BACKUP:
Fermion spectral function in a highly occupied non-Abelian plasma

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QUARK MATTER KRAKÓW 2022

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Based on:

KB, Lappi, Mace, Schlichting,
Phys. Lett. B 827, 136963 (2022) [2106.11319]

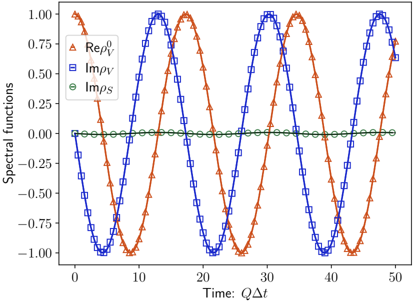
Classical-statistical lattice simulations, algorithm

1. Set initial conditions for gluons at $t' = 0$, generating a configuration with $\langle E_T^*(t{=}0,\vec{p}) E_T(t{=}0,\vec{q})\rangle \propto p\, f(t{=}0,p)\, \delta_{jk} (2\pi)^3 \delta(\vec{p}-\vec{q})$
2. Solve classical EOMs of gluonic part for $t' \leq t$, set Coulomb-type gauge $\partial^j A_j\big|_t = 0$ at $t' = t$
3. For each momentum mode $\vec{p}$ initialize $\phi^{u/v}_{\lambda,\vec{p}}$, evolve for $t' > t$
4. Use leap-frog scheme to solve classical EOMs

$$
\begin{aligned}
U_j(t',\vec{x}) &= e^{i a_t/a_s E^j(t'-a_t/2,\vec{x})} U_j(t'-a_t,\vec{x}) \\
E^j(t'+a_t/2,\vec{x}) - E^j(t'-a_t/2,\vec{x}) &= -\frac{a_t}{a_s} \sum_{j\neq i} \left[U_{ij}(t',\vec{x}) + U_{i(-j)}(t',\vec{x}) \right]_{\mathrm{ah}} \\
\phi^{u/v}_{\lambda\vec{p}}(t'+a_t,\vec{x}) - \phi^{u/v}_{\lambda\vec{p}}(t'-a_t,\vec{x}) &= -2 i a_t \gamma^0 \left(-i\gamma^j D^s_{W,j}[U] + m \right) \phi^{u/v}_{\lambda\vec{p}}(t',\vec{x})
\end{aligned}
$$

5. Calculate fermionic spectral function $\rho(t',t,\vec{p})$ for each $\vec{p}$ mode

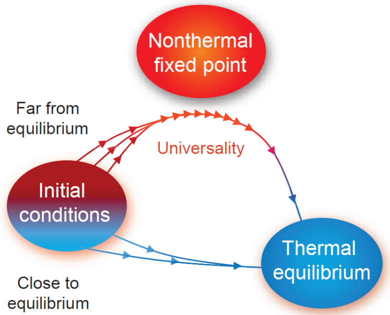
Benchmark for free fermions



- Analytically: $\rho^{\text{free}}(\Delta t, \vec{p}) = \gamma^0 \cos(E_{\vec{p}}\Delta t) + i\left(\gamma^j \frac{\hat{p}^j}{E_{\vec{p}}} - \frac{m_{\vec{p}}}{E_{\vec{p}}}\right)\sin(E_{\vec{p}}\Delta t)$
- Our method: set gauge fields to $U^i(t', \vec{x}) = 1,\ E^i(t', \vec{y}) = 0$
- Extract $\rho_V^0 = \frac{1}{4}\mathrm{Tr}(\rho\gamma^0)\,, \quad \rho_V = -\frac{E_{\vec{p}}\,p^j}{4\,p^2}\,\mathrm{Tr}(\rho\gamma^j)\,, \quad \rho_S = \frac{1}{4}\mathrm{Tr}(\rho)$
- Nice agreement!
- 64^3, $a_s\vec{p} = (0.098, 0.195, 0.29)$, mass $m\,a_s = 0.003125$;

  all other components of ρ at machine prec. $\sim 10^{-16}$

Universal (classical) attractors



Nonthermal fixed point (NTFP)

