# Resummed thermodynamics of QCD and N=4 supersymmetric Yang-Mills theory

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**References:** 2011.06938, 2105.02101, 2111.12160

# NNLO Hard Thermal Loop Perturbation Theory (HTLpt)

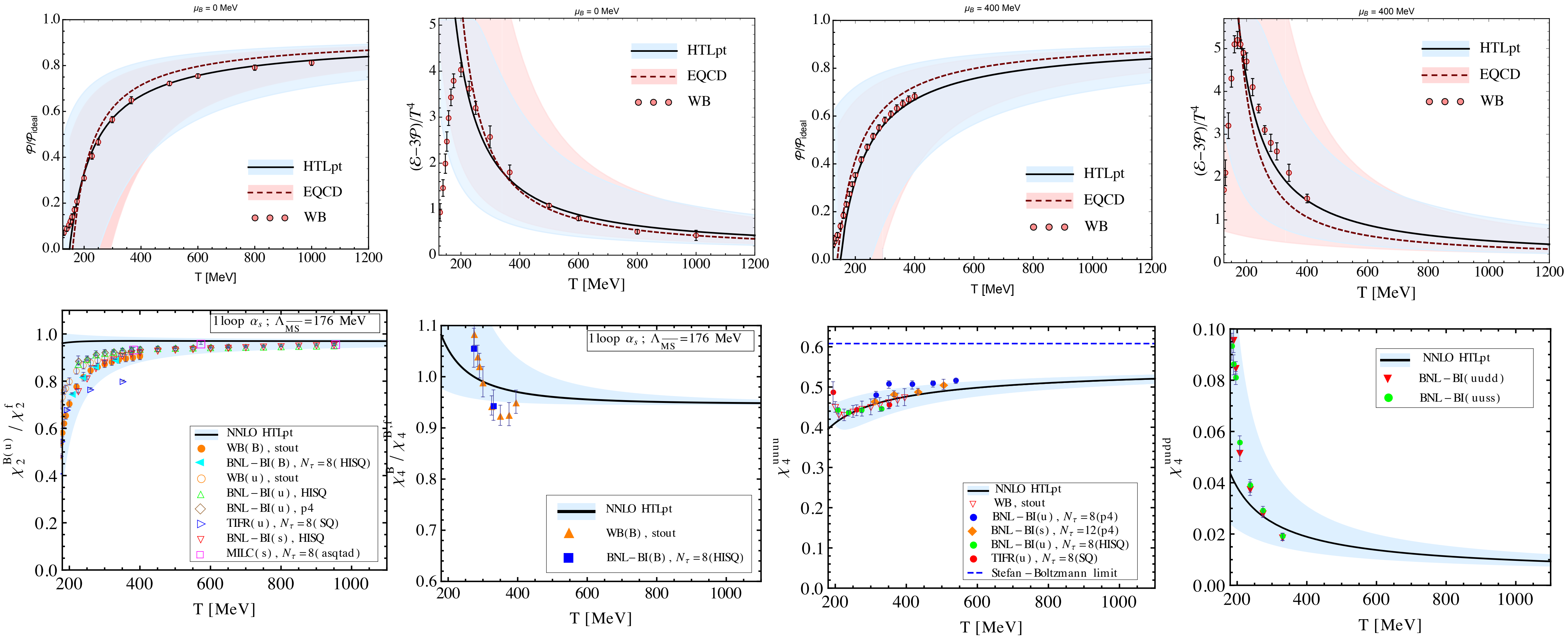
- HTLpt reorganization improves the convergence of perturbation theory compared to strict perturbation theory.
- NNLO results available at finite T and quark chemical potential(s):
  - 1309.3968
  - 1402.6907
- Result on the right is for a single quark chemical potential.
- Result is completely analytic.
- Residual dependence on the RG scales $\Lambda_g$ and $\Lambda_q$.

