Stability of classical chromodynamic fields
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- The earliest phase of heavy-ion collisions is described in terms of classical fields.
- There are longitudinal and parallel to each other chromoelectric and chromomagnetic fields.
- The early state configurations are unstable, but a character of the instability is not clear (P. Romatschke and R. Venugopalan, Phys. Rev. Lett. 96, 062302 (2006)).
- We study the problem systematically, starting with static and uniform fields.

Yang-Mills equations, linearized QCD & background gauge

**Yang-Mills equations in adjoint representation**

\[
\partial_\mu F^{\mu\nu}_a + gf^{abc} A^b_\mu F^{\mu\nu}_c = J^\nu_a, \quad F^{\mu\nu}_a = \partial_\mu A^\nu_a - \partial_\nu A^\mu_a + gf^{abc} A^\mu_b A^\nu_c
\]

**Linearized QCD**

\[
A^\mu_a (t, r) = \bar{A}^\mu_a (t, r) + a^\mu_a (t, r), \quad \text{where } |\bar{A}(t, r)| \gg |a(t, r)|
\]

**Background gauge condition**

\[
\bar{D}_{ab}^\mu a^\mu_a = \partial_\mu a^\mu_a + gf^{abc} \bar{A}^\mu_b a^\mu_c = 0
\]

**Yang-Mills equations in the background gauge**

\[
\left[ g^{\mu\nu} (\bar{D}_\rho \bar{D}^\rho)_{ac} + 2gf^{abc} \bar{F}^{\mu\nu}_b \right] a^\nu_c = J^\mu_a
\]
Chromomagnetic configuration


<table>
<thead>
<tr>
<th>Potential</th>
<th>( \vec{A}_a^\mu(t, r) = (0, 0, 0, yB)\delta^a_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>non-Abelian</td>
<td>( \vec{A}_a^\mu = \begin{bmatrix} 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; \sqrt{B/g} \ 0 &amp; \sqrt{B/g} &amp; 0 \end{bmatrix} )</td>
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Domain of instability \( \omega^2 < 0 \) of Abelian (left) and non-Abelian (right) configurations
Chromoelectric configuration


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<td>Potential</td>
<td>$\vec{A}_a^\mu(t, r) = (-xE, 0, 0, 0)\delta^a_1$</td>
<td>$\vec{A}_a^\mu = \begin{bmatrix} 0 &amp; 0 &amp; 0 &amp; 0 \ \sqrt{E/g} &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; \sqrt{E/g} &amp; 0 &amp; 0 \end{bmatrix}$</td>
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- "run-away" solutions for Abelian configuration,
- complex spectrum of modes with instability for non-Abelian configuration
Summary

- There is only Abelian configuration for uniform E&B parallel fields \( \rightarrow \) dominant behaviour of E field \( \rightarrow \) "run-away" solutions.
- The energy momentum tensor \((T^\mu_\nu)\) with fields linearized in \(a_\alpha^\mu\) is gauge invariant.
- For unstable mode \(T^\mu_\nu \sim e^{2\gamma t}\).
- Instabilities play a crucial role in temporal evolution of the system and spits up its thermalization.