- ★ Large initial occupancy
  $\Rightarrow$ may approach attractor
- ★ System 'forgets' initial conditions
- ★ Self-similar dynamics

$$f(t,p) = t^{\alpha} f_s(t^{\beta} p)$$

- ★ *Universal* α, β, $f_s(p)$

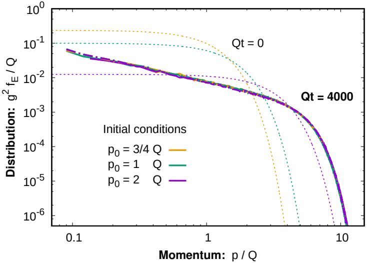
NTFP: Micha, Tkachev (2004); Berges, Rothkopf, Schmid (2008)

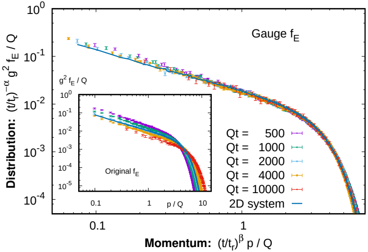
Universality: Berges, KB, Schlichting, Venugopalan (2015); Piñeiro Orioli, KB, Berges (2015)

Experimental observations: Prüfer et al., Nature 563, 217 (2018); Erne et al., Nature 563, 225 (2018)

Non-equilibrium state: self-similar turbulent attractor

Figures: attractor in 2+1D; KB, Kurkela, Lappi, Peuron, PRD 100, 094022 (2019)





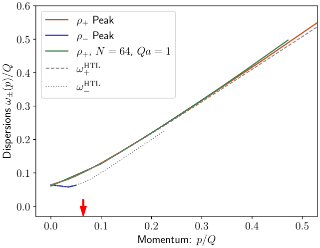
- Gluonic $f_g(t{=}0, p \lesssim Q) \sim \frac{1}{g^2} \gg 1$ approach self-similar attractor

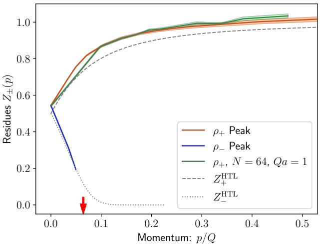
$$f(t,p) = (Qt)^{\alpha} f_s\left((Qt)^{\beta} p\right)$$

- Universal exponents insensitive to details of initial conditions
  - ✓ 2+1D: $\beta = -1/5$, $\alpha = 3\beta$, KB, Kurkela, Lappi, Peuron (2019)
  - ✓ 3+1D: $\beta = -1/7$, $\alpha = 4\beta$, Berges, Scheffler, Sexty (2009); Kurkela, Moore (2011, 2012); Berges, Schlichting, Sexty (2012); Schlichting (2012); Berges, KB, Schlichting, Venugopalan (2014); . . .
- We extract $\rho^{\alpha\beta}(t, \omega, p)$ at such a typical state in 3+1D

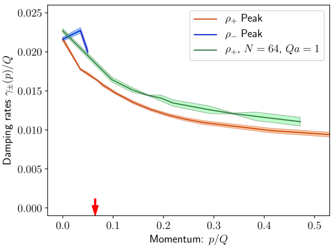
Fermion ρ in 3+1D: $\omega_\pm$, $Z_\pm$, $\gamma_\pm$

(top) $\omega_\pm$, *(bottom)* $Z_\pm$





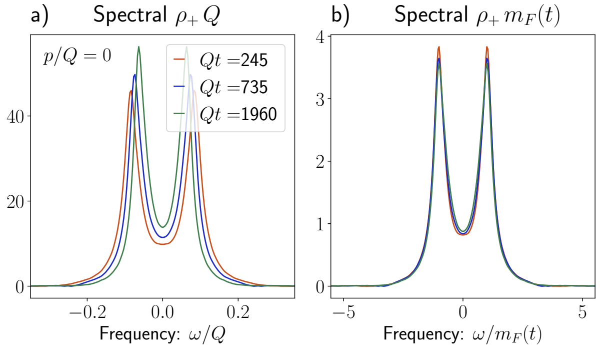
Damping rates (width) $\gamma_\pm$



- HTL dispersions and residues agree well with data
- First-principles insight into p-dependence of damping rates
- Arrows: $m_F = \left[C_F \int \frac{d^3p}{(2\pi)^3} \frac{g^2 f_g(p)}{p} \right]^{1/2}$

