Lattice simulations of the QCD chiral transition at real baryon density

Towards lattice QCD at not so small baryon densities

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Quark Matter 2022

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Based on Phys. Rev. D 105 (2022) 5, L051506

Approaches to lattice QCD at finite μ_B

$$Z = \int \mathcal{D}A_\alpha \det \mathrm{M}(\mathrm{A}_\alpha, \mu, \mathrm{m}) \mathrm{e}^{-\frac{1}{4}\int \mathrm{Tr}\, \mathrm{F}_{\alpha\beta}\mathrm{F}_{\alpha\beta}}$$

where $\mathrm{M} = \gamma_\alpha \mathrm{D}_\alpha + \mathrm{m} + \gamma_0 \mu$ is the (discretized) Dirac-operator.
Importance sampling works if $\det \mathrm{M}$ is **real and positive**:

- chemical potential $\mu = 0$
- purely imaginary chemical potentials: $\mathrm{Re}\, \mu = 0$
- isospin chemical potential: $\mu_u = -\mu_d$

If not: **complex action problem** $\rightarrow$ desperate times, desperate measures
Approaches to non-zero μ suffer from additional serious problems. E.g.

- Taylor and imaginary μ: **analytic continuation problem**
- Reweighting from $\mu = 0$: **overlap problem**
- Complex Langevin: **convergence issues**

Here: a method where the only problem is the sign problem
If the sign problem is dealt with by sufficient statistics, the results are reliable, and errors (on a fixed lattice) are statistical only.

Phase and sign reweighting

Fields: U Target theory: Z_t Simulated theory: Z_s

$$Z_t = \int \mathcal{D}U \; w_t(U) \qquad w_t(U) \in \mathbb{C}$$

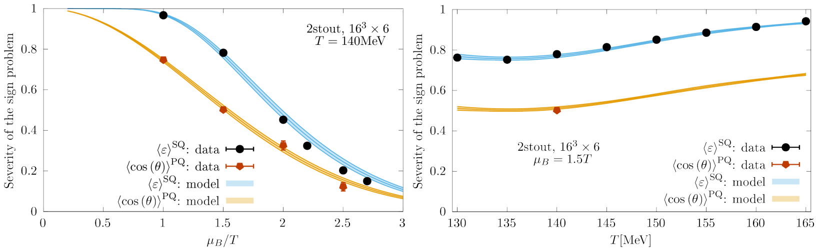
$$Z_s = \int \mathcal{D}U \; w_s(U) \qquad w_s(U) > 0$$

$$\frac{Z_t}{Z_s} = \left\langle \frac{w_t}{w_s} \right\rangle_s \quad \text{and} \quad \langle O \rangle_t = \left\langle \frac{w_t}{w_s} O \right\rangle_s \left\langle \frac{w_t}{w_s} \right\rangle_s^{-1}$$

Two problems that are exponentially hard in the volume can arise:

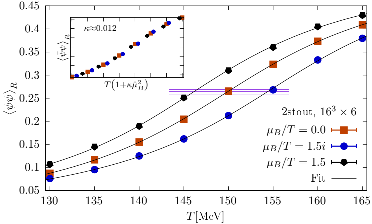
- $\frac{w_t}{w_s} \in \mathbb{C} \to$ the complex action problem became a **sign problem**
- Tails of $\rho(\frac{w_t}{w_s})$ long $\to$ **overlap problem**
- The overlap problem is avoided if the weight come from a compact space
  $\to$ phase reweighting
  $w_s = w_{PQ} = |\det M| e^{-S_g}$ and $Z_t/Z_s = \langle \cos \operatorname{Arg} M \rangle_{PQ}$
  $\to$ sign reweighting
  $w_s = w_{SQ} = |\operatorname{Re}\det M| e^{-S_g}$ and $Z_t/Z_s = \langle \operatorname{sign} \cos \operatorname{Arg} M \rangle_{SQ}$
- In these cases the severity of the sign problem is measured by Z_t/Z_s

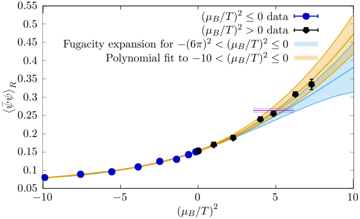
The severity of the sign problem



- Statistics required $\propto 1/(\text{strength of the sign problem})^2$
- Sign quenched ≈ 2.5 factor less statistics from this estimate
- Model: wrapped Gaussian with $\sigma^2(\mu) = -\frac{4}{9}\chi_{11}^{ud}(T)(LT)^3\hat{\mu}_B^2$
- Const. strength of the sign problem for const. $(LT)^3\left(\frac{\mu_B}{T}\right)^2$ (roughly)
- For $LT = 16/6 \approx 2.7$ (for $T = 140$MeV this is $L \approx 3.8$fm) the sign problem is managable for the entire RHIC Beam Energy Scan range

The renormalized chiral condensate





- Left: $\mu_B/T = 0, 1.5, 1.5i$ and $130\mathrm{MeV} \le \mathrm{T} \le 165\mathrm{MeV}$ (T scan)
- Similar rescalings in the imaginary μ_B direction:
  W-B: PRL 126 (2021) 23, 232001; W-B: PRL 125 (2020) 5, 052001;
- Also works at real $\mu_B \to$ no sign of a strengthening crossover
- Right: $T = 140\mathrm{MeV}$ and $0 \le \mu_B \le 380\mathrm{MeV}$ (μ_B scan)
- The direct method penetrates the region where errors from analytic continuation blow up!

Summary

- Methods to study finite density QCD are typically not bottlenecked by the sign problem itself but other effects (analytic continuation for Taylor or $\mathrm{Im}\,\mu$, overlap for reweighting from $\mu = 0$)
- Observables sensitive to criticality are unknown for $\mu_B/T \geq 1.5$
- We advocate a reweighting method that is free from the overlap problem in the weights and is therefore bottlenecked by the sign problem itself
- The sign problem is managable for the RHIC BES range
- Method penetrates the region where extrapolation methods are not that predictive
- First physics results
- Active research: cutting the costs with algorithmic tricks and a 2D scan of the $T - \mu_B$ plane