Far-from-equilibrium attractor in non-conformal plasmas

Sunil Jaiswal

Tata Institute of Fundamental Research, India

29th International Conference on Ultra–Relativistic Nucleus-Nucleus Collisions

April 7, 2022

Collaborators : Chandrodoy Chattopadhyay, Lipei Du, Ulrich Heinz, Subrata Pal

Motivation

- What is the domain of applicability of hydrodynamics?

- Textbooks: Close to local equilibrium, $\lambda_{\text{mfp}} \ll L$ or $|\nabla^\mu u^\nu|/T \ll 1$. Hence, hydrodynamics is formulated as an expansion in velocity gradients.
  Eckart, Phys. Rev.58 (1940), Landau and Lifshitz, "Fluid mechanics" (1987)

- The issue: Navier-Stokes equations, i.e., $1^{\text{st}}-$order equations in Landau or Eckart frame, imposes instantaneous response of dissipative fluxes to dissipative forces $\Longrightarrow$ acausality. Hiscock and Lindblom (1983, 1985)

- Two possible resolutions:
  1) 'Modern' hydrodynamic theories: Israel-Stewart like theories.
  Muller, Z. Phys, 198, 329 (1967), Israel and Stewart, Ann. Phys. 100, 310 (1976)

  2) $1^{\text{st}}-$order theory with a different choice of hydrodynamic frame.
  Bemfica, Disconzi and Noronha, 1708.06255 (2019), Kovtun, 1907.08191 (2019)

Motivation

- What is the domain of applicability of hydrodynamics?

- Textbooks: Close to local equilibrium, $\lambda_{\rm mfp} \ll L$ or $|\nabla^\mu u^\nu|/T \ll 1$. Hence, hydrodynamics is formulated as an expansion in velocity gradients.
  Eckart, Phys. Rev.58 (1940), Landau and Lifshitz, "Fluid mechanics" (1987)

- The issue: Navier-Stokes equations, i.e., $1^{\rm st}-$order equations in Landau or Eckart frame, imposes instantaneous response of dissipative fluxes to dissipative forces $\implies$ acausality. Hiscock and Lindblom (1983, 1985)

- Two possible resolutions:
  1) 'Modern' hydrodynamic theories: Israel-Stewart like theories.
  Muller, Z. Phys, 198, 329 (1967), Israel and Stewart, Ann. Phys. 100, 310 (1976)

  2) $1^{\rm st}-$order theory with a different choice of hydrodynamic frame.
  Bemfica, Disconzi and Noronha, 1708.06255 (2019), Kovtun, 1907.08191 (2019)

‘Modern’ hydrodynamic theories

- Promote dissipative fluxes to independent degrees of freedom.
  Muller, Z. Phys, 198, 329 (1967), Israel and Stewart, Ann. Phys. 100, 310 (1976)

$$\tau_\pi \dot{\pi}^{\mu\nu} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \cdots$$

  so that it relaxes to Navier-Stokes over a time scale $\sim \tau_\pi$.

- This introduces ‘non-hydrodynamic’ modes ($\omega \neq 0$ as $\mathrm{k} \to 0$).

- This approach has led to development of variety of theories:

  Denicol-Niemi-Molnar-Rischke Denicol et.al., 1202.4551 (2012)

  Chapman-Enskog approach Jaiswal, 1305.3480 (2013)

  anisotropic hydro Romatschke and Strickland; Martinez and Strickland; Florkowski and Ryblewski; McNelis, Bazow and Heinz; and others..

  and more..

- Are ‘modern’ formulations of hydrodynamics applicable over a broader domain than traditional hydrodynamics?

'Modern' hydrodynamic theories

- Promote dissipative fluxes to independent degrees of freedom.
  Muller, Z. Phys, 198, 329 (1967), Israel and Stewart, Ann. Phys. 100, 310 (1976)

$$\tau_\pi \dot{\pi}^{\mu\nu} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \cdots$$

  so that it relaxes to Navier-Stokes over a time scale $\sim \tau_\pi$.

- This introduces 'non-hydrodynamic' modes ($\omega \neq 0$ as $\mathrm{k} \to 0$).

- This approach has led to development of variety of theories:

  Denicol-Niemi-Molnar-Rischke Denicol et.al., 1202.4551 (2012)

  Chapman-Enskog approach Jaiswal, 1305.3480 (2013)

  anisotropic hydro Romatschke and Strickland; Martinez and Strickland; Florkowski and Ryblewski; McNelis, Bazow and Heinz; and others..

  and more..

- Are 'modern' formulations of hydrodynamics applicable over a broader domain than traditional hydrodynamics?

‘Modern’ hydrodynamic theories

- Promote dissipative fluxes to independent degrees of freedom.
  Muller, Z. Phys, 198, 329 (1967), Israel and Stewart, Ann. Phys. 100, 310 (1976)

$$\tau_\pi \dot{\pi}^{\mu\nu} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \cdots$$

  so that it relaxes to Navier-Stokes over a time scale $\sim \tau_\pi$.

- This introduces ‘non-hydrodynamic’ modes ($\omega \neq 0$ as $\mathrm{k} \to 0$).

- This approach has led to development of variety of theories:

  Denicol-Niemi-Molnar-Rischke Denicol et.al., 1202.4551 (2012)

  Chapman-Enskog approach Jaiswal, 1305.3480 (2013)

  anisotropic hydro Romatschke and Strickland; Martinez and Strickland; Florkowski and Ryblewski; McNelis, Bazow and Heinz; and others..

  and more..

