Relativistic fluid dynamics of multiple conserved charges

Jan Fotakis
University of Frankfurt
fotakis@itp.uni-frankfurt.de

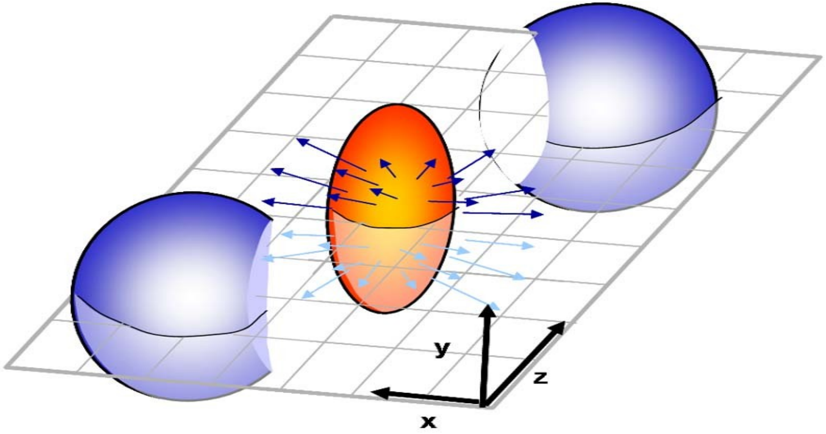
Harri Niemi, Etele Molnár, Moritz Greif, Gabriel Denicol, Dirk Rischke, Carsten Greiner

Motivation

Traditionally:

Viewed as 'blob' of one type of matter (single component) with one velocity field

- usually 'blob' of energy
  with conserved particle number



https://www.quantumdiaries.org/wp-content/uploads/2011/02/FlowPr.jpg

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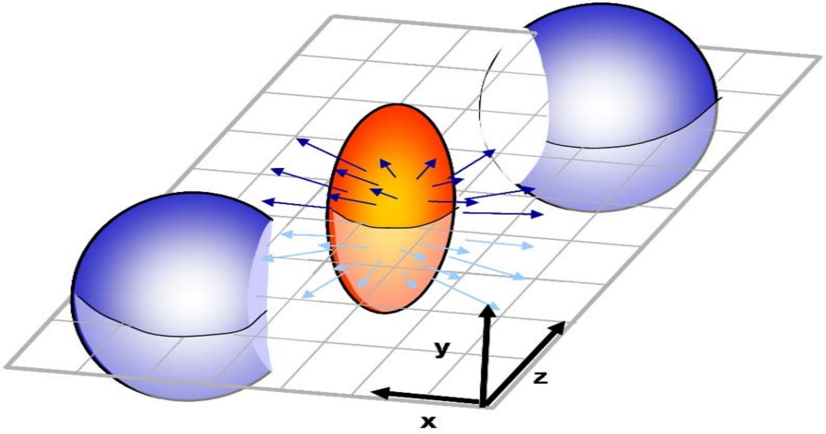

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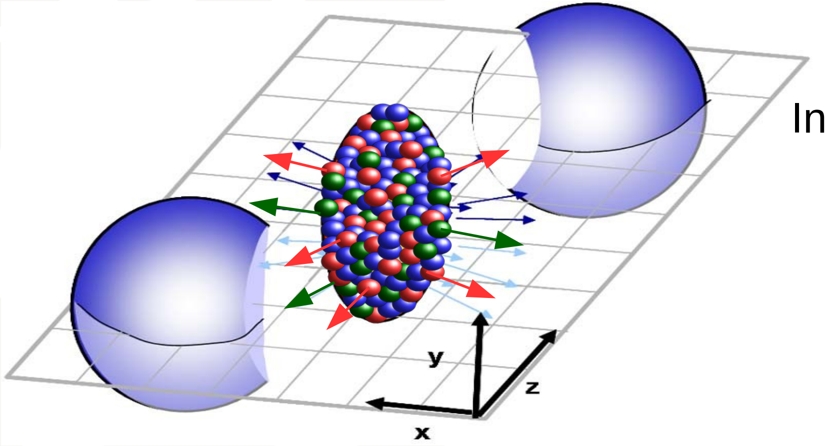

In general:

Consists of multiple components with various properties with multiple velocity fields

- with **multiple conserved quantities**
  (e.g. energy, electric charge, baryon number, strangeness, …)
- mixed chemistry → **coupled charge currents!**

Motivation

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Increasing interest in the recent years ...

Simulation with baryon number

- Denicol et al., PRC 98, 034916 (2018)
- Li et al., PRC 98, 064908 (2018)
- Du et al., Comp. Phys. Comm. 251 (2020) 107090

Diffusion coefficients with BQS

- Greif et al., PRL 120, 242301 (2018)
- Rose et al., PRD 101, 114028 (2020)
- Fotakis et al., PRD 104, 034014 (2021)
- Das et al., arXiv:2109.01543

Simulation with multiple charges

- Fotakis et al., PRD 101, 076007 (2020)
- Chen et al., arXiv:2203.04685

Theory with multiple conserved charges

- Monnai et al., Nucl. Phys. A847:283-314 (2010)
- Kikuchi et al., PRC 92, 064909 (2015)
- Fotakis et al., arXiv:2203.11549

BQS equation of state

- Noronha-Hostler et al., PRC 100, 064910 (2019)
- Monnai et al., arXiv:2101.11591

On this conference ...

- Plaschke et al., Poster Session 1 T02/T03
- Mishra et al., Poster Session 2 T03
- Almaalol et al., Poster Session 2 T14_2
- Pihan et al., Poster Session 2 T07_2
- Weickgenannt., Plenary Session VII

… and many more!

Fluid dynamics of multi-component systems

Hydrodynamics: macroscopic effective field theory of thermal matter close to local equilibrium

Here: single fluid velocity field u^μ

Conservation of Energy and Momentum: $\partial_\mu T^{\mu\nu} = 0$

Conservation of charge: $\partial_\mu N_q^\mu = 0$

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$$T^{\mu\nu} = \sum_i T_i^{\mu\nu} = \epsilon u^\mu u^\nu - (P_0 + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

q-th conserved charge (eg. B,Q,S)

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$10 + 4N_{\mathrm{ch}}$ degrees of freedom, $4 + N_{\mathrm{ch}}$ equations $\rightarrow$ $6 + 3N_{\mathrm{ch}}$ unknowns

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What needs to be known:

- Equation of state $P_0 = P_0(\epsilon, n_q), \quad T = T(\epsilon, n_q), \quad \alpha_q = \mu_q/T = \alpha_q(\epsilon, n_q)$
- Equations of motion for dissipative fields & transport coefficients $\Pi, V_q^\mu, \pi^{\mu\nu}$
- Initial state
- Final state: freeze-out and δf-correction

