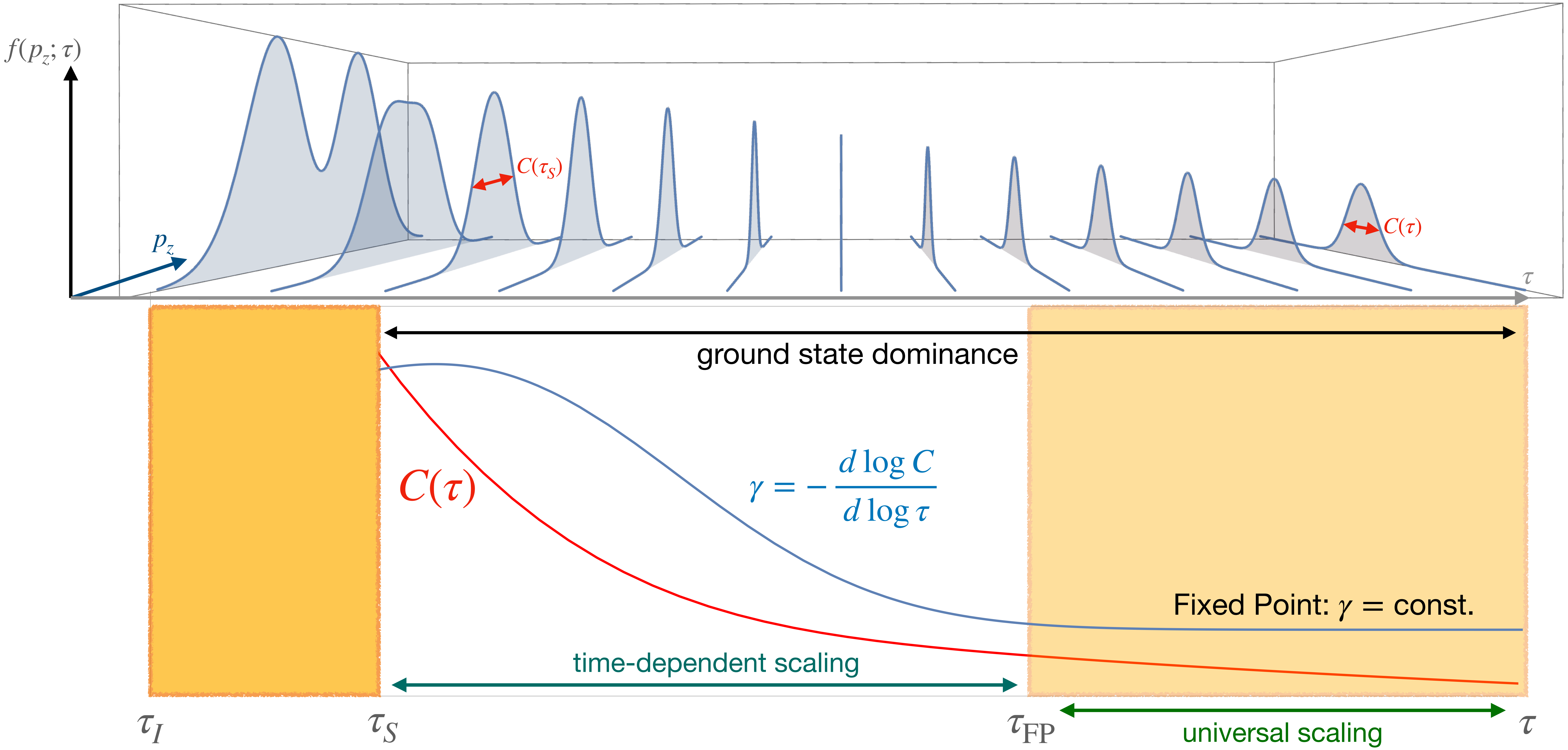
Scaling and adiabaticity in a rapidly expanding gluon plasma

Typical time evolution of the gluon occupation number in a weakly-coupled Bjorken-expanding plasma



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In collaboration with Jasmine Brewer (CERN) and Yi Yin (IMP)

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Scaling in the kinetic theory of an expanding gluon plasma