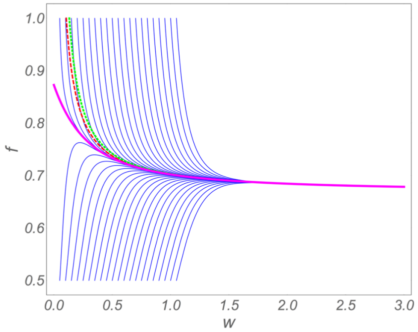
- Are ‘modern’ formulations of hydrodynamics applicable over a broader domain than traditional hydrodynamics?

Attractor in 'Modern' hydrodynamics

Bjorken flow J. D. Bjorken, PRD 27, 140 (1983)

- Boost-invariant longitudinal expansion, $v^z = \frac{z}{t}$, $v^x = v^y = 0$.
- Switch to Milne coordinate system $(\tau, x_\perp, \phi, \eta_s)$.
  - Fluid appears static, $u^\mu = (1,0,0,0)$, However, finite expansion rate, $\partial_\mu u^\mu = 1/\tau$.
  - Shear tensor is diagonal, $\pi^{\mu\nu} = \mathrm{diag}(0, \pi/2, \pi/2, -\pi/\tau^2)$.

Hydrodynamic attractor Heller and Spalinski, 1503.07514 (2015)



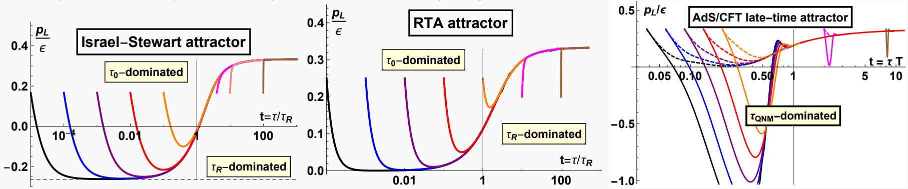
- Consider Muller-Israel-Stewart hydrodynamics.
- $f \equiv \frac{2}{3} + \frac{\pi}{4\epsilon}; \quad w \equiv \tau T \propto \tau/\tau_\pi$
- Navier-Stokes : $f_{\mathrm{NS}} = \frac{2}{3} + \frac{4\mathcal{C}_\eta}{9w}$
- Second-order : $f_2 = f_{\mathrm{NS}} + \frac{8\mathcal{C}_{\tau_\pi}\mathcal{C}_\eta}{27w^2}$
- Attractor : Stable FP : $f_0 = \frac{2}{3} + \frac{\sqrt{\mathcal{C}_\eta}}{3\sqrt{\mathcal{C}_{\tau_\pi}}}$ at $w \approx 0$

Early and late time attractors

- Late-time or near equilibrium attractor: Navier Stokes limit.
  - Expected. Not interesting.

- Early-time attractor: Universality far-from-equilibrium.

Kurkela, van der Schee, Wiedemann, and Wu 1907.08101 (2019)



Power law decay of different initializations on attractor in $\tau/\tau_R \ll 1$ regime.

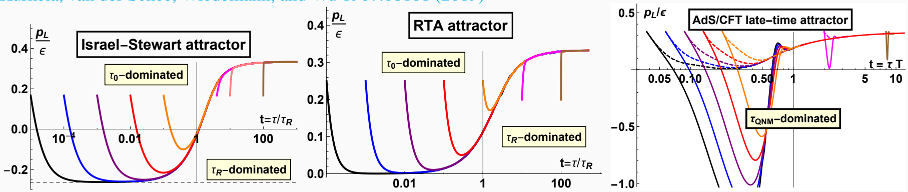
- Question: How general is this emergent ‘early-time attractor’?

Early and late time attractors

- Late-time or near equilibrium attractor: Navier Stokes limit.
  - Expected. Not interesting.

- Early-time attractor: Universality far-from-equilibrium.

Kurkela, van der Schee, Wiedemann, and Wu 1907.08101 (2019)



Power law decay of different initializations on attractor in $\tau/\tau_\mathrm{R} \ll 1$ regime.

- Question: How general is this emergent ‘early-time attractor’?

Non-conformal systems

Chattopadhyay, SJ, Du, Heinz, Pal, 2107.05500 (2021)   SJ, Chattopadhyay, Du, Heinz, Pal, 2107.10248 (2021)

- Consider Boltzmann equation in relaxation-time approximation (RTA):

$$p^\mu \partial_\mu f = -\frac{(u \cdot p)}{\tau_\mathrm{R}}(f - f_\mathrm{eq})$$

- Exact solution for Bjorken:
  G. Baym, Phys. Lett. B138, 18 (1984),   Florkowski, Maksymiuk, Ryblewski, Strickland, 1402.7348 (2014)

$$\boxed{f(\tau; p_T, w) = D(\tau, \tau_0)\, f_\mathrm{in}(\tau_0; p_T, w) + \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau_R(\tau')} D(\tau, \tau')\, f_\mathrm{eq}(\tau'; p_T, w)}$$

Relaxation time: $\tau_\mathrm{R}(\tau) = \dfrac{5C}{T(\tau)}$	Damping function: $D(\tau_2, \tau_1) = \exp\left(-\int_{\tau_1}^{\tau_2} \dfrac{d\tau'}{\tau_R(\tau')}\right)$

- Initial distribution: $\quad f_\mathrm{in} \equiv \frac{1}{\alpha_0} \exp\left(-\frac{\sqrt{m^2 + p_T^2 + (1+\xi_0)(w/\tau_0)^2}}{\Lambda_0}\right)$

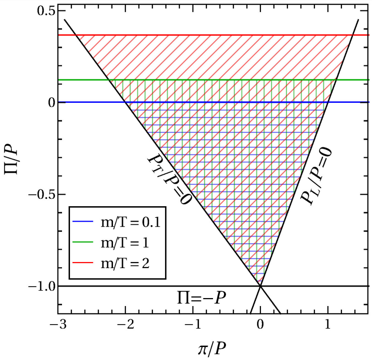
  Chattopadhyay, SJ, Du, Heinz, Pal, 2107.05500 (2021)

  - Three parameters: $\alpha_0, \Lambda_0, \xi_0$. Can be tuned for initial ϵ, P_L, P_T.
  - Allows to create a system with large initial bulk and/or shear stress.

