcal{A}(0)\cdot\nabla_x\mathcal{A}(0)\rangle\rangle = \frac{g^4}{4\pi}C_F\tilde{\mu}^2\left\{\log\left(1+\frac{\frac{\pi^2}{a_\perp^2}}{m_g^2}\right) - \frac{1}{1+\frac{a_\perp^2 m_g^2}{\pi^2}}\right\}$$



# Numerical results

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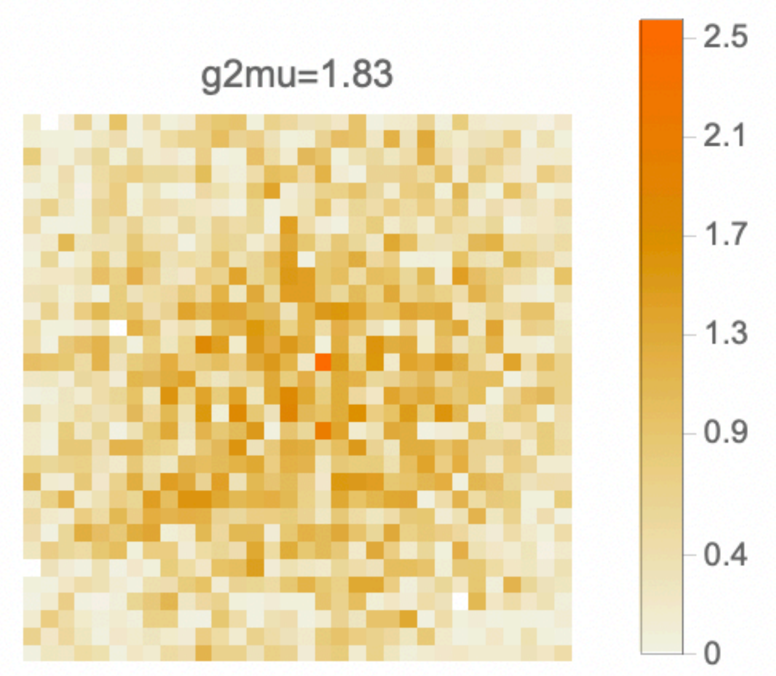
