Heavy quark-antiquark interaction in finite temperature lattice QCD

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Heavy quark-antiquark pair as a hard probe

The hot fireball of quark-gluon plasma (QGP) produced in heavy-ion collisions (HIC) is a short-lived, expanding nuclear medium out of equilibrium that evolves through various stages. **Hard probes**, i.e., jets, *heavy quarks*, and **heavy quarkonia**, are produced only in rare hard processes during the earliest stages. As their total number is conserved on the HIC's timescale, they carry information about its entire time evolution.

Pair-produced heavy quarks may initially form **quarkonia** that incur medium modification as **quasiparticle** states (the **complex** $T>0$ **potential**) later, or may propagate independently as *open heavy flavors*, and may undergo **transitions** between **hidden** or *open flavor* (HF or OF). OF interacts with QGP via collisional or radiative processes – the former more relevant for *heavy flavors*[a], the latter for light flavors resp. jets[b].

In heavy-ion phenomenology, scenarios of **strong binding** with largely unmodified attraction or **weak binding** with much weaker attraction have been considered. We calculate the **complex** $T>0$ **potential** in state-of-the-art (2+1)-flavor lattice QCD at finite temperature [1] [c]; for a broader context, see Ref. [2, 3].

[a] See T06: L. Altenkort, 04/06/2022, 12:10, Lattice QCD results for the heavy quark diffusion coefficient.
[b] See T06: A. Kumar, 04/06/2022, 12:30, Computing jet transport coefficient $\hat{q}$ in lattice QCD.
[c] **Acknowledgment:** J.H.W.'s research was funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) - Projektnummer 417533893/GRK2575 "Rethinking Quantum Field Theory".

Heavy quark-antiquark pair and screening (static picture)

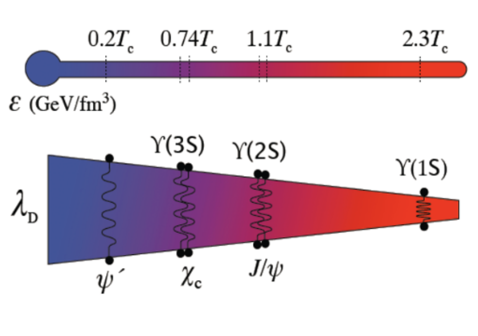

Figure 1: From [2]: the sequential melting hypothesis suggests that more compact quarkonia survive longer.

Assuming a potential model Matsui and Satz proposed in a seminal paper [4] J/Ψ **suppression due to Debye screening** of electric (A_0) gluons as fingerprint signature of QGP formation. These ideas were extended to a picture of **sequential melting** [5]: larger states melt earlier, as the screening length $1/m_D$ reaches their bound state radii already at lower temperatures, see Fig. 1.

Color screening is studied on the lattice traditionally in terms of Polyakov loop correlators. Screening lengths extracted from these correspond to a **Debye mass** of $m_D/T \approx 2.4$ for most phenomenologically interesting temperatures $(0.17-1.5\,\text{GeV})$ [6], and melting at $T(\Upsilon(1S)) \lesssim 0.4\,\text{GeV}$. Similar $T(\Upsilon(1S))$ is inferred from spatial masses of relativistic bottomonia correlators [7].

Heavy quark-antiquark pair at weak coupling

On the one hand, $V_s(T,r)$ is known for $1/r \sim m_D$ in hard-thermal-loop (HTL) approach at leading order (LO) [8]

$$V_s(r,T) = -C_F\alpha_s\left\{\frac{\exp[-rm_D]}{r} + m_D + \mathrm{i}T\phi(rm_D)\right\}, \phi(x) = 2\int_0^\infty \frac{dz\ z}{(z^2+1)^2}\left\{1 - \frac{\sin(zx)}{zx}\right\}, \tag{1}$$

i.e. $\mathrm{Re}\,[V_s] = F_S + \mathcal{O}(g^4)$ and $\mathrm{Im}\,[V_s] \sim \mathcal{O}(g^2T)$ suggesting a **weak-binding scenario**. On the other hand, $V_s(T,r)$ is known for $\Delta V \ll m_D \ll T \ll 1/r$ in potential Non-Relativistic QCD (pNRQCD) [9]

$$V_s(r,T) = \frac{-C_F\alpha_s}{r} + r^2T^3\left\{\mathcal{O}(g^4) + \mathrm{i}\mathcal{O}\left(g^4, \frac{g^6}{(rT)^2}\right)\right\}, \tag{2}$$

i.e. $\mathrm{Re}\,[V_s] = V_s + \mathcal{O}(g^4)$ and $\mathrm{Im}\,[V_s] \sim \mathcal{O}(g^4r^2T^3, g^6T)$ suggesting a **strong-binding scenario**.