Kinetic bounds on viscous stresses

- Since $f(x,p) \geq 0 \implies$
  (i) $\epsilon = \langle p_0^2 \rangle \geq 0$
  (ii) $P_L = \langle w^2 \rangle / \tau^2 \geq 0$
  (iii) $P_T = \frac{1}{2} \langle p_T^2 \rangle \geq 0$
  (iv) $P + \Pi \equiv \frac{1}{3} \langle p_T^2 + (w/\tau)^2 \rangle \geq 0$
  (v) $T^\mu_\mu \equiv m^2 \langle (1) \rangle = \epsilon - 3(P{+}\Pi) \geq 0$

  Here, $\langle (\dots) \rangle \equiv \int d^2p_T dw / [(2\pi)^3 \tau p^\tau]\, (\dots)\, f$
  $w \equiv p_\eta \equiv tp^z - zE_p$



- Therefore, bound on shear and bulk stress: $\{\overline{\Pi} \equiv \Pi/P,\ \overline{\pi} \equiv \pi/P\}$

$$\boxed{\overline{\Pi} + \frac{1}{2}\overline{\pi} \geq -1, \qquad \overline{\Pi} - \overline{\pi} \geq -1, \qquad \overline{\Pi} \geq -1, \qquad \overline{\Pi} \leq \frac{\epsilon}{3P} - 1.}$$

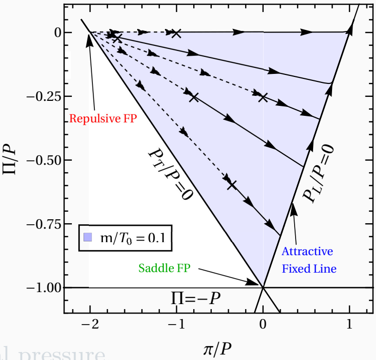
Chattopadhyay, SJ, Du, Heinz, Pal, 2107.05500 (2021)

For conformal gas of massless particles ($\Pi = 0$), allowed region shrinks to $\Pi/P = 0$; discontinues jump in allowed region for arbitrary small mass.

Collisionless regime: Free streaming

- Initial conditions:
  $m = 50\,\mathrm{MeV}$. $T_0 = 500\,\mathrm{MeV}$ at $\tau_0 = 0.1\,\mathrm{fm}/c$.

- Free streaming evolution:
  $f_{\mathrm{fs}}(\tau; p_T, p^z) = f_{\mathrm{in}}(p_T, p^z(\tau/\tau_0))$
  become sharply peaked in p_z as time evolves.

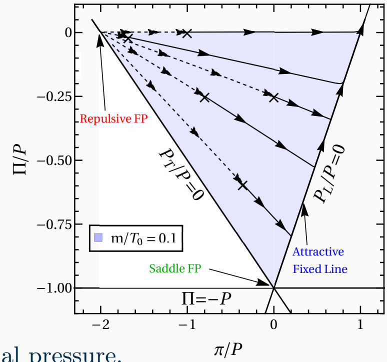


- *Attractive fixed line* : $P_L = 0$
  All curves approach this line of vanishing longitudinal pressure.

- *Repulsive fixed point* : $\{P_T/P = 0,\ \Pi/P = 0\}$
  Backward evolution of all curves approach this point.

- *Saddle point* : $\{P_T/P = 0,\ P_L/P = 0\}$
  Spherically symmetric distribution: $f \propto \delta^{(3)}(\mathbf{p}) \propto \delta(|\mathbf{p}|)/|\mathbf{p}|^2$.

Collisionless regime: Free streaming



- Initial conditions:
  $m = 50\,\text{MeV}$. $T_0 = 500\,\text{MeV}$ at $\tau_0 = 0.1\,\text{fm}/c$.

- Free streaming evolution:
  $f_{\text{fs}}(\tau; p_T, p^z) = f_{\text{in}}(p_T, p^z(\tau/\tau_0))$
  become sharply peaked in p_z as time evolves.

- *Attractive fixed line* : $P_L = 0$
  All curves approach this line of vanishing longitudinal pressure.

- *Repulsive fixed point* : $\{P_T/P = 0,\ \Pi/P = 0\}$
  Backward evolution of all curves approach this point.

- *Saddle point* : $\{P_T/P = 0,\ P_L/P = 0\}$
  Spherically symmetric distribution: $f \propto \delta^{(3)}(\mathbf{p}) \propto \delta(|\mathbf{p}|)/|\mathbf{p}|^2$.

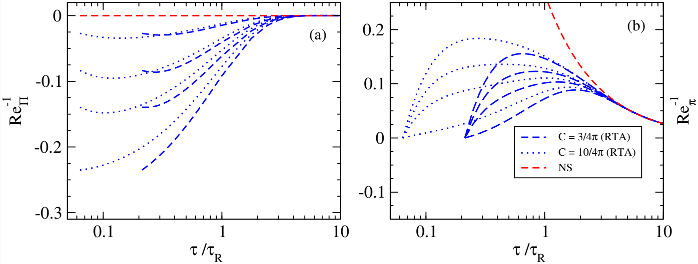
With interactions : Large Bulk, small shear

$m = 50\,\mathrm{MeV}$. $T_0 = 500\,\mathrm{MeV}$ at $\tau_0 = 0.1\,\mathrm{fm}/c$

$\mathrm{Re}_\Pi^{-1} \equiv \frac{\Pi}{\epsilon+P}$

$\mathrm{Re}_\pi^{-1} \equiv \frac{\pi}{\epsilon+P}$

$\tau_\mathrm{R} = \frac{5C}{T}$



- Absence of early-time attractor in KT in both bulk and shear inverse Reynolds number.
  - Convergence of trajectories seen only at $(\tau/\tau_R) \gtrsim 3$ with the Navier-Stokes attractor.

