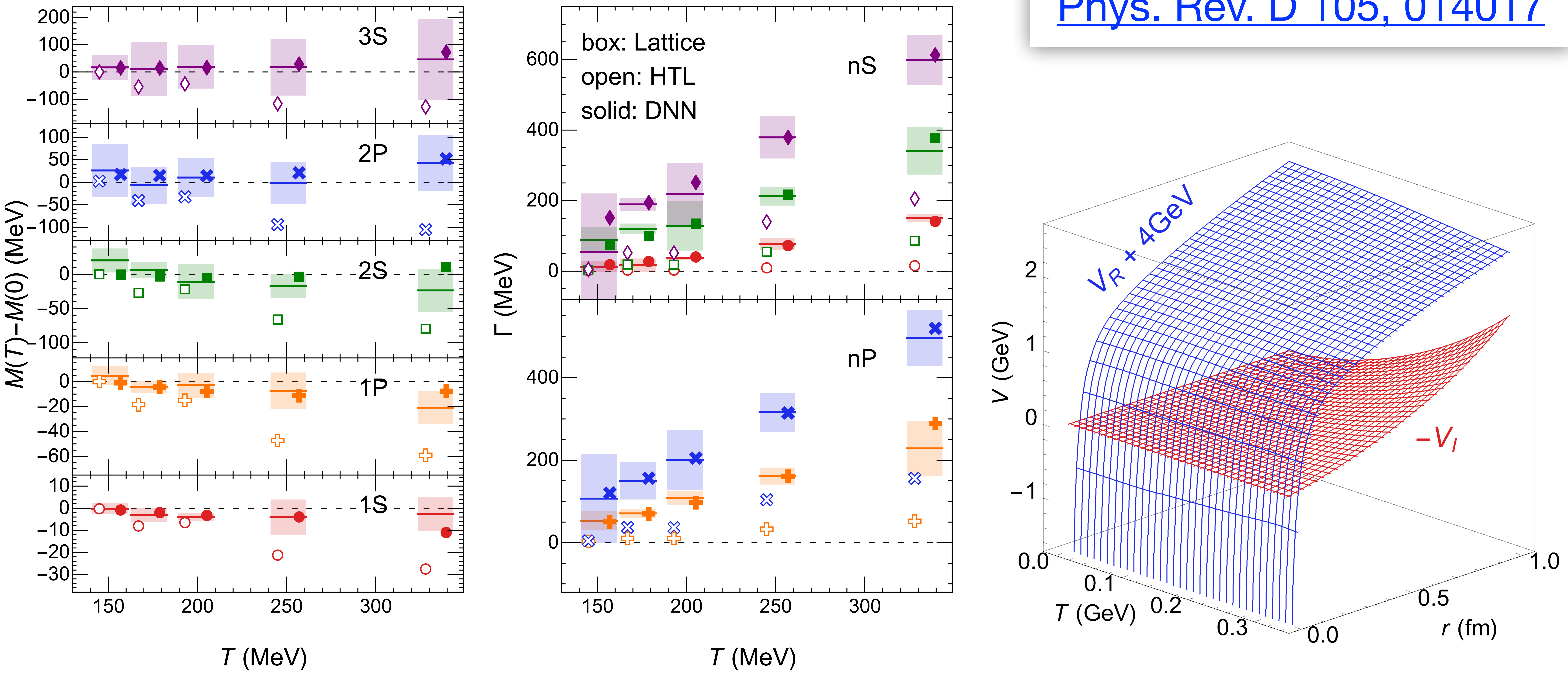
Heavy quark potential in the QGP: DNN meets LQCD