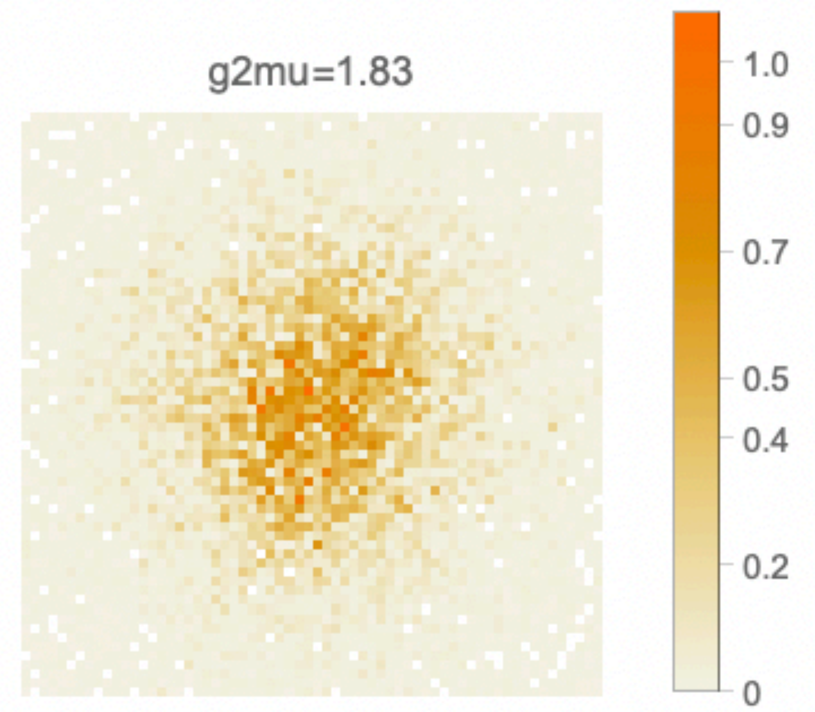
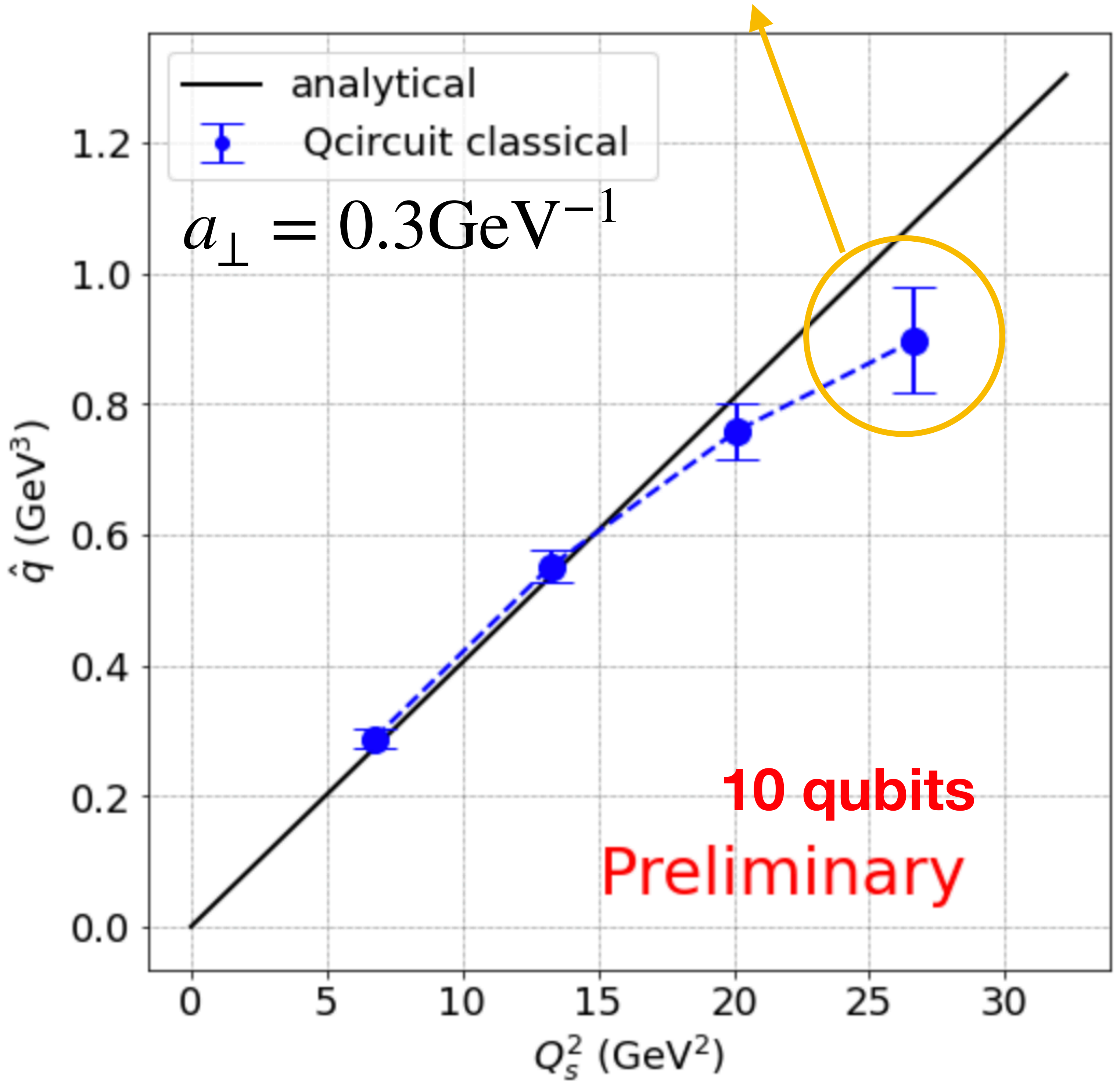
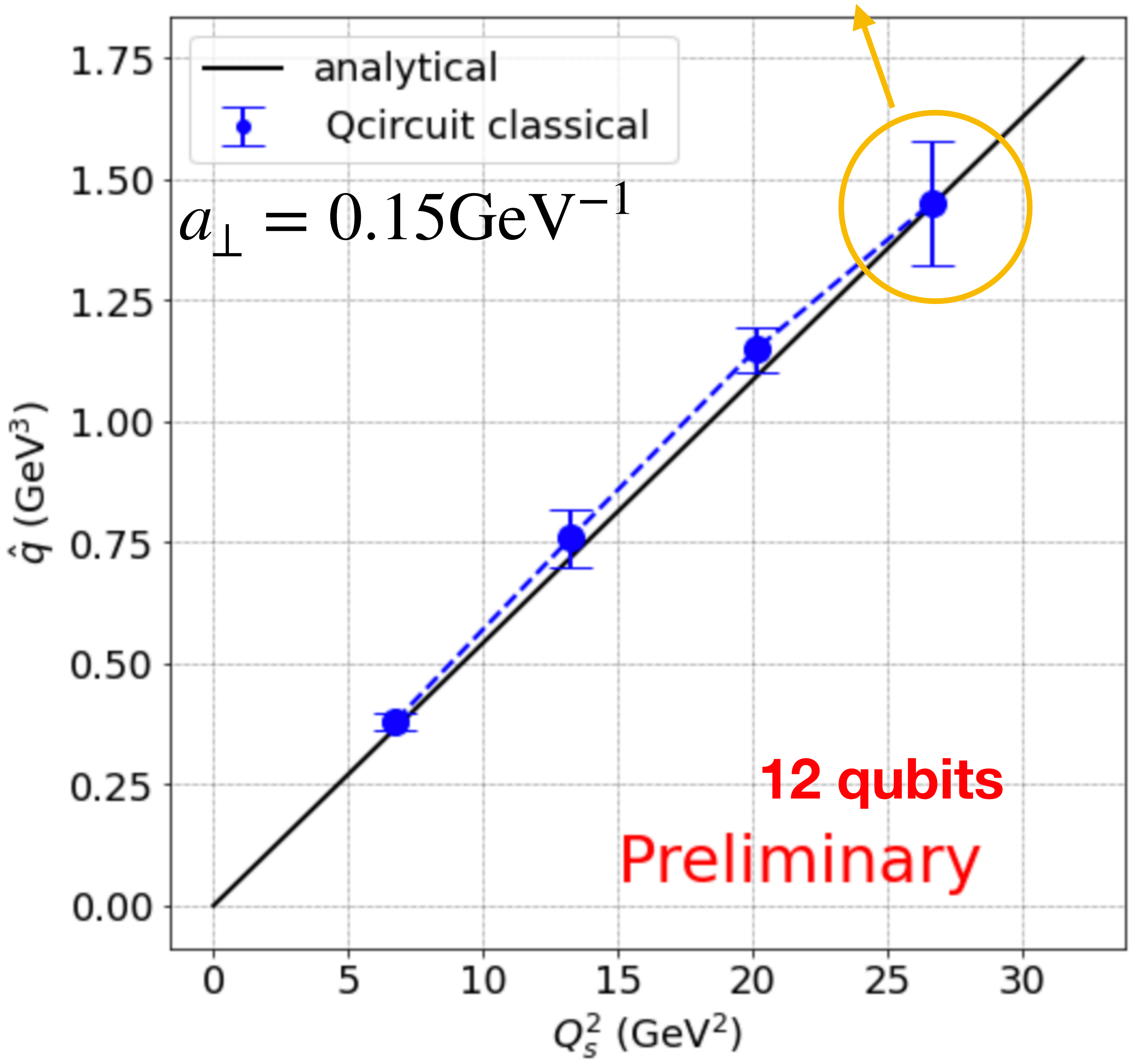
# shots=$100 \times 2^{13}$




