A fast method to compute conserved charge cumulants in hydro simulations

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Objective

Hydrodynamics

- describes bulk observables,
- allows to implement experimental kinematic cuts taking in account flow,
- incorporates event-to-event fluctuations
  (not included in lattice or pure hadron gas calculations)

BUT

- requires a large number of runs ($> 10^6$) for cumulants
- HERE: we suggest a <u>shortcut</u>.

Based on arXiv:2204.xxxx

Method

Definitions

- N (such as number of protons) measured in each collision
- Moments: $\mu_n \equiv \langle N^n \rangle$ (average on events in centrality class) can be obtained from derivatives of $\langle e^{zN} \rangle = \sum_{n=0}^{\infty} \mu_n \frac{z^n}{n!}$
- Cumulants: $C_1 = \mu_1,\ C_2 = \mu_2 - \mu_1^2,\ C_3 = \mu_3 - 3\mu_2\mu_1 + 2\mu_1^3,\ \ldots$

Two-step averaging

$$\boxed{\langle e^{zN} \rangle = \langle \langle e^{zN} \rangle_{\rm fo} \rangle_{\rm ic}}$$

1. $\langle e^{zN} \rangle_{\rm fo}$: 1 hydro event has several possible outcomes at f.out (typically 2500 MCs) $\rightarrow$ mean value $\bar{N}$, mean max. value $N_{\rm max}$
   Assume

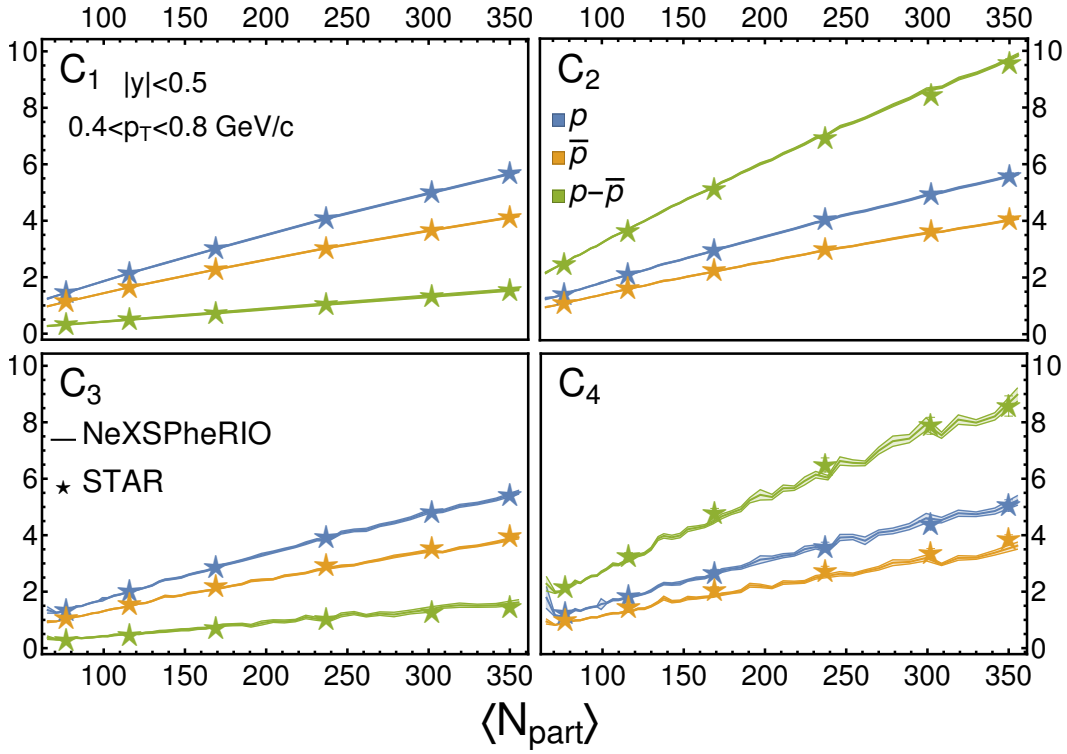
$$\boxed{\langle e^{zN} \rangle_{\rm fo} = (1 - \alpha + \alpha e^z)^{\bar{N}/\alpha} \text{ w/ } \alpha \equiv \bar{N}/N_{\rm max} \text{ (proba. proton seen in detector)}}$$

2. $\boxed{\langle\ \ \rangle_{\rm ic} \text{ average on hydro events}}$ (typically 200 in a 1% centr. class)

$C_1 = \langle \bar{N} \rangle_{\rm ic}$
$C_2 = \langle \bar{N}^2 \rangle_{\rm ic} - \langle \bar{N} \rangle_{\rm ic}^2 + (1-\alpha)\langle \bar{N} \rangle_{\rm ic}$=variance of $\bar{N}$ over IC+ average variance of f.out binomial distribution
C_3, C_4 have fluctuations in IC and at f.out intertwined

Results for p, $\bar{p}$, $p-\bar{p}$ in Au+Au at $\sqrt{s_{NN}} = 200$ GeV

NexSPheRIO predicts well dN/dy for $p-\bar{p}$ but not for p, $\bar{p}$ $\longrightarrow$ 1) height re-scaled (= C_1 is input), 2) using shapes from 3-source [1] and blast wave [2] models: $\alpha_p = 0.042$ and $\alpha_{\bar{p}} = 0.056$

