Space-time structure of 3+1D color fields in heavy-ion collisions

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Der Wissenschaftsfonds.

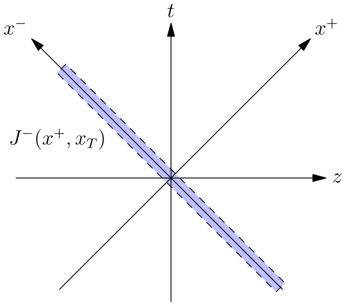
CRC-TR 211 Strong-interaction matter under extreme conditions

Color glass condensate

Color glass condensate (CGC): effective theory for high energy QCD

- Hard partons: classical color density ρ drawn from probability functional $W[\rho]$
- Soft partons: classical color fields $A_\mu \propto 1/g$ (large occupation number)

Nucleus "A" described by color current $\mathcal{J}_A^-(x^+, \mathbf{x}_\perp) = \rho_A(x^+, \mathbf{x}_\perp)$ in terms of light cone coordinates $x^\pm = (x^0 \pm x^3)/\sqrt{2}$ and transverse coordinates $\mathbf{x}_\perp = (x, y)$



analogous for "B" with $x^+ \to x^-$

- Yang-Mills eqs. $D_\mu F^{\mu\nu} = J^\nu$
- Reduce to Poisson eq. (cov. gauge $\partial_\mu A^\mu = 0$)

$$-\Delta_\perp \mathcal{A}_A^-(x^+, \mathbf{x}_\perp) = \rho_A(x^+, \mathbf{x}_\perp), \qquad \mathcal{A}_A^+ = \mathcal{A}_A^i = 0$$

- Solve with infrared regulator m

$$\mathcal{A}_A^-(x^+, \mathbf{x}_\perp) = \phi_A(x^+, \mathbf{x}_\perp) = \int \frac{d^2\mathbf{k}_\perp}{(2\pi)^2} \frac{\tilde{\rho}_A(x^+, \mathbf{k}_\perp)}{\mathbf{k}_\perp^2 + m^2} e^{-i\mathbf{k}_\perp \cdot \mathbf{x}_\perp}$$

- Extension to 3+1D: finite width and structure

Weak field expansion

Glasma: collision of two CGCs, solution to $D_\mu F^{\mu\nu} = J^\mu_A + J^\mu_B \to$ lattice simulations

Weak field expansion: small color currents $\mathcal{J}$

$$\begin{aligned} A^\mu(x) &= \mathcal{A}^\mu_A(x) + \mathcal{A}^\mu_B(x) \quad + \quad a^\mu(x) \\ J^\mu(x) &= \underbrace{\mathcal{J}^\mu_A(x) + \mathcal{J}^\mu_B(x)}_{\text{background}} \quad + \quad \underbrace{j^\mu(x)}_{\text{perturbation}} \end{aligned}$$

- Background: single nuclei color fields $\mathcal{A}^\mu_{A/B} = \mathcal{O}(\mathcal{J}^n_{A/B})$, $n \geq 1$
- Perturbation: color field of the Glasma $a^\mu = \mathcal{O}(\mathcal{J}^n_A \mathcal{J}^m_B)$, $m, n \geq 1$
- Iteratively solve YM eqs. $D_\mu F^{\mu\nu} = J^\nu$, $D_\mu J^\mu = 0$ in cov. gauge $\partial_\mu A^\mu = 0$

Energy-momentum tensor

$$T^{\mu\nu} = T^{\mu\nu}_A + T^{\mu\nu}_B + T^{\mu\nu}_{\text{mixed}} + T^{\mu\nu}_{\text{perturbation}}$$

3+1D color field solutions

Leading order limit (dilute) of color fields $a^\mu = \mathcal{O}(\rho_A \rho_B)$ with $\mathcal{A}^\pm = \phi_{A/B}$:

$$a^+(x) = \frac{g}{2} f_{abc} t^c \int\limits_{\mathbf{p}_\perp, \mathbf{q}_\perp} \int\limits_0^{+\infty} dz^+ \int\limits_0^{+\infty} dz^- \tilde{\phi}_A^a(x^+ - z^+, \mathbf{p}_\perp) \tilde{\phi}_B^b(x^- - z^-, \mathbf{q}_\perp) \frac{\left(-(\mathbf{p}_\perp + \mathbf{q}_\perp)^2 + 2\mathbf{q}_\perp^2\right) z^+}{|\mathbf{p}_\perp + \mathbf{q}_\perp| \tau_z} J_1(|\mathbf{p}_\perp + \mathbf{q}_\perp| \tau_z) e^{-i(\mathbf{p}+\mathbf{q})\cdot \mathbf{x}_\perp}$$

$$a^-(x) = \frac{g}{2} f_{abc} t^c \int\limits_{\mathbf{p}_\perp, \mathbf{q}_\perp} \int\limits_0^{+\infty} dz^+ \int\limits_0^{+\infty} dz^- \tilde{\phi}_A^a(x^+ - z^+, \mathbf{p}_\perp) \tilde{\phi}_B^b(x^- - z^-, \mathbf{q}_\perp) \frac{\left(+(\mathbf{p}_\perp + \mathbf{q}_\perp)^2 - 2\mathbf{p}_\perp^2\right) z^-}{|\mathbf{p}_\perp + \mathbf{q}_\perp| \tau_z} J_1(|\mathbf{p}_\perp + \mathbf{q}_\perp| \tau_z) e^{-i(\mathbf{p}+\mathbf{q})\cdot \mathbf{x}_\perp}$$

$$a^i(x) = \frac{g}{2} f_{abc} t^c \int\limits_{\mathbf{p}_\perp, \mathbf{q}_\perp} \int\limits_0^{+\infty} dz^+ \int\limits_0^{+\infty} dz^- \tilde{\phi}_A^a(x^+ - z^+, \mathbf{p}_\perp) \tilde{\phi}_B^b(x^- - z^-, \mathbf{q}_\perp) i(\mathbf{p}_\perp^i - \mathbf{q}_\perp^i)\, J_0(|\mathbf{p}_\perp + \mathbf{q}_\perp| \tau_z) e^{-i(\mathbf{p}+\mathbf{q})\cdot \mathbf{x}_\perp}$$