Deriving fluid dynamics from kinetic theory

Denicol et al., PRD 85, 114047 (2012)

On basis of DNMR theory: derivation from the Boltzmann equation with method of moments

Fotakis et al., arXiv:2203.11549

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relativistic Boltzmann eq.

$$k_i^{\mu}\partial_{\mu}f_{i,\mathbf{k}} = \mathcal{C}_i[f_i]$$

→

2nd-order (multi-component) hydro

$$\dot{\Pi},\ \dot{V}_q^{\langle\mu\rangle},\ \dot{\pi}^{\langle\mu\nu\rangle}$$

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equilibrium off-equilibrium

$$f_{i,\mathbf{k}} = \boxed{f_{i,\mathbf{k}}^{(0)}} + \boxed{\delta f_{i,\mathbf{k}}}$$

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Irreducible off-equilibrium moments obey Boltzmann eq.:

$$f_{i,\mathbf{k}} = \underbrace{f^{(0)}_{i,\mathbf{k}}}_{\text{equilibrium}} + \underbrace{\delta f_{i,\mathbf{k}}}_{\text{off-equilibrium}}$$

$$\rho^{\mu\nu}_{i,n} = \sum_{i=1}^{N_{\text{species}}} \int \frac{\mathrm{d}^3\mathbf{k}_i}{(2\pi)^3 E_{i,\mathbf{k}}} E^n_{i,\mathbf{k}} k_i^{\langle\mu} k_i^{\nu\rangle} \delta f_{i,\mathbf{k}}$$

Problem: infinitely many coupled PDEs.

Aim: Truncate in a well-defined manner (“perturbation theory”)

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"Order-of-magnitude approximation":
relate them to the dissipative fields with constituent's transport coefficients

$$\rho_{i,n}^{\mu\nu} = \frac{\eta_{i,n}}{\eta}\pi^{\mu\nu} + \mathcal{O}(2)$$

Counting scheme:

Gradients in velocity, temperature etc. $\sigma^{\mu\nu} \sim \mathcal{O}(1), \mathcal{O}(\mathrm{Kn})$
Dissipative fields $\pi^{\mu\nu} \sim \mathcal{O}(1), \mathcal{O}(\mathrm{Rn}^{-1})$

Equations of motion with multiple conserved charges

2nd-order (multi-component) hydro

$$\dot{\Pi},\ \dot{V}_q^{\langle\mu\rangle},\ \dot{\pi}^{\langle\mu\nu\rangle}$$

upcoming publication!

$$\tau_\Pi \dot{\Pi} + \Pi = S_\Pi$$
$$\sum_{q'} \tau_{qq'} \dot{V}_{q'}^{\langle\mu\rangle} + V_q^\mu = S_q^\mu$$
$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = S_\pi^{\mu\nu}$$

Relaxation equations (Israel-Stewart-type causal theory)

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$$S_q^\mu = \sum_{q'} \kappa_{qq'} \nabla^\mu \alpha_{q'} - \sum_{q'} \tau_{qq'} V_{q',\nu} \omega^{\nu\mu} - \sum_{q'} \delta_{VV}^{(q,q')} V_{q'}^\mu \theta - \sum_{q'} \lambda_{VV}^{(q,q')} V_{q',\nu} \sigma^{\mu\nu}$$
$$- \ell_{V\Pi}^{(q)} \nabla^\mu \Pi + \ell_{V\pi}^{(q)} \Delta^{\mu\nu} \nabla_\lambda \pi_\nu^\lambda + \tau_{V\Pi}^{(q)} \Pi \dot{u}^\mu - \tau_{V\pi}^{(q)} \pi^{\mu\nu} \dot{u}_\nu$$
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Mixed chemistry couples diffusion currents (coupled charge-transport); already present in 1st order term

2nd order terms: couples all currents to each other; depend on all gradients!

Explicit expressions for transport coefficients!

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Diffusion coefficients: extensively studied!

Greif, Fotakis et al., PRL 120, 242301 (2018)
Fotakis, Greif et al., PRD 101, 076007 (2020)
Fotakis, Soloveva et al, PRD 104, 034014 (2021)

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Coupled charge-transport

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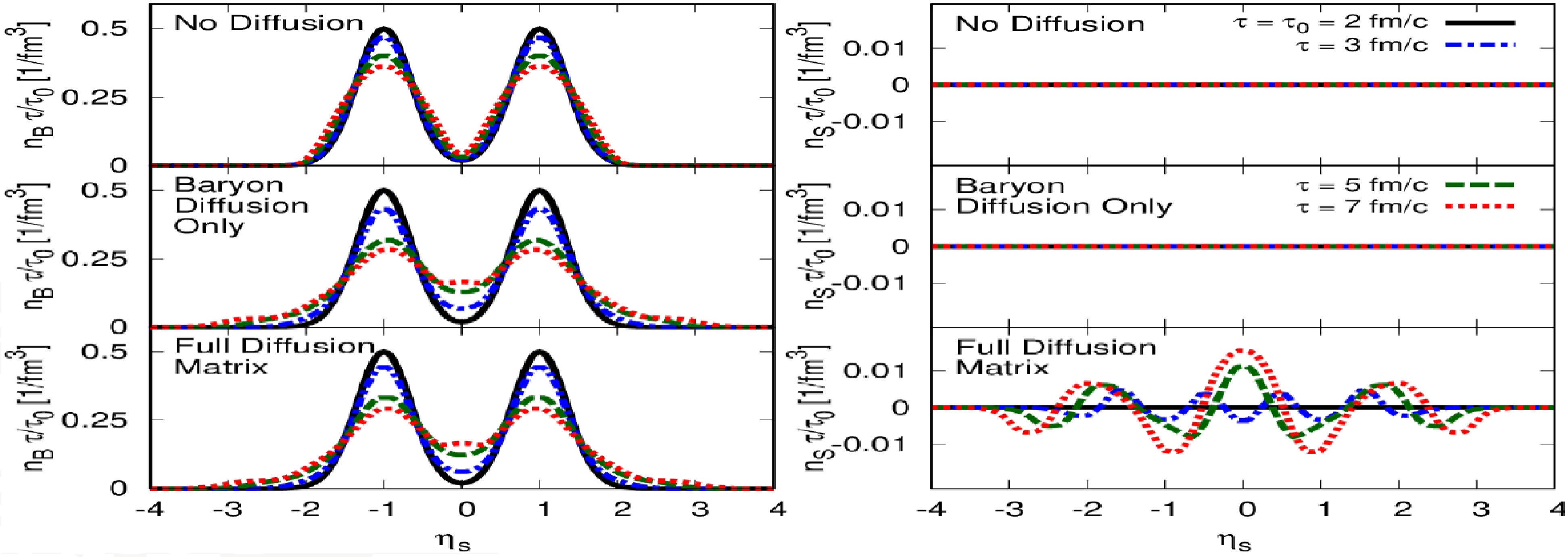

