# Heavy quark-antiquark interaction in finite temperature lattice QCD

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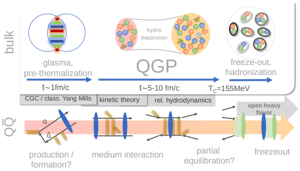
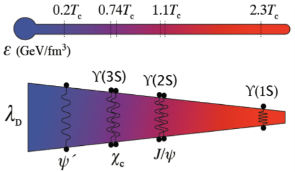
# Heavy $q\bar{q}$ pairs as hard probes in heavy-ion collisions?



source: *Rothkopf, Phys.Rept. 858 (2020) 1-117*



source: *USQCD WP 2018, EPJ A 55 (2019)*

- Hard probes are produced in **hard processes** in initial stages
- Important: *jets*[a], open heavy flavor[b] & **heavy quarkonia**

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[a] T06: A. Kumar, 04/06/2022, 12:30

[b] T06: L. Altenkort, 04/06/2022, 12:10

- Idea to look at **quarkonia** in QGP is old and famous *Matsui, Satz, PLB 178 (1986)*
- **Debye screening** length $^1/_{m_D}$ of electric gluons ($A_0$) limits the bound state radii
- Fingerprint of QGP formation ⇔ sequential **quarkonia suppression**

Spatial meson correlators in LQCD confirm such melting temperatures

*Bazavov et al., PRD 91 (2015); Petreczky et al., PRD 104 (2021)*

Debye-screened static free energies suggest similar melting points

*Bazavov et al., PRD 98 (2018); Petreczky et al., PoS(LAT2021) 471*

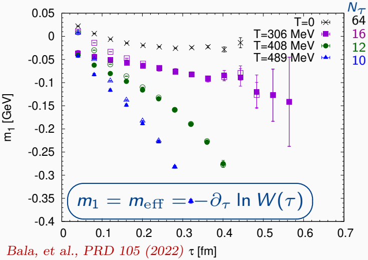
# Screening is not the whole story... (at weak coupling)

Matsui & Satz's idea of the **quarkonium suppression mechanism** was turned inside out by **weak-coupling EFT results** emerging 15 years ago

- For ${}^{1}/_{r} \sim m_D \sim gT \ll T$: $\mathrm{Re}\,[V_s] = F_S + \mathcal{O}(g^4)$ & $\mathrm{Im}\,[V_s] \sim \mathcal{O}(g^2 T)$ **HTL@LO**
  *Laine, et al., JHEP 03 (2007)*
- For $\Delta V \ll m_D \ll T \ll {}^{1}/_{r}$: $\mathrm{Re}\,[V_s] = V_s + \mathcal{O}(g^4)$ & $\mathrm{Im}\,[V_s] \sim \mathcal{O}(g^4 r^2 T^3, g^6 T)$
  *Brambilla, et al., PRD 78 (2008)*
- **Weak- vs strong-binding** $\mathrm{Re}\,[V_s]$ depending on the hierarchies
- **Imaginary parts** leading to **dissociation** – no stable ground state

- Does a potential even exist at $T > T_c$ for thermal coupling $g(T) \gtrsim 1$?
- Shorter time scales for **dissociation** than **screening** ${}^{1}/_{g^{2n}T} \ll {}^{1}/_{m_D}$
- Dissociation into open heavy flavor ⇒ **open quantum system**
  TUM/KSU or Stavanger/Osaka coll.; complex $T > 0$ potential is input
- We study the non-perturbative **complex $T > 0$ potential** in LQCD

# Static $q\bar{q}$ pair at $T > 0$ on the lattice



$$m_1 = m_{\mathrm{eff}} = -\partial_\tau \ln W(\tau)$$

$a \approx 0.04$ fm, $r \approx 0.25$ fm

source: *Bala, et al., PRD 105 (2022)*

- **Static $q\bar{q}$** interaction is encoded in (real-time) **Wilson loops**[a]

$$W_{[r,T]}(t) = \left\langle e^{\,ig \oint_{r\times t} dz^\mu A_\mu} \right\rangle_{\mathrm{QCD},T}$$

