Bayesian inference on quark matter from observations of neutron stars

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Quark matter inside neutron stars

In the center of heavy neutron stars, at a few times normal nuclear density (n_0), **quark matter** might appear. This possibility has already been explored in previous studies [1]. These objects are then called **hybrid stars** and consist of several layers.

Ideally, we would have a single model describing all the components, but in reality we use separate **hadronic** and **quark** models, as well as some **concatenation method** to connect the two.

Hadronic model

Models for hadronic matter are roughly constrained up to **1-2 n_0**, however there are still some debate about some parameters, such as the **slope of symmetry energy**, which largely influences the stiffness at densities above $2n_0$. We mainly use the **SFHo** [2] equation of state (EoS), which is a soft one, but we also apply the **DD2** [3] EoS to explore possibilities with stiffer EoS's.

Constituent quark model

For quark matter we use an **(axial)vector meson extended linear sigma model**, introduced in [4]. The parameters are set by meson vacuum phenomenology and finite temperature behavior. The model shows an **excellent agreement** with **lattice QCD simulations** at zero chemical potential.

There are two somewhat free parameters: the constituent quark–vector meson coupling (**g_V**), and the mass of the sigma meson (**m_σ**), since it is a very broad resonance.

Phase transition

It is necessary to use a concatenation method to connect the two phases. One can use a simple Maxwell construction, resulting in a first order phase transition. However, in our approach the phase transition is of **crossover type**. The **energy density** in the crossover region is defined as:

$$\varepsilon(n_B) = \varepsilon_H(n_B) f_-(n_B) + \varepsilon_Q(n_B) f_+(n_B),$$

where f_- and f_+ exponentially suppress the hadronic and quark EoS's, respectively. The **width** and **center** of these functions is set by Γ and $\bar{n}$.

4 parameters: g_V, m_σ, $\bar{n}$, Γ

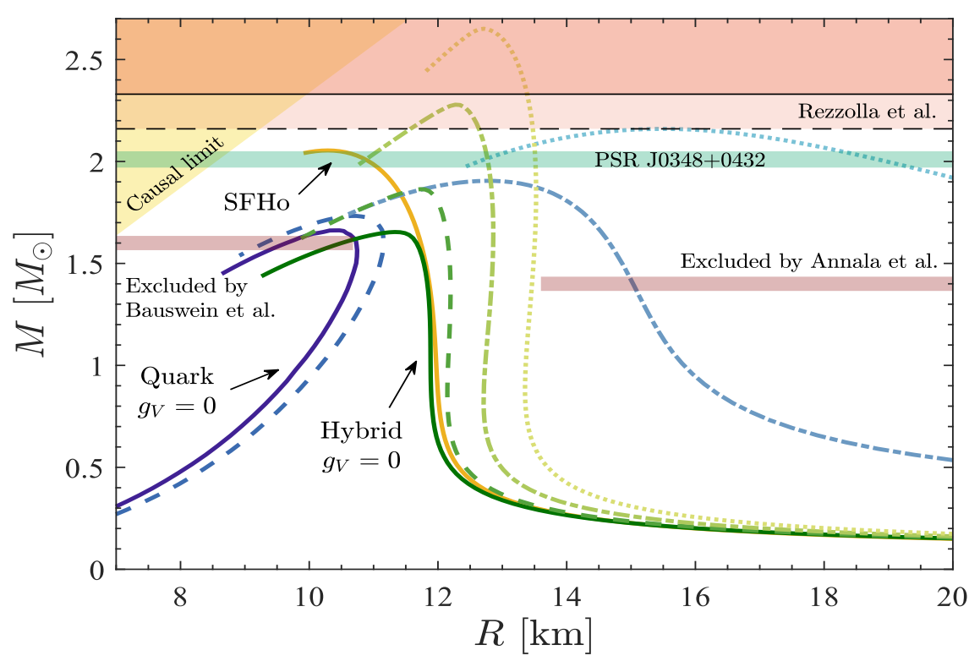
Mass-radius relations with different parameters

