Spin Boltzmann equation and global polarization in HIC

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Abstract

We derive Boltzmann equations for massive spin-1/2 fermions with local and nonlocal collision terms from the Kadanoff--Baym equation in the Schwinger-Keldysh formalism, properly accounting for the spin degrees of freedom. The Boltzmann equations are expressed in terms of matrix-valued spin distribution functions (MVSDs).

Introduction

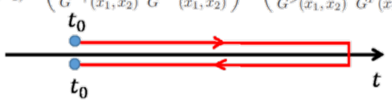
Noncentral heavy-ion collisions have a large orbital angular momentum, which may polarize the spin of particles created in the fireball. Microscopically. this is achieved through spin-orbit coupling in nonlocal collisions.

Our goal is to derive Boltzmann equations that are suitable for the simulation of spin transport processes involving the spin-orbit coupling.

Closed-time path formalism

Two-point Green function on the closed-time path (CTP)

$$G(x_1,x_2)=\begin{pmatrix} G^{++}(x_1,x_2) & G^{+-}(x_1,x_2)\\ G^{-+}(x_1,x_2) & G^{--}(x_1,x_2)\end{pmatrix}=\begin{pmatrix} G^{F}(x_1,x_2) & G^{<}(x_1,x_2)\\ G^{>}(x_1,x_2) & G^{\bar F}(x_1,x_2)\end{pmatrix}$$



Kadanoff-Baym equation is derived from Dyson-Schwinger equation. Taking a Fourier transform of the relative position, we obtain kinetic equation for the Wigner function

$$\begin{aligned}(\gamma\cdot K-m)\,G^<(x,p) &= -\frac{i\hbar}{2}\left[\Sigma^<(x,p)G^>(x,p)-\Sigma^>(x,p)G^<(x,p)\right]\\ &\quad -\frac{\hbar^2}{4}\left[\{\Sigma^<(x,p),G^>(x,p)\}_{\rm PB}-\{\Sigma^>(x,p),G^<(x,p)\}_{\rm PB}\right]\end{aligned}$$

Matrix-valued spin distribution

Clifford decomposition

$$G^<(x,p)=\frac{1}{4}\left(\mathcal{F}+i\gamma^5\mathcal{P}+\gamma^\mu\mathcal{V}_\mu+\gamma^5\gamma^\mu\mathcal{A}_\mu+\frac{1}{2}\sigma^{\mu\nu}\mathcal{S}_{\mu\nu}\right)$$

Semiclassical expansion for Wigner function. In general, Wigner function at first order can be written as the sum of a *quasi-classical* term, a *gradient and collision* term, and an *off-shell* term,

$$G^{\lessgtr(1)}=G^{\lessgtr(1)}_{\rm qc}+G^{\lessgtr(1)}_{\nabla}+G^{\lessgtr(1)}_{\rm off}$$

Quasi-classical contribution can be expressed in terms of MVSDs

$$\begin{aligned}G^<_{{\rm qc},\alpha\beta}(x,p) &= -2\pi\hbar\,\theta(p_0)\delta\left(p^2-m^2\right)\sum_{r,s}u_\alpha(r,p)\,\overline{u}_\beta(s,p)\;f^{(+)}_{sr}(x,p)\\ &\quad -2\pi\hbar\,\theta(-p_0)\delta\left(p^2-m^2\right)\sum_{r,s}v_\alpha(s,\overline{p})\,\overline{v}_\beta(r,\overline{p})\left[\delta_{sr}-f^{(-)}_{sr}(x,\overline{p})\right]+\mathcal{O}(\hbar^2)\end{aligned}$$

Scalar component of Wigner function

$$\mathcal{F}_{\rm qc}(x,p)=-2\pi\hbar\frac{m}{E_p}\left\{\delta(p_0-E_p)\,{\rm tr}\left[f^{(+)}(x,p)\right]+\delta(p_0+E_p)\,{\rm tr}\left[f^{(-)}(x,\overline{p})-1\right]\right\}+\mathcal{O}(\hbar^2)$$

Axial-vector component

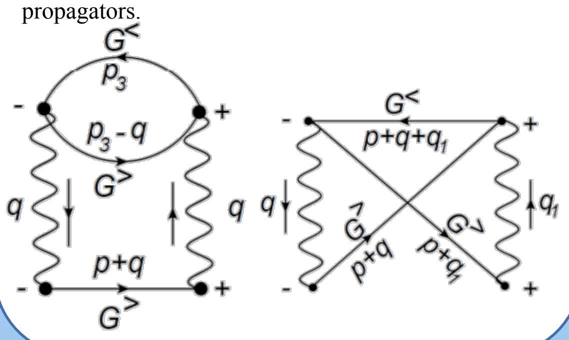
$$\begin{aligned}\mathcal{A}^\mu_{\rm qc} &= -2\pi\hbar\frac{m}{E_p}\left\{\delta(p_0-E_p)\,n_j^{(+)\mu}\,{\rm tr}\left[\tau_j^T f^{(+)}(x,p)\right]\right.\\ &\qquad\left.+\delta(p_0+E_p)\,n_j^{(-)\mu}\,{\rm tr}\left[\tau_j^T f^{(-)}(x,\overline{p})\right]\right\}+\mathcal{O}(\hbar^2)\end{aligned}$$

Other components

$$\mathcal{P}_{\rm qc}=0\quad,\quad \mathcal{V}^\mu_{\rm qc}=\frac{1}{m}p^\mu\mathcal{F}_{\rm qc}\quad,\quad \mathcal{S}^{\mu\nu}_{\rm qc}=-\frac{1}{m}\epsilon^{\mu\nu\alpha\beta}p_\alpha\mathcal{A}_{{\rm qc},\beta}$$

Self-energy

Feynman diagrams for $\Sigma^>(x,p)$. Solid lines represent fermion propagators, wavy lines represent boson propagators.