$$
\begin{aligned}
\frac{\Omega_{\rm NNLO}}{\Omega_0} = {}& \frac{7}{4}\frac{d_F}{d_A}\left(1+\frac{120}{7}\hat\mu^2+\frac{240}{7}\hat\mu^4\right) + \frac{s_F\alpha_s}{\pi}\Bigg[-\frac{5}{8}\left(1+12\hat\mu^2\right)\left(5+12\hat\mu^2\right)+\frac{15}{2}\left(1+12\hat\mu^2\right)\hat m_D \\
&+\frac{15}{2}\left(2\ln\frac{\hat\Lambda_q}{2}-1-\aleph(z)\right)\hat m_D^3 - 90\hat m_q^2\hat m_D\Bigg] + s_{2F}\left(\frac{\alpha_s}{\pi}\right)^2\Bigg[\frac{15}{64}\Bigg\{35-32\left(1-12\hat\mu^2\right)\frac{\zeta'(-1)}{\zeta(-1)}+472\hat\mu^2 \\
&+1328\hat\mu^4+64\Big(-36i\hat\mu\aleph(2,z)+6(1+8\hat\mu^2)\aleph(1,z)+3i\hat\mu(1+4\hat\mu^2)\aleph(0,z)\Big)\Bigg\} - \frac{45}{2}\hat m_D\left(1+12\hat\mu^2\right)\Bigg] \\
&+\left(\frac{s_F\alpha_s}{\pi}\right)^2\Bigg[\frac{5}{4\hat m_D}\left(1+12\hat\mu^2\right)^2+30\left(1+12\hat\mu^2\right)\frac{\hat m_q^2}{\hat m_D}+\frac{25}{12}\Bigg\{\left(1+\frac{72}{5}\hat\mu^2+\frac{144}{5}\hat\mu^4\right)\ln\frac{\hat\Lambda_q}{2} \\
&+\frac{1}{20}\left(1+168\hat\mu^2+2064\hat\mu^4\right)+\frac{3}{5}\left(1+12\hat\mu^2\right)^2\gamma_E-\frac{8}{5}(1+12\hat\mu^2)\frac{\zeta'(-1)}{\zeta(-1)}-\frac{34}{25}\frac{\zeta'(-3)}{\zeta(-3)} \\
&-\frac{72}{5}\Big[8\aleph(3,z)+3\aleph(3,2z)-12\hat\mu^2\aleph(1,2z)+12i\hat\mu\left(\aleph(2,z)+\aleph(2,2z)\right)-i\hat\mu(1+12\hat\mu^2)\,\aleph(0,z) \\
&-2(1+8\hat\mu^2)\aleph(1,z)\Big]\Bigg\} - \frac{15}{2}\left(1+12\hat\mu^2\right)\left(2\ln\frac{\hat\Lambda_q}{2}-1-\aleph(z)\right)\hat m_D\Bigg] \\
&+\left(\frac{c_A\alpha_s}{3\pi}\right)\left(\frac{s_F\alpha_s}{\pi}\right)\Bigg[\frac{15}{2\hat m_D}\left(1+12\hat\mu^2\right)-\frac{235}{16}\Bigg\{\left(1+\frac{792}{47}\hat\mu^2+\frac{1584}{47}\hat\mu^4\right)\ln\frac{\hat\Lambda_q}{2}-\frac{144}{47}\left(1+12\hat\mu^2\right)\ln\hat m_D \\
&+\frac{319}{940}\left(1+\frac{2040}{319}\hat\mu^2+\frac{38640}{319}\hat\mu^4\right)-\frac{24\gamma_E}{47}\left(1+12\hat\mu^2\right)-\frac{44}{47}\left(1+\frac{156}{11}\hat\mu^2\right)\frac{\zeta'(-1)}{\zeta(-1)}-\frac{268}{235}\frac{\zeta'(-3)}{\zeta(-3)} \\
&-\frac{72}{47}\Big[4i\hat\mu\aleph(0,z)+\left(5-92\hat\mu^2\right)\aleph(1,z)+144i\hat\mu\aleph(2,z)+52\aleph(3,z)\Big]\Bigg\}+90\frac{\hat m_q^2}{\hat m_D}+\frac{315}{4}\Bigg\{\left(1+\frac{132}{7}\hat\mu^2\right)\ln\frac{\hat\Lambda_q}{2} \\
&+\frac{11}{7}\left(1+12\hat\mu^2\right)\gamma_E+\frac{9}{14}\left(1+\frac{132}{9}\hat\mu^2\right)+\frac{2}{7}\aleph(z)\Bigg\}\hat m_D\Bigg] + \frac{\Omega^{\rm YM}_{\rm NNLO}(\Lambda_g)}{\Omega_0}\,. \qquad (5)
\end{aligned}
$$