- Stable (ground) state $\Omega_r$ exists if

$$\Omega_{[r,T]} \equiv -\mathrm{i} \lim_{t\to\infty} \partial_t W_{[r,T]}(t)$$

- Same **spectral functions** yield real- or imaginary-time correlators

$$W_{[r,T]} \begin{pmatrix} t \\ \tau \end{pmatrix} = \int d\omega \begin{pmatrix} e^{+i\omega t} \\ e^{-\omega\tau} \end{pmatrix} \rho_{[r,T]}(\omega)$$

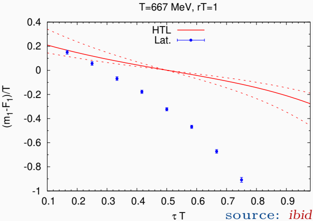
- Motivates **generic decomposition**

$$\rho_{[r,T]}(\omega) = \rho^{\mathrm{tail}}_{[r,T]}(\omega) + \rho^{\{\Omega;\Gamma\}}_{[r,T]}(\omega) + \rho^{\mathrm{UV}}_{[r,T]}(\omega)$$

- Premise: UV continuum is far above **quasiparticle feature** $\{\Omega, \Gamma\}$

$\Rightarrow$ Guess $\rho^{\mathrm{UV}}_{[r,T]}(\omega)$ via $\rho^{\mathrm{UV}}_{[r,0]}(\omega)$ $\Rightarrow$ subtract

**Note**: "tail" due to backward propagating UV physics (vacuum excited states) at $\tau \lesssim 1/T$.



$N_\tau = 12$, UV-subtracted

source: *ibid.*

- Antisymmetry of **HTL@LO** $(m_1 - F_S)$ not ruled out for $\tau T \approx 1/2$ in $(m_1 - F_S)$
- **HTL@LO** $(m_1 - F_S) = 0 > (m_1 - F_S)$ for $\tau T = 1/2$: $-\partial_\tau m_1 \gg -\partial_\tau m_1$

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[a] We use Wilson line correlators in Coulomb gauge.

# Strategy: quasiparticle feature from four different methods

We can constrain only few parameters with limited number of data ($N_\tau$), and infer $\Omega$, $\Gamma$ of the **quasiparticle** via **four conceptually different methods**.

*Bala, et al., PRD 105 (2022)*

1. UV-subtraction + **Gaussian** + delta peak: *Larsen, et al. PRD 100 (2019)*

$$W^{\mathrm{sub}}_{[r,T]}(\tau) = A^{G}_{[r,T]} e^{-\Omega^{G}_{[r,T]}\tau + (\Gamma^{G}_{[r,T]})^2 \frac{\tau^2}{2}} + A^{\mathrm{tail}}_{[r,T]} e^{-\omega^{\mathrm{tail}}_{[r,T]}\tau}, \quad \omega^{\mathrm{tail}}_{[r,T]} \ll \Omega^{G}_{[r,T]} .$$

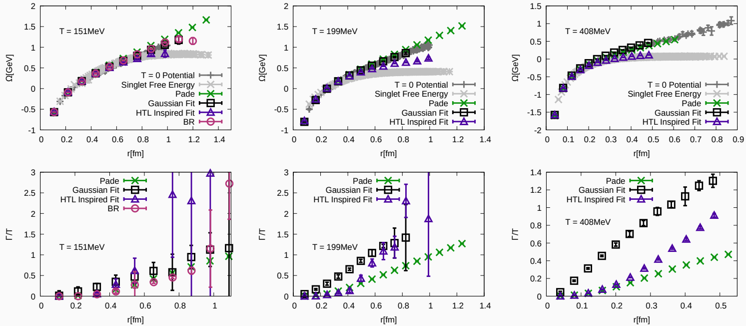
2. Fit via **HTL-inspired Ansatz** in window $\tau T \approx 1/2$: *Bala, Datta, PRD 101 (2020)*

$$W_{[r,T]}(\tau) = A^{BD}_{[r,T]} e^{-\Omega^{BD}_{[r,T]}\tau - \frac{\mathrm{i}}{\pi}\Gamma^{BD}_{[r,T]} \log\sin(\pi\tau T)} .$$

