Relativistic spin-magnetohydrodynamics

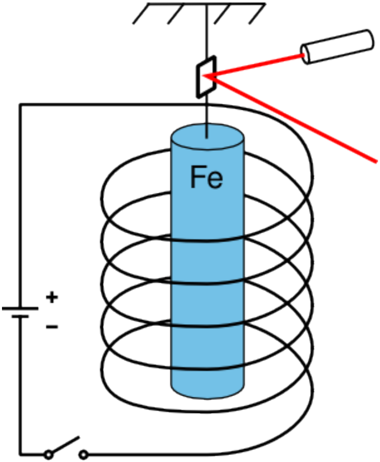
Amaresh Jaiswal

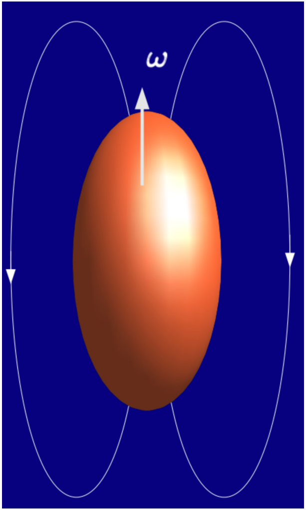
Quark Matter 2022

S. Bhadury, W. Florkowski, A. Jaiswal, A. Kumar, R. Ryblewski

arXiv:2204.01357 [nucl-th]

Einstein-de Haas effect   Barnett effect





Electron spins aligned in ext. magnetic field compensated by rotation. Spontaneous magnetization when spun around. Large **B** & **J** in HIC.

Equations of motion

- The particle four-current and its conservation is given by

$$N^\mu = nu^\mu + n^\mu, \qquad \partial_\mu N^\mu = 0$$

- The stress-energy tensor of the system of fluid and electromagnetic field can be expressed as $T^{\mu\nu} = T_{\rm f}^{\mu\nu} + T_{\rm int}^{\mu\nu} + T_{\rm em}^{\mu\nu}$, where

$$T_{\rm f}^{\mu\nu} = \epsilon u^\mu u^\nu - (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}, \qquad T_{\rm int}^{\mu\nu} = -\Pi^\mu u^\nu - F^\mu{}_\alpha M^{\nu\alpha}$$

$$T_{\rm em}^{\mu\nu} = -F^{\mu\alpha} F^\nu{}_\alpha + \frac{1}{4} g^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta}$$

- Using Maxwell's equation for magnetizable medium, $\partial_\mu H^{\mu\nu} = J^\nu$, and the relation $H^{\mu\nu} = F^{\mu\nu} + M^{\mu\nu}$,

$$\partial_\nu T_{\rm em}^{\mu\nu} = F^\mu{}_\alpha J^\alpha$$

- Define current generating external electromagnetic field, $J^\mu = J_{\rm f}^\mu + J_{\rm ext}^\mu$ where $J_{\rm f}^\mu = \mathfrak{q} N^\mu$, we have

$$\partial_\mu T^{\mu\nu} = -f_{\rm ext}^\nu, \qquad f_{\rm ext}^\nu = F^\nu{}_\alpha J_{\rm ext}^\alpha$$

Equations of motion contd.

- Divergence of matter part of energy-momentum tensor,

$$\partial_\nu T_{\mathrm{f}}^{\mu\nu} = F^{\mu}{}_{\alpha} J_{\mathrm{f}}^{\alpha} + \frac{1}{2} \left(\partial^\mu F^{\nu\alpha} \right) M_{\nu\alpha}$$

- Next, consider total angular momentum conservation:

$$J^{\lambda,\mu\nu} = L^{\lambda,\mu\mu} + S^{\lambda,\mu\nu}$$

- In presence of external torque its divergence leads to,

$$\partial_\lambda J^{\lambda,\mu\nu} = -\tau_{\mathrm{ext}}^{\mu\nu}, \qquad \tau_{\mathrm{ext}}^{\mu\nu} = x^\mu f_{\mathrm{ext}}^{\nu} - x^\nu f_{\mathrm{ext}}^{\mu}$$

- The orbital part of angular momentum and its divergence is

$$L^{\lambda,\mu\nu} = x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu}, \qquad \partial_\lambda L^{\lambda,\mu\nu} = -\tau_{\mathrm{ext}}^{\mu\nu}$$

- Spin part of the total angular momentum is conserved

$$\partial_\lambda S^{\lambda,\mu\nu} = 0$$

- Along with particle four-current conservation, $\partial_\mu N^\mu = 0$.