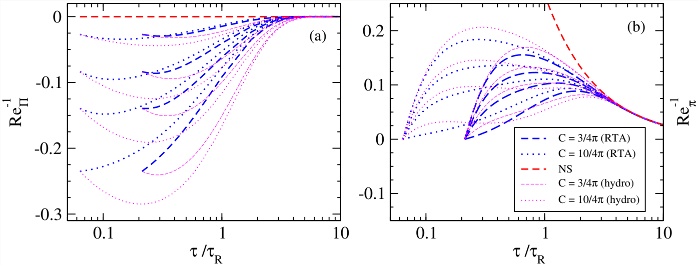
With interactions : Large Bulk, small shear

$$m = 50\,\mathrm{MeV}.\; T_0 = 500\,\mathrm{MeV} \text{ at } \tau_0 = 0.1\,\mathrm{fm}/c$$

$$\mathrm{Re}_\Pi^{-1} \equiv \frac{\Pi}{\epsilon+P}$$

$$\mathrm{Re}_\pi^{-1} \equiv \frac{\pi}{\epsilon+P}$$

$$\tau_\mathrm{R} = \frac{5C}{T}$$



- Absence of early-time attractor in KT in both bulk and shear inverse Reynolds number.
  - Convergence of trajectories seen only at $(\tau/\tau_R) \gtrsim 3$ with the Navier-Stokes attractor.
- Above statement holds for 2nd order non-conformal hydrodynamics.
  Denicol, Jeon and Gale, 1403.0962 (2014), Jaiswal, Ryblewski and Strickland, 1407.7231 (2014)
- Can one recover the early-time attractor in shear stress as seen in conformal system for small bulk viscous pressure?

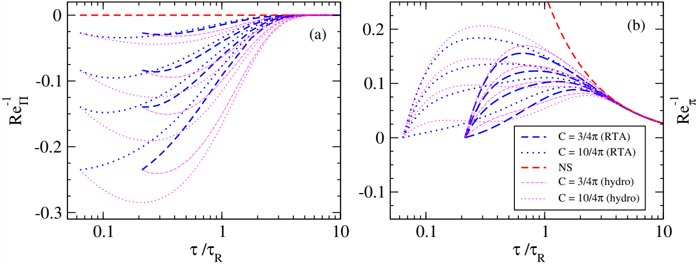
With interactions : Large Bulk, small shear

$m = 50\,\mathrm{MeV}$. $T_0 = 500\,\mathrm{MeV}$ at $\tau_0 = 0.1\,\mathrm{fm}/c$

$$\mathrm{Re}_\Pi^{-1} \equiv \frac{\Pi}{\epsilon+P}$$

$$\mathrm{Re}_\pi^{-1} \equiv \frac{\pi}{\epsilon+P}$$

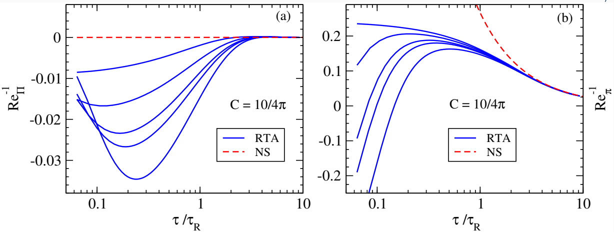
$$\tau_\mathrm{R} = \frac{5C}{T}$$



- Absence of early-time attractor in KT in both bulk and shear inverse Reynolds number.
  - Convergence of trajectories seen only at $(\tau/\tau_R) \gtrsim 3$ with the Navier-Stokes attractor.
- Above statement holds for 2^{nd} order non-conformal hydrodynamics.
  Denicol, Jeon and Gale, 1403.0962 (2014), Jaiswal, Ryblewski and Strickland, 1407.7231 (2014)
- Can one recover the early-time attractor in shear stress as seen in conformal system for small bulk viscous pressure?

With interactions : Small Bulk, large shear

$T_0 = 500\,\text{MeV}$ at $\tau_0 = 0.1\,\text{fm}/c$

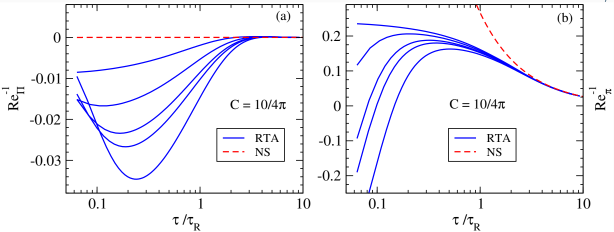


$m = 50\,\text{MeV}$

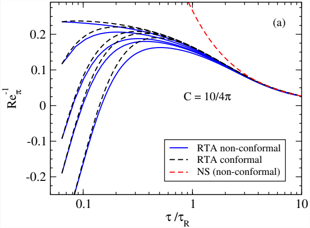
- Early-time attractor still absent in bulk channel for KT.
  - Convergence of trajectories seen only at $(\tau/\tau_R) \gtrsim 3$.