We consider the small-angle scattering approximation [1] of QCD EKT [2]:

$$\partial_\tau f - \frac{p_z}{\tau}\partial_{p_z} f = 4\pi\alpha_s^2 N_c^2 l_{\mathrm{Cb}}[f]\left[\mathcal{I}_a[f]\,\nabla_{\mathbf{p}}^2 f + \mathcal{I}_b[f]\,\nabla_{\mathbf{p}}\cdot\left(\hat{p}(1+f)f\right)\right],$$

[1] A.H. Mueller, "The Boltzmann equation for gluons at early times after a heavy ion collision," Physics Lett. B 475 (2000) 220
[2] P.B. Arnold, G.D. Moore and L.G. Yaffe, "Effective kinetic theory for high temperature gauge theories," JHEP 01 (2003) 030

where $\mathcal{I}_a[f] = \int_{\mathbf{p}} (1+f)f$, $\mathcal{I}_b[f] = \int_{\mathbf{p}} \frac{2}{p} f = \frac{m_D^2}{2N_c g_s^2}$, $l_{\mathrm{Cb}}[f] = \ln\left(\frac{p_{\mathrm{UV}}}{p_{\mathrm{IR}}}\right) \approx \ln\left(\frac{\sqrt{\langle p_\perp^2\rangle}}{m_D}\right)$.

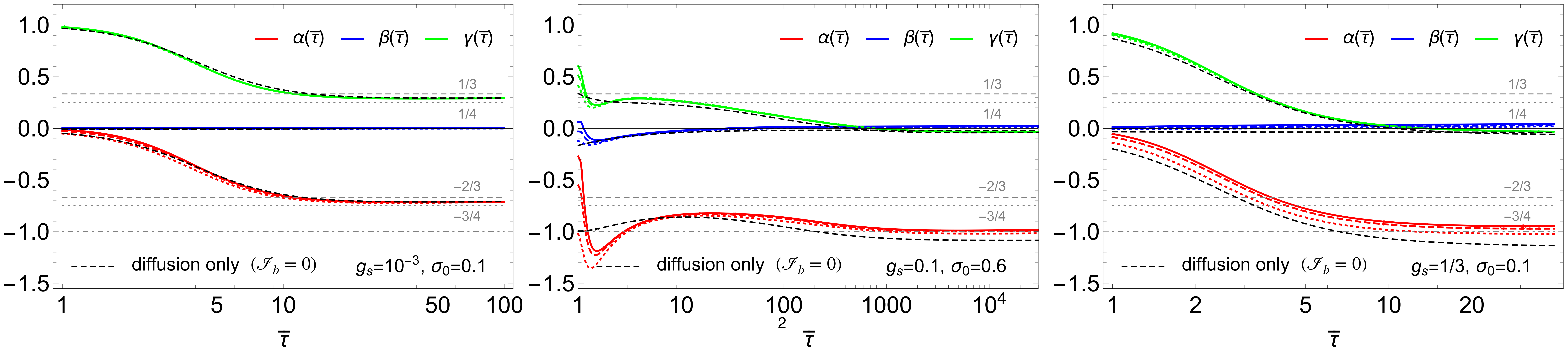
Form of initial condition ∀plots:
$$f(\tau_I) = \frac{\sigma_0}{g_s^2}\exp\left(-\frac{p_\perp^2 + \xi^2 p_z^2}{Q_s^2}\right)$$

$\xi = 2$

[3] J. Brewer, B. Scheihing-Hitschfeld and Y. Yin, "Scaling and adiabaticity in a rapidly expanding gluon plasma," arXiv:2203.02427
[4] A. Mazeliauskas and J. Berges, "Prescaling and far-from-equilibrium hydrodynamics in the quark-gluon plasma," Phys. Rev. Lett. 122 (2019) 122301

We have verified [3] that this theory exhibits scaling: $f(p_\perp, p_z; \tau) = A_S(\tau)\, w_S\left(p_\perp/B_S(\tau), p_z/C_S(\tau)\right)$,

with scaling exponents $\dot{A}_S \equiv \frac{\tau\partial_\tau A_S}{A_S} = \alpha_S(\tau)$, $\dot{B}_S = -\beta_S(\tau)$, $\dot{C}_S = -\gamma_S(\tau)$, similar to QCD EKT [4].



Adiabaticity

Let us now derive why scaling will appear for a generic initial condition [3].

