The rigidity of ultradense matter: understanding the peak in the speed of sound

PRD 104, arXiv:2105.04535

Mauricio Hippert[1], Jorge Noronha[1] and Eduardo S. Fraga[2]

[1] Illinois Center for Advanced Studies of the Universe,
Department of Physics, University of Illinois at Urbana-Champaign
[2] Instituto de Física, Universidade Federal do Rio de Janeiro

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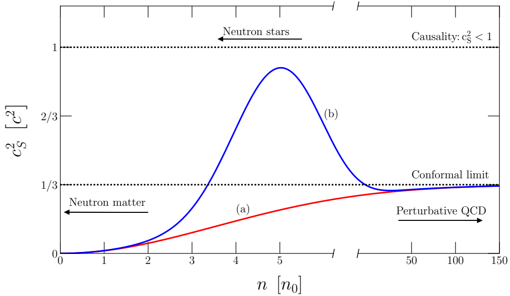
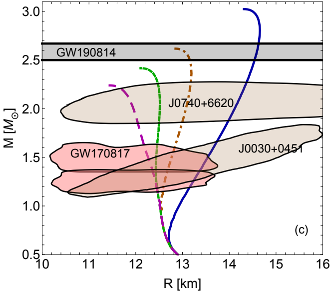
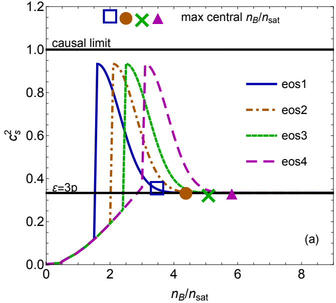
Stiffness of ultradense matter

- Constrained by neutron-star observations
- Speed of sound c_s must rapidly increase with density

P. Bedaque and A. W. Steiner, PRL **114** (2015)
Tews, Carlson, Gandolfi and Reddy, Astrophys. J. **860** (2018)







Figures: H. Tan *et al.* PRD **105** (2022)
Tews, Carlson, Gandolfi and Reddy, Astrophys. J. **860** (2018)

Dimensional analysis

- Non-perturbative scale $|\Phi|$ needed!
- For $p \sim (|\Phi|/\mu)^{\alpha}\mu^{d+1}$,

$$c_s^2 \sim (d-\alpha)^{-1}$$

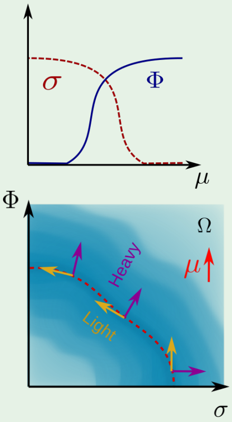
- Peak $c_s^2 \lesssim 1$ if $\alpha \sim 2$
- Condensate carrying baryon number q_B:

$$p = p_0(\mu) + q_B^2|\Phi|^2\mu^2/2$$

- $U(1)_B$ breaking $\rightarrow$ superfluid $\rightarrow$ transport
- $|\Phi|$ must be $\sim$ constant

Mechanism

- Mixing with neutral VEV: $|\Phi|$ saturates!
- Similar to QC2D or μ_{isospin}



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Toy model

- Simplified by symmetry under $\Phi \leftrightarrow \sigma$
- Mixing between neutral and charged condensates: "particle-hole" symmetry
- Spontaneous and explicit breakdown $SU(2)_{GM} \to U_B(1)$
- Integrating out heavy directions,

$$F U = F\, e^{i\,\gamma_5\,\vec{\phi}\cdot\vec{\tau}} = \sigma\,\mathbb{1} + i\,\gamma_5\,\vec{\Phi}\cdot\vec{\tau}\,,$$

"Particle-hole" symmetry

- Transformation

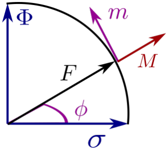
$$\psi \to \psi' = \psi + i\,\epsilon\,\gamma_5\,\psi^C$$