Andersen, Leganger, Su, and MS 1009.4644, 1103.2528
Haque, Andersen, Mustafa, MS, N. Su, 1309.3968

# Comparisons with lattice QCD data

Andersen, Leganger, Su, and MS 1009.4644, 1103.2528
Haque, Andersen, Mustafa, MS, N. Su, 1309.3968



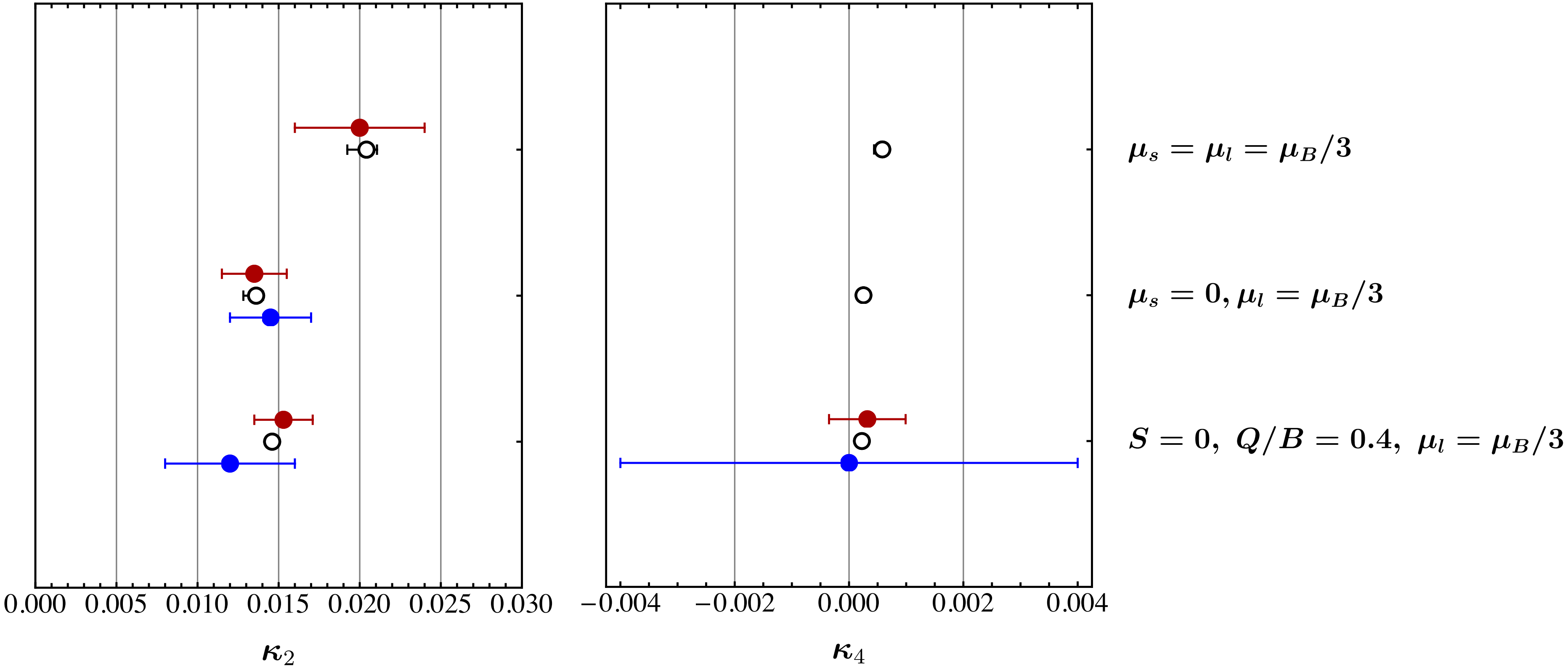

NNLO HLpt can reproduce lattice measurements. An incomplete selection of results is shown above.

# Curvature(s) of the phase transition line

- By finding the point at which P=0 as a function of $\kappa_2$, we can extract the quadratic and quartic curvatures of the QCD phase transition line.
- Plots below show comparisons with LQCD measurements of $\kappa_2$ (left) and $\kappa_4$ (right).
- Uncertainties reported for the HTLpt results come from RG scale variation.
- The rows from top to bottom show different physics cases. Open circles = HTLpt predictions, closed = lattice results.

$$\frac{T_c^\mu}{T_c^0} = 1 - \kappa_2 \left(\frac{\mu_B}{T_c^\mu}\right)^2 - \kappa_4 \left(\frac{\mu_B}{T_c^\mu}\right)^4 - \ \ldots$$