[3] J. Brewer, B. Scheihing-Hitschfeld and Y. Yin, "Scaling and adiabaticity in a rapidly expanding gluon plasma," arXiv:2203.02427

Writing $f(p_z, p_\perp; \tau) = A(\tau)\, w\left(p_\perp/B(\tau), p_z/C(\tau); \tau\right)$, with $A(\tau), B(\tau), C(\tau)$ arbitrary rescalings, and setting $\mathcal{I}_b = 0$, $w(\zeta, \xi; \tau)$ undergoes time evolution $\partial_{\ln \tau} w = -\mathcal{H} w$ with

$$\mathcal{H} = \alpha - (1-\gamma)\left[\tilde{q}\,\partial_\xi^2 + \xi\,\partial_\xi\right] + \beta\left[\tilde{q}_B\,(\partial_\zeta^2 + \frac{1}{\zeta}\partial_\zeta) + \zeta\,\partial_\zeta\right],$$

$q = 4\pi\alpha_s^2 N_c^2 l_{\mathrm{Cb}}[f]\mathcal{I}_a[f]\,\tau$

where $\tilde{q} = \dfrac{q}{C^2(1-\gamma)}$, $\tilde{q}_B \equiv -\dfrac{q}{B^2\beta}$. This Hamiltonian has eigenstates & eigenvalues

$$\phi^L_{n,m} = \mathrm{He}_{2n}\left(\frac{\xi}{\sqrt{\tilde{q}}}\right) {}_1F_1\left(-2m, 1, \frac{\zeta^2}{2\tilde{q}_B}\right), \qquad \mathcal{E}_{n,m} = 2n(1-\gamma) - 2m\beta \qquad n, m = 0, 1, 2, \ldots$$

$$\phi^R_{n,m} = \frac{1}{\sqrt{2\pi\tilde{q}}(2n)!}\frac{1}{\tilde{q}_B}\mathrm{He}_{2n}\left(\frac{\xi}{\sqrt{\tilde{q}}}\right) {}_1F_1\left(-2m, 1, \frac{\zeta^2}{2\tilde{q}_B}\right) e^{-\frac{\xi^2}{2\tilde{q}}-\frac{\zeta^2}{2\tilde{q}_B}}$$

We have chosen $\alpha = \gamma + 2\beta - 1$ to set the ground state energy $\mathcal{E}_{0,0} = 0$

- The evolution of w is made adiabatic by finding $B(\tau), C(\tau)$ such that $\tilde{q} = \tilde{q}_B = 1$.
- Energy gap $\Longrightarrow$ the ground state will dominate after a transient time.

Flow equations for the scaling exponents

This allows one to derive evolution equations for the scaling exponents:

over − occupied ($A_S \gg 1 \iff$ "$f \gg 1$") :

$$\partial_y \beta_S = \left(\gamma_S + 4\beta_S - 1 + i_{\mathrm{Cb}}\right)\beta_S\,,$$

