# Neutron stars with a crossover equation of state

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# Crossover Equation of State

Neutron star masses have an upper bound that is strongly dependent on the thermodynamics of their interior. This makes neutron stars an excellent laboratory for exploring the QCD equation of state. Modern estimates place this bound just above $2M_{\odot}$.

Discoveries of increasingly heavy neutron stars put strains on this bound:

- J1640-2230 : $1.928 \pm 0.017\ M_{\odot}$
- J0348+0432 : $2.01 \pm 0.04\ M_{\odot}$
- J0740+6620 : $2.14 \pm 0.10\ M_{\odot}$

Here we present a crossover model that connects the thermodynamics of hadrons with that of quarks and gluons, and show that we can obtain neutron stars above $2M_{\odot}$ for particular choices of parameters. The equation of state has been modeled using a switching function $S(\mu)$, ranging from 0 to 1, to transition smoothly from a hadronic gas to quark–gluon plasma.[1] At $T = 0$ the equation of state is given by:

$$P(\mu) = S(\mu)P_q(\mu) + (1 - S(\mu))P_h(\mu)\ , \qquad S(\mu) = \exp\left[-\left(\frac{\mu_0}{\mu}\right)^4\right]$$

Here $P_h(\mu)$ is the hadronic pressure and $P_q(\mu)$ is the quark pressure as functions of the baryon chemical potential $\mu$.

[1] M. Albright, J. Kapusta, and C. Young, Phys. Rev. C 90, 024915 (2014); 92, 044904 (2015).

# Mean Field Theory

Interactions between baryons are mediated by meson fields. If we consider a simple, mean field model with scalar $\sigma$, and vectors $\omega$ and $\rho$, and express the meson fields as their mean-field values.

$$\sigma = \bar{\sigma}, \qquad \omega_\mu = \bar{\omega}_0 \delta_{\mu 0}, \qquad \rho_\mu^a = \delta_{\mu 0}\delta_{a3}\bar{\rho}_0^3$$

In the Lagrangian, these produce terms which can be absorbed by the baryon mass and chemical potential to get their effective values.

$$m_j^* = m_j - g_{\sigma j}\bar{\sigma}, \qquad \mu_j^* = \mu_j - g_{\omega j}\bar{\omega}_0 - I_{3j} g_{\rho j}\bar{\rho}_0^3$$

$$\mathcal{L} = \sum_{j \in Baryons} \bar{\psi}_j \left( i\partial\!\!\!/ - m_j^* + \gamma_0 \mu_j^* \right) \psi_j - \frac{m_\sigma^2}{2}\bar{\sigma}^2 - \frac{b m_N}{3}\left(g_{\sigma N}\bar{\sigma}\right)^3 - \frac{c}{4}\left(g_{\sigma N}\bar{\sigma}\right)^4 + \frac{m_\omega^2}{2}\bar{\omega}_0^2 + \frac{m_\rho^2}{2}\left(\bar{\rho}_0^3\right)^2$$

The constants $g_{\sigma N}, g_{\omega N}, g_{\rho N}, b$, and $c$ are determined by fixing the properties of isospin-symmetric matter. For simplicity, we will assume a neutron star consisting purely of zero temperature neutron matter. This gives:

$$P_h = P_{FG}(\mu_n^*, m_N^*) - \frac{m_\sigma^2}{2}\bar{\sigma}^2 - \frac{b m_N}{3}\left(g_{\sigma N}\bar{\sigma}\right)^3 - \frac{c}{4}\left(g_{\sigma N}\bar{\sigma}\right)^4 + \frac{m_\omega^2}{2}\bar{\omega}_0^2 + \frac{m_\rho^2}{2}\left(\bar{\rho}_0^3\right)^2$$

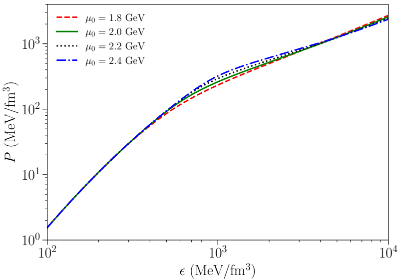
where $P_{FG}$ is the pressure of a $T = 0$ Fermi gas, and the mean field values are determined by finding the extrema of the pressure.