- No assumptions made about ϕ_A and ϕ_B, except localization around $x^+ = 0$ and $x^- = 0$
- Not accounting for terms that appear near the light cone boundary
- Expressions describe full 3+1D structure, no boost-invariance!

Nuclear model with longitudinal structure

Simple generalization of McLerran-Venugopalan (MV) model with non-trivial longitudinal structure

$$\left\langle \rho^a_{A/B}(x^\pm, \mathbf{x}_\perp)\rho^b_{A/B}(x'^\pm, \mathbf{x}'_\perp)\right\rangle = \underbrace{g^2\mu^2_{A/B}}_{\substack{\text{strength of}\\ \text{color charges}\\ Q_s \propto g^2\mu}} \; \delta^{ab} \; \underbrace{T_R\left(\frac{x^\pm + x'^\pm}{2}\right)}_{\substack{\text{longitudinal profile}\\ \text{Gaussian of width } R}} \; \underbrace{U_\xi(x^\pm - x'^\pm)}_{\substack{\text{long. correlations}\\ \text{Gaussian of width } \xi}} \; \underbrace{\delta^{(2)}(\mathbf{x}_\perp - \mathbf{x}'_\perp)}_{\substack{\text{transverse}\\ \text{correlations}}}$$

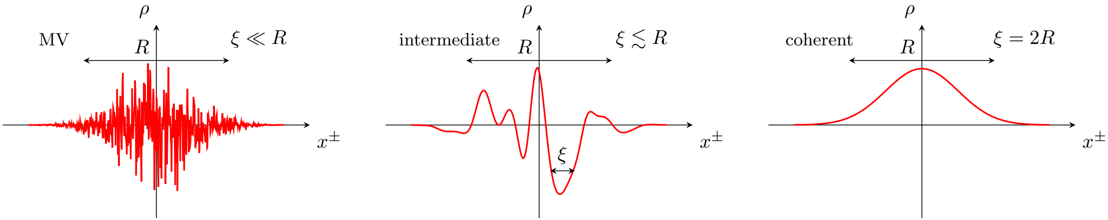
strength of color charges $Q_s \propto g^2\mu$

longitudinal profile Gaussian of width R

long. correlations Gaussian of width ξ

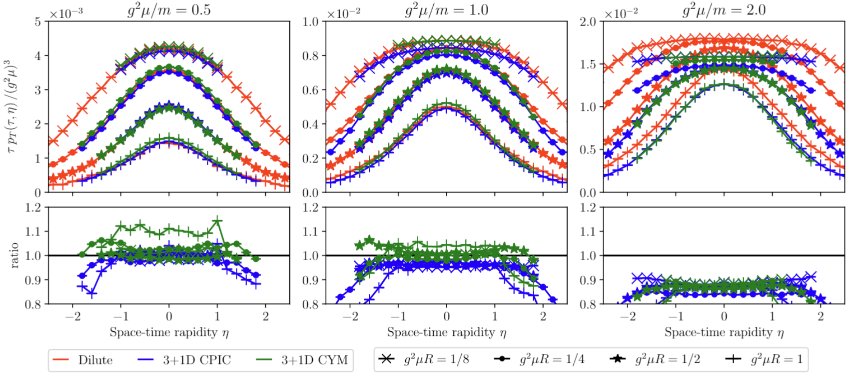
transverse correlations

longitudinal structure



(lattice simulations)

Comparison to 3+1D lattice simulations

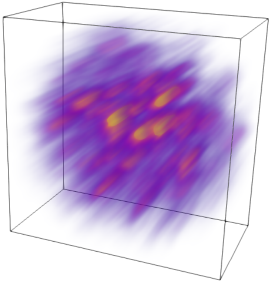


- Transverse pressure $p_T(\tau, \eta)$ as a function of space-time rapidity η and proper time τ
- Dilute approximation: a few hours on a desktop CPU (Monte Carlo integrator)
- Lattice simulations: days/weeks on a supercomputer (arXiv:1804.01995 and arXiv:2010.11172)
- Dilute approximation overestimates at higher $g^2\mu/m$, rapidity profile less affected

Conclusions and outlook

What we have published so far (arXiv:2109.05028)

- 3+1D solutions to Yang-Mills equations valid at small ρ
- Orders of magnitude faster numerical evaluation vs. lattice simulations
- Investigate nuclear models with longitudinal structure
  $\rightarrow$ Glasma with rapidity η dependence



Next steps

- Simplified Glasma field strength tensor ✓
- GPU-based Monte Carlo integrator for more complicated nuclear models ✓
  - Full Glasma energy-momentum tensor $T^{\mu\nu}$ ✓
  - Transverse and longitudinal geometry, impact parameter $\mathbf{b}_\perp$ ✓
- 3D nuclear model based on parton distribution functions (work in progress) ✓
- Coupling to effective kinetic theory and hydrodynamics for phenomenological applications