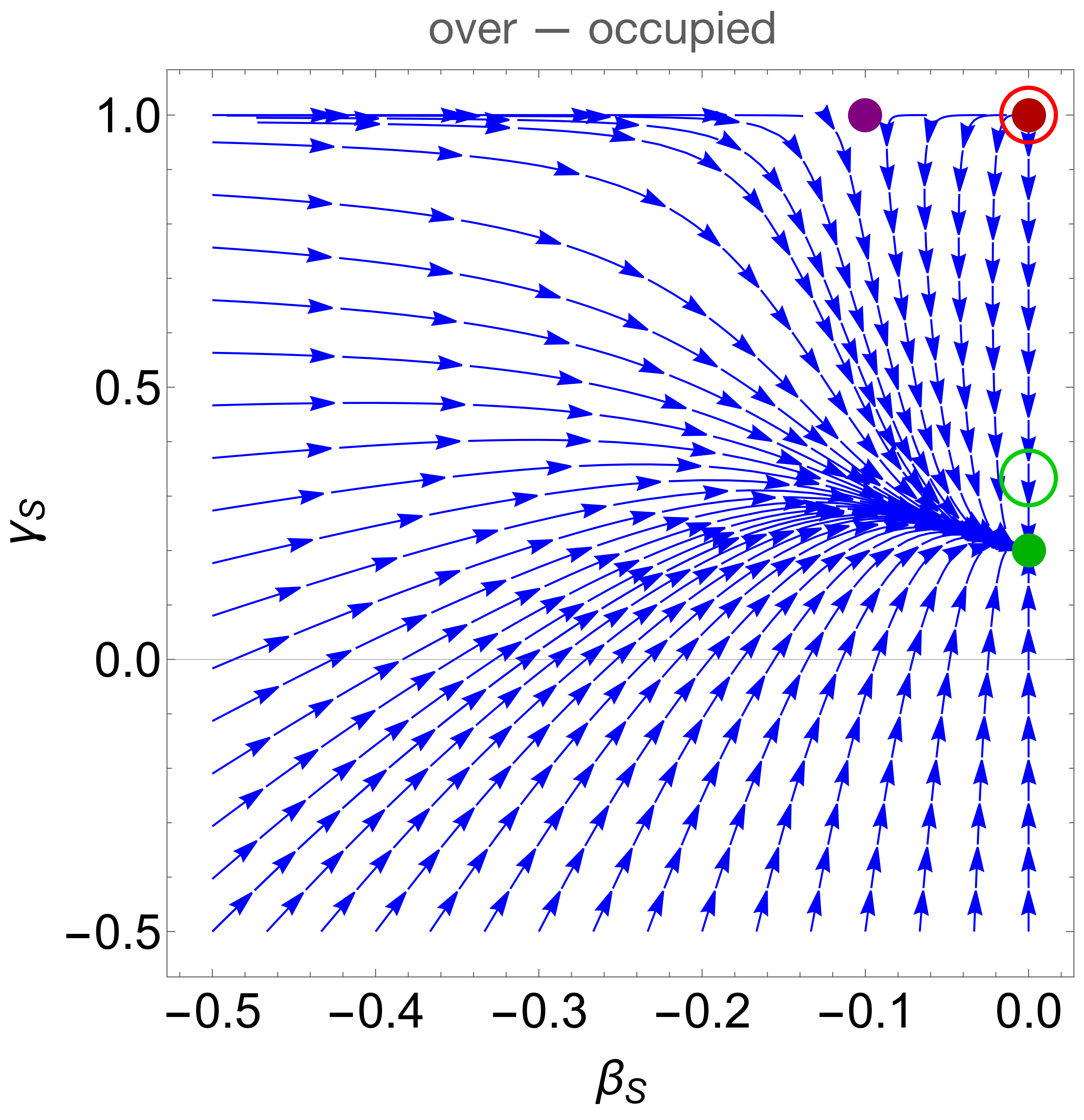
$$\partial_y \gamma_S = \left(3\gamma_S + 2\beta_S - 1 + i_{\mathrm{Cb}}\right)(\gamma_S - 1)\,.$$

dilute ($A_S \ll 1 \iff$ "$f \ll 1$") :