The presence of an **imaginary part** scaling as $\mathrm{Im}\,[V_s] \sim g^{2n}T$ suggests **dynamic dissociative processes** matter, which were not considered by Matsui and Satz [4]. Since at phenomenologically interesting temperatures $g(T) \gtrsim 1$, time scales of **dissociation** $\tau_{\text{diss}} \sim 1/\mathrm{Im}\,[V_s]$ may be much shorter than those of **screening** $\tau_{\text{diss}} \sim 1/m_D$.

Heavy quark-antiquark pair as an open quantum system

Transitions between HF and OF imply that the pair should be treated as an **open quantum system** (OQS) with QGP as environment [10]. Its master equation has Lindblad form in the dilute limit

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_n\left[C_n\,\rho\,C_n^\dagger - \frac{1}{2}\{C_n^\dagger C_n, \rho\}\right], \tag{3}$$

where the density matrix ρ is diagonal between color-singlet or -octet, and between different angular momentum quantum numbers. The system is coupled to QGP (evolved via hydrodynamics) by a set of collapse operators C_n, while H is the vacuum Hamiltonian. In the hierarchy $m \gtrsim 1/a_0 \gg \pi T \sim m_D \gg E$ the C_n are fixed through **heavy-quark transport coefficients** $\kappa(T)$ and $\gamma(T)$ – amenable to LQCD calculation and related to **thermal width** or **mass shift**, repectively [11]. Other parametrizations use $V_s(r,T)$ directly, see [10]. A 1st-principles prediction of bottomonia R_{AA} obtained via quantum trajectories algorithm [12] is shown in Fig. 2.

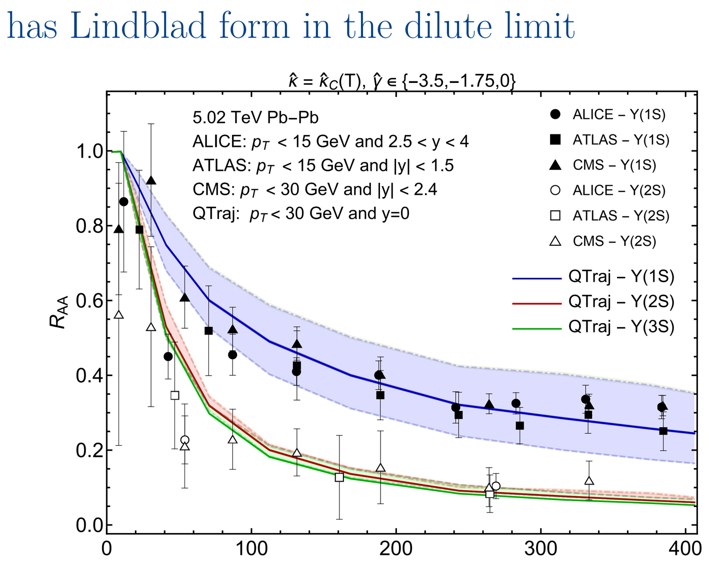

Figure 2: From [13]: R_{AA} of S-wave bottomonia obtained in an OQS+pNRQCD approach using the `QTraj` package [12]. Given the data $\gamma \approx 0$ **(small mass shift)** seems to be favored [13].