Shuzhe Shi, Kai Zhou, Jiaxing Zhao, Swagato Mukherjee, and Pengfei Zhuang

Phys. Rev. D 105, 014017



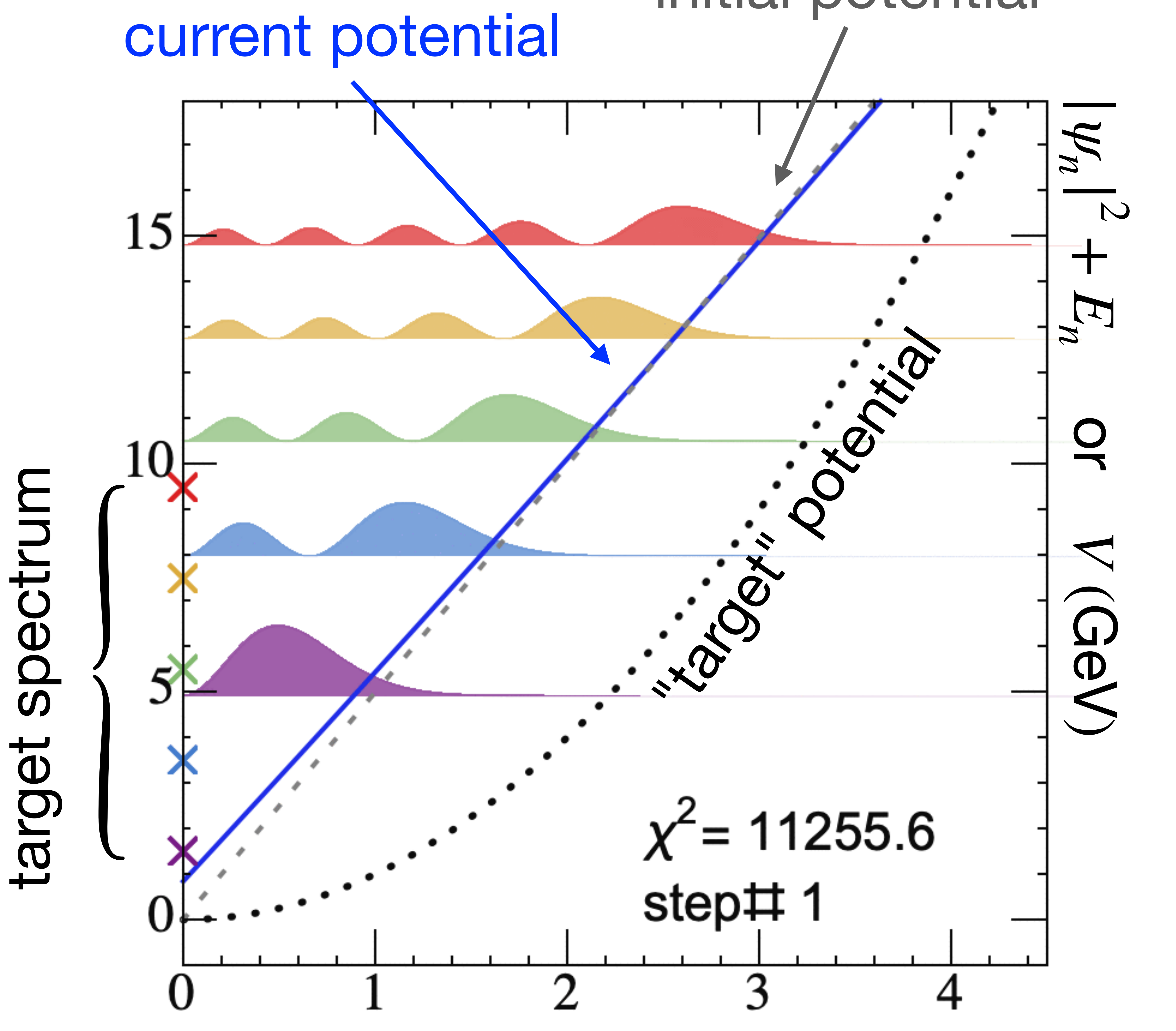
Proof of Concept - Can we really learn $V(r)$ from $\{E_n\}$?

$$\hat{H}\psi_n = -\frac{\nabla^2}{2m}\psi_n + V(r)\psi_n = E_n\psi_n$$

- $V(r)$ known $\Longrightarrow \{E_n, \psi_n(r)\}$: ☑
- $\psi_n(r)$ known $\Longrightarrow V(r)$: $\dfrac{\nabla^2\psi_n}{2m\,\psi_n} = V(r) - E_n$
- $\{E_n\}$ known $\Longrightarrow V(r)$: ???

learn $V(r)$ according to

$$\{E_n\} = \left\{\frac{3}{2}, \frac{7}{2}, \frac{11}{2}, \frac{15}{2}, \frac{19}{2}\right\}\ \text{GeV}$$



