Stability of classical chromodynamic fields

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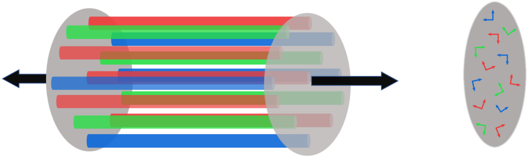
- The earliest phase of heavy-ion collisions is described in terms of classical fields.



- There are longitudinal and parallel to each other chromoelectric and chromomagnetic fields.
- The early state configurations are unstable, but a character of the instability is not clear (P. Romatschke and R. Venugopalan, Phys. Rev. Lett. 96, 062302 (2006)).
- We study the problem systematically, starting with static and uniform fields.

Based on S.Bazak, S.Mrówczyński, Phys. Rev. D **105**, 034023 (2022).

Yang-Mills equations, linearized QCD & background gauge

Yang-Mills equations in adjoint representation

$$\partial_\mu F_a^{\mu\nu} + g f^{abc} A_\mu^b F_c^{\mu\nu} = J_a^\nu, \qquad F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g f^{abc} A_b^\mu A_c^\nu$$

Linearized QCD

$$A_a^\mu(t,\mathbf{r}) = \bar{A}_a^\mu(t,\mathbf{r}) + a_a^\mu(t,\mathbf{r}), \qquad \text{where } |\bar{A}(t,\mathbf{r})| \gg |a(t,\mathbf{r})|$$

Background gauge condition

$$\bar{D}^\mu_{ab} a^a_\mu = \partial^\mu a^a_\mu + g f^{abc} \bar{A}_b^\mu a^c_\mu = 0$$

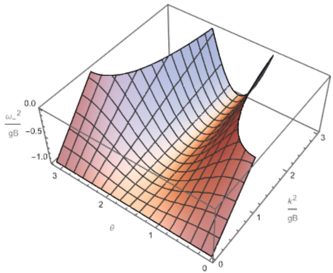
Yang-Mills equations in the background gauge

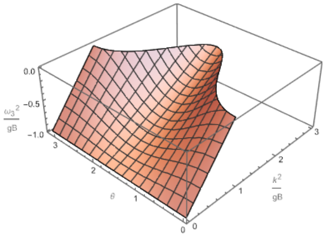
$$\left[g^{\mu\nu}\left(\bar{D}_\rho \bar{D}^\rho\right)_{ac} + 2 g f^{abc} \bar{F}_b^{\mu\nu}\right] a_\nu^c = J_a^\mu$$

Chromomagnetic configuration

S. J. Chang and N. Weiss, Phys. rec. D **20**, 869 (1979), P. Sikivie, Phys. Rev. D **22**, 877 (1979)

	Abelian	non-Abelian
Potential	$\bar{A}^{\mu}_{a}(t,\mathbf{r}) = (0,0,0,yB)\delta^{a1}$	$\bar{A}^{\mu}_{a} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{B/g} \\ 0 & 0 & \sqrt{B/g} & 0 \end{bmatrix}$
YM Current	$J^{\nu}_{a} = 0$	$J^{\nu}_{a} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{gB^3} \\ 0 & 0 & \sqrt{gB^3} & 0 \end{bmatrix}$





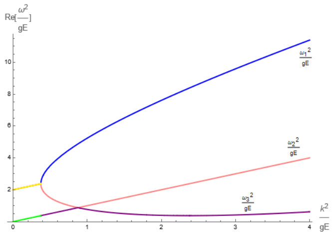
Domain of instability $\omega^2 < 0$ of Abelian (left) and non-Abelian (right) configurations

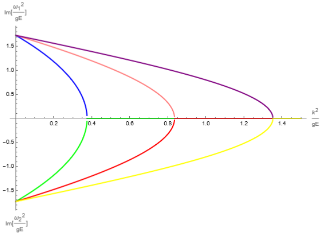
Chromoelectric configuration

S. J. Chang and N. Weiss, Phys. rec. D **20**, 869 (1979), P. Sikivie, Phys. Rev. D **22**, 877 (1979)

	Abelian	non-Abelian
Potential	$\bar{A}^{\mu}_{a}(t,\mathbf{r}) = (-xE,0,0,0)\delta^{a1}$	$\bar{A}^{\mu}_{a} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \sqrt{E/g} & 0 & 0 & 0 \\ 0 & \sqrt{E/g} & 0 & 0 \end{bmatrix}$
YM Current	$J^{\nu}_{a} = 0$	$J^{\nu}_{a} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \sqrt{gE^3} & 0 & 0 & 0 \\ 0 & -\sqrt{gE^3} & 0 & 0 \end{bmatrix}$

- "run-away" solutions for Abelian configuration,
- complex spectrum of modes with instability for non-Abelian configuration





Summary

- There is only Abelian configuration for uniform E&B parallel fields $\longrightarrow$ dominant behaviour of E field $\longrightarrow$ "run-away" solutions.
- The energy momentum tensor $(T^{\mu\nu})$ with fields linearized in a_a^μ is gauge invariant.
- For unstable mode $T^{\mu\nu} \sim e^{2\gamma t}$.
- Instabilities play a crucial role in temporal evolution of the system and spits up its thermalization.