Heavy quark-antiquark pair on the lattice

The interaction of a static quark-antiquark pair is encoded in real-time Wilson loops

$$W_{[r,T]}(t) = \langle e^{\mathrm{i}g\oint_{r\times t} dz^\mu A_\mu}\rangle_{\text{QCD},T}\,. \tag{4}$$

If a stable ground state $\Omega_{[r,T]}$ (aka the **potential**) exists

$$\Omega_{[r,T]} \equiv -\mathrm{i}\lim_{t\to\infty}\partial_t W_{[r,T]}(t) = \text{ constant}\,. \tag{5}$$

On the lattice we must use imaginary time τ, whose finite extent defines the **inverse temperature** $aN_\tau = \beta$. A stable ground state implies a plateau at large τ in the **1st cumulant** m_1 (aka effective mass: m_{eff})

$$m_1 \equiv -\frac{\partial\ln W(\tau)}{\partial\tau},\ m_{n+1} \equiv -\frac{\partial m_n}{\partial\tau},\ n>1. \tag{6}$$

A state with **non-zero width** may satisfy Eq. (5) but retain **non-zero** m_{n+1} defined as in Eq. (6), see Fig. 3.

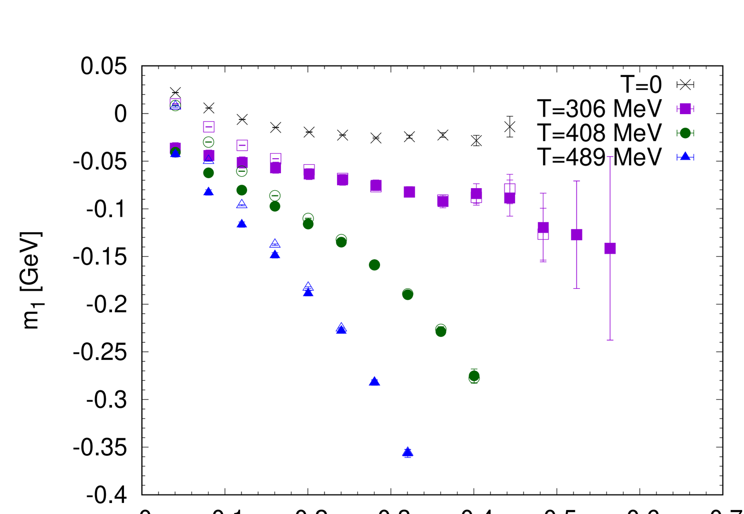

Figure 3: From [1]: m_1 at $r=0.24\,\text{fm}$ for $a=0.04\,\text{fm}$, $N_\tau = 64$, 16, 12, 10 has a plateau for $T=0$ (×). On the one hand, $\partial_\tau m_1$ is rather T-independent at small τ (open); after subtracting the $T=0$ UV part, see Eq. (8), m_1 is quite linear (filled). On the other hand, m_1 steepens at large τ for larger T.

Spectral structure of a heavy quark-antiquark pair

While **real-time dynamics** is not directly accessible in imaginary time, it is encoded in **spectral functions**

$$W_{[r,T]}\begin{pmatrix} t \\ \tau \end{pmatrix} = \int d\omega \begin{pmatrix} e^{+\mathrm{i}\omega t} \\ e^{-\omega\tau} \end{pmatrix}\rho_{[r,T]}(\omega). \tag{7}$$

Assuming well-separated spectral features we may put

$$\rho_{[r,T]} = \rho^{\text{tail}}_{[r,T]} + \rho^{\{\Omega;\Gamma\}}_{[r,T]} + \rho^{\text{UV}}_{[r,T]}\,, \tag{8}$$

as first employed for bottomonia in lattice NRQCD [14]. $\rho^{\{\Omega;\Gamma\}}_{[r,T]}$ encodes what may be a **quasiparticle state** (**potential** $\Omega + \mathrm{i}\Gamma$). $\rho^{\text{UV}}_{[r,T]}$ – dominating at small τ – is rather T-independent, we **estimate it at** $T=0$ and **subtract** it. Lastly, the **low-energy tail** $\rho^{\text{tail}}_{[r,T]}$ exists for extended operators due to an interplay of the system with an excited vacuum. $\rho^{\{\Omega;\Gamma\}}_{[r,T]}$ is independent from details of the operator, whereas $\rho^{\text{UV}}_{[r,T]}$ or $\rho^{\text{tail}}_{[r,T]}$ depend on these [1]. Using Wilson line correlators in Coulomb gauge for $N_\tau = 16, 12, 10$ we have access to m_n for $n \le 3$.