Fermion ρ in 3+1D: Time evolution

KB, Lappi, Mace, Schlichting, arXiv:2106.11319

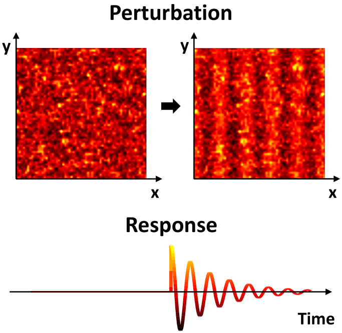


- $\rho_+(t, \omega, p{=}0)$ scales with fermion mass $m_F(t) \equiv \omega_\pm(t, p{=}0)$
- Expected from HTL: $\gamma^{\mathrm{HTL}}(t, p{=}0) \propto g^2 T^*(t) \sim Q(Qt)^{-3/7}$
  $\Rightarrow$ observed $\gamma(t, p{=}0) \sim m_F(t)$ is surprising

Nonperturbative computation of gluonic ρ

Classical-statistical $\mathrm{SU}(N_c)$ simulations + linear response theory

KB, Kurkela, Lappi, Peuron, *PRD 98, 014006 (2018)*, Editors' suggestion



- Similar algorithm as for fermions
- Split $A(t,\vec{x}) \mapsto A(t,\vec{x}) + \delta A(t,\vec{x})$ at t, perturb with plane wave $j_0(\vec{p})\,\delta\,(t'-t)$
- Response $\langle \delta A(t',\vec{p})\rangle = G_R(t',t,\vec{p})\,j_0(\vec{p})$
- Linearized EOM for $\delta A(t,\vec{x})$ such that Gauss law conserved (also in gauge-cov. formulation)

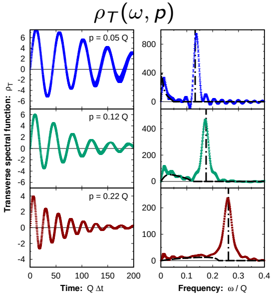
  Kurkela, Lappi, Peuron, *EUJC 76 (2016) 688*
- $\theta(t'-t)\ \boxed{\rho(t',t,p)} = G_R(t',t,p)$

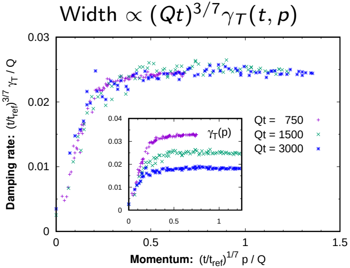
Similar methods for scalars:

Aarts (2001); Piñeiro Orioli, Berges (2019); Schlichting, Smith, von Smekal (2020); KB, Piñeiro Orioli (2020);

Gluon spectral function in isotropic 3+1D plasmas

KB, Kurkela, Lappi, Peuron, PRD 98, 014006 (2018)





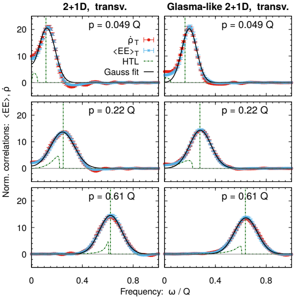
- Narrow Lorentzian q.p. peaks (position $\omega(p)$, width $\gamma(p)$)
- HTL at LO (black dashed) describes main features well
- Landau cut ($\omega < p$) and q.p. peak distinguishable

- Peak width $\gamma(t,p) \ll \omega(t,p)$
- Full p dep., $\gamma_T(t,p) \approx \gamma_L(t,p)$
- $\frac{\gamma_T(t,p)}{\omega(t,p=0)} \sim (Qt)^{-2/7}$ decreases

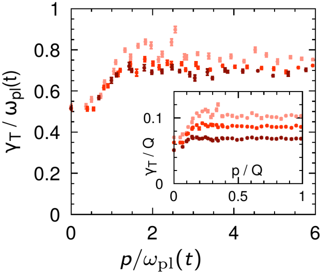
Vs. Gluon spectral function in 2+1D plasmas

KB, Kurkela, Lappi, Peuron, *JHEP 05, 225 (2021)*

$\dot{\rho}_T(\omega, p) \approx \omega \rho_T(\omega, p)$



Width $\gamma_T(t, p)/\omega_{\rm pl}(t)$



- Broad non-Lorentzian peaks
- HTL curves (green) agree poorly
- Landau cut and q.p. peak not distinguishable

- Peak width $\gamma(t, p) \sim \omega_{\rm pl}(t)$

$(\omega_{\rm pl} \equiv \omega_T(p{=}0))$

$\Rightarrow$ $\boxed{\text{no quasiparticles for } p \lesssim \omega_{\rm pl}!}$