Edge leads to mismatch with analytical result



# Numerical results

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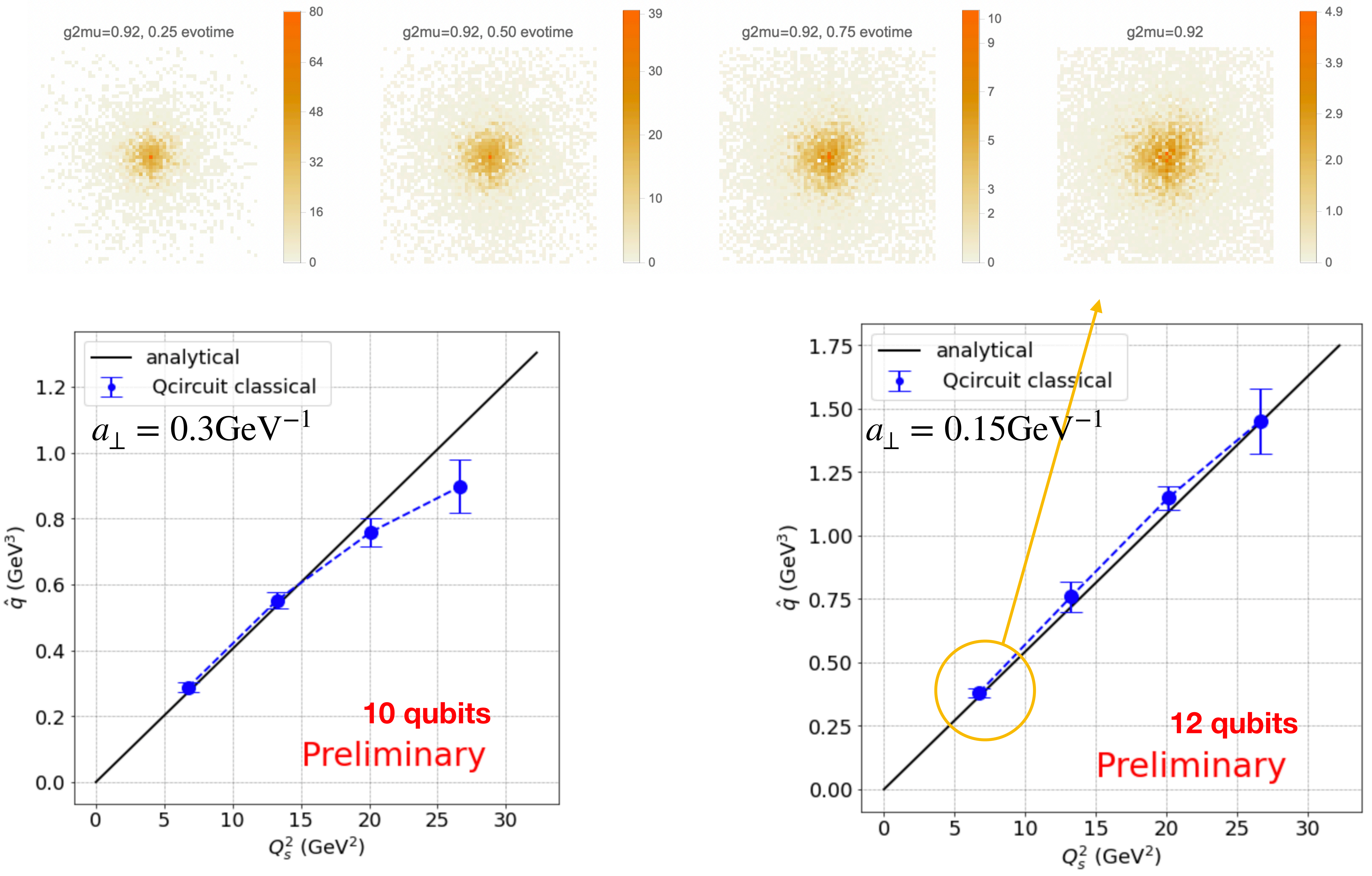

# Conclusion and Outlook



- We proposed and implemented circuit to benchmark jet broadening in a Qcomputer

- Numerical results will appear (shortly) for time evolution through colored medium

