Our Model

- **IP-Glasma**: boost-invariant saturation-based model (strong classical gluon fields, CGC) (Schenke, Tribedy, Venugopalan, PRL 108, 252301 (2012))
- **MUSIC**: 2+1D Relativistic viscous hydrodynamics (Schenke, Jeon, Gale, PRC 82, 014903 (2010))
- **iS3D+UrQMD**: Cooper-Frye Monte Carlo particle sampling + hadronic transport, interactions and final state resolution (McNelis, Everett, Heinz, arXiv 1912, 08271 (2019) & Bass, Belkacem, Bleicher et al., PPNP 41, 225-370 (1998))

Sample MUSIC evolution, from $\tau = 0.4$ fm to 13 fm
Why Collide Deformed Nuclei?

- **Non-trivial initial geometries in ultra-central regime**
  - Elliptic flow ($v_2$)
  - Skewed $p_T$ distributions
  - Jet quenching
- Diversity in initial matter distributions at $b \approx 0$ fm
- Test understanding of multiplicity-generating mechanisms

Figure from Giacalone, PRL 124, 202301 (2020)
Figure 1A: 2-particle elliptic flow and eccentricity vs. scaled multiplicity ($|\eta| < 1, 0.2 \leq p_T \leq 2.0$ GeV)
Results A

Figure 1A: 2-particle elliptic flow and eccentricity vs. scaled multiplicity

Figure 2A: 2-particle and 4-particle elliptic flow vs. $N_{CH}$ ($|\eta| < 1, 0.2 \leq p_T \leq 2.0$ GeV)
Results A

Figure 1A: 2-particle elliptic flow and eccentricity vs. scaled multiplicity

Figure 2A: 2-particle and 4-particle elliptic flow vs. charged particle multiplicity

Figure 3A: $p_T$ distribution variance and skewness vs. centrality ($|\eta| < 1, 0.2 \leq p_T \leq 2.0$ GeV)
Figure 1B: 2-particle elliptic flow and eccentricity vs. scaled multiplicity ($|\eta| < 1, 0.2 \leq p_T \leq 2.0 \text{ GeV}$)
Results B

Figure 1B: 2-particle elliptic flow and eccentricity vs. scaled multiplicity

Figure 2B: 2-particle and 4-particle elliptic flow vs. $N_{CH}$ ($|\eta| < 1, 0.2 \leq p_T \leq 2.0$ GeV)
Figure 1B: 2-particle elliptic flow and eccentricity vs. scaled multiplicity

Figure 2B: 2-particle and 4-particle elliptic flow vs. charged particle multiplicity

Figure 3B: $p_T$ distribution variance and skewness vs. centrality ($|\eta| < 1, 0.2 \leq p_T \leq 2.0$ GeV)
Conclusion & Next Steps

Conclusion

▶ Presented results from the first comprehensive $v_2$ and $p_T$ analysis of initial state + hydro + hadronic cascade simulations of deformed systems at $\sqrt{s} = 193$ GeV

▶ IP-Glasma’s QCD-based approach is successful at reproducing key observables of deformed systems, especially in the ultra-central regime, thanks to
   - Initial state fluctuations
   - Pre-equilibrium flow
   - Multiplicity dependence on $Q_s^2 S_\perp$

▶ Our data shows that the unique geometry of deformed systems has a large effect on key observables

▶ $p_T$ distributions are robust to changes in $\delta f$ correction

Next Steps

▶ Analyze full set of observables to further compare 14-moment and Chapman-Enskog $\delta f$ correction

▶ Select events in 10-30% centrality and determine if lattice artifacts in hydro phase are a factor

▶ Compute and release identified particle spectra
Backup
Chapman-Enskog vs. 14-moment

Both are popular methods of linearizing the phase-space distribution function $f_n(x, p)$ around its equilibrium $f_{eq,n}$, which is relevant since it appears in the Cooper-Frye formula

$$E_p \frac{dN_n}{d^3p} = \frac{1}{(2\pi\hbar)^3} \int p \cdot d^3\sigma(x)f_n(x, p)$$