Boltzmann equations

Boltzmann equations for particle number density and spin polarization density

$$\frac{1}{E_p}p\cdot\partial_x{\rm tr}\left[f^{(1)}(x,p)\right]=\mathcal{C}_{\rm scalar}\left(\Delta I^{(1)}_{\rm coll,qc}\right)+\mathcal{C}_{\rm scalar}\left(\Delta I^{(1)}_{\rm coll,\nabla}\right)+\mathcal{C}_{\rm scalar}\left(I^{(0)}_{\rm coll,PB}\right)+\mathcal{C}_{\rm scalar}\left(\partial_x I^{(1)}_{\rm coll,qc}\right)$$

$$\frac{1}{E_p}p\cdot\partial_x{\rm tr}\left[n_j^{(+)\mu}\tau_j^T f^{(1)}(x,p)\right]=\mathcal{C}_{\rm pol}\left(\Delta I^{(1)}_{\rm coll,qc}\right)+\mathcal{C}_{\rm pol}\left(\Delta I^{(1)}_{\rm coll,\nabla}\right)+\mathcal{C}_{\rm pol}\left(I^{(0)}_{\rm coll,PB}\right)+\mathcal{C}_{\rm pol}\left(\partial_x I^{(1)}_{\rm coll,qc}\right)$$

Local collision terms do not depend on space-time gradients of distributions, e.g.,

$$\begin{aligned}&\mathscr{C}_{\rm scalar}\left(\Delta I^{(1)}_{\rm coll,qc}\right)\\ &=\frac{1}{2E_p}\int\frac{d^3\mathbf{p}_1}{(2\pi\hbar)^3 2E_1}\frac{d^3\mathbf{p}_2}{(2\pi\hbar)^3 2E_2}\frac{d^3\mathbf{p}_3}{(2\pi\hbar)^3 2E_3}(2\pi\hbar)^4\delta^{(4)}(p+p_3-p_1-p_2)\\ &\quad\times{\rm Re}\left[\left\{f_1^{(1)}\left[f_2^{(0)}\left(1-f^{(0)}-f_3^{(0)}\right)+f_3^{(0)}f^{(0)}\right]+f_2^{(1)}\left[f_1^{(0)}\left(1-f_3^{(0)}-f^{(0)}\right)+f_3^{(0)}f^{(0)}\right]\right.\right.\\ &\qquad\left.-f^{(1)}\left[f_3^{(0)}\left(1-f_1^{(0)}-f_2^{(0)}\right)+f_1^{(0)}f_2^{(0)}\right]-f_3^{(1)}\left[f^{(0)}\left(1-f_1^{(0)}-f_2^{(0)}\right)+f_1^{(0)}f_2^{(0)}\right]\right\}\\ &\qquad\left.\times\left(M_a^{\rm scalar}+M_b^{\rm scalar}\right)\right]\ ,\end{aligned}$$

where we suppressed spin indices for simplicity

Nonlocal collision terms depend on space-time gradients of distributions, e.g.,

$$\begin{aligned}&\mathscr{C}^\mu_{\rm pol}\left(\partial_x I^{(1)}_{\rm coll}\right)\\ &=\frac{1}{4mE_p}\int\frac{d^3\mathbf{p}_1}{(2\pi\hbar)^3 2E_1}\frac{d^3\mathbf{p}_2}{(2\pi\hbar)^3 2E_2}\frac{d^3\mathbf{p}_3}{(2\pi\hbar)^3 2E_3}(2\pi\hbar)^4\delta^{(4)}(p+p_3-p_1-p_2)\\ &\quad\times{\rm Im}\left\{\partial_x^\mu\left[f_1^{(0)}f_2^{(0)}\left(1-f_3^{(0)}\right)\left(1-f^{(0)}\right)-f_3^{(0)}f^{(0)}\left(1-f_1^{(0)}\right)\left(1-f_2^{(0)}\right)\right]\left(M^{\rm pol}_{a,5}+M^{\rm pol}_{b,5}\right)\right\}\end{aligned}$$

* Explicit expressions for collision terms are given in Ref. [1]

Summary

Derived Kadanoff-Baym equation for massive spin-1/2 fermions from Dyson-Schwinger equation on CTP contour.

Divided Wigner function into three parts, expressed quasi-classical term using MVSDs.

Derived Boltzmann equation for MVSD, which includes nonlocal collision terms.

References:

[1] X.-L. Sheng, N. Weickgenannt, E. Speranza, D. H. Rischke, Q. Wang, Phys. Rev. D 104, 016029 (2021).
[2] N. Weickgenannt, E. Speranza, X.-L. Sheng, Q. Wang, D. H. Rischke, Phys. Rev. D 104, 016022 (2021).