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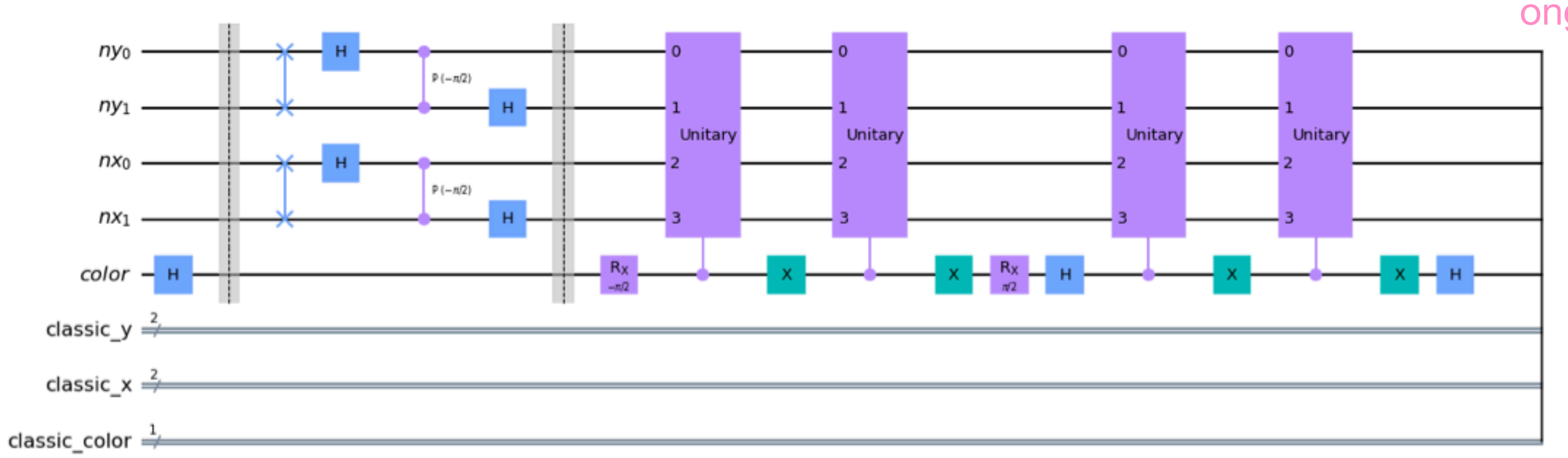
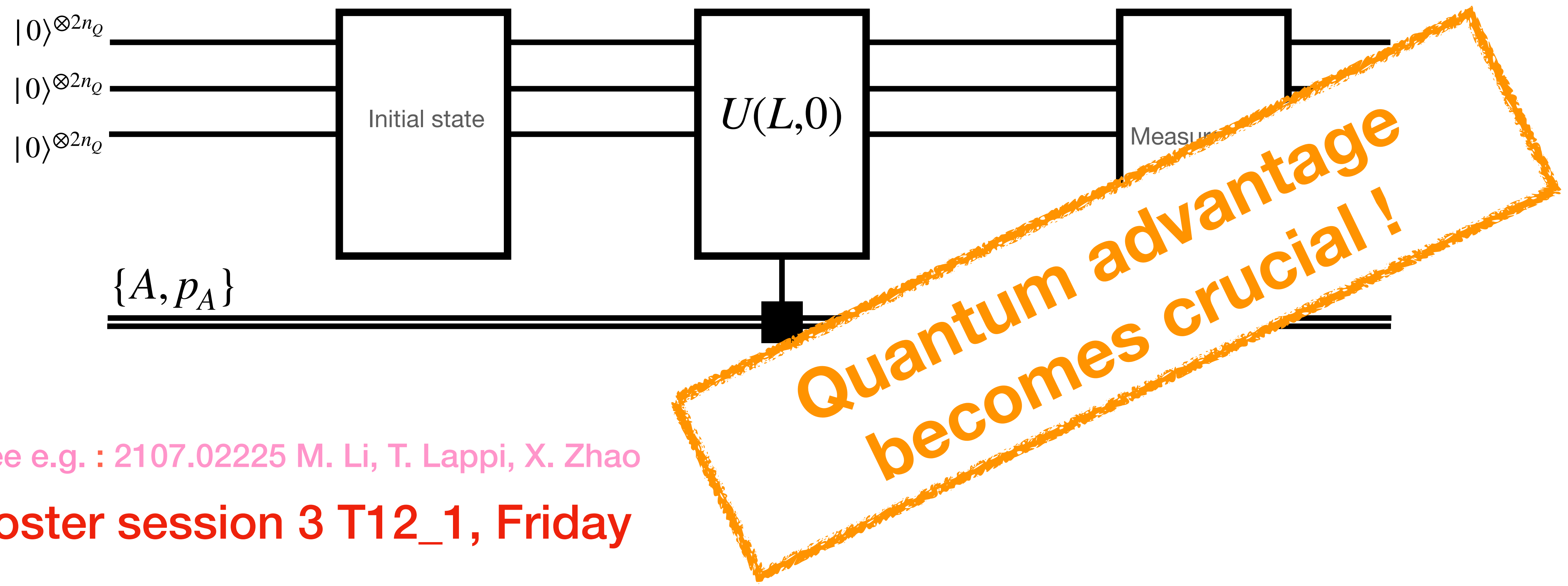