- With $U_B(1)$, forms $SU(2)_{GM}$
- In Nambu-Gorkov basis $\Psi = (\psi, \psi^C)$, generators

$$\vec{G} = (\gamma_5\tau_x, \gamma_5\tau_y, \tau_z)$$

- Taking $\Psi \to e^{i\,\gamma_5\,\vec{\phi}\cdot\vec{\tau}/2}\Psi$,

$$\mathcal{L} = \overline{\Psi}\,\left(i\,\gamma^\mu\partial_\mu + \gamma^\mu\,V_\mu + i\,\gamma^\mu\,\gamma_5\,A_\mu - g\,F\right)\,\Psi + \\ + \frac{F^2}{4}\,\mathrm{tr}\left[\partial^\mu U^\dagger\partial_\mu U\right] + \frac{F^2\,m_\Phi^2}{4}\,\mathrm{tr}\left(U^\dagger + U\right)$$



Results ($T = 0$)

- Rotation along light direction ✓
- Speed of sound peaks after rotation ✓

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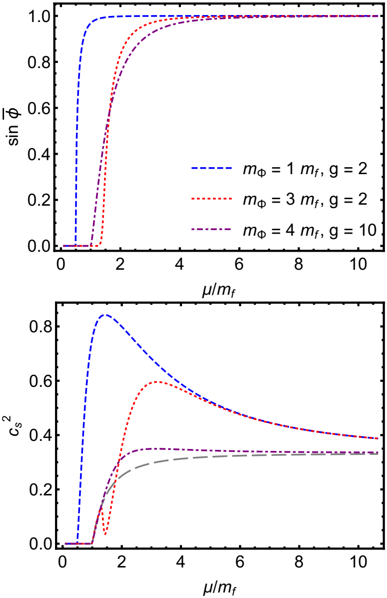


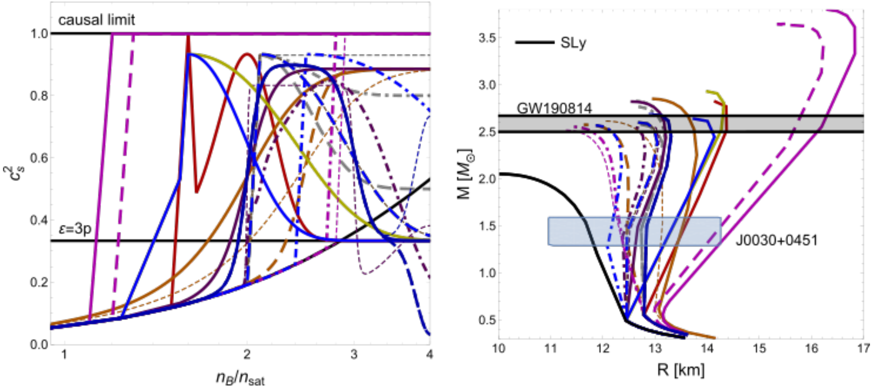
Conclusions

- General mechanism for $c_s^2 > 1/3$ peak. Tested in toy model.
  MH, E. S. Fraga and J. Noronha, PRD **104** (2021)
- Emerges naturally from the interplay between baryon-charged and neutral condensates.
- Supports importance of diquarks to the EoS of ultradense matter.
  T. Kojo, P. D. Powell, Y. Song and G. Baym, PRD **91** (2015)
  A. Ayriyan *et al*, EPJ A **57** (2021)

Backup slides...

Stiffness of ultradense matter

- Pressure *vs* gravity: stiffness of dense matter → masses, radii...
- Soft: smaller maximum mass, smaller radii at fixed mass.
- Stiff: larger maximum mass, larger radii at fixed mass.



Figures: H. Tan, J. Noronha-Hostler and N. Yunes, PRL **125** (2020)

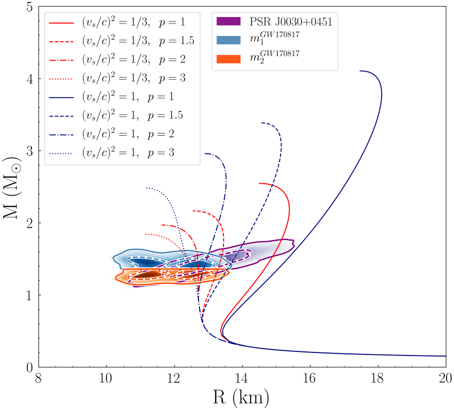
Speed of Sound as Function of Density

- Evidence from gravitational waves, pulsars, low-energy effective field theory → rapid stiffening of the EoS.