With interactions : Small Bulk, large shear

$T_0 = 500\,\mathrm{MeV}$ at $\tau_0 = 0.1\,\mathrm{fm}/c$



$m = 50\,\mathrm{MeV}$

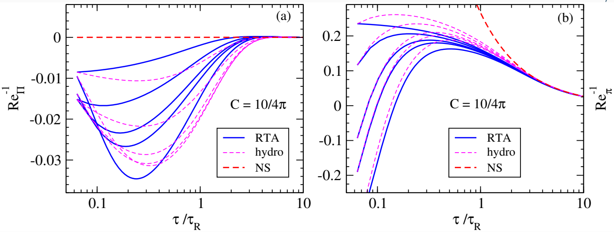


$m = 0\,\mathrm{MeV}$

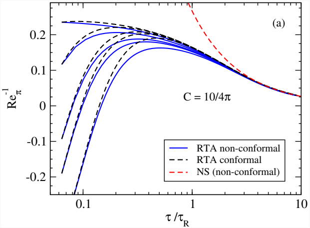
- Early-time attractor still absent in bulk channel for KT.
  - Convergence of trajectories seen only at $(\tau/\tau_R) \gtrsim 3$.
- In shear channel: initial power law decay for non-conformal KT is lost quickly when compared to conformal KT evolution.
  - Conformal KT converges $\approx (\tau/\tau_R) \gtrsim 0.5$; No ETA in non-conformal KT in shear.

With interactions : Small Bulk, large shear

$T_0 = 500\,\text{MeV}$ at $\tau_0 = 0.1\,\text{fm}/c$



$m = 50\,\text{MeV}$



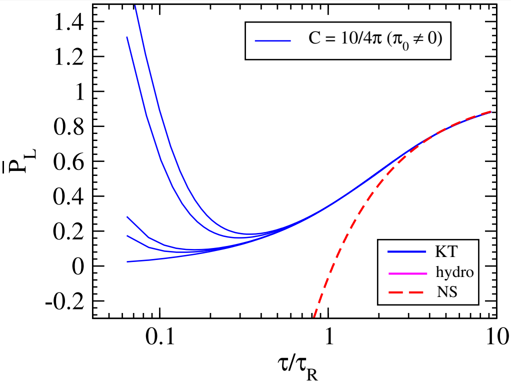
$m = 0\,\text{MeV}$

- Early-time attractor still absent in bulk channel for KT.
  - Convergence of trajectories seen only at $(\tau/\tau_R) \gtrsim 3$.
- In shear channel: initial power law decay for non-conformal KT is lost quickly when compared to conformal KT evolution.
  - Conformal KT converges $\approx (\tau/\tau_R) \gtrsim 0.5$; No ETA in non-conformal KT in shear.
- ETA also absent in non-conformal hydrodynamics in both channels.

Is there an early-time attractor?

Yes!

- $\overline{P}_L \equiv P_L/P\,, \quad \tau_{\mathrm{R}} = 5C/T.$
- Attractor reappears in P_L/P !

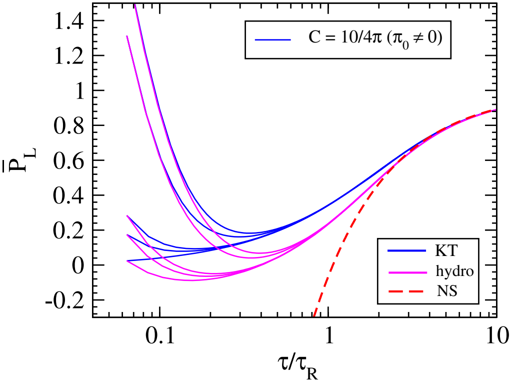


Is there an early-time attractor?

Yes!

- $\overline{P}_L \equiv P_L/P\,, \quad \tau_{\rm R} = 5C/T.$

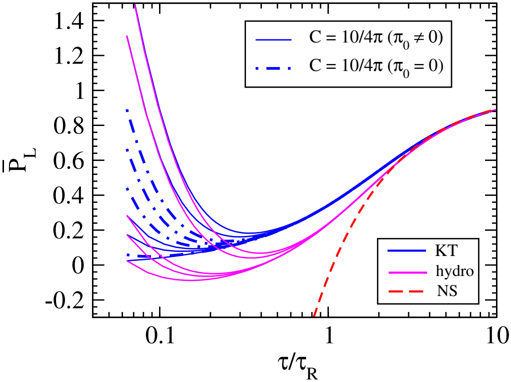
- Attractor reappears in P_L/P !



Is there an early-time attractor?

Yes!

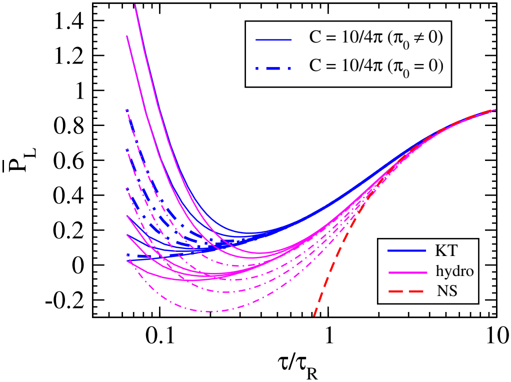
- $\overline{P}_L \equiv P_L/P\,, \quad \tau_{\mathrm{R}} = 5C/T.$
- Attractor reappears in P_L/P !



Is there an early-time attractor?

Yes!

- $\overline{P}_L \equiv P_L/P\,, \quad \tau_{\rm R} = 5C/T.$
- Attractor reappears in P_L/P !

