Heavy quark potential in the QGP: DNN meets LQCD

Shuzhe Shi, Kai Zhou, Jiaxing Zhao, Swagato Mukherjee, and Pengfei Zhuang

Phys. Rev. D 105, 014017
Proof of Concept - Can we really learn $V(r)$ from $\{E_n\}$?

$$\hat{H} \psi_n = -\frac{\nabla^2}{2m} \psi_n + V(r) \psi_n = E_n \psi_n$$

- $V(r)$ known $\Rightarrow \{E_n, \psi_n(r)\}$: ✔
- $\psi_n(r)$ known $\Rightarrow V(r)$: $\frac{\nabla^2 \psi_n}{2m \psi_n} = V(r) - E_n$
- $\{E_n\}$ known $\Rightarrow V(r)$: ???

learn $V(r)$ according to

$$\{E_n\} = \left\{ \frac{3}{2}, \frac{7}{2}, \frac{11}{2}, \frac{15}{2}, \frac{19}{2} \right\} \text{ GeV}$$

$$\delta E_n = \langle \psi_n | \delta V(r) | \psi_n \rangle$$

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<thead>
<tr>
<th>r (GeV$^{-1}$)</th>
<th>0</th>
<th>1</th>
<th>2</th>
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<tbody>
<tr>
<td>$</td>
<td>\psi_n</td>
<td>^2 + E_n$ or $V(r)$</td>
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<td>target spectrum</td>
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<td>target potential</td>
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<td>(\chi^2 = 11255.6)</td>
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step #1
How to learn potential using DNN?

\[ V(T, r) = V_R(T, r) + i \, V_I(T, r) \]

\[ \hat{H} \psi_n = -\frac{\nabla^2}{2m_\mu} \psi_n + V(r) \psi_n = E_n \psi_n \]

\[ \delta E_n = \langle \psi_n | \delta V(r) | \psi_n \rangle \]

\[ \chi^2 = \sum_{T, i} \left( \frac{m_{T,i} - m_{T,i}^{\text{lattice}}}{\delta m_{T,i}^{\text{lattice}}} \right)^2 + \left( \frac{\Gamma_{T,i} - \Gamma_{T,i}^{\text{lattice}}}{\delta \Gamma_{T,i}^{\text{lattice}}} \right)^2, \]
Closure Test - Can we recover a known complex $V(T, r)$?

- Start with a known potential (solid line)
  $V_R(T, r) = \frac{\sigma}{\mu_D} \left( 2 - (2 + \mu_D r)e^{-\mu_D r} \right) - a \left( \mu_D T + \frac{e^{-\mu_D r}}{r} \right) + B$.
  $V_I(T, r) = -\sqrt{\frac{\pi}{4}} \mu_D T \sigma r^3 G^2 \frac{\mu_D r}{2} + \alpha T \phi(\mu_D r)$.

- Compute $\{m_n, \Gamma_n\}$ at five different $T$-points

- Learn the potential using DNN (dash + band)
What physics we have learned from $V_{\text{DNN}}(T, r)$?

- ☑ color screening
- ○ weaker than existing LQCD result
- ☑ increase with $T$ and $r$
- ○ stronger than HTL