Quantum statistical fluctuation of energy and baryon number in subsystems of hot and dense relativistic gas

Arpan Das

Institute of Nuclear Physics Polish Academy of Sciences Krakow, Poland

29TH INTERNATIONAL CONFERENCE ON ULTRA – RELATIVISTIC NUCLEUS - NUCLEUS COLLISIONS
Quark Matter 2022, Krakow, Poland

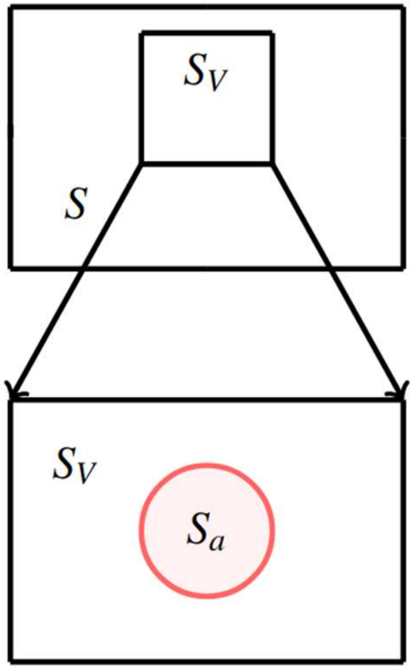
Collaborators: Wojciech Florkowski, Radoslaw Ryblewski, Rajeev Singh

Journal References: Acta Phys. Pol. B **52**, 1395 (2021); Phys.Rev.D 103 (2021) 9, L091502; arXiv:2105.02125.
Funding information: Polish National Science Center Grant No:2018/30/E/ST2/00432

Motivation

- Initial stages → Hydrodynamic evolution → Freeze-out of hadrons

Hydrodynamic description ↔ Classical concepts of energy density and pressure ↔ Coarse-graining and Fluid cell



How well the classical concepts are defined over a fluid cell ??

Concepts of energy density, number density for a finite system size.

→ **Possible application to small systems produced in heavy-ion collisions**

We study the quantum statistical fluctuation of energy and baryon number within a small Gaussian subsystem S_a

S is a closed/isolated system: microcanonical ensemble

S_V is a subsystem of S : canonical ensemble

Ref: P. Romatschke and U. Romatschke, 1712.05815

Framework

Ref: Lectures of Sidney Coleman on Quantum Field theory

Gaussian smeared/ space averaged quantum field theory (QFT) operator

$$\hat{\mathcal{O}}_a = \frac{1}{(a\sqrt{\pi})^3} \int d^3x \, \hat{\mathcal{O}}(x) \, \exp\left(-\frac{\boldsymbol{x}^2}{a^2}\right)$$

Measure of quantum fluctuation: **variance**

$$\sigma^2(a, m, T) = \langle : \hat{\mathcal{O}}_a :: \hat{\mathcal{O}}_a : \rangle - \langle : \hat{\mathcal{O}}_a : \rangle^2$$

Normalized standard deviation:

$$\sigma_n(a, m, T) = \frac{(\langle : \hat{\mathcal{O}}_a :: \hat{\mathcal{O}}_a : \rangle - \langle : \hat{\mathcal{O}}_a : \rangle^2)^{1/2}}{\langle : \hat{\mathcal{O}}_a : \rangle}$$

QFT operator can be the energy-momentum tensor operator, baryon number operator, etc.

Noether theorem → No unique energy-momentum tensor (EMT)

$$\hat{T}'^{\mu\nu} = \hat{T}^{\mu\nu} + \partial_\lambda \hat{\Phi}^{\nu\mu\lambda} \qquad \hat{\Phi}^{\nu\mu\lambda} = -\hat{\Phi}^{\nu\lambda\mu} \qquad \partial_\mu \hat{T}^{\mu\nu} = 0 \qquad \partial_\mu \hat{T}'^{\mu\nu} = 0.$$