<u>Simplistic case study:</u> no viscosity, diffusion only, no 2nd-order terms, conserved B and S charge, classical, hadronic system (19 species), realistic binary elastic cross sections

Hydrodynamic (1+1)D-simulation with SHASTA

$$\Pi \equiv 0, \quad \pi^{\mu\nu} \equiv 0, \quad \tau_q \dot{V}_q^{\langle\mu\rangle} + V_q^\mu = \sum_{q'} \kappa_{qq'} \nabla^\mu \left(\frac{\mu_{q'}}{T}\right)$$

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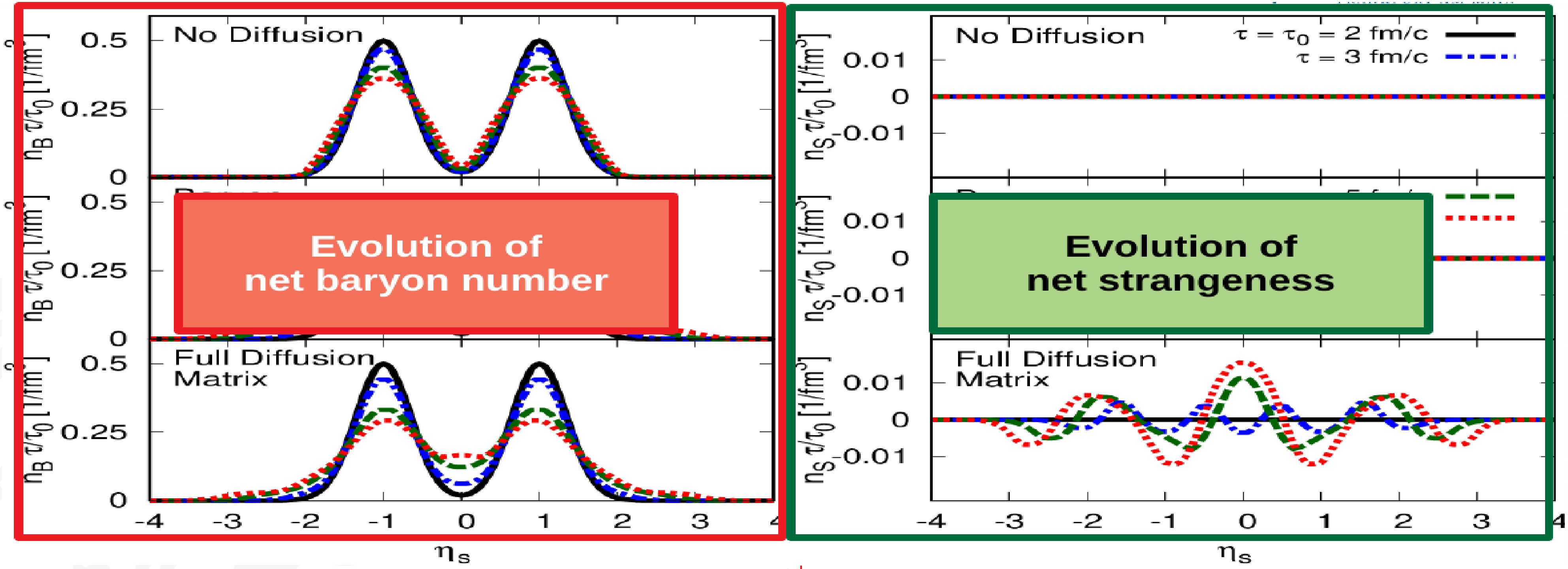

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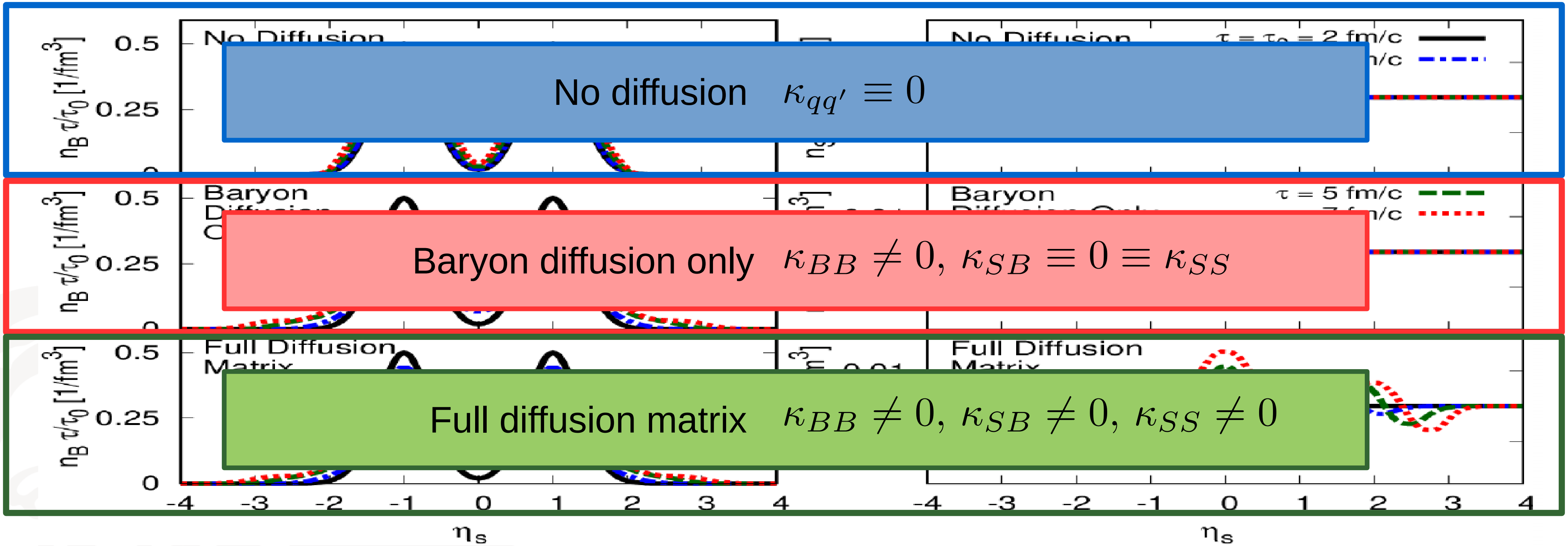

No diffusion $\kappa_{qq'} \equiv 0$

Baryon diffusion only $\kappa_{BB} \neq 0,\ \kappa_{SB} \equiv 0 \equiv \kappa_{SS}$

Full diffusion matrix $\kappa_{BB} \neq 0,\ \kappa_{SB} \neq 0,\ \kappa_{SS} \neq 0$