$$\partial_y \beta_S = \left(2\beta_S + i_{\mathrm{Cb}}\right)\beta_S\,,$$

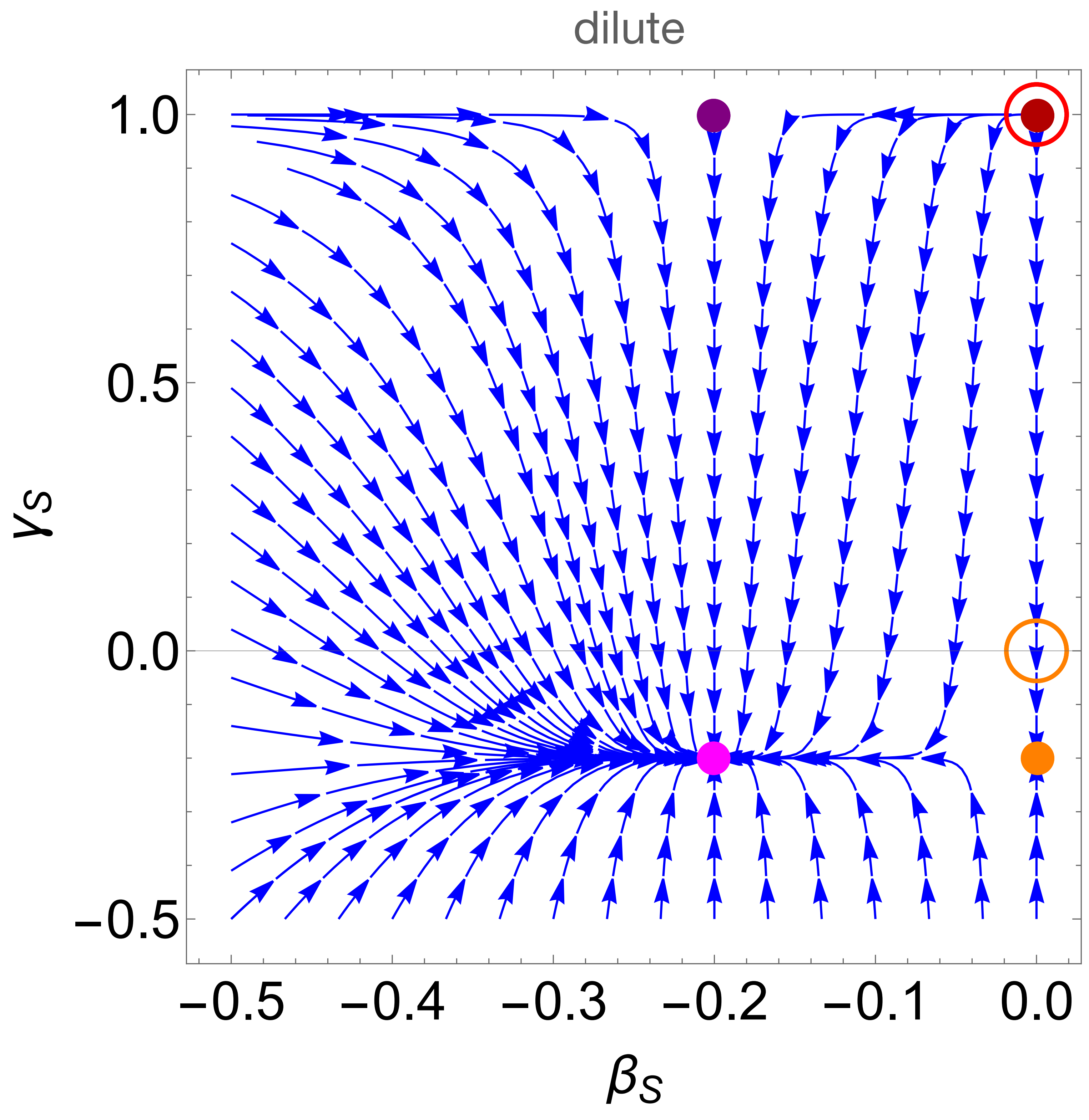
$$\partial_y \gamma_S = \left(2\gamma_S + i_{\mathrm{Cb}}\right)(\gamma_S - 1)\,.$$