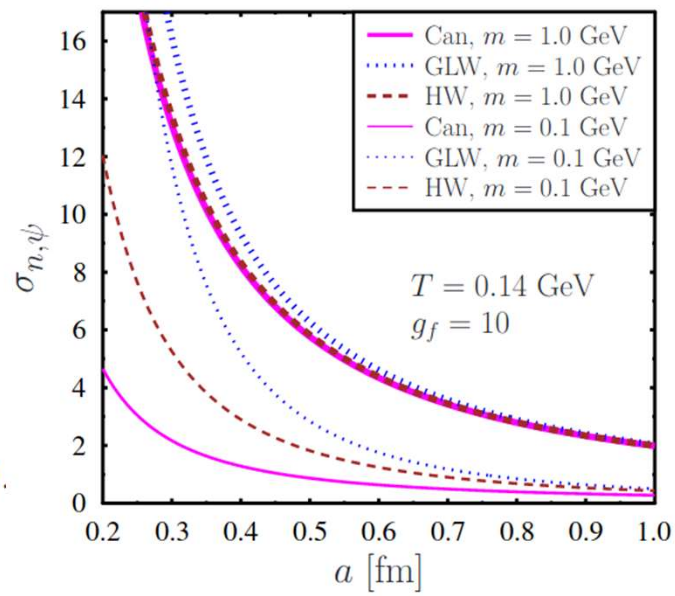
Pseudo-gauge choices: Effect of pseudo-gauge on quantum fluctuations.

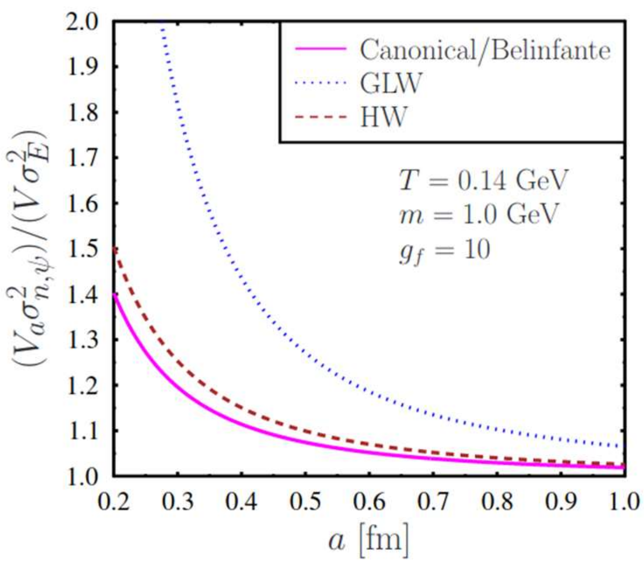
Refs: A. Das et.al. Phys.Rev.D 103 (2021) 9, L091502; A. Das et.al. Acta Phys. Pol. B **52**, 1395 (2021); L. Tinti et.al. 2007.04029; W. Florkowski et.al. Prog. Part. Nucl. Phys. 108 (2019) 103709.

Canonical EMT (CAN), Belinfante-Rosenfeld EMT (BR), de Groot-van Leeuwen-van Weert EMT (GLW), Hilgevoord- Wouthuysen EMT (HW)

Main results: ★ $$\sigma^2_{\psi,Can}(a,m,T) = 2\int dP\, dP' \mathfrak{f}_f(\omega_{\boldsymbol{p}})(1-\mathfrak{f}_f(\omega_{\boldsymbol{p}'})) \times \Big[(\omega_{\boldsymbol{p}}+\omega_{\boldsymbol{p}'})^2(\omega_{\boldsymbol{p}}\omega_{\boldsymbol{p}'}+\boldsymbol{p}\cdot\boldsymbol{p}'+m^2)e^{-\frac{a^2}{2}(\boldsymbol{p}-\boldsymbol{p}')^2} - (\omega_{\boldsymbol{p}}-\omega_{\boldsymbol{p}'})^2(\omega_{\boldsymbol{p}}\omega_{\boldsymbol{p}'}+\boldsymbol{p}\cdot\boldsymbol{p}'-m^2)e^{-\frac{a^2}{2}(\boldsymbol{p}+\boldsymbol{p}')^2}\Big]$$

★ $$\langle :\hat{\mathcal{T}}^{tt}_{\psi,Can,a}:\rangle = 4\int \frac{d^3p}{(2\pi)^3}\,\omega_{\boldsymbol{p}}\,\mathfrak{f}_f(\omega_{\boldsymbol{p}}) \equiv \varepsilon_{Can}(T,m) = \langle :\hat{\mathcal{T}}^{tt}_{\psi,BR,a}:\rangle = \langle :\hat{\mathcal{T}}^{tt}_{\psi,GLW,a}:\rangle = \langle :\hat{\mathcal{T}}^{tt}_{\psi,HW,a}:\rangle.$$