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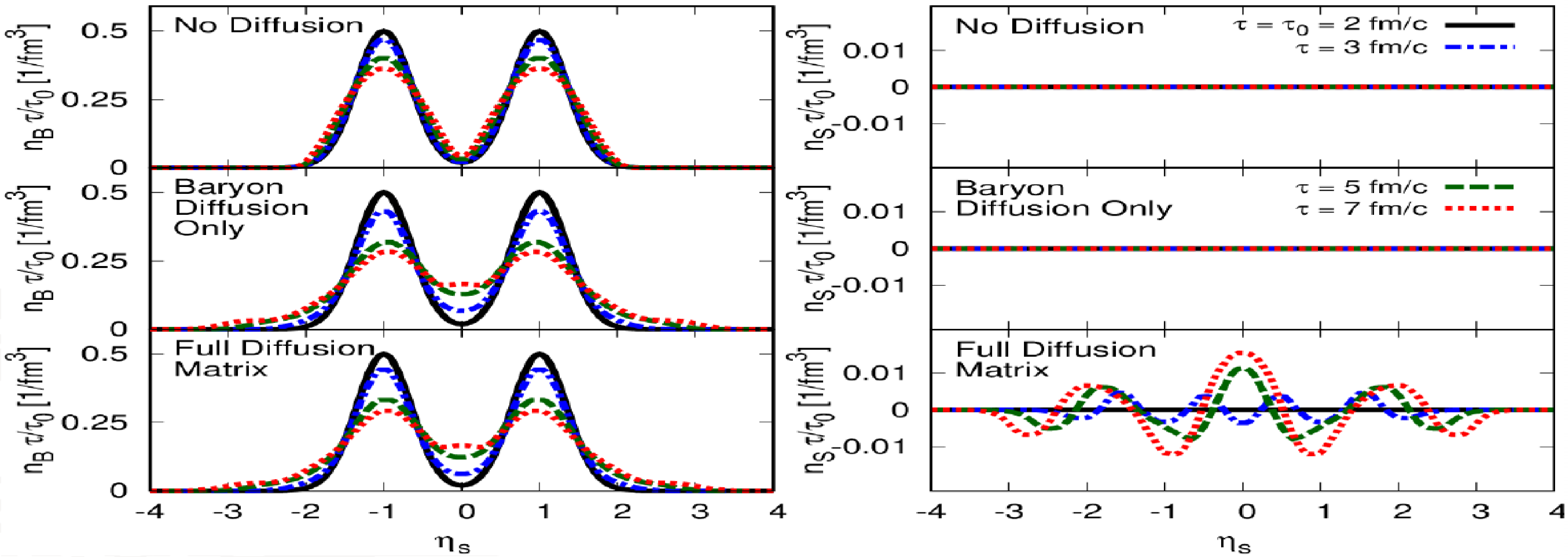

Mixed chemistry couples diffusion currents and introduces charge-correlation through EoS

e.g.: $\mu_S \equiv \mu_S(\epsilon, n_B, n_S)$

$\nabla^\mu \alpha_S \sim \nabla^\mu n_B$

→ Generation of domains of non-vanishing local net charge (here net strangeness)!

Single-component vs. Multi-component system

A potentially problematic term in single-component systems $$S_q^\mu = (\dots) + \ell_{V\pi}^{(q)} \Delta^{\mu\nu}\nabla_\lambda \pi_\nu^\lambda + (\dots)$$

Ultrarelativistic, classical system with hard-sphere interactions:

Denicol et al., PRD 85, 114047 (2012)

Used in simulations of heavy-ion collisions!

TABLE I. The coefficients for the particle diffusion for a classical gas with constant cross section in the ultrarelativistic limit, in the 14-moment approximation. The transport coefficient $\tau_{n\pi}$ was incorrectly listed as being zero in Ref. [1]

κ	$\tau_n[\lambda_{\mathrm{mfp}}]$	$\delta_{nn}[\tau_n]$	$\lambda_{nn}[\tau_n]$	$\lambda_{n\pi}[\tau_n]$	$\ell_{n\pi}[\tau_n]$	$\tau_{n\pi}[\tau_n]$
$3/(16\sigma)$	$9/4$	1	$3/5$	$\beta_0/20$	$\beta_0/20$	$\beta_0/80$

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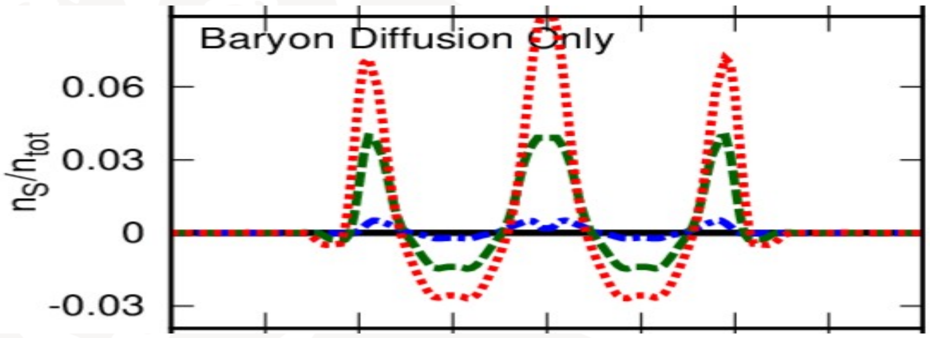


Second-order transport coefficients not consistent with assumed system

→ generation of unphysical charge currents

Consistency is important in charge transport!
Use multi-component expressions.

Conclusion

- Derived 2nd-order relativistic fluid dynamic theory
  for multicomponent systems from the Boltzmann equation
- Transport coefficients given explicitly containing all information about particle interactions
- Mixed chemistry couples diffusion currents to each other ⟶ coupled charge-transport
- Consistency of EoS, 1st and 2nd transport coefficients is important!
- Thermal features from LQCD can be adapted in transport coefficients with quasi-particle models
- Implemented derived fluid dynamic theory in (3+1)D-hydro code