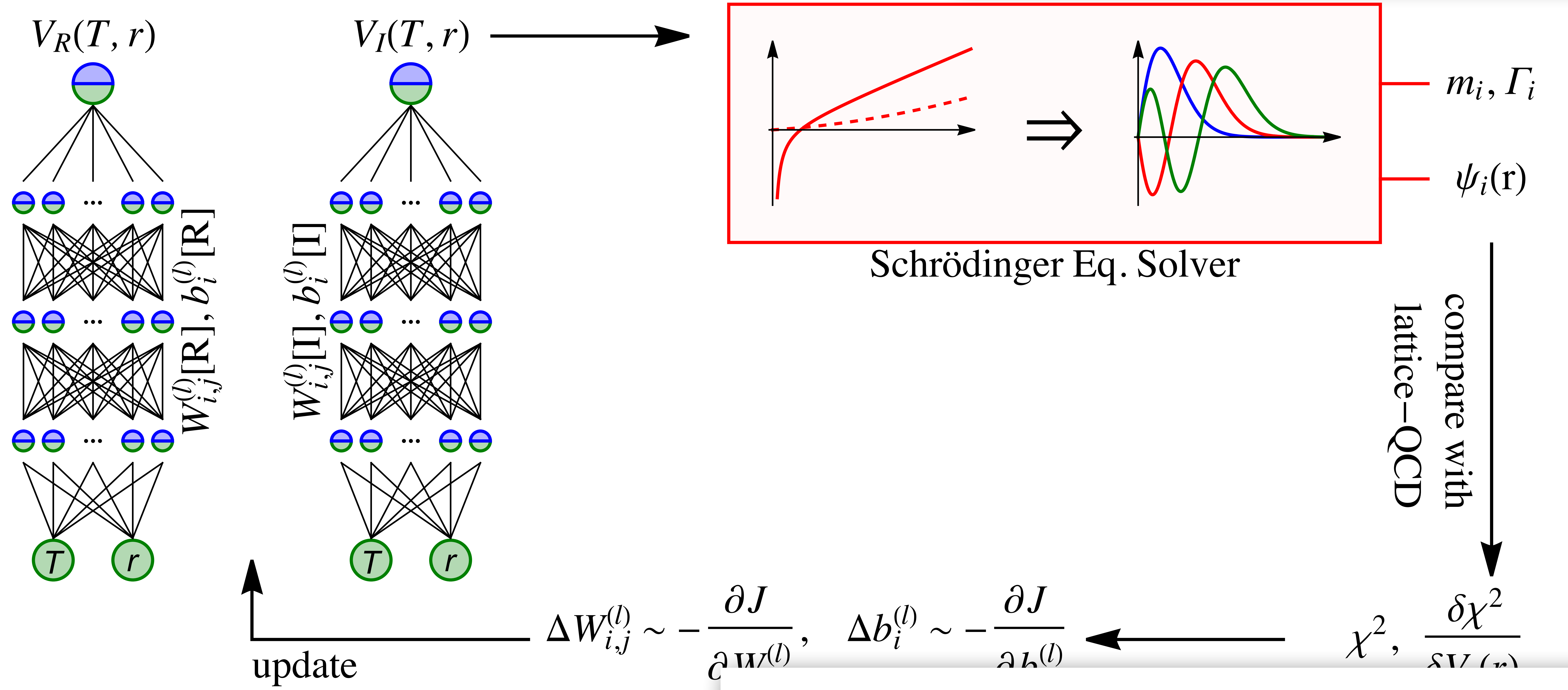
$\delta E_n = \langle \psi_n | \delta V(r) | \psi_n \rangle$

(gif on show)

How to learn potential using DNN?

$$V(T,r) = V_R(T,r) + i\, V_I(T,r)$$

$$\hat{H}\psi_n = -\frac{\nabla^2}{2m_\mu}\psi_n + V(r)\,\psi_n = E_n\psi_n$$



$$\delta E_n = \langle\psi_n|\,\delta V(r)\,|\psi_n\rangle$$

$$\chi^2 = \sum_{T,i}\left(\frac{m_{T,i} - m^{\text{lattice}}_{T,i}}{\delta m^{\text{lattice}}_{T,i}}\right)^2 + \left(\frac{\Gamma_{T,i} - \Gamma^{\text{lattice}}_{T,i}}{\delta\Gamma^{\text{lattice}}_{T,i}}\right)^2,$$

Closure Test - Can we recover a known complex $V(T, r)$?