$y = \ln \tau/\tau_I$



Flow of γ, β under time evolution

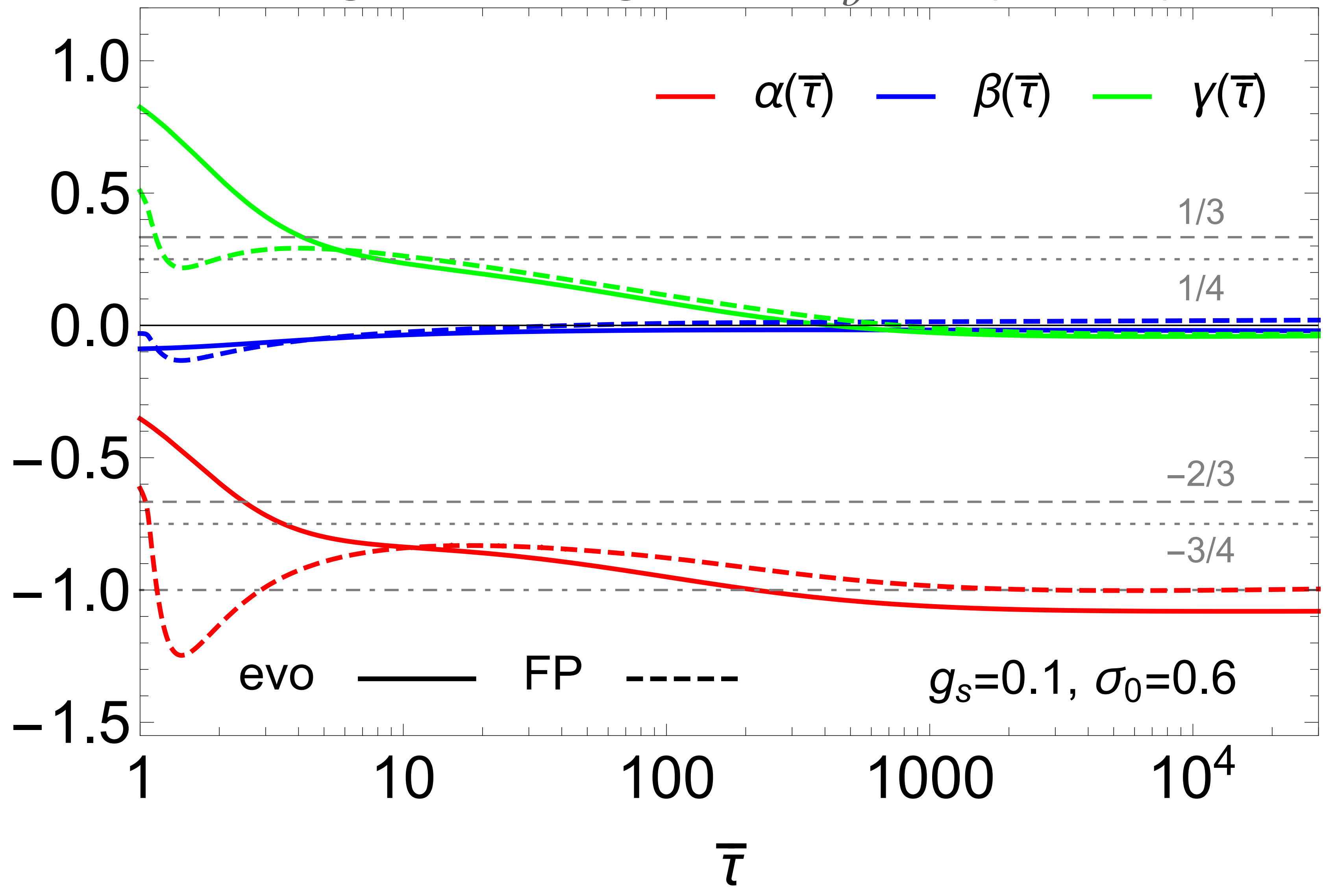
- Open circles: fixed points with $i_{\mathrm{Cb}} = 0$
- Filled circles: fixed points with $i_{\mathrm{Cb}} = 0.4$