Outlook

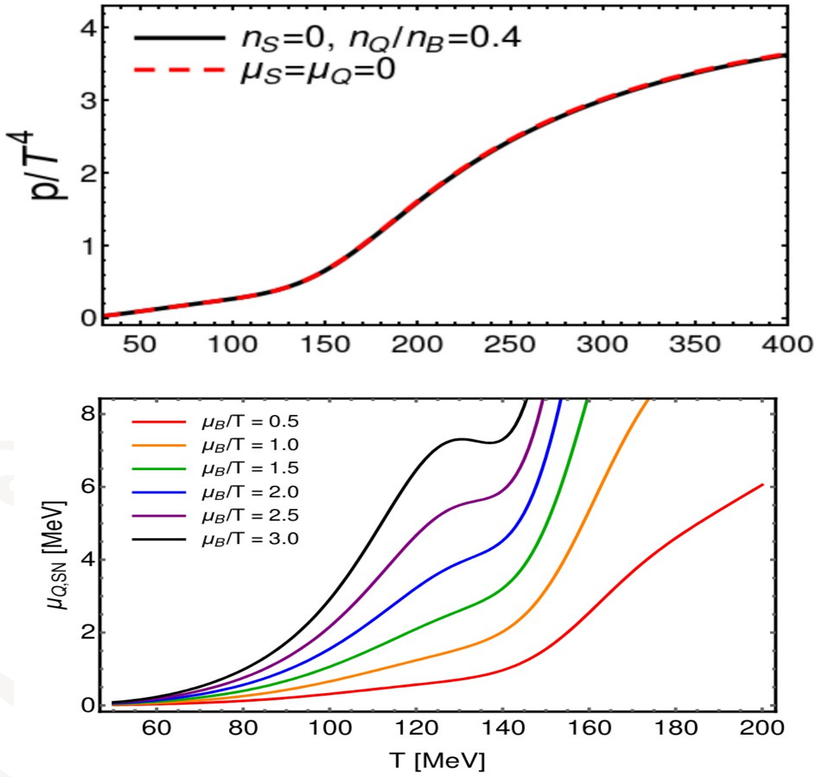
- Evaluate 2nd order transport coefficients for more realistic systems
- Use more realistic initial state and equation of state
- Apply freeze-out routines, take δf-correction
- Find observables sensitive to charge-coupling ⟶ investigate impact

Backup

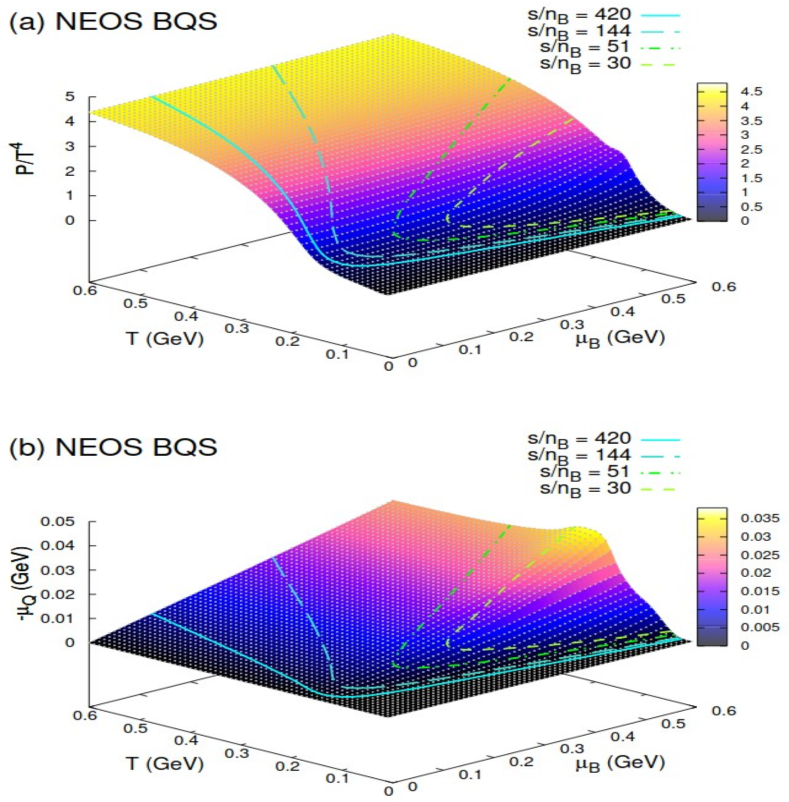
Equation of state with multiple conserved charges

$$P_0(T) \quad \rightarrow \quad P_0(T, \mu_\mathrm{B}, \mu_\mathrm{Q}, \mu_\mathrm{S})$$

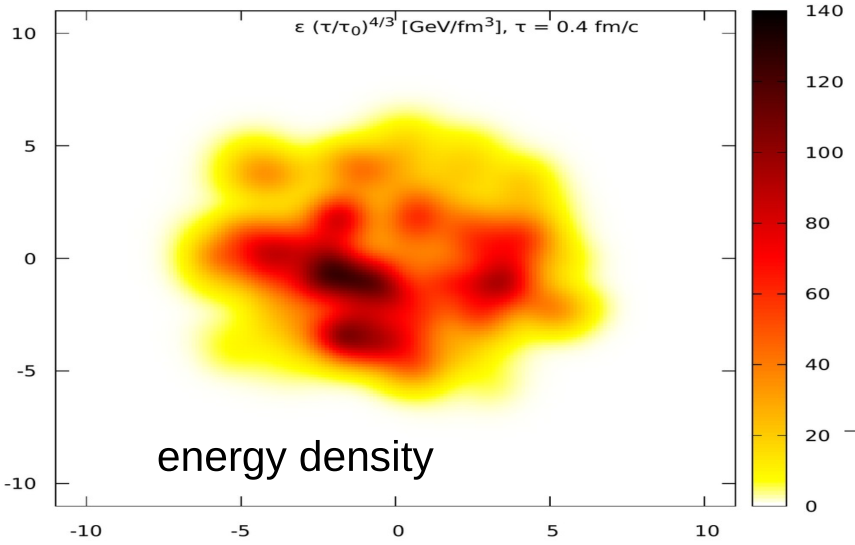
Noronha-Hostler et al., PRC 100, 064910 (2019)