Haque and MS, 2011.06938

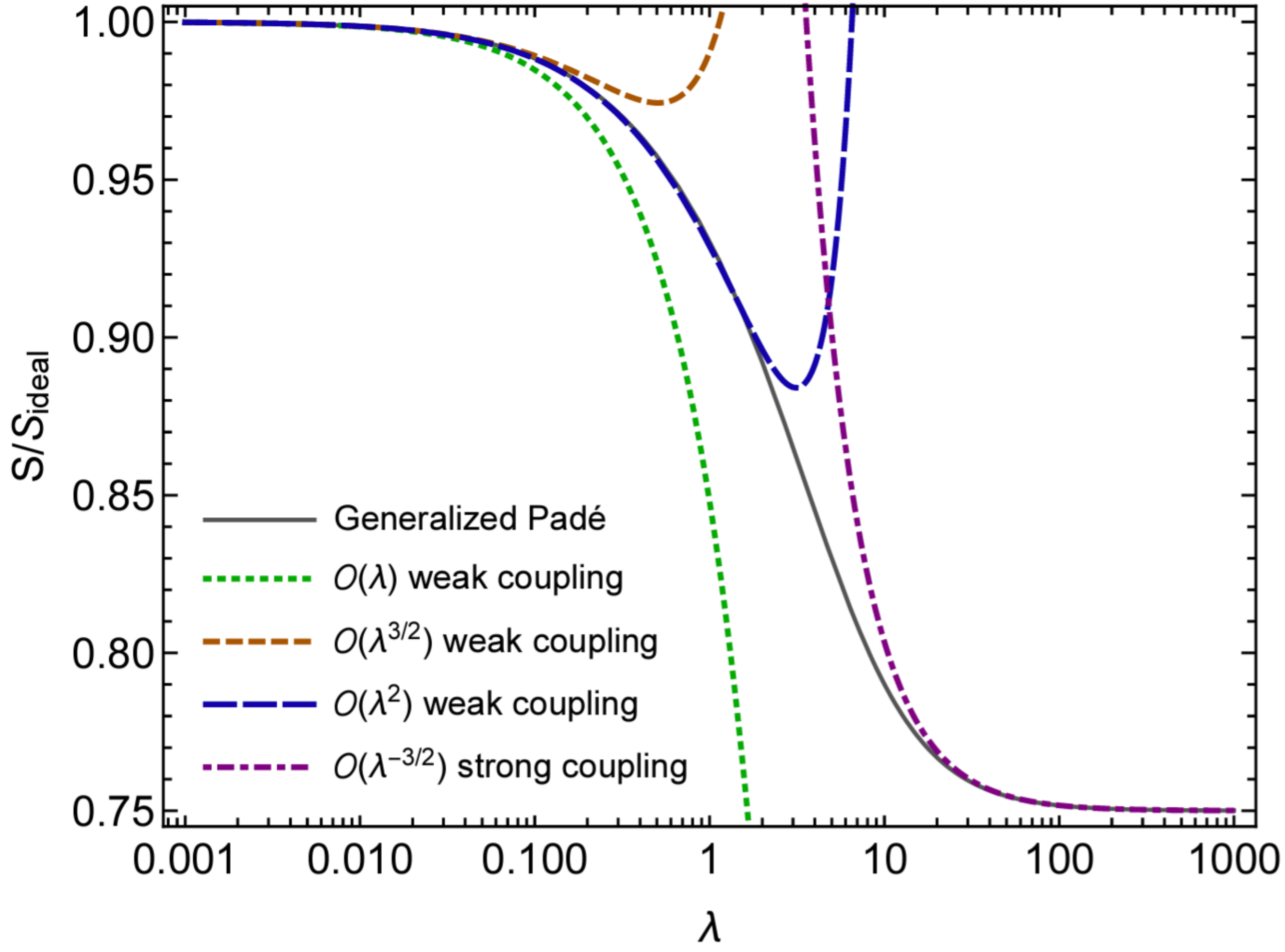


# N=4 Supersymmetric Yang Mills Thermodynamics

Du, MS, and Tantary, 2105.02101; Andersen, Du, MS, Tantary, 2111.12160

strong $\dfrac{\mathcal{S}}{\mathcal{S}_{\text{ideal}}} = \dfrac{3}{4}\left[1 + \dfrac{15}{8}\zeta(3)\lambda^{-3/2} + \mathcal{O}(\lambda^{-3})\right]$ Gubser, Klebanov,and Tseytlin, hep-th/9805156