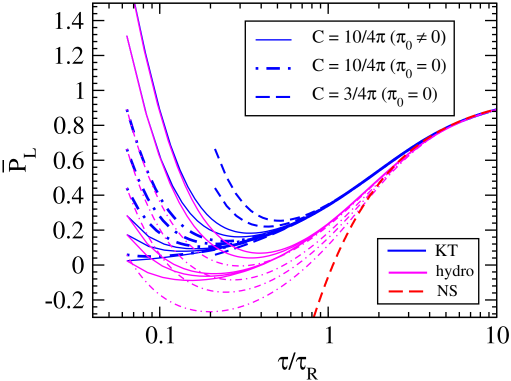


Is there an early-time attractor?

Yes!

- $\overline{P}_L \equiv P_L/P\,, \quad \tau_{\rm R} = 5C/T.$

- Attractor reappears in P_L/P !



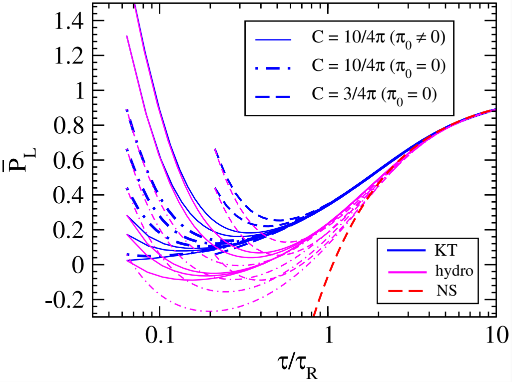
- All KT solutions seen to join at early times $(\tau/\tau_R) \lesssim 0.5$.

Is there an early-time attractor?

Yes!

- $\overline{P}_L \equiv P_L/P\,, \quad \tau_{\rm R} = 5C/T.$

- Attractor reappears in P_L/P !



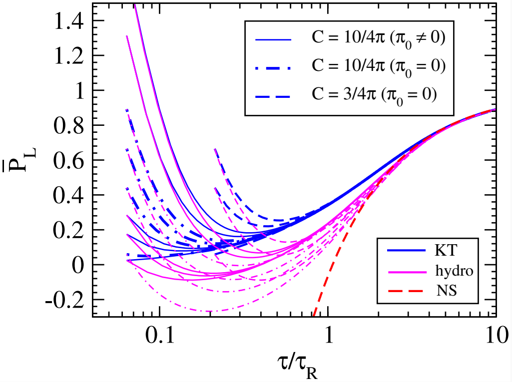
- All KT solutions seen to join at early times $(\tau/\tau_R) \lesssim 0.5$.

- Early time attractor is absent in non-conformal hydrodynamics.

- Reason for ETA: Fast expansion + weak coupling $\Longrightarrow$ Approximate free-streaming at early times $\Longrightarrow$ $P_L = 0$ line acts as an attractor.

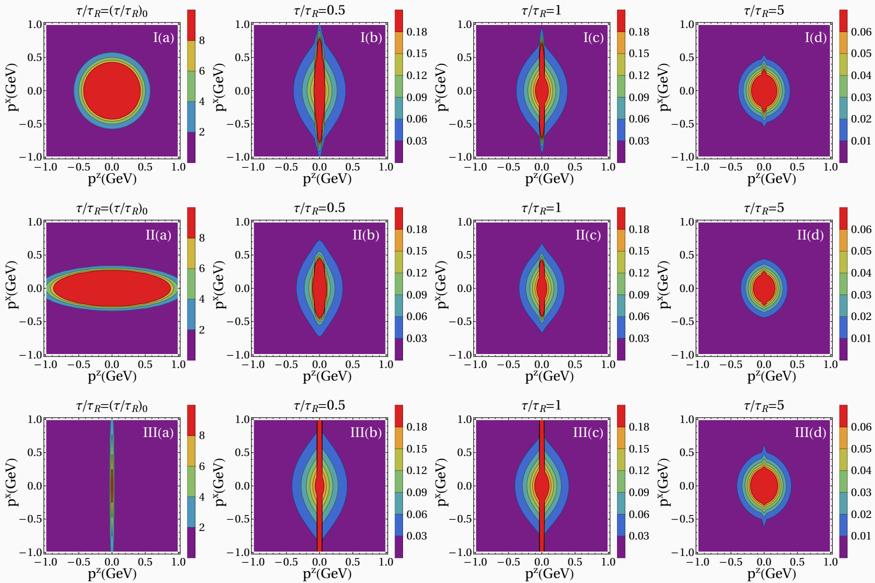
Is there an early-time attractor?

Yes!



- $\overline{P}_L \equiv P_L/P\,, \quad \tau_{\rm R} = 5C/T.$
- Attractor reappears in P_L/P !
- All KT solutions seen to join at early times $(\tau/\tau_R) \lesssim 0.5$.
- Early time attractor is absent in non-conformal hydrodynamics.
- Reason for ETA: Fast expansion + weak coupling $\Longrightarrow$ Approximate free-streaming at early times $\Longrightarrow$ $P_L = 0$ line acts as an attractor.

Evolution of the distribution function



Some remarks..

Early-time attractor: feature of free-streaming, not interactions.

- Possible confusion: ETA exists for normalized shear for conformal systems. Governed by interactions?
  - For conformal systems undergoing Bjorken flow, P_L and π are equivalent quantities : $P_L/P = 1 - \pi/P$. Share a common attractor.
  - Not the case for non-conformal system : $P_L/P = 1 + (\Pi - \pi)/P$.
- Is there a relation between the divergence of hydrodynamic gradient series and far-from-equilibrium hydrodynamic attractor?

Some remarks..

Early-time attractor: feature of free-streaming, not interactions.