Large volume limit: Quantum statistical fluctuation reproduces statistical fluctuations

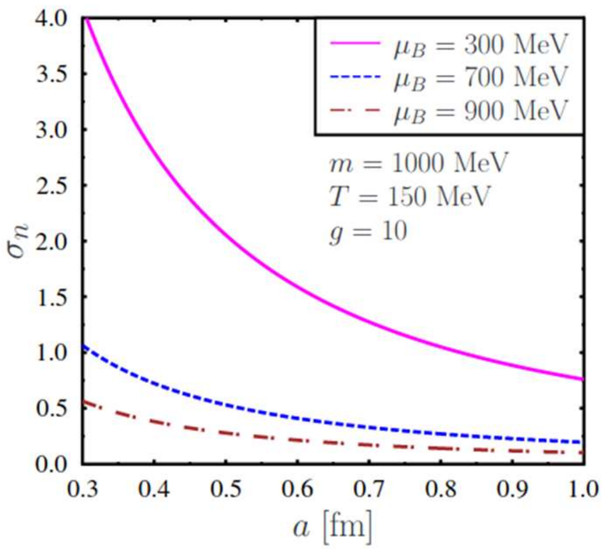
$$V_a\sigma^2_{n,\psi,Can} = V_a\sigma^2_{n,\psi,GLW} = \frac{T^2 C_{V,\psi}}{\varepsilon^2_{Can}} = V\frac{\langle E^2\rangle - \langle E\rangle^2}{\langle E\rangle^2} \equiv V\sigma^2_E.$$

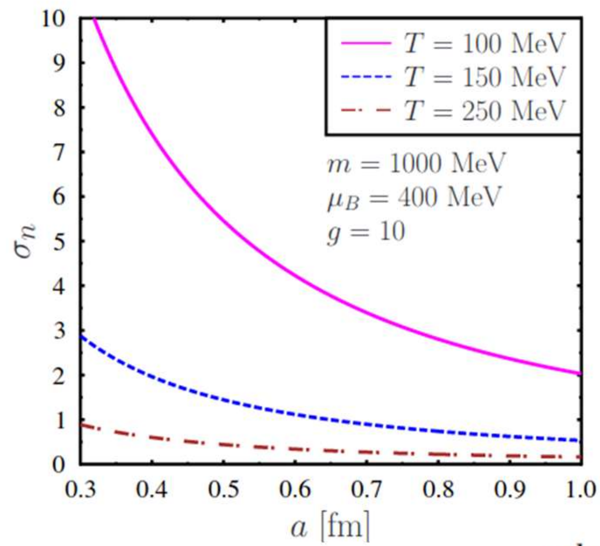
★ **Coarse-graining scale should be large to eliminate quantum effects**

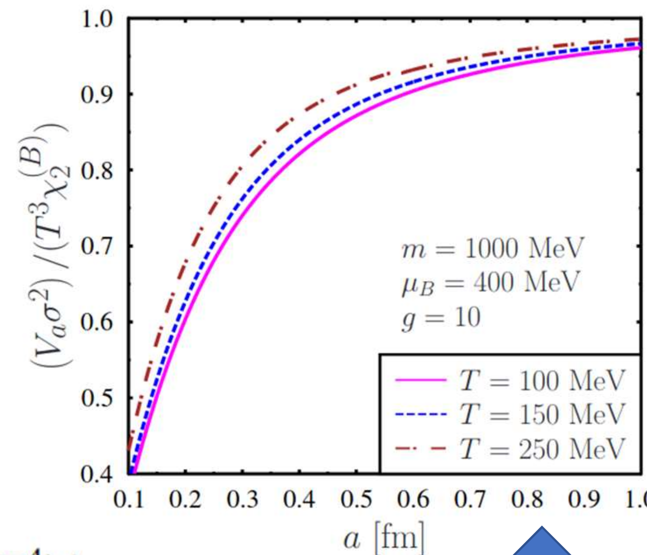
Quantum statistical fluctuation of Baryon number

- Hunt for the QCD critical point is one of the main goals of HIC experiments.
- Signal for quark-hadron transition: Event-by-event fluctuations of conserved net baryon number.