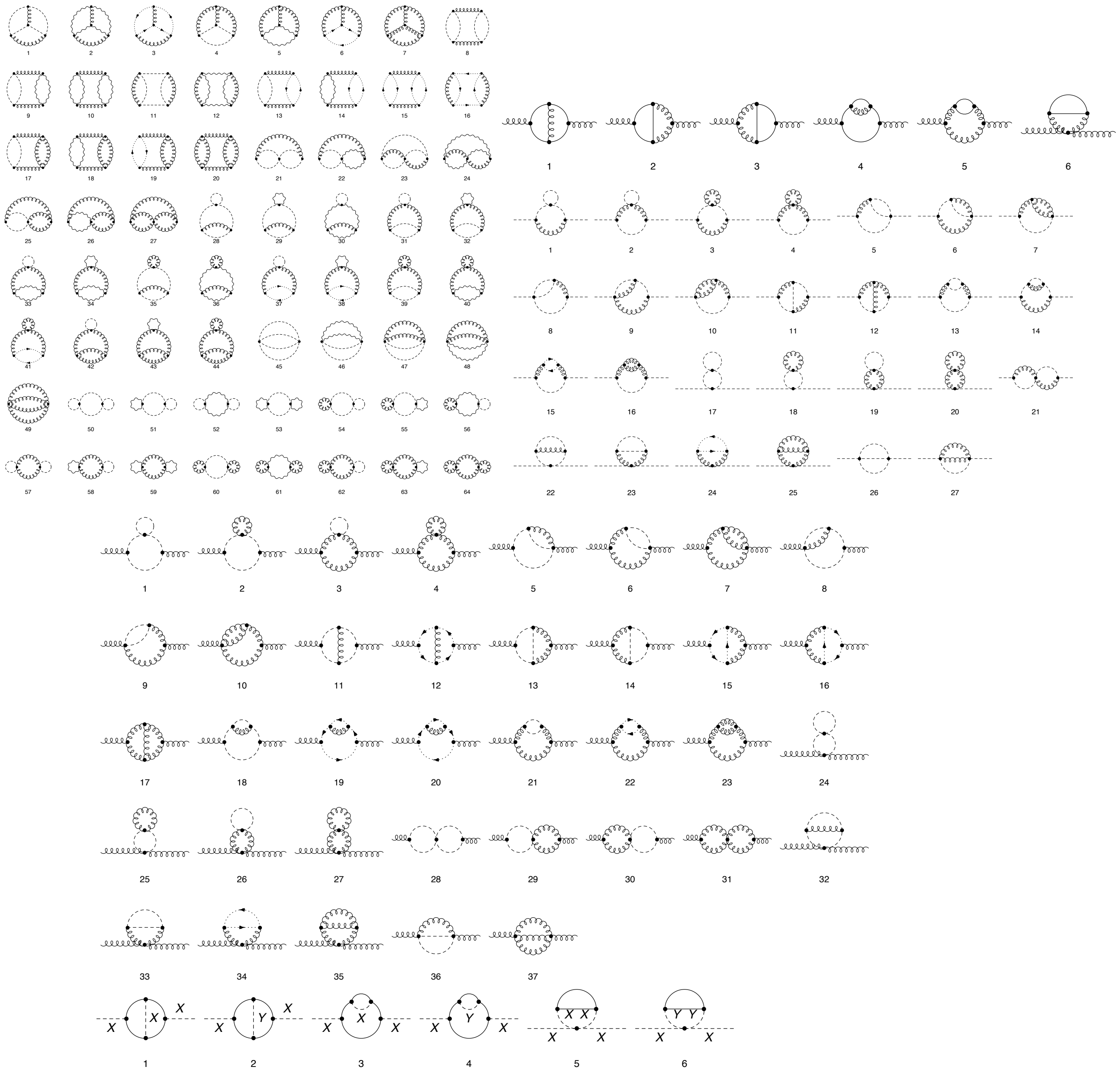
weak $\dfrac{\mathcal{S}}{\mathcal{S}_{\text{ideal}}} = 1 - \dfrac{3}{2}\dfrac{\lambda}{\pi^2} + \left(3+\sqrt{2}\right)\left(\dfrac{\lambda}{\pi^2}\right)^{3/2} + \left[-\dfrac{21}{8} - \dfrac{9\sqrt{2}}{8} + \dfrac{3}{2}\gamma_E + \dfrac{3}{2}\dfrac{\zeta'(-1)}{\zeta(-1)} - \dfrac{25}{8}\log 2 + \dfrac{3}{2}\log\dfrac{\lambda}{\pi^2}\right]\left(\dfrac{\lambda}{\pi^2}\right)^2$




- Our new result extends the perturbative calculation to order $\lambda^2$.
- Contains $\lambda^2\log(\lambda)$.
- This has potentially deep implications for the strong coupling limit.
- With knowledge of the strong and weak coupling results one can construct a Padé approximant that interpolates between the two limits (solid line in the figure to the left).
- Upon adding each successive perturbative order computed, agreement with the Padé extends from 0.02 → 0.2 → 2.
- Finite and large radius of convergence.

# Conclusions

- NNLO HTLpt result for the QCD EoS is purely analytic and describes lots of lattice data reasonably well.
- For some quantities, e.g. the susceptibilities and curvature coefficients, the residual scale dependence is small and HTLpt provides quantitative predictions which agree quite well with lattice measurements.
- Together with my students we have completed the order $\lambda^2$ correction to N=4 SUSY thermodynamics.
- Results were confirmed using EFT techniques.
- For N=4 SUSY, the perturbative expansion seems to have a finite and large radius of convergence.
- Work is underway using EFT techniques to calculate the order $\lambda^{5/2}$ coefficient (see diagrams on right).
- If the pattern continues, the result should be accurate for $\lambda < 20$!



Diagrams necessary to compute the $\lambda^{5/2}$ coefficient