IBM qiskit implementation for $SU(2)$

- Future plans:

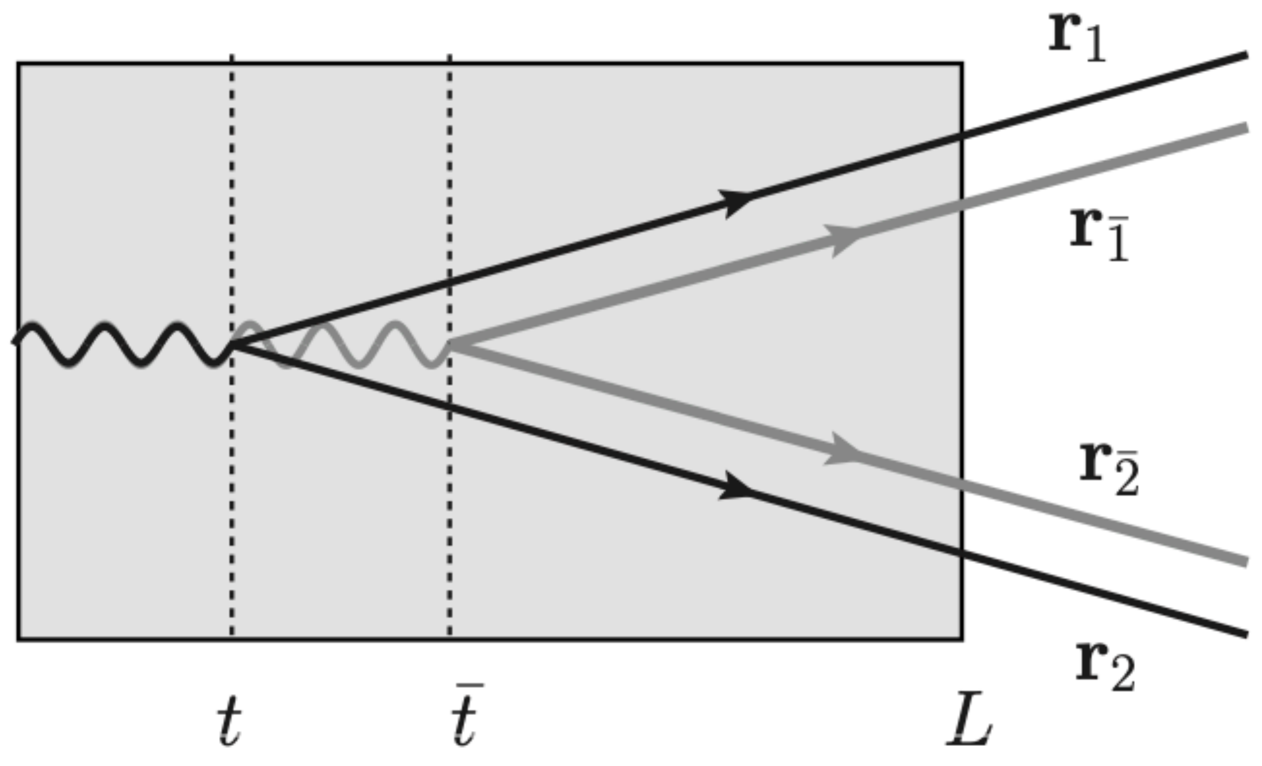
$$|\psi\rangle = c_1|q\rangle + c_2|qg\rangle + c_3|qgg\rangle + \cdots$$




See e.g. : 2107.02225 M. Li, T. Lappi, X. Zhao

Poster session 3 T12_1, Friday

**Multiple particle quantum effects already accessible**




See e.g. : 1907.03653 F. Dominguez, J. G. Milhano, C. Salgado, K. Tywoniuk, V. Vila