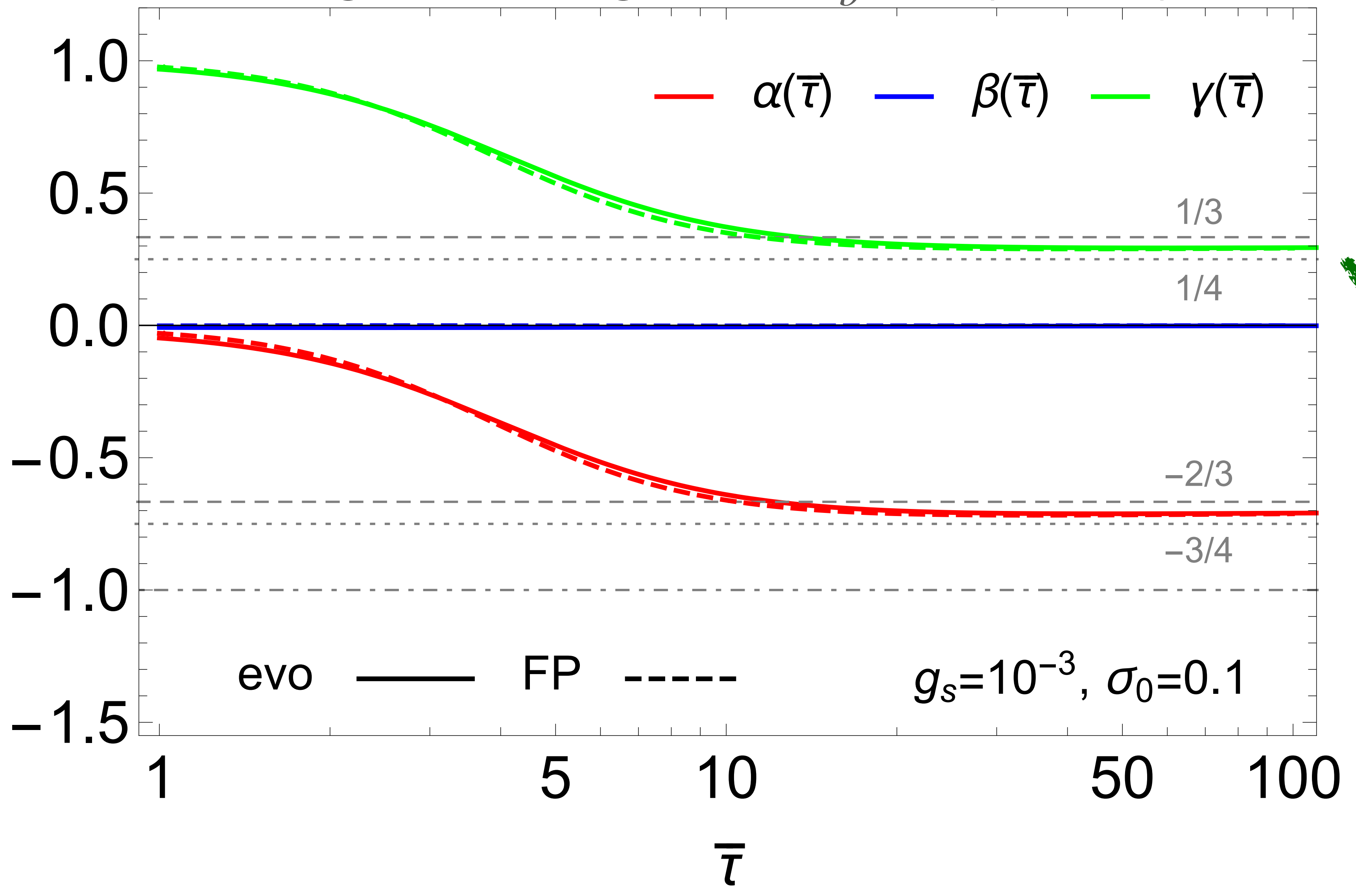
Comparison with numerical solutions to kinetic theory

Comparison with small-angle scatterings with $\mathscr{I}_b \neq 0$

Flow equations (solid) versus small-angle scatterings with $\mathscr{I}_b \neq 0$ (dashed)



Flow equations (solid) versus small-angle scatterings with $\mathscr{I}_b \neq 0$ (dashed)



