Berry flux and topological aspects of color superconductor

Will nature fabric topological matter inside the neutron star?

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Noriyuki Sogabe, YY, arXiv: 2022.05XX

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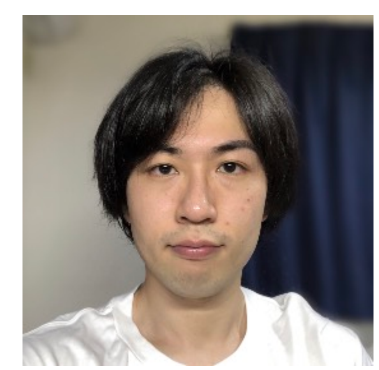


Noriyuki Sogabe, postdoc@IMP -> UIC

Single flavor pairing in color superconducting phase

Baryonic matter at asymptotically high density is a color superconductor in color-flavor locked (CFL) phase. *Alford, Schmitt, Rajagopal, Schaffer, Rev. Mod. Phys. 2008*

Single flavor pairing might be favored at environments in stars. (mismatch between Fermi surface at different flavors induced by charge neutrality condition).

Gap function is spin-one triplet.

pairing between quarks with opposite chirality is energetically preferred.

Extensively studied but why we revisit? *e.g. works by Mei Huang, Defu Hou, T. Schaffer, A. Schmitt, Qun Wang, Pengfei Zhuang and many others*

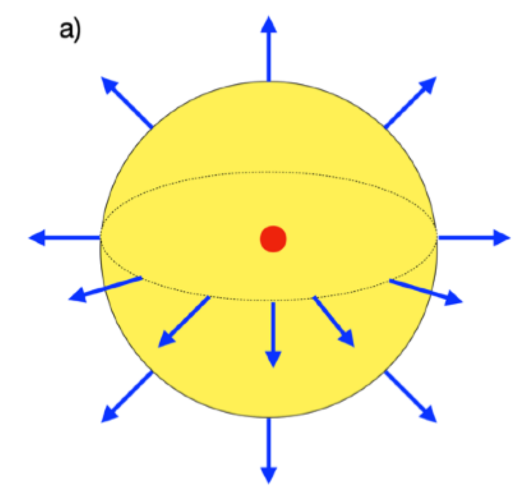
Topological aspects has been overlooked for years

Topological structure of the chiral Fermion surface (F.S.)

- Consider positive helicity spinor $\xi_R(\hat{k})$ and define Berry connection and curvature.

$$\vec{A}(\vec{k}) \equiv i\xi_R^\dagger(\hat{k})\, \vec{\nabla}_k \xi_R(\hat{k}) \qquad \vec{b} \equiv \vec{\nabla} \times \vec{A} = \frac{\vec{k}}{2k^2}$$

Similar for left helicity



Berry flux of F.S. = $4\pi q_{R/L}$

(with monopole charge $q_{R/L} = \pm 1/2$)

- The presence of monopole implies that helicity spinor can not be continuous on the whole F. S. .

- What happens when F.S. collapses in super-conducting phase?

Berry structure in Cooper pair

Murakami-Nagaosa, PRL 03; Li-Haldane, PRL 18';

- Consider $\uparrow\uparrow$ component of gap matrix in spin space.

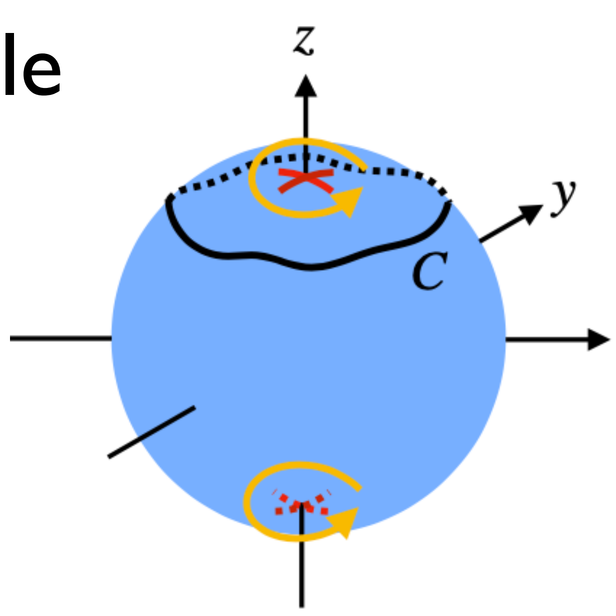
$$H_{int} \propto \Delta_{\uparrow\uparrow}(\vec{k}) \left[\xi_{R,\uparrow}(\hat{k})\xi_{L,\uparrow}(-\hat{k})c_R^\dagger(\vec{k})c_L^\dagger(-\vec{k})\right]$$