P. Demorest *et al.*, Nature **467** (2010) and J. Antoniadis *et al.*, Science **340** (2013)
R. Abbott *et al.* [LIGO Scientific and Virgo], Astrophys. J. Lett. **896** (2020) and PRL **121** (2018)
T. E. Riley *et al.*, Astrophys. J. Lett. **887** (2019) and Astrophys. J. Lett. **918** (2021)
M. C. Miller *et al.*, Astrophys. J. Lett. **887** (2019) and Astrophys. J. Lett. **918** (2021) S. Gandolfi, J. Carlson and S. Reddy, PRC **85** (2012)

- Low c_s^2 at moderate densities, $c_s^2 \gtrsim 1/3$ at large densities

P. Bedaque and A. W. Steiner, PRL **114** (2015)
I. Tews *et al.*, Astrophys. J. **860** (2018)
B. Reed and C. J. Horowitz, PRC **101** (2020)
Y. Fujimoto, K. Fukushima and K. Murase, JHEP **03** (2021)



Const. c_s^2, match to APR,
$n_{\rm match} = p\, n_{\rm sat}$

Figure: A. Kanakis-Pegios, P. S. Koliogiannis and C. C. Moustakidis, PRC **102** (2020)

Very Large Densities

- Asymptotic freedom: $c_s^2 \to 1/3$ at $\mu_B \gg \Lambda_{\mathrm{QCD}}$.
- Perturbative QCD: conformal limit approached from below.

A. Kurkela, P. Romatschke and A. Vuorinen, PRD **81** (2010)
A. Kurkela, E. S. Fraga, J. Schaffner-Bielich and A. Vuorinen, Astrophys. J. **789** (2014)

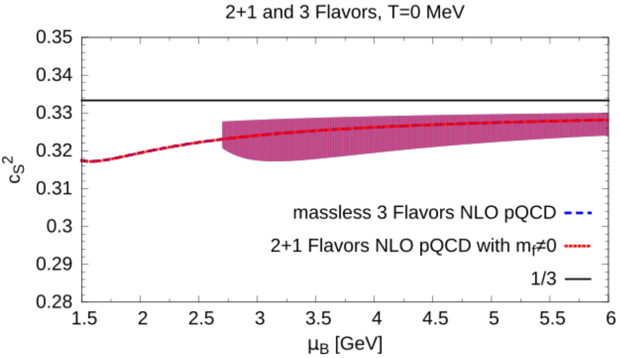


Figure: T. Graf, J. Schaffner-Bielich and E. S. Fraga, EPJ A **52** (2016)

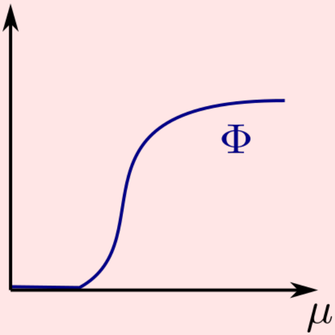
Problems

- Only works if Φ is approximately constant, but it must grow at some point: Φ must "saturate".
- For an effective potential $\Omega(\Phi,\mu) = \Omega_0(\Phi) - q^2\,\mu^2\,\Phi^2/2$,

$$\frac{d\Phi}{d\mu} = 2q^2\frac{\mu\,\Phi}{m_\Phi^2}$$

  where $m_\Phi^2 \equiv d^2\Omega_0/d\Phi^2$

- Does not work for $\Omega_0 \sim \Phi^\gamma$.



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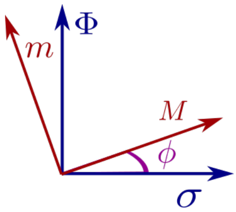
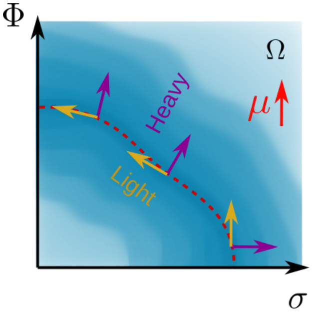
Saturating Φ

- Extra condensate with zero baryon number: "σ"
- "Heavy" direction for $\mu = 0$, "light" direction for $\mu \gtrsim \mu_{\rm sat}$