# Quark-Gluon equation of state

For three massless quarks with equal chemical potential $\mu_q = \mu/3$, the pressure at zero temperature is:

$$P_q = \frac{N_f \mu_q^4}{4\pi^2} \left\{ \frac{N_c}{3} - N_g \frac{\alpha_s(M)}{4\pi} + N_g \left(\frac{\alpha_s(M)}{4\pi}\right)^2 \left[ \frac{11N_c - 2N_f}{3} \ln\left(\frac{\mu_q^2}{M^2}\right) - 2.25N_c - 2.81N_f - \frac{4.236}{N_c} - 2N_f \ln\left(\frac{N_f \alpha_s(M)}{4\pi}\right) \right] \right\}$$

$N_c = N_f = 3$ and $N_g = 8$. We let the renormalization scale $M$ equal to $\mu$.




Pressure versus energy density for different choices of $\mu_0$.

The figure shows the pressure as a function of energy density of the full crossover equation of state for various valus of $\mu_0$. The bend in the curve indicates the point where the quark equation of state becomes relevant.

# Speed of Sound

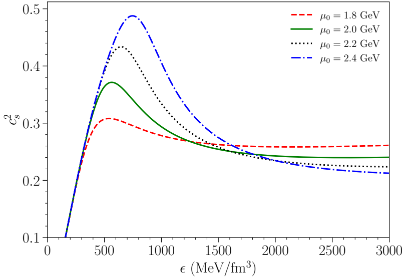
At large $\mu$, $P_h$ and $\epsilon_h$ approach the same limit.

$$P_h \to \epsilon_h \to \frac{1}{2}\left[\frac{g_\omega^2}{m_\omega^2} + \frac{g_\rho^2}{4m_\rho^2}\right]^{-1}\mu^2$$

Which implies that $c_s^2 = \frac{\partial P}{\partial \epsilon} = 1$. Meanwhile, for the quark equation of state:

$$c_s^2 = \frac{1}{3}\left(1 - \frac{33 - 2N_f}{9\pi^2}\alpha_s^2\right)$$

Which gives the limit of $1/3$ approached from below, which is the expected theoretical behavior for quark matter.

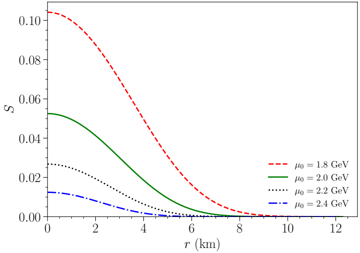



Speed of sound squared versus energy density for different choices of $\mu_0$.

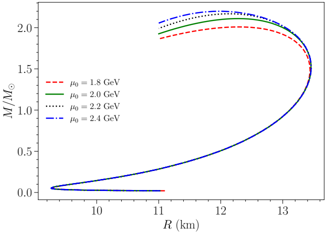
# Neuton Star Mass and Radius

We can get mass and radius by solving the Tolman–Oppenheimer–Volkoff (TOV) Equation:

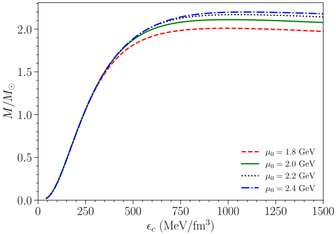
$$\frac{dP}{dr} = \frac{-G\left(\epsilon(r) + P(r)\right)\left(m(r) + 4\pi r^3 P(r)\right)}{r^2 - Grm(r)}$$




Switching function vs. radius of the maximum neutron star mass for different choices of $\mu_0$. It represents the fraction of the total pressure contributed by quark matter.




Neutron star mass vs. radius




Neutron star mass vs. central energy density