Quantum statistical fluctuation of energy and baryon number in subsystems of hot and dense relativistic gas

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Motivation

• Initial stages \(\rightarrow\) Hydrodynamic evolution \(\rightarrow\) Freeze-out of hadrons

Hydrodynamic description  \(\longleftrightarrow\) Classical concepts of energy density and pressure  \(\longleftrightarrow\) Coarse-graining and Fluid cell

How well the classical concepts are defined over a fluid cell ??

Concepts of energy density, number density for a finite system size.

Possible application to small systems produced in heavy-ion collisions

We study the quantum statistical fluctuation of energy and baryon number within a small Gaussian subsystem \(S_a\)

\(S\) is a closed/isolated system: microcanonical ensemble

\(S_V\) is a subsystem of \(S\): canonical ensemble

Ref: P. Romatschke and U. Romatschke, 1712.05815
Framework

Gaussian smeared/ space averaged quantum field theory (QFT) operator

Measure of quantum fluctuation: variance

Normalized standard deviation:

QFT operator can be the energy-momentum tensor operator, baryon number operator, etc.

Noether theorem → No unique energy-momentum tensor (EMT)

Pseudo-gauge choices: Effect of pseudo-gauge on quantum fluctuations.

 Canonical EMT (CAN), Belinfante-Rosenfeld EMT (BR), de Groot-van Leeuwen-van Weert EMT (GLW), Hilgevoord-Wouthuysen EMT (HW)

Main results:

\[
\sigma^2_{\psi,Can}(a, m, T) = 2 \int dP \int dP' f_f(\omega_p)(1 - f_f(\omega_{p'})) \times \left[ (\omega_p + \omega_{p'})^2 (\omega_p \omega_{p'} + p \cdot p' + m^2) e^{-\frac{a^2}{2}(p-p')^2} - (\omega_p - \omega_{p'})^2 (\omega_p \omega_{p'} + p \cdot p' - m^2) e^{-\frac{a^2}{2}(p+p')^2} \right]
\]

\[
\langle : \hat{T}^{tt}_{\psi,Can,a} : \rangle = 4 \int \frac{d^3p}{(2\pi)^3} \omega_p f_f(\omega_p) \equiv \varepsilon_{Can}(T, m) = \langle : \hat{T}^{tt}_{\psi,BR,a} : \rangle = \langle : \hat{T}^{tt}_{\psi,GLW,a} : \rangle = \langle : \hat{T}^{tt}_{\psi,HW,a} : \rangle.
\]

Large volume limit: Quantum statistical fluctuation reproduces statistical fluctuations

\[
V \sigma^2_{n,\psi,Can} = V \sigma^2_{n,\psi,GLW} = \frac{T^2 C_{V,\psi}}{\varepsilon^2_{Can}} = V \frac{\langle E^2 \rangle - \langle E \rangle^2}{\langle E \rangle^2} \equiv V \sigma_E^2.
\]

Coarse-graining scale should be large to eliminate quantum effects
Quantum statistical fluctuation of Baryon number

- Hunt for the QCD critical point is one of the main goals of HIC experiments.
- Signal for quark-hadron transition: Event-by-event fluctuations of conserved net baryon number.

Baryon number operator: \( \hat{O} = \bar{\psi} \gamma^0 \psi \)

Susceptibilities: 
\[
V\langle (n_B - \langle n_B \rangle)^2 \rangle = T^3 \chi_2^{(B)}, \quad \chi_{l}^{(B)} = \frac{\partial^l (P/T^4)}{\partial (\mu_B/T)^l} \bigg|_T
\]

Quantum nature of fluctuations should be considered for small scales/ small systems.

Refs: Das et.al. 2105.02125; M. Nahrgang et.al. EPJC 75 (12) (2015) 573
Conclusions:

A novel feature of quantum statistical fluctuations of energy is that they depend on the form of the energy-momentum tensor.

A generic feature of quantum fluctuations is that they decrease with the system size and for a small length scale such fluctuations can be significant.

These analyses also give us a practical way to determine the fluid cell or coarse-graining size.

Results on the quantum baryon number fluctuations may shed new light on the heavy-ion experimental data.