- Possible confusion: ETA exists for normalized shear for conformal systems. Governed by interactions?
  - For conformal systems undergoing Bjorken flow, P_L and π are equivalent quantities : $P_L/P = 1 - \pi/P$. Share a common attractor.
  - Not the case for non-conformal system : $P_L/P = 1 + (\Pi - \pi)/P$.
- Is there a relation between the divergence of hydrodynamic gradient series and far-from-equilibrium hydrodynamic attractor?

Modified aHydro

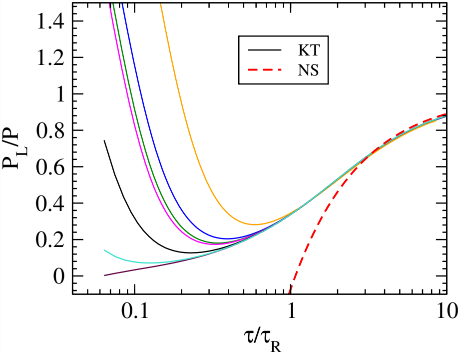
- Hydrodynamics derived from KT: expansion of the distribution function around the thermal distribution.
  - Does not capture well the effects of strong longitudinal expansion.

- anisotropic hydrodynamics captures the free-streaming effects.
  Romatschke and Strickland, hep-ph/0304092 (2003); Martinez and Strickland, 1007.0889 (2010); Florkowski and Ryblewski; McNelis, Bazow and Heinz; and others..

Modified aHydro expand

$$f = \frac{1}{\alpha(\tau)} \exp\left(-\frac{\sqrt{m^2 + p_T^2 + (1+\xi(\tau))(w/\tau)^2}}{\Lambda(\tau)}\right)$$

SJ, Chattopadhyay, Du, Heinz, Pal, 2107.10248 (2021)



Modified aHydro

- Hydrodynamics derived from KT: expansion of the distribution function around the thermal distribution.
  - Does not capture well the effects of strong longitudinal expansion.

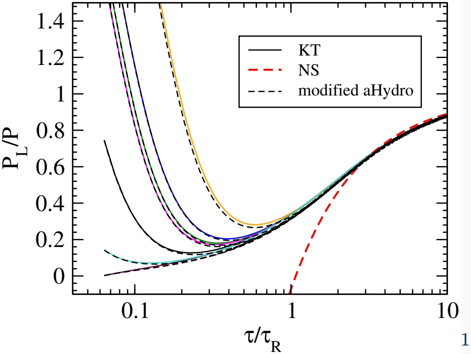
- anisotropic hydrodynamics captures the free-streaming effects.

  Romatschke and Strickland, hep-ph/0304092 (2003); Martinez and Strickland, 1007.0889 (2010); Florkowski and Ryblewski; McNelis, Bazow and Heinz; and others..

Modified aHydro expand

$$f = \frac{1}{\alpha(\tau)} \exp\left(-\frac{\sqrt{m^2 + p_T^2 + (1+\xi(\tau))(w/\tau)^2}}{\Lambda(\tau)}\right)$$

SJ, Chattopadhyay, Du, Heinz, Pal, 2107.10248 (2021)



From moments of distribution function

R. S. Bhalerao, J.-P. Blaizot, Z. Chen, A. Jaiswal, SJ and L. Yan. Forthcoming..

Consider a system with massive particle and vanishing chemical potential undergoing Bjorken expansion.

- Goal : Study transition from free-streaming to hydrodynamic regime.

$$\mathcal{L}_n \equiv \int_p p_0^2 \; P_{2n}(p_z/p_0) \; f(\tau,p), \qquad \mathcal{M}_n \equiv m^2 \int_p P_{2n}(p_z/p_0) \; f(\tau,p)$$

where $\int_p \equiv \frac{d^3p}{(2\pi)^3 \, p_0}$ and P_{2n} is the Legendre polynomial of order $2n$.

- Coupled equations for $\mathcal{L}_n$ and $\mathcal{M}_n$-moments:

$$\frac{\partial \mathcal{L}_n}{\partial \tau} = -\frac{1}{\tau}\left(a_n \mathcal{L}_n + b_n \mathcal{L}_{n-1} + c_n \mathcal{L}_{n+1}\right) - (1-\delta_{n,0})\frac{(\mathcal{L}_n - \mathcal{L}_n^{\rm eq})}{\tau_R}$$

$$\frac{\partial \mathcal{M}_n}{\partial \tau} = -\frac{1}{\tau}\left(a'_n \mathcal{M}_n + b'_n \mathcal{M}_{n-1} + c'_n \mathcal{M}_{n+1}\right) - \frac{(\mathcal{M}_n - \mathcal{M}_n^{\rm eq})}{\tau_R}$$

where the coefficients $a_n, b_n, c_n, a'_n, b'_n, c'_n$ are pure numbers.

- Only three moments are hydro quantities: $(\mathcal{L}_0 = \varepsilon,\ \mathcal{L}_1,\ \mathcal{M}_0 = T^\mu_\mu)$

$$\epsilon = \mathcal{L}_0, \quad P_L = \frac{1}{3}\left(\mathcal{L}_0 + 2\mathcal{L}_1\right), \quad P_T = \frac{1}{3}\left(\mathcal{L}_0 - \mathcal{L}_1 - \frac{3}{2}\mathcal{M}_0\right).$$

Conclusions

- Absence of early-time attractor in the normalized shear and bulk stress channel (in both non-conformal kinetic theory and 'modern' hydrodynamics).
- In KT, normalized longitudinal pressure P_L/P continues to exhibit universal early-time attractive behavior driven by rapid longitudinal expansion.
- Rapid free-streaming expansion (and not interactions) is the reason for early-time attractor. Should be tested in other expansion geometries.
- 2^{nd} order non-conformal hydrodynamics fails to capture this early-time attractor, but modified aHydro reproduces it very well.