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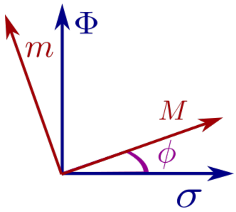

For $M \gg m$:

$$\frac{d\Phi}{d\mu} \approx \frac{2\,q^2\,\mu\,\Phi\,\cos^2\phi}{m^2 - q^2\,\mu^2\,\cos^2\phi}$$

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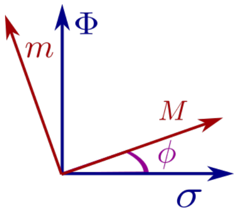

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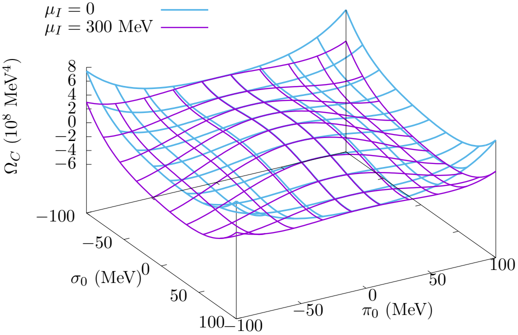
$$\phi \to \pi/2\,, \qquad \frac{d\Phi}{d\mu} \to 0 \qquad (1)$$

Relationship to QCD with isospin chemical potential



$$\Omega_C = -\frac{1}{2}\,\mu_I^2\,\pi_0^2 + \frac{\lambda}{4}\,(\sigma_0^2 + \pi_0^2 - v^2)^2 - h\,\sigma_0$$

D. T. Son and M. A. Stephanov, PRL **86** (2001)
J. B. Kogut and D. K. Sinclair, PRD **66** (2002)
S. Carignano *et al.* EPJ A **53** (2017)
MH, E. S. Fraga, J. Schaffner-Bielich, work in progress

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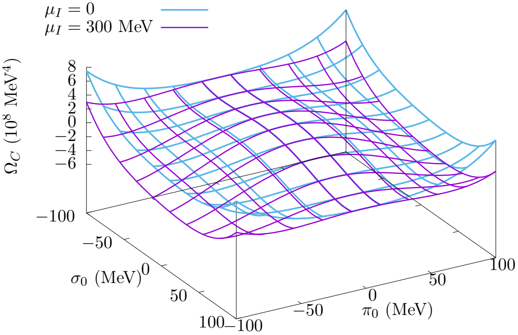

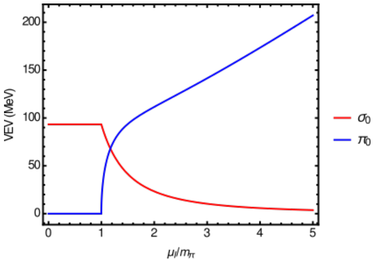

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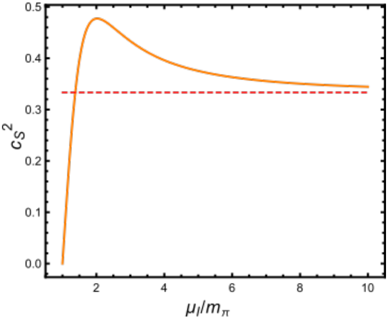

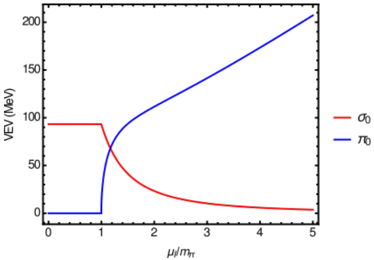

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S. Carignano *et al.* EPJ A **53** (2017)
MH, E. S. Fraga, J. Schaffner-Bielich, work in progress

Particle-Hole Symmetry

- Simplifying assumption for model building — "particle-hole" symmetry:

$$\psi \to \psi' = \psi + i\,\epsilon\,\gamma_5\,\psi^C$$

- Rotates baryon-charged diquark qq into neutral meson $\bar{q}q$, Dirac mass into Majorana mass.
- With $U_B(1)$, forms a $SU(2)_{GM}$ algebra
- In the Nambu-Gorkov basis,

$$\begin{aligned} \Lambda_C : &\quad \Psi \to \Psi + i\,\epsilon\,\gamma_5\,\tau_x\,\Psi \\ \Lambda_B : &\quad \Psi \to \Psi + i\,\epsilon\,\tau_z\,\Psi \\ \Lambda_{C^*} : &\quad \Psi \to \Psi + i\,\epsilon\,\gamma_5\,\tau_y\,\Psi \end{aligned}$$