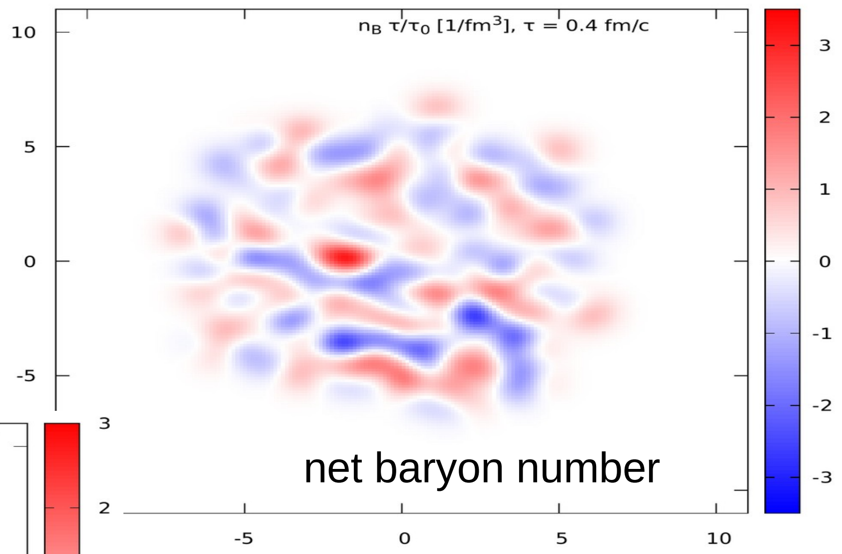


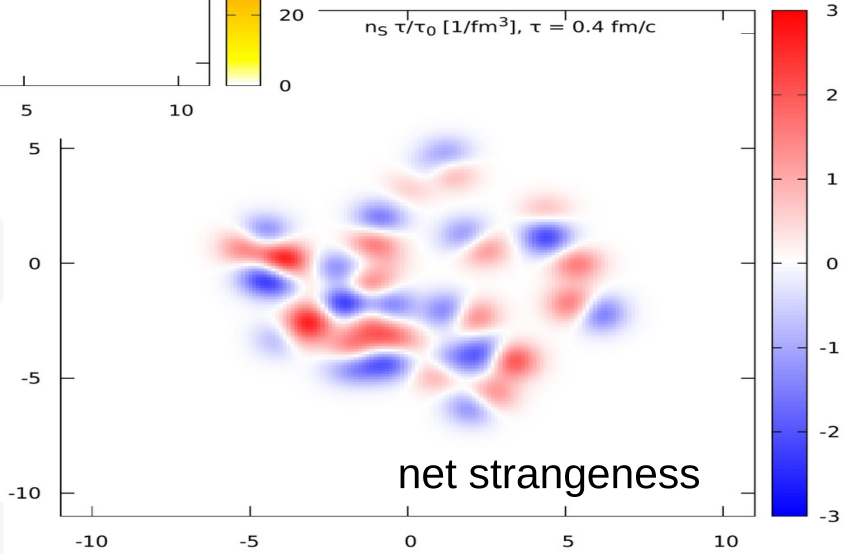
Monnai et al., PRC 100, 024907 (2019)



Initial state with multiple conserved charges







Smoothed initial condition for Pb Pb @ 5 TeV from ICCING

Martinez et al., arXiv:1911.10272, arXiv:1911.12454

Plots not published!

Computation of transport coefficients (Example: diffusion coefficients)

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On basis of DNMR theory: derivation from the Boltzmann equation with method of moments

Fotakis et al., arXiv:2203.11549

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2nd-order (multi-component) hydro

$$\dot{\Pi},\ \dot{V}_q^{\langle\mu\rangle},\ \dot{\pi}^{\langle\mu\nu\rangle}$$

$$\mathcal{C}_{i,n-1}^{\langle\mu\rangle} \equiv \int \frac{\mathrm{d}^3\mathbf{k}_i}{(2\pi)^3 E_{i,\mathbf{k}}} E_{i,\mathbf{k}}^{n-1} k_i^{\langle\mu\rangle} \mathcal{C}_i[f_i]$$

$$= -\sum_{m=0}^{\infty}\sum_j \mathcal{C}_{ij,nm}^{(1)} \rho_{j,m}^{\mu} + \text{non-linear terms}$$

Entries of „collision matrix“ (for diffusive moments)

Computation of transport coefficients (Example: diffusion coefficients)

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

On basis of DNMR theory: derivation from the Boltzmann equation with method of moments

Fotakis et al., arXiv:2203.11549

relativistic Boltzmann eq.

$$k_i^\mu \partial_\mu f_{i,\mathbf{k}} = \mathcal{C}_i[f_i]$$

→

2nd-order (multi-component) hydro

$$\dot{\Pi},\ \dot{V}_q^{\langle\mu\rangle},\ \dot{\pi}^{\langle\mu\nu\rangle}$$

$$\mathcal{C}_{i,n-1}^{\langle\mu\rangle} \equiv \int \frac{\mathrm{d}^3\mathbf{k}_i}{(2\pi)^3 E_{i,\mathbf{k}}} E_{i,\mathbf{k}}^{n-1} k_i^{\langle\mu\rangle} \mathcal{C}_i[f_i]$$

$$= -\sum_{m=0}^{\infty} \sum_j \mathcal{C}_{ij,nm}^{(1)} \rho_{j,m}^\mu + \text{non-linear terms}$$

Entries of „collision matrix" (for diffusive moments)

$$\kappa_{qq'} = \sum_{n=0}^{\infty} \sum_{i,j=1}^{N_{\text{species}}} \left(\mathcal{C}^{(1)}\right)^{-1}_{ij,0n} q_i \left(q_j' J_{j,n+1,1} - \frac{n_{q'}}{\epsilon + P_0} J_{j,n+2,1}\right)$$

Diffusion coefficient matrix! (equivalent to our PRL and PRD expression)

Greif, Fotakis et al., PRL 120, 242301 (2018)
Fotakis, Greif et al., PRD 101, 076007 (2020)
Fotakis, Soloveva et al, PRD 104, 034014 (2021)