3. Fourier transform → **Padé** rational interpolation → analytic continuation → **lowest pole**. *Tripolt, et al., PLB 774 (2017); Tripolt, et al., CPC 237 (2019)*

4. **Bayesian reconstruction** (BR) works only at lowest temperature (needs positive weights). *Burnier, Rothkopf, PRL 111 (2013)*

# Comparison: complex $T > 0$ potential from four different methods



source: *Bala, et al., PRD 105 (2022)*

- $T \approx 150\,\mathrm{MeV}$ conclusive: $\Omega_{[r,T]} \approx F_S(r,T) \approx V_s(r)$ for $r \lesssim 0.8\,\mathrm{fm}$
- $T \lesssim 250\,\mathrm{MeV}$: all three methods yield $\Omega_{[r,T]} \gg F_S(r,T)$
- $T \approx 400\,\mathrm{MeV}$ inconclusive: $\Omega^{BD}_{[r,T]} \approx F_S(r,T)$ vs $\Omega^{G}_{[r,T]} \approx \Omega^{P}_{[r,T]} \approx V_s(r)$
- All methods find for all $T$ nontrivial $\Gamma_{[r,T]}$ that increases with $r$ or $T$

# Heavy $q\bar{q}$ interaction in finite temperature lattice QCD

i. **Spatial** correlation functions using **relativistic heavy quarks**
   $\Rightarrow$ Model-independent lattice studies of quarkonia melting
ii. **Polyakov loop correlators** (static picture → Debye mass $m_D/T \approx 2.4$)
   $\Rightarrow$ Compatible with realistic melting temperatures
iii. **Static quarkonia (static $q\bar{q}$ pair)** *Bala, et al., PRD 105 (2022), PoS(LAT2021) 515*
   - Robust **quasiparticle feature** $\{\Omega;\Gamma\}$ + tail + UV continuum
   - Model-independent cumulant analysis → robust evidence for **large thermal width** $\Gamma$ ($\gg \Gamma^{\mathrm{HTL}}$) being important for quarkonia melting
   - Still inconclusive wrt. thermally modified or vacuum-like real part $\Omega$
iv. **Nonrelativistic bottomonia** in lattice NRQCD *Larsen et al. PRD 100 (2019); …*
   - Extended sources or BS wave functions boost resolving power of LQCD
   - **Large widths** $\Gamma$ ($\gg \Gamma^{\mathrm{HTL}}$), but no large mass shifts
   - ML reconstructed potential: **large width** $\Gamma$ ($\gg \Gamma^{\mathrm{HTL}}$), no screening
v. **OQS+pNRQCD** (TUM/KSU): 1st-principles, non-Abelian evolution
   - heavy-quark transport coefficients couple to hydrodynamic medium
   - mildly sensitive to **width** ($\propto \kappa$), but prefers **small mass shift** ($\propto \gamma$)

LQCD predicts a **width**, but inconclusive wrt. weak vs strong binding.

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# At which $T$ are there either bound states or melted $q\bar{q}$ pairs?

- **Spatial $q\bar{q}$ pair correlators** are a **model-independent** analysis tool