Baryon number operator: $\hat{O} = \bar{\psi}\gamma^0\psi$







Susceptibilities: $V\langle(n_B - \langle n_B\rangle)^2\rangle = T^3\chi_2^{(B)}$, $\chi_l^{(B)} = \left.\dfrac{\partial^l(P/T^4)}{\partial(\mu_B/T)^l}\right|_T$

Large volume limit

★ **Quantum nature of fluctuations should be considered for small scales/ small systems.**

Refs: Das et.al. 2105.02125; M. Nahrgang et.al. EPJC 75 (12) (2015) 573

Conclusions:

★ **A novel feature of quantum statistical fluctuations of energy is that they depend on the form of the energy-momentum tensor.**

★ **A generic feature of quantum fluctuations is that they decrease with the system size and for a small length scale such fluctuations can be significant.**

★ **These analyses also give us a practical way to determine the fluid cell or coarse-graining size.**

★ **Results on the quantum baryon number fluctuations may shed new light on the heavy-ion experimental data.**

Thank you.

Backup slides

The thermal average for a bosonic operator

$$\langle \mathfrak{a}_{\boldsymbol{p}}^\dagger \mathfrak{a}_{\boldsymbol{p}'} \rangle = \delta^{(3)}(\boldsymbol{p} - \boldsymbol{p}')\mathfrak{f}_b(\omega_{\boldsymbol{p}}),$$

$$\langle \mathfrak{a}_{\boldsymbol{p}}^\dagger \mathfrak{a}_{\boldsymbol{p}'}^\dagger \mathfrak{a}_{\boldsymbol{k}} \mathfrak{a}_{\boldsymbol{k}'} \rangle = \Big(\delta^{(3)}(\boldsymbol{p} - \boldsymbol{k})\ \delta^{(3)}(\boldsymbol{p}' - \boldsymbol{k}') + \delta^{(3)}(\boldsymbol{p} - \boldsymbol{k}')\ \delta^{(3)}(\boldsymbol{p}' - \boldsymbol{k}) \Big) \mathfrak{f}_b(\omega_{\boldsymbol{p}})\mathfrak{f}_b(\omega_{\boldsymbol{p}'}).$$

The thermal average for a fermionic operator

$$\langle a_r^\dagger(\boldsymbol{p}) a_s(\boldsymbol{p}') \rangle = (2\pi)^3 \delta_{rs} \delta^{(3)}(\boldsymbol{p} - \boldsymbol{p}')\mathfrak{f}_f(\omega_{\boldsymbol{p}}),$$

$$\langle a_r^\dagger(\boldsymbol{p}) a_s^\dagger(\boldsymbol{p}') a_{r'}(\boldsymbol{k}) a_{s'}(\boldsymbol{k}') \rangle$$

$$= (2\pi)^6 \Big(\delta_{rs'}\delta_{r's}\delta^{(3)}(\boldsymbol{p} - \boldsymbol{k}')\ \delta^{(3)}(\boldsymbol{p}' - \boldsymbol{k}) - \delta_{rr'}\delta_{ss'}\delta^{(3)}(\boldsymbol{p} - \boldsymbol{k})\ \delta^{(3)}(\boldsymbol{p}' - \boldsymbol{k}') \Big) \mathfrak{f}_f(\omega_{\boldsymbol{p}})\mathfrak{f}_f(\omega_{\boldsymbol{p}'}).$$

Canonical Energy Momentum tensor (Can)

$$\hat{\mathcal{T}}^{\mu\nu}_{\psi,Can} = \frac{i}{2}\bar{\psi}\gamma^{\mu}\mathcal{D}^{\nu}\psi, \quad \mathcal{D}^{\mu} \equiv \overrightarrow{\partial}^{\mu} - \overleftarrow{\partial}^{\mu}$$

Belinfante-Rosenfeld EMT (BR)

$$\hat{\mathcal{T}}^{\mu\nu}_{\psi,BR} = \frac{i}{2}\bar{\psi}\gamma^{\mu}\mathcal{D}^{\nu}\psi - \frac{i}{16}\partial_{\lambda}\Big(\bar{\psi}\Big\{\gamma^{\lambda},\Big[\gamma^{\mu},\gamma^{\nu}\Big]\Big\}\psi\Big).$$

de Groot-van Leeuwen-van Weert EMT (GLW):