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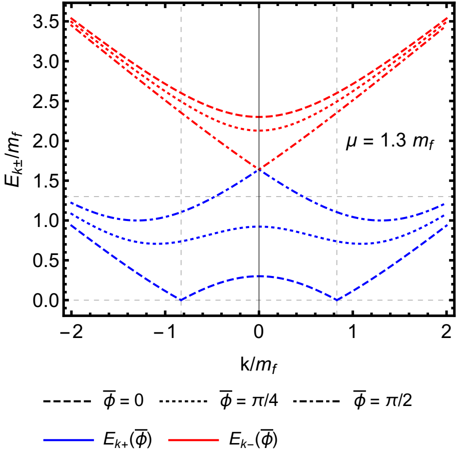
Dispersion relations

Value of $\bar{\phi} \equiv \langle \phi \rangle$: interpolation between Dirac mass and superfluid gap

$$\begin{cases} \bar{\phi} = 0\,, & \text{mass } m = gF \\ \bar{\phi} = \pi/2\,, & \text{gap } |\Delta| = gF \end{cases}$$



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Potential barrier

A barrier could be present, in which case a phase transition should exist.

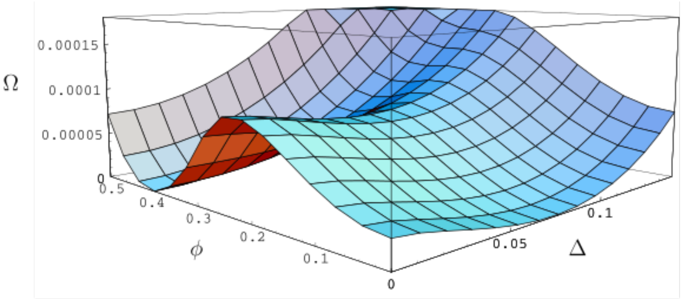
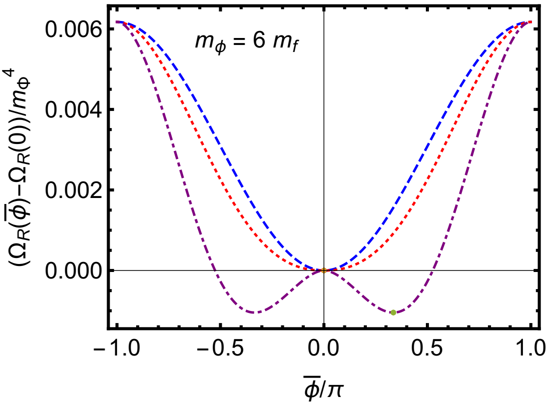


FIG. 1. The thermodynamic potential Ω (in GeV^4) as a function of ϕ and Δ at $T = 0$ and $\mu = 0.292$ GeV. The two degenerate minima have $\phi = \phi_0^{vac} = 0.4$ GeV, $\Delta = 0$ and $\phi = 0$, $\Delta = 0.072$ GeV.

Figure: J. Berges and K. Rajagopal, Nucl. Phys. B **538**, 215-232 (1999)

Effective Potential in Toy Model



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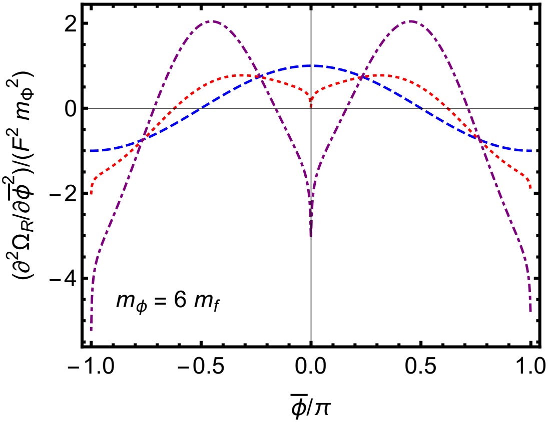
Second Derivative



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