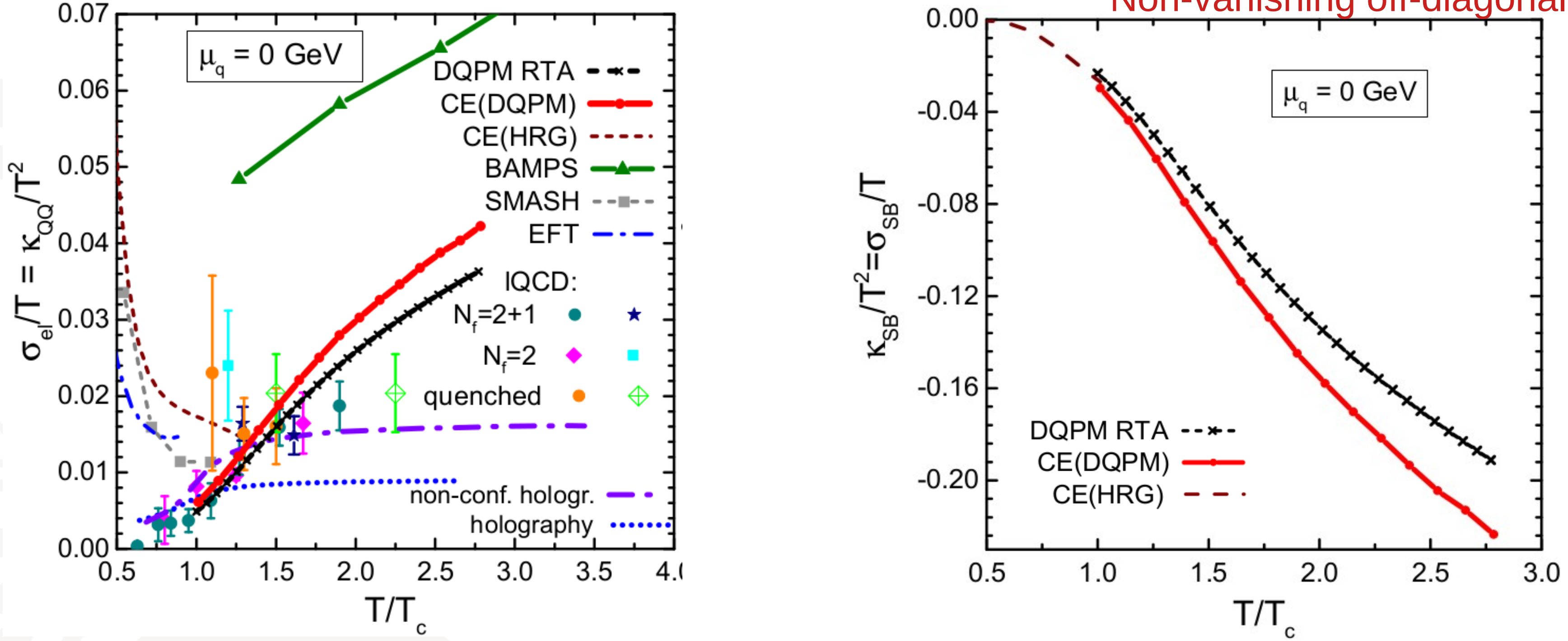
Coupled charge-transport

$$\kappa_{qq'} = \sum_{n=0}^{\infty} \sum_{i,j=1}^{N_{\text{species}}} \tau_{ij,0n}^{(1)} q_i \left(q'_j J_{j,n+1,1} - \frac{n_{q'}}{\epsilon + P_0} J_{j,n+2,1} \right)$$

Greif, Fotakis et al., PRL 120, 242301 (2018)
Fotakis, Greif et al., PRD 101, 076007 (2020)
Fotakis, Soloveva et al, PRD 104, 034014 (2021)

Example: introduction of features from LQCD via the usage of DQPM

Non-vanishing off-diagonal entries!



Equation of State - details

- Hadronic system including lightest 19 species

$$\pi^\pm, \pi^0, K^\pm, K^0, \bar{K}^0, p, \bar{p}, n, \bar{n}, \Lambda^0, \bar{\Lambda}^0, \Sigma^0, \bar{\Sigma}^0, \Sigma^\pm, \bar{\Sigma}^\pm$$

- Assume classical statistics and non-interacting limit

$$P_0(T, \{\mu_q\}) \equiv \frac{1}{3} \sum_{i=1}^{N_\text{species}} \int \frac{\mathrm{d}p^3}{(2\pi)^3 E_{i,p}} \left(E_{i,p}^2 - m_i^2\right) g_i \exp(-E_{i,p}/T + \sum_q q_i \alpha_q)$$

- Only assume baryon number and strangeness, neglect electric charge

- Tabulate state variables over energy density and net charge densities

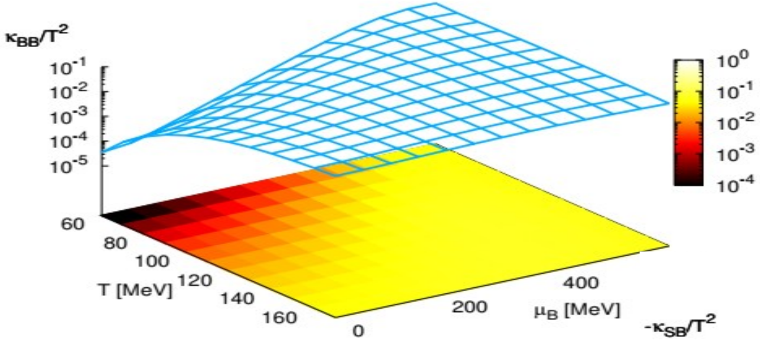
$$T \equiv T(\epsilon, \{n_q\}), \quad \mu_q \equiv \mu_q(\epsilon, \{n_q\}), \quad P_0 \equiv P_0(\epsilon, \{n_q\})$$

Diffusion coefficient matrix - details

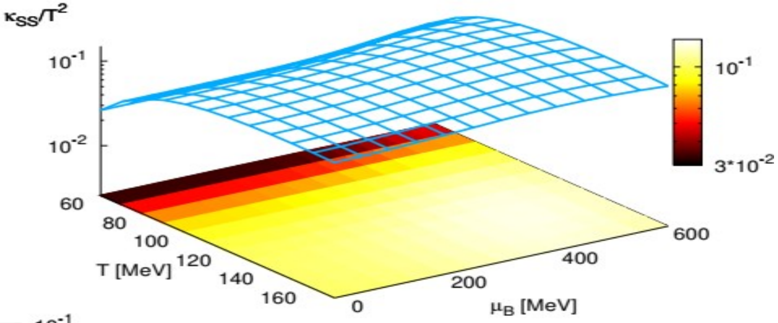
$$\begin{pmatrix} V_B^\mu \\ V_S^\mu \end{pmatrix} \sim \begin{pmatrix} \kappa_{BB} & \kappa_{BS} \\ \kappa_{SB} & \kappa_{SS} \end{pmatrix} \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_S \end{pmatrix}$$