**Chapman-Enskog**

Boltzmann-based gradient expansion which uses RTA

$$\delta f_n = -\frac{p \cdot \delta f_{eq,n}}{p \cdot u/\tau_r}$$

Conservation of net baryon number, energy & momentum are applied together with the first-order Navier-Stokes relations

$$\delta f_n = f_{eq,n} \bar{f}_{eq,n} \left[ \frac{\beta_B}{\beta_V} \left( b_n G + \frac{(u \cdot p) F}{T^2} + \frac{(-p \cdot \Delta \cdot p)}{3(u \cdot p) T} \right) \right]$$

$$\frac{V_B^\mu}{\beta_V} \left( \frac{n_B}{\varepsilon + \mathcal{P}_{eq}} - \frac{b_n}{(u \cdot p)} \right) + \frac{\pi_{\mu\nu}^{(\mu)} p^{(\nu)}}{2\beta_V(u \cdot p) T}$$

Coefficients are adjusted to reproduce $T^{\mu\nu}$

**14-moment**

Truncated at hydro level ($p^\mu$ & $p^\mu p^\nu$)

$$\delta f_n = f_{eq,n} \bar{f}_{eq,n} \left( b_n c_\mu p^\mu + c_\mu \nu p^{\mu\nu} \right)$$

Assume $c_\mu$ & $c_\mu \nu$ to be species independent

$$\delta f_n = f_{eq,n} \bar{f}_{eq,n} \left( c_T m_n^2 + b_n \left( c_B (u \cdot p) + c_\nu^{(\mu)} p^{(\mu)} \right) \right)$$

$$+ c_E (u \cdot p)^2 + c_Q^{(\mu)} (u \cdot p) p^{(\mu)} + c_{\mu\nu}^{(\mu)} p^{(\mu)} p^{(\nu)}$$

Fix coefficients s.t. Landau matching condition is respected and they reproduce $\pi^{\mu\nu}$, $\Pi$ and $V_B^\mu$
Comparing WS Parametrizations

McGill parametrization

\[ R_0 = 6.874 \text{ fm}; a = 0.556 \text{ fm} \]
\[ \beta_2 = 0.2802; \beta_4 = -0.0035 \]

\[ \varepsilon_2 \sim 0.16 \]

BNL parametrization

\[ R_0 = 6.81 \text{ fm}; a = 0.55 \text{ fm} \]
\[ \beta_2 = 0.28; \beta_4 = 0.093 \]

\[ \varepsilon_2 \sim 0.21 \]
The IP-Glasma model is based on the CGC framework. $N_{coll}$ and $N_{part}$ do not enter explicitly. Multiplicities are governed by the combination

$$\frac{Q_s^2 S_\perp}{\alpha_s}$$

with $Q_s$ the saturation scale, $S_\perp$ the overlap area in the transverse plane and $\alpha_s$ the QCD coupling constant.
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For symmetric nuclei, $Q_s^2 \propto A^{1/3}$, while for deformed nuclei, $A^{1/3}$ is replaced with the average number of nucleons along the beam direction!
Importance of nuclear structure
Importance of nuclear structure

Nuclear structure determines the nuclear thickness function $T_{A(B)}$, which is a crucial early-stage quantity in initial state models like TRENTO and IP-Glasma.

- **TRENTO:**
  
  $s(x, y) \propto \left[ \frac{T_A^p + T_B^p}{2} \right]^\frac{1}{p}$

  $\langle N_{CH} \rangle \propto \int s(x, y) dx dy$

- **IP-Glasma:**

  $\pi^2 x g N_c \alpha_s T_A = Q_s^2$

  $\langle N_{CH} \rangle \propto Q_s^2 S_\perp \alpha_s$
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  - $\frac{xg}{N_c}$ is the gluon momentum density
  - $\alpha_s$ is the strong coupling constant
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Au+Au

U+U

body+body  tip+tip  body+tip
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Figure from Giacalone, PRL 124, 202301 (2020)
Deformed Woods-Saxon Basics