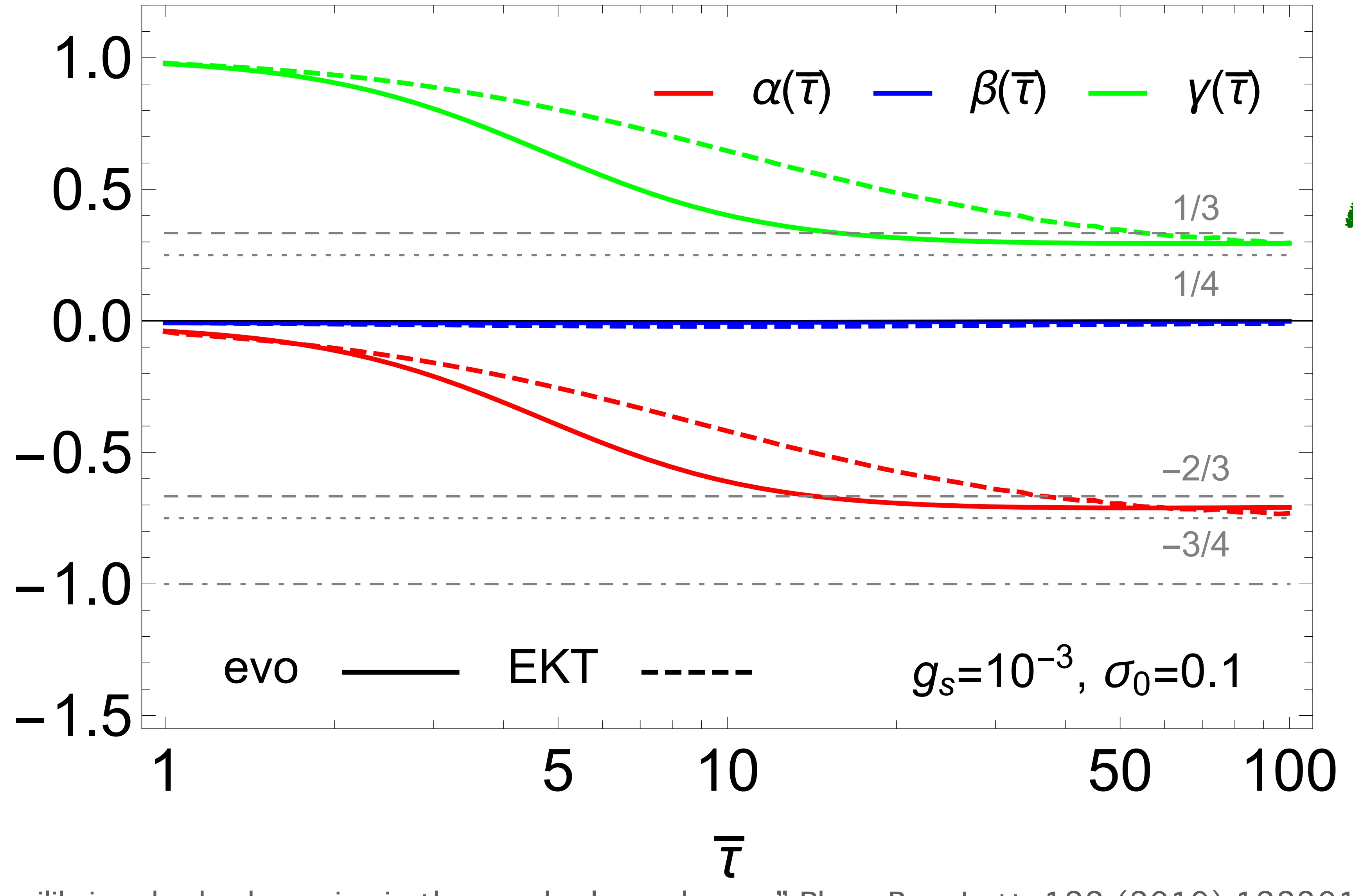
Form of initial condition ∀plots:

$$f(\tau_I) = \frac{\sigma_0}{g_s^2} \exp\left(-\frac{p_\perp^2 + \xi^2 p_z^2}{Q_s^2}\right)$$

$\xi = 2$

Comparison with QCD EKT [4]

Flow equations (solid) versus QCD EKT (dashed)



Consistent deviation from BMSS [5] scaling exponent $\gamma_{\rm BMSS} = 1/3$ in all theories!

Our prediction:

$$\delta\gamma \equiv \gamma - \frac{1}{3} = -\frac{1}{3\ln\left(\frac{4\pi\tau}{N_c \tau_I \sigma_0}\right)}$$

For $\sigma_0 = 0.1$, we get $\gamma \approx 0.29$ at $\bar{\tau} \equiv \tau/\tau_I = 100$.

[4] A. Mazeliauskas and J. Berges, "Prescaling and far-from-equilibrium hydrodynamics in the quark-gluon plasma," Phys. Rev. Lett. 122 (2019) 122301
[5] R. Baier, A.H. Mueller, D. Schiff and D.T. Son, "'Bottom-up' thermalization in heavy ion colllisions," Physics Lett. B 502 (2001) 51-58

Summary and conclusions

- Adiabatic hydrodynamization [6] is an extremely useful paradigm to describe the early stages of the 'bottom-up' thermalization scenario [5] in a heavy ion collision.

[6] J. Brewer, L. Yan and Y. Yin, "Adiabatic hydrodynamization in rapidly-expanding quark-gluon plasma," Physics Lett. B 816 (2021) 136189

  - The analysis of the system is greatly simplified in terms of the slow modes of the system.
  - It allows one to capture deviations from the fixed point scaling exponents in the BMSS 'bottom-up' scenario [5] systematically.

- These results, together with the relaxation-time approximation analysis of [6], suggest that the entire hydrodynamization process of a weakly-coupled gluon plasma can be understood in terms of adiabatic evolution.

- Further generalizations in sight: more general collision kernels; including radial expansion in the kinetic equation. Also, identify the adiabatic aspects of the hydrodynamization process in strongly-coupled theories.