Thank you.
**Backup slides**

The thermal average for a bosonic operator

\[
\langle a_p^\dagger a_{p'} \rangle = \delta^{(3)}(p - p') f_b(\omega_p),
\]

\[
\langle a_p^\dagger a_p^\dagger a_k a_{k'} \rangle = \left( \delta^{(3)}(p - k) \delta^{(3)}(p' - k') + \delta^{(3)}(p - k') \delta^{(3)}(p' - k) \right) f_b(\omega_p) f_b(\omega_{p'}). \]

The thermal average for a fermionic operator

\[
\langle a_r^\dagger(p) a_s(p') \rangle = (2\pi)^3 \delta_{rs} \delta^{(3)}(p - p') f_f(\omega_p),
\]

\[
\langle a_r^\dagger(p) a_s^\dagger(p') a_{r'}(k) a_{s'}(k') \rangle
\]

\[
= (2\pi)^6 \left( \delta_{rs'} \delta_{rs} \delta^{(3)}(p - k') \delta^{(3)}(p' - k) - \delta_{rr'} \delta_{ss'} \delta^{(3)}(p - k) \delta^{(3)}(p' - k') \right) f_f(\omega_p) f_f(\omega_{p'}). \]
Canonical Energy Momentum tensor (Can)

\[ \hat{T}_{\psi,Can}^{\mu\nu} = \frac{i}{2} \bar{\psi} \gamma^\mu D^\nu \psi, \quad D^\mu \equiv \overrightarrow{\partial}^\mu - \overleftarrow{\partial}^\mu \]

Belinfante-Rosenfeld EMT (BR)

\[ \hat{T}_{\psi,BR}^{\mu\nu} = \frac{i}{2} \bar{\psi} \gamma^\mu D^\nu \psi - \frac{i}{16} \partial_\lambda \left( \bar{\psi} \left\{ \gamma^\lambda, [\gamma^\mu, \gamma^\nu] \right\} \psi \right) \]

de Groot-van Leeuwen-van Weert EMT (GLW):

\[ \hat{T}_{\psi,GLW}^{\mu\nu} = -\frac{1}{4m} \bar{\psi} D^\mu D^\nu \psi - g^{\mu\nu} \mathcal{L}_D \]

\[ = \frac{1}{4m} \left[ - \bar{\psi} (\partial^\mu \partial^\nu \psi) + (\partial^\mu \bar{\psi})(\partial^\nu \psi) + (\partial^\nu \bar{\psi})(\partial^\mu \psi) \right. \]
\[ \quad \left. - (\partial^\mu \partial^\nu \bar{\psi})\psi \right] . \]
Fluctuations for the scalar field:

\[
\sigma_{\phi}^2(a, m, T) = \int dP \, dP' \, f_b(\omega_p) (1 + f_b(\omega_{p'})) \\
\times \left[ \left( \omega_p \omega_{p'} + \mathbf{p} \cdot \mathbf{p}' + m^2 \right)^2 e^{-\frac{a^2}{2} (\mathbf{p} - \mathbf{p}')^2} \\
+ \left( \omega_p \omega_{p'} - \mathbf{p} \cdot \mathbf{p}' - m^2 \right)^2 e^{-\frac{a^2}{2} (\mathbf{p} + \mathbf{p}')^2} \right],
\]

Fluctuations for the fermion field:

\[
\sigma_{\psi, GLW}^2(a, m, T) = \frac{1}{2m^2} \int dP \, dP' \, f_f(\omega_p) (1 - f_f(\omega_{p'})) \\
\times \left[ \left( \omega_p + \omega_{p'} \right)^4 \left( \omega_p \omega_{p'} - \mathbf{p} \cdot \mathbf{p}' + m^2 \right) e^{-\frac{a^2}{2} (\mathbf{p} - \mathbf{p}')^2} \\
- \left( \omega_p - \omega_{p'} \right)^4 \left( \omega_p \omega_{p'} - \mathbf{p} \cdot \mathbf{p}' - m^2 \right) e^{-\frac{a^2}{2} (\mathbf{p} + \mathbf{p}')^2} \right],
\]