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\[ \rho(r, \theta) = \frac{\rho_0}{1 + \exp\left(\frac{R_0 - R_0'}{a}\right)} \]

\[ R(\theta) = R_0(1 + \beta_2 Y^{20}(\theta) + \beta_3 Y^{30}(\theta) + \ldots) \]
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▶ Spherically symmetric: \( \beta_2 = \beta_3 = 0 \)
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- **Spherically symmetric**: \( \beta_2 = \beta_3 = 0 \)
- **Deformed**: \( 0.05 \leq \beta_2 \leq 0.4 \); \( 0.01 \leq \beta_3 \leq 0.3 \); \( 0 \leq \beta_4 \leq 0.1 \)
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Example Parametrization\(^1\): \( R_0 = 6.784; a = 0.0556; \beta_2 = 0.2802; \beta_4 = -0.0035 \)

\(^1\) H. De Vries, C.W. De Jager, C. Devries, Atomic Data (1987)
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Deformed Woods-Saxon Basics

Other WS deformation parameters exist, such as the triaxiality parameter $\gamma$:

$$\rho(r, \theta) = \frac{\rho_0}{1 + \exp\left(\frac{R(\theta) - R_0'}{a}\right)}$$

$$R(\theta) = R_0(1 + \beta_2 \left[ \cos \gamma Y^{20}(\theta) + \sin \gamma Y^{22}(\theta, \varphi) \right] + \beta_3 Y^{30}(\theta))$$
Deformed Woods-Saxon Basics

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$$R(\theta) = R_0(1 + \beta_2 \left[ \cos \gamma Y_{20}^0(\theta) + \sin \gamma Y_{22}^0(\theta, \phi) \right] + \beta_3 Y_{30}^0(\theta))$$

Figure from Jiangyong Jia, arXiv 2109, 00604 (2021)
Initial Condition

- Large-$x$ partons serve as $\delta$-function colour sources on the light cone ($x^+ & x^-$)

$$J^\mu_{A(B)}(x) = \delta^{\mu-(+)} \delta(x^{-(+)}) \rho_{A(B)}(x)$$

Fields in regions 2 & 4 are pure gauge

Solution-matching on boundaries 1-4 and 1-2 provides solution for gauge fields in region 1
Initial Condition

- Large-$x$ partons serve as $\delta$-function colour sources on the light cone ($x^+ & x^-$)
- Fields in regions 2 & 4 are pure gauge

\[ A_j^{A(B)} = \frac{i}{g} V \partial_j V^\dagger \]
\[ A_\eta^{A(B)} = 0 \]
Initial Condition

- Large-x partons serve as $\delta$-function colour sources on the light cone ($x^+ & x^ -$)  
- Fields in regions 2 & 4 are pure gauge  
- Solution-matching on boundaries 1-4 and 1-2 provides solution for gauge fields in region 1

\[
E_\eta = -ig \left[ A^i_{(A)}, A^i_{(B)} \right]
\]

\[
A^i = A^i_{(A)} + A^i_{(B)}
\]

\[
A_\eta = 0
\]
Hydrodynamics & Hadronization

Hydrodynamics: MUSIC

▶ Relativistic viscous hydrodynamics code
▶ Parametrization based on previous work (S. McDonald, C. Shen, F. Fillion-Gourdeau, S. Jeon, C. Gale, PRC (2017)):
  ▶ Equation of state: HotQCD
  ▶ $\tau_{sw} = 0.4$ fm
  ▶ Constant shear viscosity $\frac{\eta}{S} = 0.12$
  ▶ Bulk viscosity depends on temperature
  ▶ Freeze-out temperature set to $T_{\text{freeze}} = 145$ MeV
  ▶ Once every cell is frozen, move to hadronization

Hadronization: iSS + UrQMD

▶ Standard iSS + UrQMD parameters
Selecting Collision types

Figure from Giacalone, PRL124 (2020)
Selecting Collision types

▶ Strong cuts in ZDC & scaled multiplicity enable body-body and tip-tip event selection
▶ Larger $v_2^2$ (and overlap area) at smaller $\langle p_T \rangle$ means energy density is primary generator of $p_T$

N. Fortier, S. Jeon and C. Gale
Deformed Initial States
April 6th 2022