- Matrix is symmetric

*L. Onsager, Phys. Rev. **37**, 405 (1931) & Phys. Rev. **38**, 2265 (1931)*

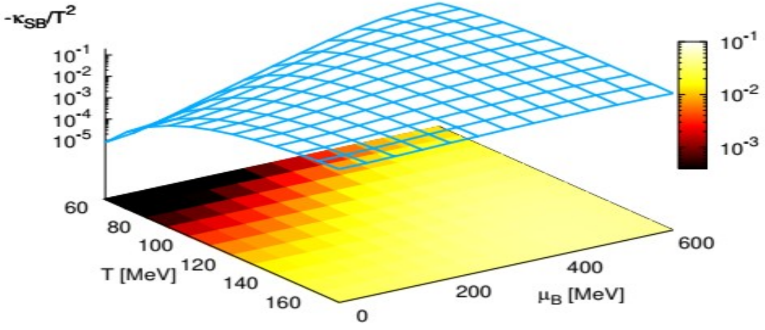


- Elastic isotropic cross sections from PDG, SMASH, GiBUU, UrQMD



$\kappa_{\rm SB}$ is negative and has similar magnitude as $\kappa_{\rm BB}$

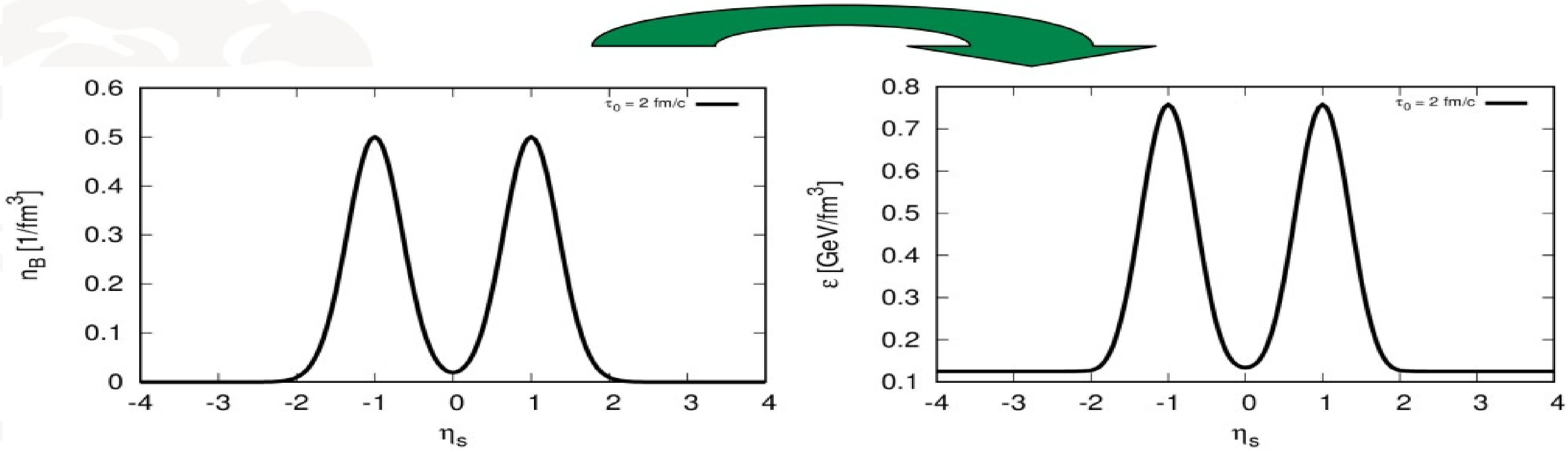
$\Longrightarrow$ significant coupling?



- Tabulate coefficient matrix over $T,\ \mu_B,\ \mu_S$
- $\mu_Q = 0$

Initial conditions - details

- $\tau_0 = 2\ \mathrm{fm/c}$
- Initially: no dissipation and only **Bjorken scaling flow**
- Temperature = 160 MeV
- Double-gaussian profile in net baryon number
- From **EoS**: get energy density



Single-component vs. Multi-component system

A potentially problematic term in single-component systems $S_q^\mu = (\ldots) + \ell_{V\pi}^{(q)} \Delta^{\mu\nu} \nabla_\lambda \pi_\nu^\lambda + (\ldots)$

Ultrarelativistic, classical system with hard-sphere interactions:

Denicol et al., PRD 85, 114047 (2012)

Used in simulations of heavy-ion collisions!

TABLE I. The coefficients for the particle diffusion for a classical gas with constant cross section in the ultrarelativistic limit, in the 14-moment approximation. The transport coefficient $\tau_{n\pi}$ was incorrectly listed as being zero in Ref. [1]

κ	$\tau_n[\lambda_{\text{mfp}}]$	$\delta_{nn}[\tau_n]$	$\lambda_{nn}[\tau_n]$	$\lambda_{n\pi}[\tau_n]$	$\ell_{n\pi}[\tau_n]$	$\tau_{n\pi}[\tau_n]$
$3/(16\sigma)$	$9/4$	1	$3/5$	$\beta_0/20$	$\beta_0/20$	$\beta_0/80$

Multi-component system:

$$\ell_{V\pi}^{(q)} = \frac{9}{80\sigma P} \sum_{i=1}^{N_{\text{spec}}} q_i \frac{P_i}{P} \overset{\text{single}}{\rightarrow} \ell_{n\pi} = \frac{\tau_n}{20T}$$

Fotakis et al., arXiv:2203.11549

The problem: system with conserved net-charge and if **each** constituent has anti-particle partner then at **vanishing** chemical potential it is:

$$\ell_{V\pi}^{(q)} = 0 \neq \ell_{n\pi} = \frac{\tau_n}{20T}$$

Single-component vs. Multi-component system

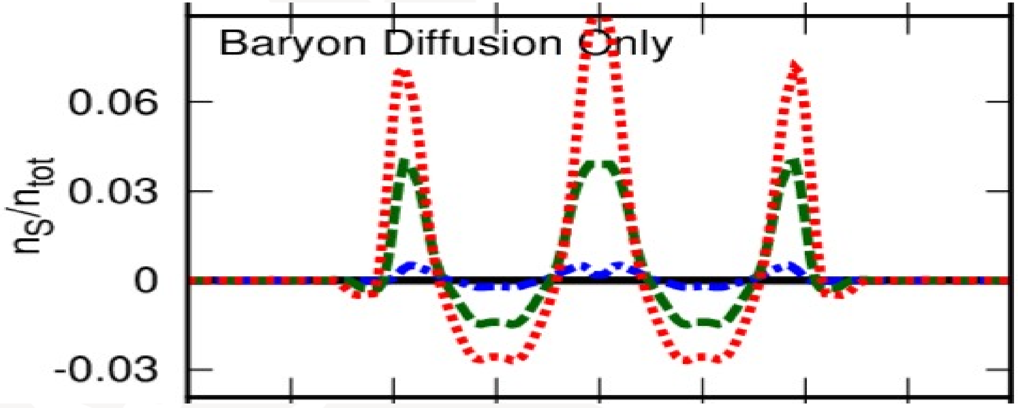
Run simulation of system with conserved baryon number and strangeness with shear viscosity and diffusion; account for second-order terms

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Second-order coefficients in ultrarelativistic, single-component limit from Denicol (2012)

$$\tau_n \dot{V}_q^{\langle\mu\rangle} + V_q^\mu = \sum_{q'} \kappa_{qq'} \nabla^\mu \alpha_{q'} - V_{q,\nu}\omega^{\nu\mu} - \tau_n V_{q'}^\mu \theta - \frac{3\tau_n}{5} V_{q,\nu}\sigma^{\mu\nu} + \boxed{\frac{\tau_n}{20T}\Delta^{\mu\nu}\nabla_\lambda \pi_\nu^\lambda - \frac{\tau_n}{20T}\pi^{\mu\nu}\dot{u}_\nu} - \frac{\tau_n}{20T}\pi^{\mu\nu}\nabla_\nu \alpha_q$$

Here (plot): baryon diffusion only: $\kappa_{\mathrm{BB}} \neq 0, \qquad \kappa_{\mathrm{SS}} = 0 = \kappa_{\mathrm{SB}}$



Second-order transport coefficients not consistent with assumed system

→ generation of unphysical charge currents

Consistency is important in charge transport!