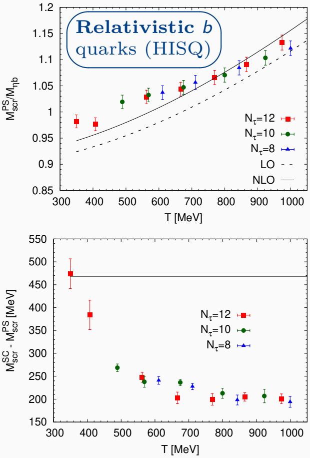
*charm sector* ⇒ *Bazavov, et al., PRD 91 (2015)*

$$G_T(z) = \int_0^{1/T} d\tau \int d^2x_\perp \left\langle \mathcal{J}(\tau, \boldsymbol{x}_\perp, z)\mathcal{J}^\dagger(0)\right\rangle$$

$$= \int_0^{\infty} \frac{2d\omega}{\omega} \int_{-\infty}^{\infty} dp_z e^{ip_z z} \rho_T(\omega, p_z) \overset{z\to\infty}{\sim} e^{-M(T)z}$$

with **spectral function** $\rho_T(\omega, p_z)$

$$\sim \begin{cases} \delta\left[\omega^2 - p_z^2 - M_0^2\right] & \text{mesons} \\ \delta\left[\omega - \sum_{q_i} \sqrt{m_{q_i}^2 + [\pi T]^2}\right] & \text{free quarks} \end{cases}$$



source: *Petreczky, et al., PRD 104 (2021)*

- Survival of $\eta_b$ & $\Upsilon(1S)$ until $T \approx 400\,\mathrm{MeV}$; cf. $\eta_c$ & $J/\Psi$ until $T \approx 200\,\mathrm{MeV}$
- Survival of $\chi_{b0}$ & $h_b$ until $T \approx 350\,\mathrm{MeV}$; cf. $\chi_{c0}$ & $\chi_{c1}$ until $T \sim T_{\mathrm{pc}}$
- How can we understand the **melting mechanism** at work?

# Screening from Polyakov loop correlators

- **Color screening** usually studied via **Polyakov loop correlator**

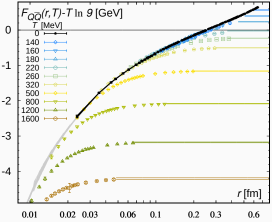
$$C_P(r,T) = \langle P(0)P^\dagger(\boldsymbol{r})\rangle_T^{\mathrm{ren}} = e^{-F_{Q\bar{Q}}(r,T)/T}$$

- $rT \ll 1$: **singlet**/**octet** decomposition

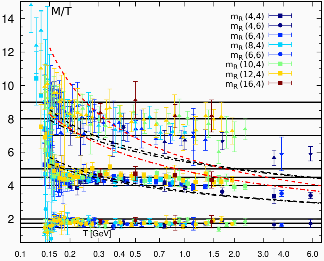
$$C_P(r,T) = {}^1/_9 e^{-F_S(r,T)/T} + {}^8/_9 e^{-F_O(r,T)/T}$$

- $rT \lesssim 0.4$: via $T=0$ potentials and adjoint Polyakov loop: no screening!

$$C_P(r,T) = {}^1/_9 e^{-V_s(r)/T} + L_A(T)\ {}^8/_9 e^{-V_o(r)/T} + \mathcal{O}(\alpha_s^3)$$



source: *Brambilla, et al., PRD 98 (2018)*



⇒ *Petreczky, et al., PoS(LATTICE2021) 471*

- $rm_D \gtrsim 1$: screening regime; decompose

$$\begin{aligned} C_P(r,T) &= C_R(r,T) + C_I(r,T) \\ C_R(r,T) &= \langle \mathrm{Re}\,P(0)\mathrm{Re}\,P(\boldsymbol{r})\rangle_T^{\mathrm{ren}} \\ C_I(r,T) &= \langle \mathrm{Im}\,P(0)\mathrm{Im}\,P(\boldsymbol{r})\rangle_T^{\mathrm{ren}} \end{aligned} \quad \to \mathcal{C} \ \begin{matrix}\text{even}\\ \text{odd}\end{matrix}$$

- Asymptotically 1PE: $C_{R,I}(r,T) \sim e^{-m_{R,I}r}/rT$
- EQCD: $m_R \sim 2m_D$, $m_I \sim 3m_D$ ($m_D$ per $A_0$)
  LQCD: $\frac{m_R}{T} \approx 4.5$, $\frac{m_I}{T} \approx 8$, $\frac{m_I}{m_R} \approx 1.75$