Thank You!

Backup Slides

Second-order non-conformal hydrodynamics

- Hydrodynamic equations for gas undergoing Bjorken expansion:
  Denicol, Jeon and Gale, 1403.0962, (2014), Jaiswal, Ryblewski and Strickland, 1407.7231, (2014)

$$\boxed{\frac{d\epsilon}{d\tau} = -\frac{1}{\tau}\left(\epsilon + P + \Pi - \pi\right) \quad \Big| \quad \frac{d\Pi}{d\tau} + \frac{\Pi}{\tau_R} = -\frac{\beta_\Pi}{\tau} - \delta_{\Pi\Pi}\frac{\Pi}{\tau} + \lambda_{\Pi\pi}\frac{\pi}{\tau}}$$

$$\boxed{\frac{d\pi}{d\tau} + \frac{\pi}{\tau_R} = \frac{4}{3}\frac{\beta_\pi}{\tau} - \left(\frac{1}{3}\tau_{\pi\pi} + \delta_{\pi\pi}\right)\frac{\pi}{\tau} + \frac{2}{3}\lambda_{\pi\Pi}\frac{\Pi}{\tau}}$$

Free streaming fixed points

- Set $\tau_R \to \infty$, evaluate transport coefficients in $m/T \to 0$ limit:

$$\{P_L/P,\ P_T/P\} = \{-0.212, 1.61\};\ \{3.64, -0.32\};\ \{-1, -3.33\}.$$

$$\{\Pi/P,\ \pi/P\} = \{0, 1.214\};\ \{0, -2.64\};\ \{-3.56, -1.56\}.$$

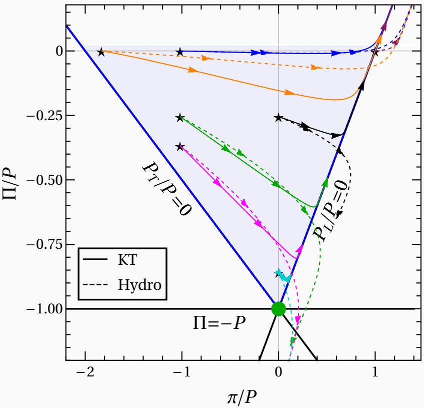
- Blue and Red fixed points are precisely the stable and unstable fixed points of second-order *conformal* hydrodynamics.

- Green fixed point– absent in conformal hydrodynamics: a feature of *non-conformal* hydrodynamics.

Collisionless regime: comparing KT and hydro

Free-streaming evolution of KT and 2^{nd}-order hydrodynamics:

- Pushing hydro in free streaming, $\tau_R \to \infty$



- All free-streaming KT trajectories evolves upwards after reaching $P_L/P = 0$ line. Not the same for hydro.
  - Reason: incorrect description of the saddle point; acts like attractive fixed point (according to hydro equations) for perturbations near this FP.

Modified aHydro

SJ, Chattopadhyay, Du, Heinz, Pal, 2107.10248 (2021)

- Hydrodynamics derived from KT: expansion of the distribution function around the thermal distribution.
  - Does not capture the rapid shrinking at early times of the p_z distribution caused by the rapid longitudinal Bjorken expansion.

- anisotropic hydrodynamics: Standard derivation assumes the form for distribution function (for Bjorken flow):

Romatschke and Strickland, hep-ph/0304092 (2003), L. Tinti, 1506.07164 (2016)

$$f(\tau; p_T, w) \approx f_a \equiv \exp\left(-\frac{\sqrt{m^2 + p_T^2/\alpha_T^2(\tau) + (w/\tau)^2/\alpha_L^2(\tau)}}{\Lambda(\tau)}\right)$$

  - However.. unable to span the entire allowed region: fails to generate large bulk.

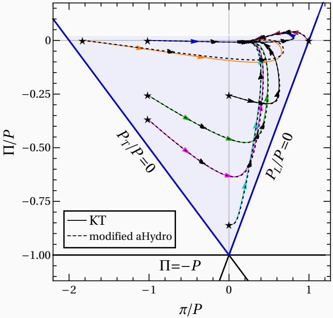
- Modified ansatz for the anisotropic distribution:

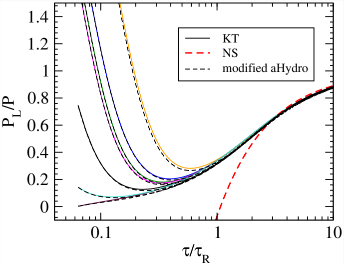
$$f \approx \tilde{f}_a = \frac{1}{\alpha(\tau)} \exp\left(-\frac{\sqrt{m^2 + p_T^2 + (1+\xi(\tau))(w/\tau)^2}}{\Lambda(\tau)}\right)$$

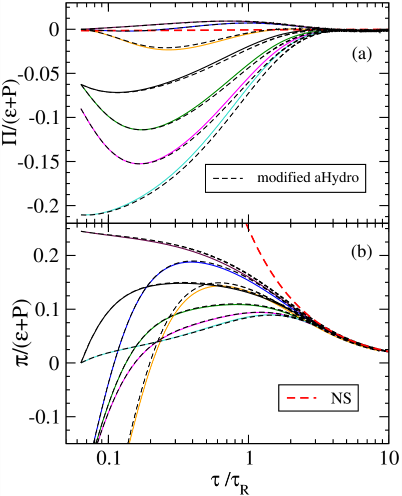
Spans entire solution space allowed by KT.

back

Modified aHydro: Results







Excellent agreement with KT. Reproduces the far-from-equilibrium attractor in P_L/P.

back