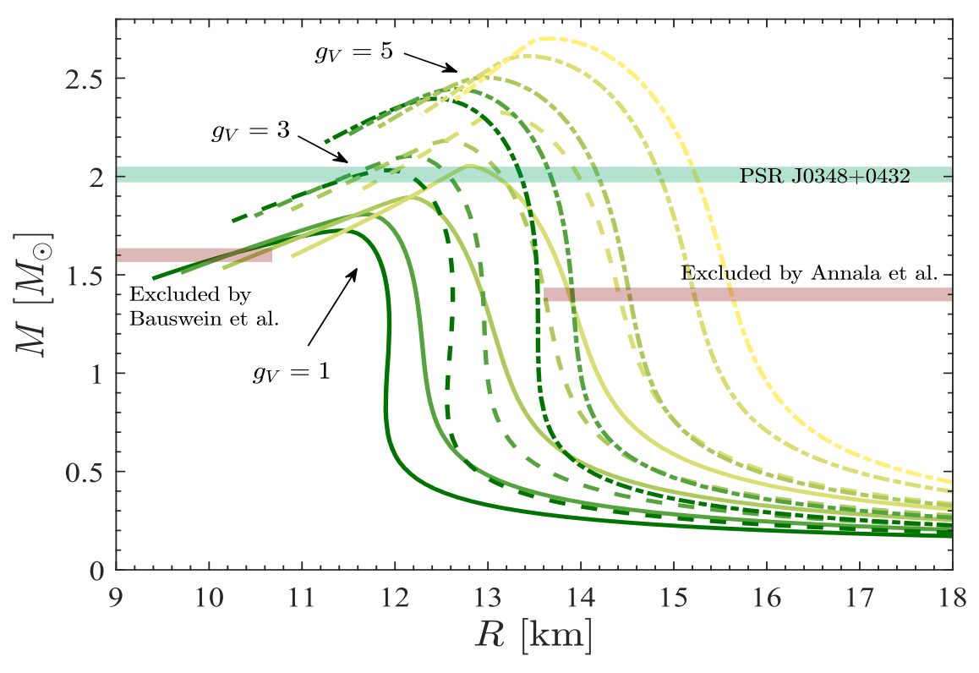
The **quark EoS** greatly influences the mass–radius relation of hybrid stars.

- Increasing the **vector coupling (g_V)** increases the maximum mass of hybrid stars by making the EoS stiffer (top figure, brighter tones)
- Larger **sigma meson masses** increase both masses and radii significantly (bottom figure, brighter tones)
- Hybrid stars can have larger masses than both pure hadronic and quark stars

Several **astrophysical constraints** can be applied:

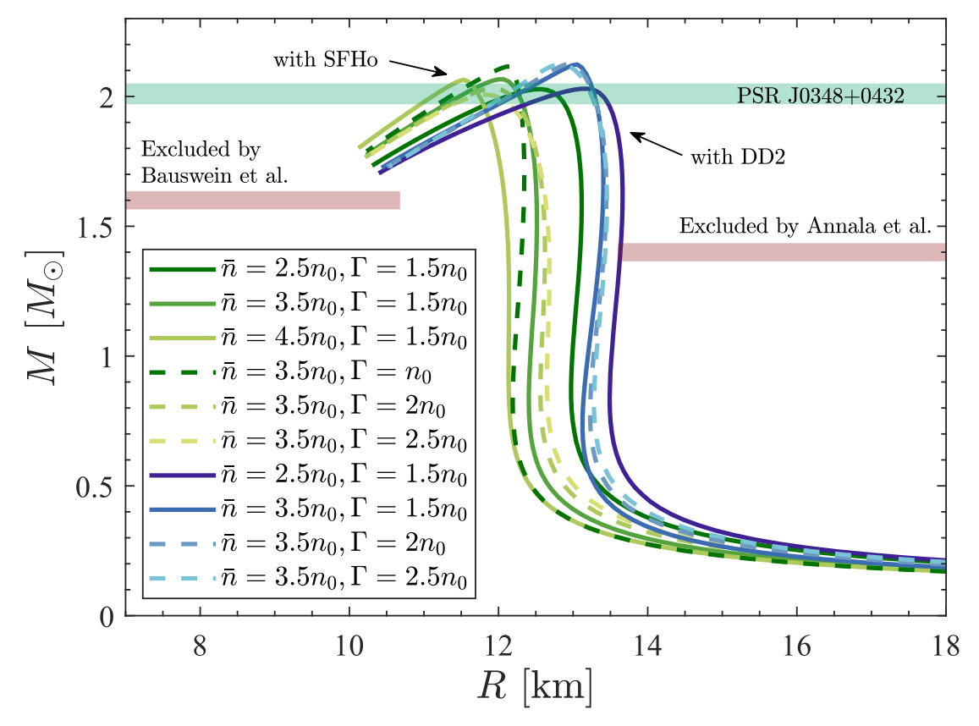
- Lower limit for maximum mass (PSR J0348+0432)
- Upper mass constraint from GW170817 and gamma-ray observation GRB170817a [5]
- Lower and upper radius limits for mid-mass stars from GW170817 [6,7]