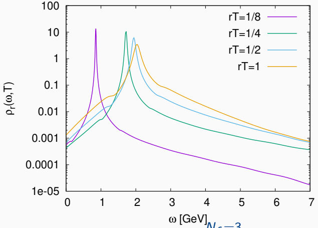
$$\Rightarrow \quad {}^1/_{m_D} \approx {}^2/_{m_R} \approx {}^3/_{m_I} = \{0.38 - 0.44\}/T$$

$r_{\Upsilon(1S)} \approx 0.21$ fm survives until $T \sim 380$ MeV

$r_{J/\Psi} \approx 0.43$ fm survives until $T \sim 190$ MeV

# Comparison: lattice QCD vs HTL

**HTL spectral function** for $T = 667$ MeV



[NLO, 2-loop $\alpha_s(2\pi T)$, $\Lambda^{N_f=3}_{\overline{\mathrm{MS}}} = 332$ MeV]

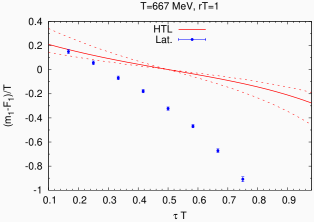
source: *Bala, et al., PRD 105 (2022)*

- **HTL** is an attractive proposition: **motivated & regularized BW**
- **HTL** result is **antisymmetric** around the midpoint $\tau = {}^1\!/_{2T}$:

$$\log W_{[r,T]}(\tau) = -\mathrm{Re}\, V_s(r,T) \times \tau + \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \left\{ e^{-\omega\tau} + e^{-\omega(1/T-\tau)} \right\} \times \{1 + n_B(\omega)\}\, \sigma_{[r,T]}(\omega)$$

- Leading **singularity** of $\sigma_{[r,T]}(\omega)$ (transv. gluon spec. fun.) fixes $\mathrm{Im}\, V_s(r,T)$

$N_\tau = 12$, ${}^r\!/_a = 12$, subtracted correlator

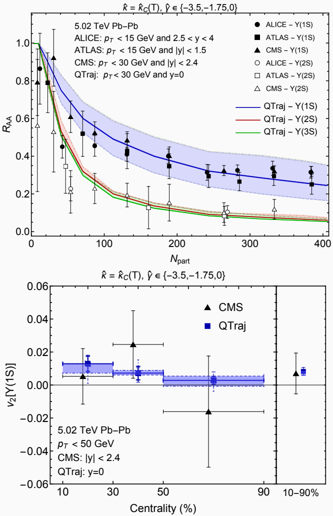


source: *Bala, et al., PRD 105 (2022)*

- **HTL** should work at $r \sim {}^1\!/_{m_D}$
- *Subtleties* due to renormalons and regulators: consider $(m_1 - F_S)/T$

  Reminder: $\mathrm{Re}\,[V_s] = F_S + \mathcal{O}(g^4)$ in **HTL**
- No large UV component in HTL, compare UV-subtracted result
- $m_1$ at midpoint lower than **HTL**, $m_2 = -\partial_\tau m_2$ is more negative

# Quarkonium melting in an Open Quantum System approach



source: *Brambilla, et al., PRD 104 (2021)*

If $M \gtrsim {}^1/_{a_0} \gg \pi T \sim m_D \gg E$

$\Rightarrow$ master equation has a Lindblad form, is discretized and solved stochastically[a] $\longrightarrow$ `QTraj`

- Temperature dependence from hydrodynamics evolution using lattice QCD **equation of state**
- For strongly-coupled plasma: $T$ dependence via **heavy-quark transport coefficients** $\kappa$, $\gamma$
- Lattice transport coefficients & EoS in OQS+pNRQCD approach: **quarkonium suppression**

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[a] *Brambilla, et al., JHEP 05 (2021) + PRD 104 (2021); Ba Omar, et al., CPC 273 (2022)*