Corrections to the hadron resonance gas from lattice QCD and their effect on fluctuation-ratios at finite density*

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Ideal Hadron Resonance Gas (HRG) and fugacity expansion

The ideal Hadron Resonance Gas (HRG) model simply sums over **contributions from all known hadronic resonances:**

$$P\left(T,\,\hat{\mu}\right)=\sum_{i\in\text{hadrons}}P_i\left(T,\,\hat{\mu}\right)=-\frac{T}{V}\sum_{i\in\text{hadrons}}\ln\mathcal{Z}_i\left(T,\,\hat{\mu}\right)$$

While overall successful in describing QCD thermodynamics, increased precision in lattice results suggest small deviations.

Due to charge conjugation symmetry, an **expansion in the fugacity** $e^{\mu}=e^{(B\mu_B+S\mu_S)}$ yields:

$$P(T,\,\hat{\mu}_B,\,\hat{\mu}_S)\simeq\sum_{i\in\text{hadrons}}P_i(T)\,e^{(B_i\,\hat{\mu}_B+S_i\,\hat{\mu}_S)}=\sum_{j,k}P^{BS}_{jk}(T)\,\cosh(j\,\hat{\mu}_B-k\,\hat{\mu}_S)$$

In an ideal hadron gas, only nonzero contributions come from:

- P^{BS}_{00}: π, f, ω, ρ, ...
- P^{BS}_{01}: K, ...
- P^{BS}_{10}: p, n, Δ, ...
- P^{BS}_{11}: Λ, Σ, ...
- P^{BS}_{12}: Ξ, ...
- P^{BS}_{13}: Ω, ...

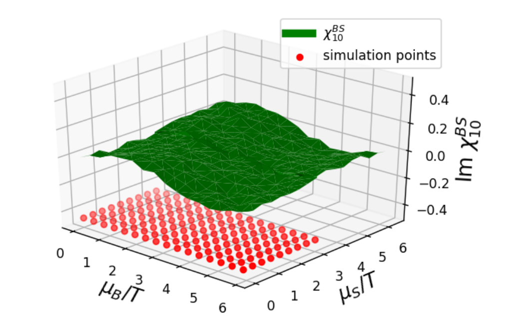
In general, additional terms exist, due to composite or exotic objects, or interactions

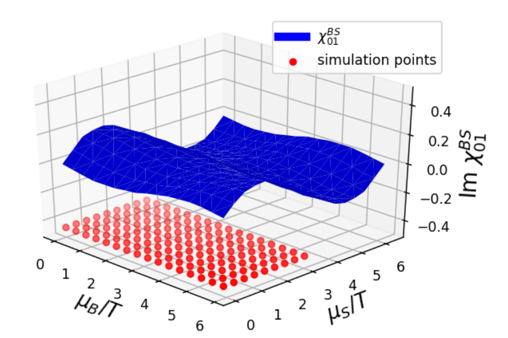
- P^{BS}_{20}: p-p, ...
- P^{BS}_{21}: p-Λ, ...
- P^{BS}_{02}: K-K, ...
- P^{BS}_{22}: Λ-Λ, p-Ξ ...
- $P^{BS}_{1,-1}$: pentaquark (e.g. $udd\bar{d}s$), ...
- and so on...

From fugacity expansion to Fourier expansion

Lattice simulations, not currently possible at finite (real) chemical potential, can be done at imaginary chemical potential ($\hat{\mu}_a = i\,\hat{\mu}_a^I$). In this case, one gets a Fourier expansion:

$$P(T,\, \hat{\mu}_B,\, \hat{\mu}_S) = \sum_{j,k} P_{jk}^{BS}(T)\, \cos(j\,\hat{\mu}_B^I - k\,\hat{\mu}_S^I)$$





We performed a 2D scan in the chemical potentials plane at $(\mu_B, \mu_S) = \frac{\pi}{8}(i, j)$, with $i = 0, ..., 15$, $j = 0, ..., 8$ (144 points) for $T = 145, 150, 155, 160\,\mathrm{MeV}$

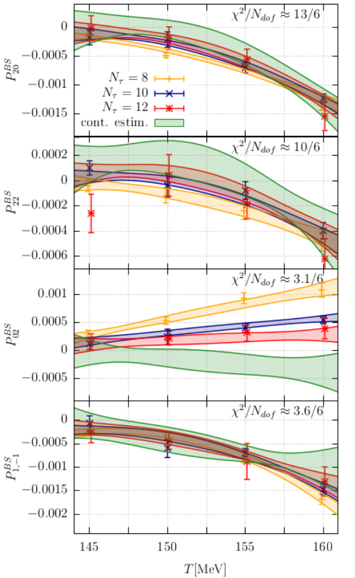
The fugacity/Fourier expansion coefficients

We calculated different observables containing the $P^{BS}_{ij}(T)$, *temperature-by-temperature*, at all simulation points:

$$\begin{aligned}
\mathrm{Im}\,\chi^{BS}_{10} &= \textstyle\sum_{j,k} j P^{BS}_{jk}(T)\sin(j\hat\mu^I_B - k\hat\mu^I_S)\\
\mathrm{Im}\,\chi^{BS}_{01} &= \textstyle\sum_{j,k} (-k) P^{BS}_{jk}(T)\sin(j\hat\mu^I_B - k\hat\mu^I_S)\\
\chi^{BS}_{20} &= \textstyle\sum_{j,k} j^2 P^{BS}_{jk}(T)\cos(j\hat\mu^I_B - k\hat\mu^I_S)\\
\chi^{BS}_{11} &= \textstyle\sum_{j,k} (-jk) P^{BS}_{jk}(T)\cos(j\hat\mu^I_B - k\hat\mu^I_S)\\
\chi^{BS}_{02} &= \textstyle\sum_{j,k} k^2 P^{BS}_{jk}(T)\cos(j\hat\mu^I_B - k\hat\mu^I_S)
\end{aligned}$$

At each T, we perform a global fit to determine the $P^{BS}_{ij}(T)$.

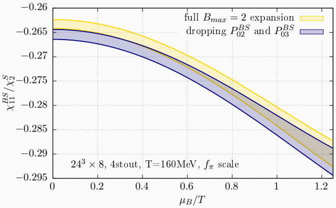
- We see in general a mild N_τ- dependence, meaning discretization errors are under control
- At most temperatures, most contributions are non-zero. **Interactions and exotic states do play a role.**
- We can then reconstruct the pressure and its derivatives **also at finite (real)** $\hat\mu_B$

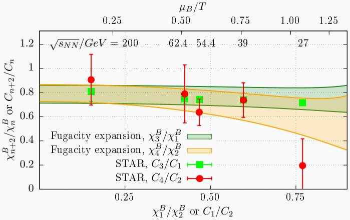


Fugacity expansion: fluctuation ratios at real $\hat{\mu}_B$

i. **A proxy for baryon-strangeness correlation:** In **PRD 101 034506** we proposed $\sigma^2_\Lambda / \left(\sigma^2_K + \sigma^2_\Lambda\right)$ to be a good proxy for χ^{BS}_{11}/χ^S_2, even after including some experimental effects. The inclusion of multi-kaon interaction terms P^{BS}_{02}, P^{BS}_{03} has a small effect, reinforcing predictions from ideal hadronic model

ii. **Comparison with STAR data**: We are able to compare to χ^B_3/χ^B_1 and χ^B_4/χ^B_2 at different energies. Results are consistent with Taylor expansions (**PRD 101 074502** from HotQCD). Notably, fluctuations in the experiment seem not to deviate largely from QCD matter in equilibrium





(Extrapolated fixing $n_S = 0$)