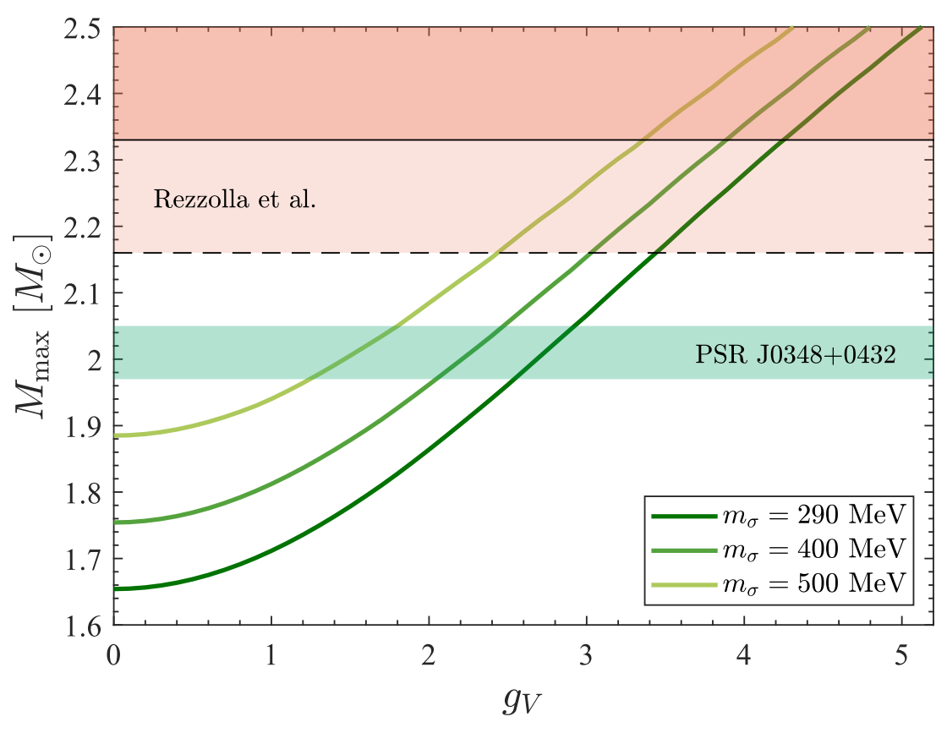




Quark matter properties from mass constraints

The **hadronic EoS** and the parameters of the **phase transition** will also affect the mass–radius relation and other neutron star properties:





- Varying the concatenation parameters and the hadronic EoS modifies radii
- The **maximum mass** neutron star is only **weakly affected** by both (left figure)
- We can use **mass constraints** to set boundaries for parameters of our **quark model**

Low sigma meson mass and a narrow phase transition are preferred

The center of the phase transition is optimally between $2.5\,n_0$ and $3.5\,n_0$