In Fig. 4 we compare m_1 on the lattice to HTL, where m_1 is antisymmetric unter $\tau \to \beta - \tau$[a].

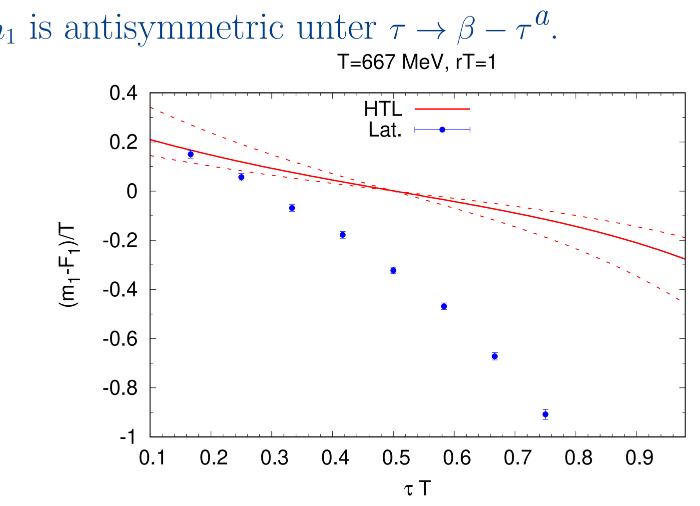

Figure 4: From [1]: $(m_1 - F_S)/T$ at $r=0.29\,\text{fm}$ on the lattice for $a=0.024\,\text{fm}$, $N_\tau = 12$ is much steeper than in HTL. Antisymmetry in a narrow window $\tau \approx \beta/2$ is not ruled out.

[a] $\sigma^{\text{HTL}}_{[r,T]}(\omega)$ is the HTL transverse gluon spectral function.

Quasiparticle feature of a heavy quark-antiquark pair

With limited number of independent lattice data, we can constrain only few parameters. We infer **position** and **width** of the **quasiparticle** feature shown in Fig. 5 via **four conceptually different methods** [1].

I UV-subtraction + **Gaussian** as regularized Breit-Wigner for $\rho^{\{\Omega;\Gamma\}}_r(\omega)$ + delta peak for $\rho^{\text{tail}}_r(\omega)$: (cf. [14])

$$W^{\text{sub}}_{[r,T]}(\tau) = A^G_{[r,T]}\exp\left[-\Omega^G_{[r,T]}\tau + (\Gamma^G_{[r,T]})^2\tau^2/2\right] + A^{\text{tail}}_{[r,T]}\exp\left[-\omega^{\text{tail}}_{[r,T]}\tau\right],\quad \omega^{\text{tail}}_{[r,T]} \ll \Omega_{[r,T]}\,. \tag{9}$$

II Fit via **HTL-inspired Ansatz** for lowest spectral feature in narrow window around $\tau \approx \beta/2$: (cf. [15])

$$W_{[r,T]}(\tau) = A^{BD}_{[r,T]}\exp\left[-\Omega^{BD}_{[r,T]}\tau - \tfrac{1}{\pi}\Gamma^{BD}_{[r,T]}\log\sin(\pi\tau T)\right]\,. \tag{10}$$

III Fourier transform → **Padé** rational interpolation → analytic continuation → **lowest pole**. (cf. [16])

IV **Bayesian reconstruction** (BR) works only at lowest temperature (needs positive weights). (cf. [17])