Kinetic theory

- In terms of the distribution function, $f(x, p, s)$,

$$S^{\lambda,\mu\nu} = \int dP dS \, p^\lambda s^{\mu\nu} \left(f + \bar{f}\right), \quad T_{\rm f}^{\mu\nu} = \int dP dS \, p^\mu p^\nu \left(f + \bar{f}\right),$$

$$N^\mu = \int dP dS p^\mu \left(f - \bar{f}\right), \; dP \equiv \frac{d^3p}{E_p (2\pi)^3}, \; dS \equiv m \frac{d^4 s}{\pi \, \mathfrak{s}} \, \delta(s \cdot s + \mathfrak{s}^2) \, \delta(p \cdot s)$$

- One can also define the polarization-magnetization tensor as

$$M^{\mu\nu} = m \int dP dS \, m^{\mu\nu} \left(f + \bar{f}\right)$$

- Boltzmann equation (BE) in relaxation-time approximation (RTA)

$$\left(p^\alpha \frac{\partial}{\partial x^\alpha} + m \, \mathcal{F}^\alpha \frac{\partial}{\partial p^\alpha}\right) f = C[f] = -\left(u \cdot p\right) \frac{f - f_{\rm eq}}{\tau_{\rm eq}}$$

- The force term is:

$$\mathcal{F}^\alpha = \frac{\mathfrak{q}}{m} F^{\alpha\beta} p_\beta + \frac{1}{2} \left(\partial^\alpha F^{\beta\gamma}\right) m_{\beta\gamma}, \qquad m^{\alpha\beta} = \chi s^{\alpha\beta}$$

Kinetic theory approach

- Impose Landau frame and extended matching conditions

$$u_\mu T^{\mu\nu} = \epsilon u^\nu, \quad \epsilon = \epsilon_{\rm eq}, \quad n = n_{\rm eq}, \quad u_\lambda \delta s^{\lambda,\mu\nu} = 0$$

- Zeroth, first and "spin" moment of the RTA collision vanishes

$$\int dP dS\, C[f] = \int dP dS\, p^\mu\, C[f] = \int dP dS\, s^{\mu\nu} C[f] = 0$$

- Using definitions of hydro quantities, these moments of BE gives

$$\partial_\mu N^\mu = 0, \quad \partial_\nu T_{\rm f}^{\mu\nu} = F^\mu{}_\alpha J_{\rm f}^\alpha + \frac{1}{2}\left(\partial^\mu F^{\nu\alpha}\right) M_{\nu\alpha}, \quad \partial_\lambda S^{\lambda,\mu\nu} = 0$$

- The spin matching condition results in

$$\dot\omega^{\mu\nu} = \mathcal{D}_\Pi^{[\mu\nu]}\,\theta + \mathcal{D}_{\rm n}^{[\mu\nu]\gamma}(\nabla_\gamma \xi) + \mathcal{D}_a^{[\mu\nu]\gamma}\,\dot u_\gamma + \mathcal{D}_\pi^{[\mu\nu]\rho\kappa}\,\sigma_{\rho\kappa} + \mathcal{D}_\Omega^{[\mu\nu]\rho\kappa}\,\Omega_{\rho\kappa} + \mathcal{D}_B^{[\mu\nu]\rho\kappa}(\nabla_\rho B_\kappa) + \mathcal{D}_\Sigma^{[\mu\nu]\phi\rho\kappa}\left(\nabla_\phi \omega_{\rho\kappa}\right)$$

- Expressions for dissipative quantities obtained [arXiv:2204.01357].

Thank You!