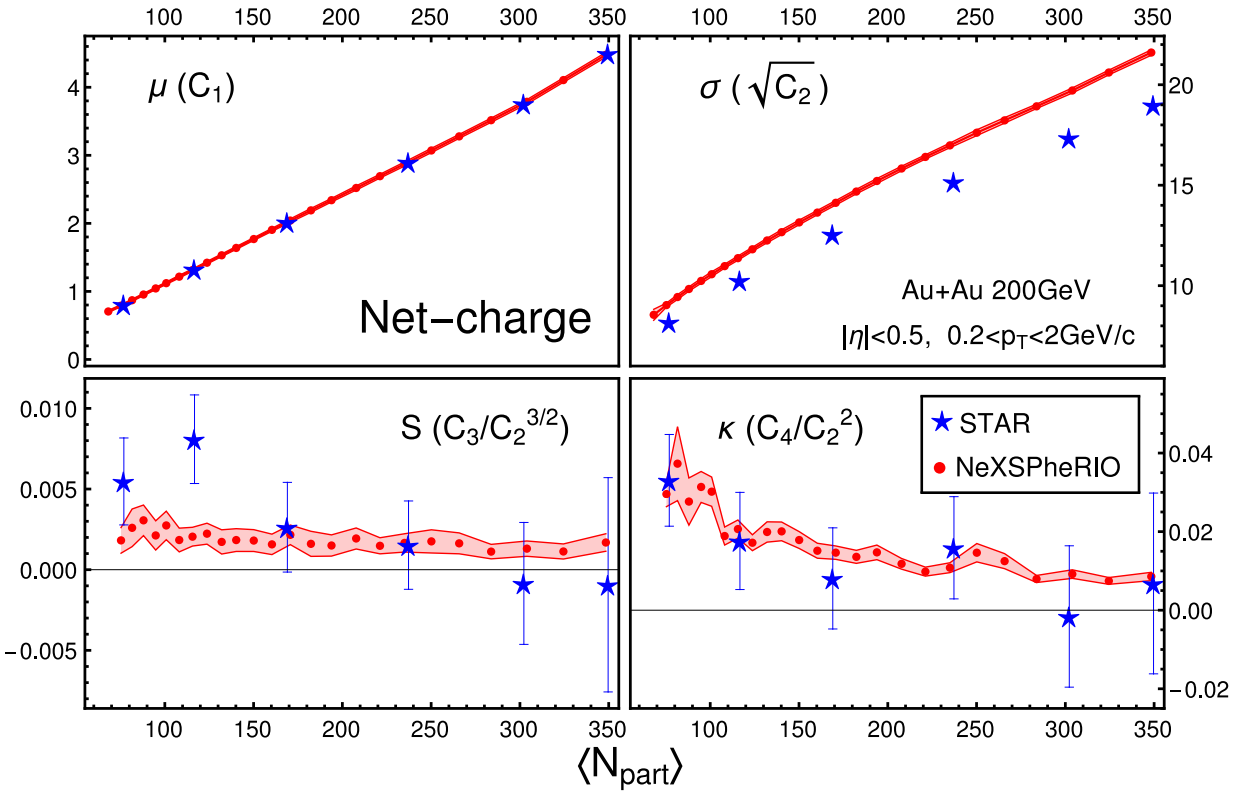


Excellent agreement [3]

- p, $\bar{p}$ almost Poisson ($C_1 \sim C_2 \sim C_3 \sim C_4$) and $p-\bar{p}$ almost Skellam ($C_1 \sim C_3$, $C_2 \sim C_4$)
- BUT non-trivial canceling between freeze out fluctuations (5% effect on C_2 to 30% on C_4) and initial-state fluctuations.

Results for net charge

NexSPheRIO underestimates the charge excess by ~ 1 particle $\longrightarrow$ 1) height re-scaled ($= C_1$ is input), 2) using data [4] and assuming $\alpha_+ \sim \alpha_-$: $\alpha_\pm \sim 0.09$



Reasonable agreement [5]

- C_2 slightly overestimated, $C_3 \sim C_4 \sim 0$ i.e net-charge fluctuations almost gaussian
- BUT non-trivial canceling between freeze out fluctuations and initial-state fluctuations.

Conclusion

- First (as far as we know) hydro calculations of cumulants including both initial-state and freeze out fluctuations: obtained with two-step factorization
- Here: (Non-trivial) agreement with data at top RHIC energies
- W.Serenone: Improved calculation (no rescaling, direct calculation of α) with AMPT+MUSIC agrees with data at various energies
- Assumptions: initial-state and freeze out fluctuations dominate, freeze out distribution is binomial, etc $\longrightarrow$ confirm or improve
- Two-step factorization may allow to explore effects of a critical point in the equation of state, initial conditions, etc, for hydro cumulants.

[1]L. N. Gao, F. H. Liu, Y. Sun, Z. Sun and R. A. Lacey, Eur. Phys. J. A **53** (2017) 61; [2] E. Schnedermann, J. Sollfrank and U. W. Heinz, PRC **48** (1993) 2462; [3] STAR PRC **104**, (2021) 024902; [4] BRAHMS PRL **88** (2002) 202301; [5] STAR PRL **113** (2014) 092301