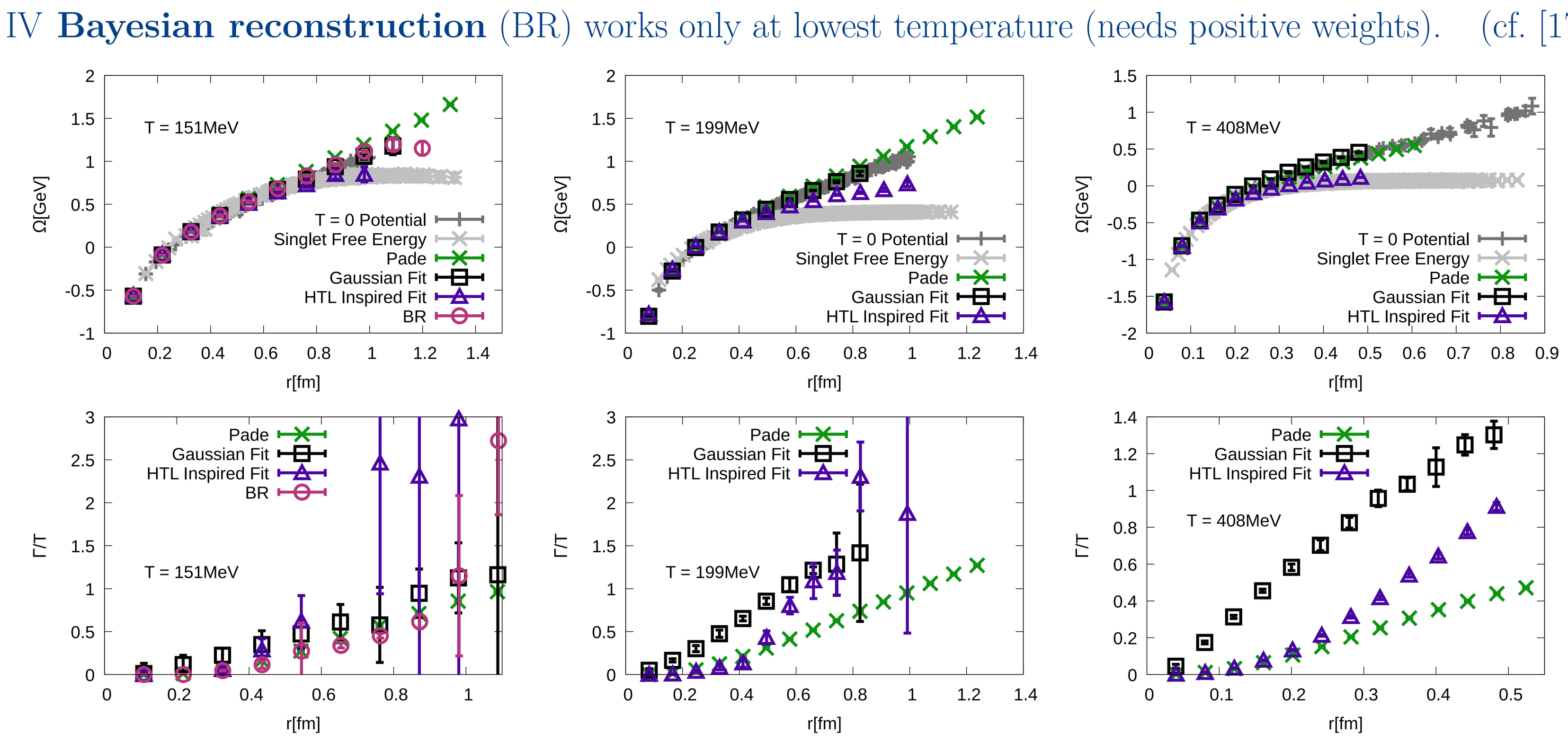

Figure 5: From [1]: All methods are consistent at $T=151\,\text{MeV}$, where $\Omega \approx V_s \approx F_S$. At higher temperatures, $\Omega \approx V_s$ for Gaussian fit or Padé (I&III), with $V_s > \Omega \gtrsim F_S$ for the HTL-inspired fit (II). All methods identify at all temperatures a **non-zero width** Γ **that increases with** r **and with** T, although at higher temperatures differently in all approaches.

Conclusions and Outlook

The perturbative [8, 9] or non-perturbative [1] evidence for **dissociation and thermal widths** is overwhelming – the original idea of Matsui and Satz [4] is missing this key ingredient. Although various conceptually independent results [1, 13, 14] seem to be pointing towards **small mass shifts and large widths**, this is still **under investigation** [1, 15]. A simple explanation for similar melting temperatures inferred from **color screening** [6] or model independent analyses [7] is not available given that a **width** could only expedite it.

The presented study [1] has a resolution of imaginary time that is insufficient to identify cumulants higher than m_3. A study that remedies this through finer lattices and larger $N_\tau = 20, \ldots, 56$ is in preparation [18].

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The 29th International Conference on Ultra-relativistic Nucleus-Nucleus Collisions