- Since spinor can not be continuous on the whole F.S., neither can the phase of $\Delta_{\uparrow\uparrow}$, $\phi_\Delta(\hat{k})$.

$\Rightarrow \Delta_{\uparrow\uparrow}$ must have nodes where $\phi_\Delta(\hat{k})$ is ambiguous.

- The winding number for the "velocity field" $\vec{v}^\Delta = \nabla_k \phi^\Delta$ around the nodes inherits the monopole charge of single particle F.S.

$$g_w = \oint d\vec{k} \cdot \vec{v}^\Delta = q_R - q_L$$



The color structure has not been taken into account.

Single flavor pairing

- Polar and A-phase:
  - Gap function has nodes associated with topological winding number ± 1.
  - Gapless excitations (corresponding to green quark which does not involve in pairing) carry the same Berry monopole charge $\pm 1/2$ as chiral fermions.
- New feature arises in planar and color spin locking phase:
  - Gap function has no nodes.
  - Gapless excitations (mixing among red, blue, green quarks) carry an unusual Berry monopole charge $\pm 3/2$

    Gapless excitation inherit the topology of Berry structure.

Summary and outlook

- We find non-trivial topological structure in the same flavor pairing with opposite chirality: either in gap functions or in excitations.
- Among the above two scenarios, the second one is energetically favorable since gap function without nodes will lower the free energy.
- Outlook: anomaly matching? Hadron-quark continuity? Observational consequence?

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Back-up

BdG Hamiltonian

$$\mathcal{H}_{\rm BdG} = \begin{pmatrix} -kh(\hat{\boldsymbol{k}}) - \mu & 0 & 0 & \Delta_L \\ 0 & kh(\hat{\boldsymbol{k}}) - \mu & \Delta_R & 0 \\ 0 & \Delta_R^{+} & -kh^*(\boldsymbol{k}) + \mu & 0 \\ \Delta_L^{\dagger} & 0 & 0 & kh^*(\hat{\boldsymbol{k}}) + \mu \end{pmatrix}$$

Color structure of gap matrices

$$\begin{pmatrix} 0 & -ic_3 & 0 \\ ic_3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$ *Polar phase*

$$\begin{pmatrix} 0 & -ic_3 & ic_2 \\ ic_3 & 0 & -ic_1 \\ -ic_2 & ic_1 & 0 \end{pmatrix}$$ *CSL phase*

$$c_1 = e^{i\phi}(\cos\theta\cos\phi - i\sin\phi)\,, \qquad c_2 = e^{i\phi}(\cos\theta\sin\phi + i\cos\phi)\,, \qquad c_3 = -e^{i\phi}\sin\theta\,.$$

Gapless excitations

Polar phase: gapless excitations carry Berry monopole charge 1/2.

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \otimes e_+(\hat{\boldsymbol{k}})\,. \quad \Rightarrow \quad \int d\boldsymbol{S} \cdot \boldsymbol{B}_{R,0} = 2\pi\,.$$

CSL phase: gapless excitations carry Berry monopole charge 3/2.

$$\frac{1}{\sqrt{2}} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} \otimes e_+\,, \quad \Rightarrow \quad A_{R,0} = \frac{i}{2} \sum_{i=1}^{3} (c_i^* \nabla c_i) + i \left(e_+^\dagger \nabla e_+ \right)$$

So

$$\boldsymbol{B}_{R,0} = (1 + \frac{1}{2})\hat{\boldsymbol{k}}\,, \qquad \int d\boldsymbol{S} \cdot \boldsymbol{B}_{R,0} = 6\pi\,.$$