The vector coupling: $2.5 < g_V < 4$

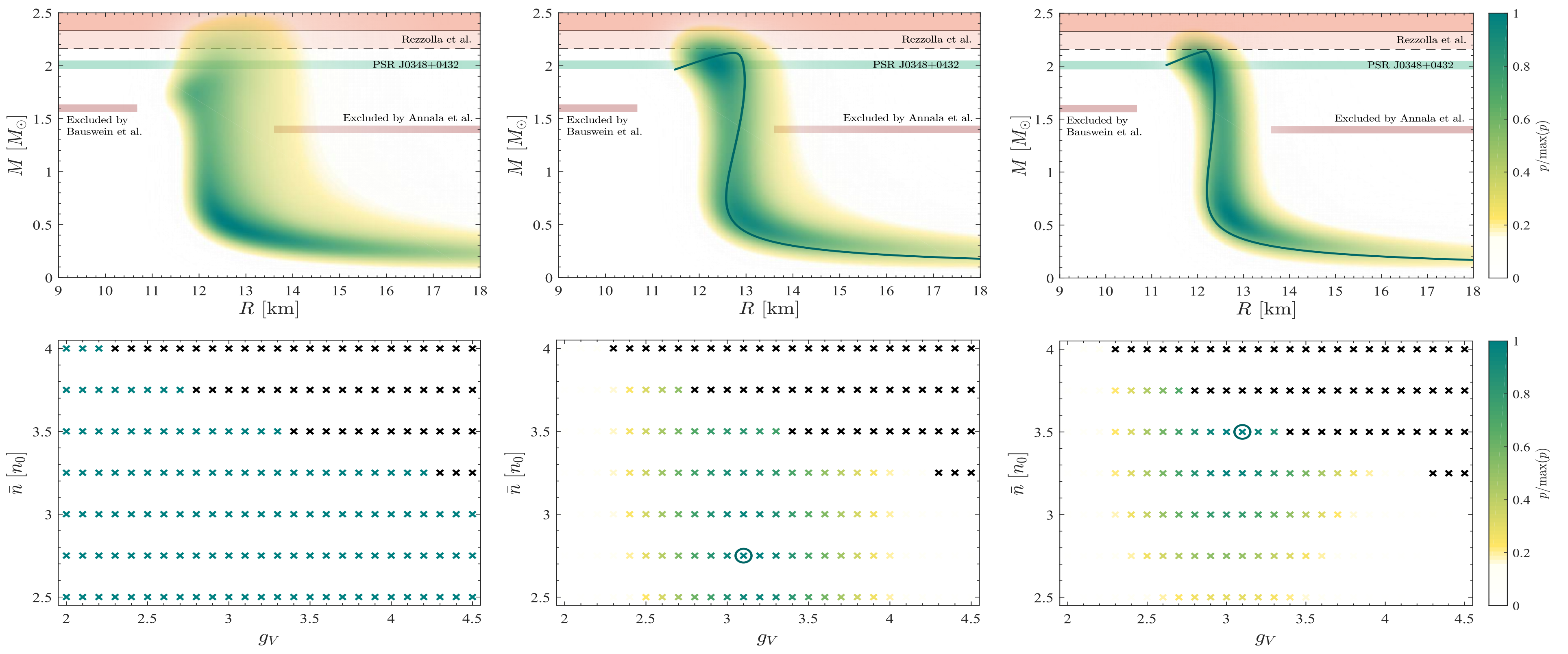
Bayesian analysis using astrophysical measurements

We performed a full **Bayesian analysis** using multiple astrophysical constraints: GW170817, NICER measurements and mass constraints.

The bottom figures show the a priori and posterior probabilities of different parameter sets. The top figures show the corresponding probability densities on the mass–radius diagram.

❖ A priori probabilities (left panel):

Even without any astrophysical constraints some EoS's are ruled out due to being unstable or violating causality (black crosses). A broad region is covered on the M–R diagram.



❖ NICER measurements (middle panel):

Using the probability distributions from the two NICER measurements combined with the mass constraints, the maximum probability will correspond to $g_V = 3.1$, $\bar{n} = 2.75\,n_0$ (blue circle).

❖ NICER + GW170817 (right panel):

With GW170817 also considered the allowed region shrinks and the preferred phase transition density is higher ($\bar{n} = 3.5\,n_0$). In both cases $m_\sigma = 290$ MeV and $\Gamma = 1\,n_0$ is preferred.

Conclusion

The goal of our study was to investigate whether models of quark matter can be constrained by observations of neutron stars.

❖ M–R relations of hybrid stars

Since they significantly increase the radii of hybrid stars, **large sigma meson masses are excluded** – consistently with our purely particle physics-based parametrization. The **vector coupling can be constrained** by mass limits since the maximum mass of hybrid stars is only weakly dependent on the phase transition.

❖ The Bayesian analysis

Our Bayesian analysis was based on recent **X-ray and gravitational wave observations**. The results for the model parameters are consistent with our simple M–R relation-based investigation, but in addition, we found that observations are best accommodated by a **narrow phase transition** with a **center at ~3 n_0**.

❖ Discussion: a self-consistent approximation

We can improve our approximation and use a **self-consistent approach** when solving our **constituent quark model**. This way the vector coupling is also set by particle physics, for which we get **$g_V \approx 5$** [8]. This suggests a very high maximum hybrid star mass. Interestingly, we get a **similar value from astrophysics** when we **ignore upper mass limits**.

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