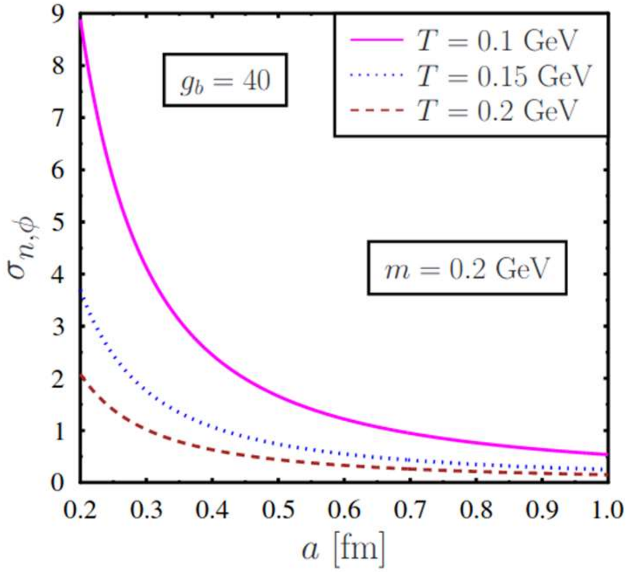
$$\begin{aligned}\hat{\mathcal{T}}^{\mu\nu}_{\psi,GLW} &= -\frac{1}{4m}\bar{\psi}\mathcal{D}^{\mu}\mathcal{D}^{\nu}\psi - g^{\mu\nu}\mathcal{L}_{D} \\ &= \frac{1}{4m}\Big[-\bar{\psi}(\partial^{\mu}\partial^{\nu}\psi) + (\partial^{\mu}\bar{\psi})(\partial^{\nu}\psi) + (\partial^{\nu}\bar{\psi})(\partial^{\mu}\psi) \\ &\qquad - (\partial^{\mu}\partial^{\nu}\bar{\psi})\psi\Big].\end{aligned}$$

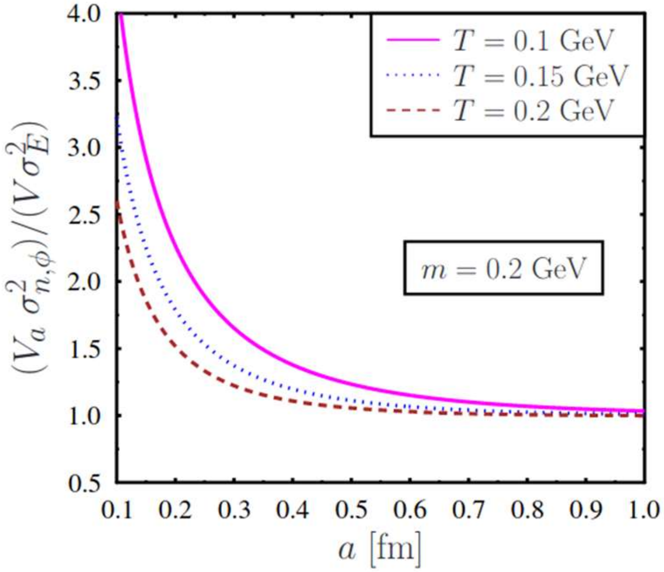
Fluctuations for the scalar field:

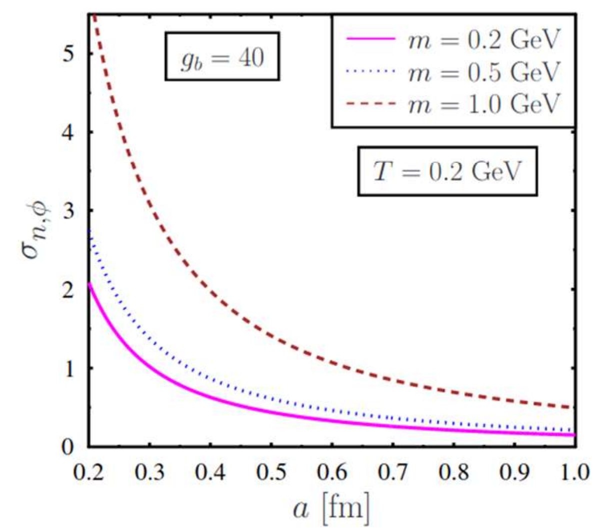
$$\sigma_{\phi}^{2}(a,m,T)=\int dP\ dP'\mathfrak{f}_{b}(\omega_{\boldsymbol{p}})(1+\mathfrak{f}_{b}(\omega_{\boldsymbol{p}'}))$$
$$\times\left[(\omega_{\boldsymbol{p}}\omega_{\boldsymbol{p}'}+\boldsymbol{p}\cdot\boldsymbol{p}'+m^{2})^{2}e^{-\frac{a^{2}}{2}(\boldsymbol{p}-\boldsymbol{p}')^{2}}\right.$$
$$\left.+(\omega_{\boldsymbol{p}}\omega_{\boldsymbol{p}'}+\boldsymbol{p}\cdot\boldsymbol{p}'-m^{2})^{2}e^{-\frac{a^{2}}{2}(\boldsymbol{p}+\boldsymbol{p}')^{2}}\right],$$

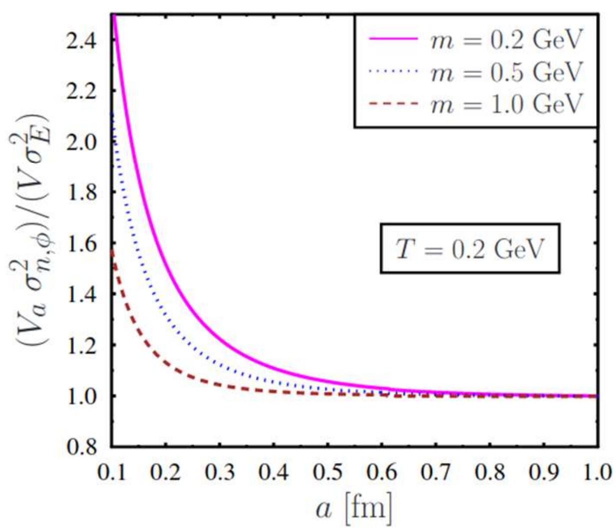
Fluctuations for the fermion field:

$$\sigma_{\psi,GLW}^{2}(a,m,T)=\frac{1}{2m^{2}}\int dP\ dP'\mathfrak{f}_{f}(\omega_{\boldsymbol{p}})(1-\mathfrak{f}_{f}(\omega_{\boldsymbol{p}'}))$$
$$\times\left[(\omega_{\boldsymbol{p}}+\omega_{\boldsymbol{p}'})^{4}\left(\omega_{\boldsymbol{p}}\omega_{\boldsymbol{p}'}-\boldsymbol{p}\cdot\boldsymbol{p}'+m^{2}\right)e^{-\frac{a^{2}}{2}(\boldsymbol{p}-\boldsymbol{p}')^{2}}\right.$$
$$\left.-(\omega_{\boldsymbol{p}}-\omega_{\boldsymbol{p}'})^{4}\left(\omega_{\boldsymbol{p}}\omega_{\boldsymbol{p}'}-\boldsymbol{p}\cdot\boldsymbol{p}'-m^{2}\right)e^{-\frac{a^{2}}{2}(\boldsymbol{p}+\boldsymbol{p}')^{2}}\right],$$


$T = 0.1$ GeV
$T = 0.15$ GeV
$T = 0.2$ GeV
$g_b = 40$
$m = 0.2$ GeV
$\sigma_{n,\phi}$
a [fm]


$T = 0.1$ GeV
$T = 0.15$ GeV
$T = 0.2$ GeV
$m = 0.2$ GeV
$(V_a\ \sigma^2_{n,\phi})/(V\ \sigma^2_E)$
a [fm]


$m = 0.2$ GeV
$m = 0.5$ GeV
$m = 1.0$ GeV
$g_b = 40$
$T = 0.2$ GeV
$\sigma_{n,\phi}$
a [fm]


$m = 0.2$ GeV
$m = 0.5$ GeV
$m = 1.0$ GeV
$T = 0.2$ GeV
$(V_a\ \sigma^2_{n,\phi})/(V\ \sigma^2_E)$
a [fm]