A fast method to compute conserved charge cumulants in hydro simulations

R.Hirayama (FIAS-Germany)
F.Grassi & W.M.Serenone (Univ. de São Paulo-Brazil)
J.-Y. Ollitrault (Univ. Paris Saclay-France)

Objective

Hydrodynamics

▶ describes bulk observables,
▶ allows to implement experimental kinematic cuts taking in account flow,
▶ incorporates event-to-event fluctuations
  (not included in lattice or pure hadron gas calculations)

BUT

▶ requires a large number of runs (> $10^6$) for cumulants
▶ HERE: we suggest a shortcut.
Method

Definitions

- $N$ (such as number of protons) measured in each collision
- Moments: $\mu_n \equiv \langle N^n \rangle$ (average on events in centrality class)
  
  can be obtained from derivatives of $\langle e^{zN} \rangle = \sum_{n=0}^{\infty} \mu_n \frac{z^n}{n!}$

- Cumulants: $C_1 = \mu_1$, $C_2 = \mu_2 - \mu_1^2$, $C_3 = \mu_3 - 3\mu_2\mu_1 + 2\mu_1^3$, ...

Two-step averaging

$\langle e^{zN} \rangle = \langle \langle e^{zN} \rangle_{fo} \rangle_{ic}$

1. $\langle e^{zN} \rangle_{fo}$: 1 hydro event has several possible outcomes at f.out
   
   (typically 2500 MCs) $\rightarrow$ mean value $\bar{N}$, mean max. value $N_{\text{max}}$

   Assume

   $\langle e^{zN} \rangle_{fo} = (1 - \alpha + \alpha e^z)\bar{N}/\alpha$ w/ $\alpha \equiv \bar{N}/N_{\text{max}}$ (proba. proton seen in detector)

2. $\langle \rangle_{ic}$ average on hydro events (typically 200 in a 1% centr. class)

$C_1 = \langle \bar{N} \rangle_{ic}$

$C_2 = \langle \bar{N}^2 \rangle_{ic} - \langle \bar{N} \rangle_{ic}^2 + (1 - \alpha)\langle \bar{N} \rangle_{ic}$ = variance of $\bar{N}$ over IC + average variance of f.out binomial distribution

$C_3$, $C_4$ have fluctuations in IC and at f.out intertwined
Results for $p$, $\bar{p}$, $p - \bar{p}$ in Au+Au at $\sqrt{s_{NN}} = 200$ GeV

NeXSPheRIO predicts well $dN/dy$ for $p - \bar{p}$ but not for $p$, $\bar{p}$ → 1) height re-scaled ($= C_1$ is input), 2) using shapes from 3-source [1] and blast wave [2] models: $\alpha_p = 0.042$ and $\alpha_{\bar{p}} = 0.056$

Excellent agreement [3]

- $p$, $\bar{p}$ almost Poisson ($C_1 \sim C_2 \sim C_3 \sim C_4$) and $p - \bar{p}$ almost Skellam ($C_1 \sim C_3$, $C_2 \sim C_4$)
- BUT non-trivial canceling between freeze out fluctuations (5% effect on $C_2$ to 30% on $C_4$) and initial-state fluctuations.
Results for net charge

NexSPheRIO underestimates the charge excess by $\sim 1$ particle $\rightarrow$ 1) height re-scaled ($= C_1$ is input), 2) using data [4] and assuming $\alpha_+ \sim \alpha_- : \alpha_\pm \sim 0.09$

Reasonable agreement [5]
- $C_2$ slightly overestimated, $C_3 \sim C_4 \sim 0$ i.e net-charge fluctuations almost gaussian
- BUT non-trivial canceling between freeze out fluctuations and initial-state fluctuations.
Conclusion

First (as far as we know) hydro calculations of cumulants including both initial-state and freeze out fluctuations: obtained with two-step factorization

Here: (Non-trivial) agreement with data at top RHIC energies

W. Serenone: Improved calculation (no rescaling, direct calculation of $\alpha$) with AMPT+MUSIC agrees with data at various energies

Assumptions: initial-state and freeze out fluctuations dominate, freeze out distribution is binomial, etc $\rightarrow$ confirm or improve

Two-step factorization may allow to explore effects of a critical point in the equation of state, initial conditions, etc, for hydro cumulants.