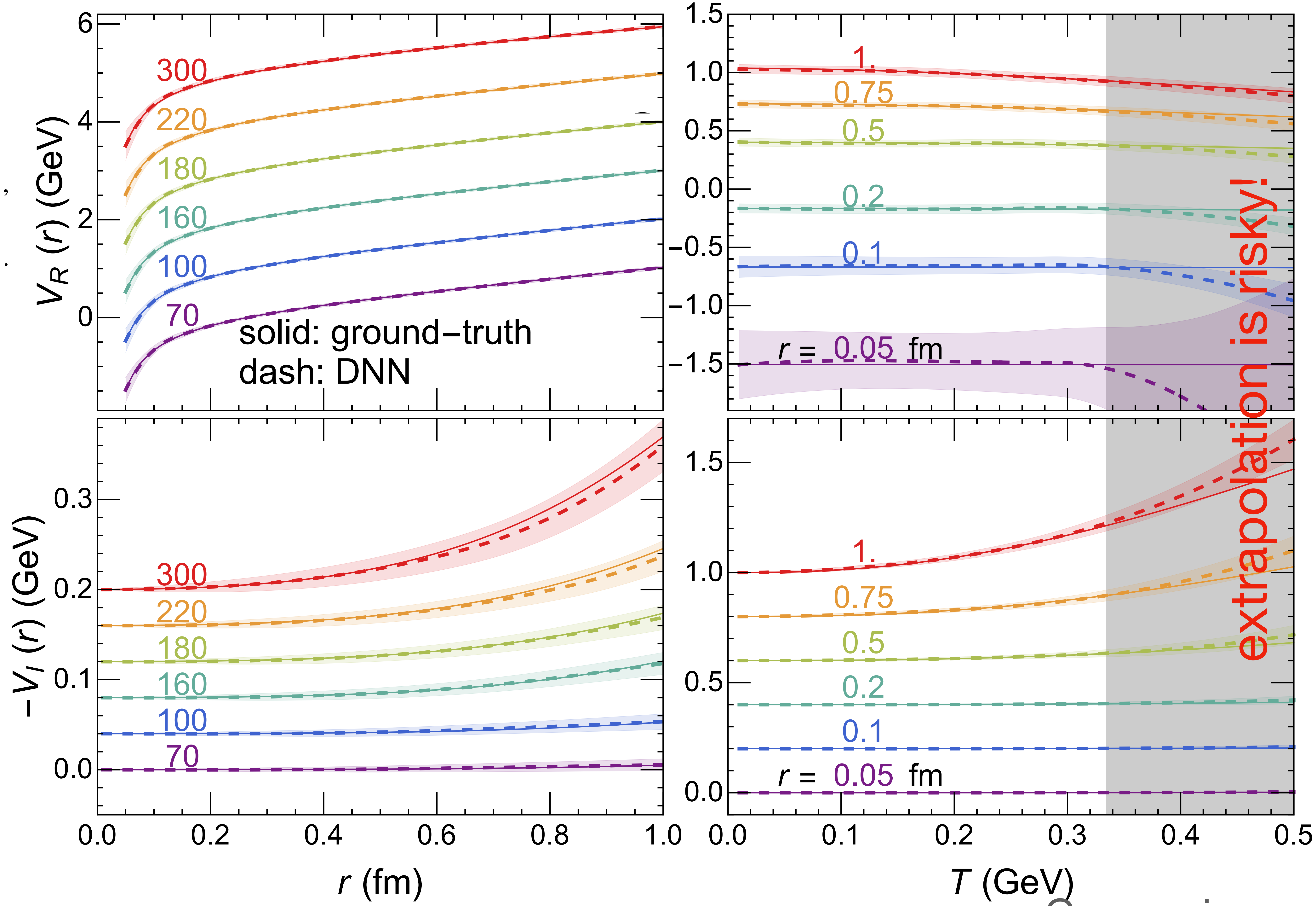
- Start with a known potential (solid line)

$$V_R(T,r) = \frac{\sigma}{\mu_D}\left(2-(2+\mu_D r)e^{-\mu_D r}\right) - \alpha\left(\mu_D + \frac{e^{-\mu_D r}}{r}\right) + B\,,$$

$$V_I(T,r) = -\frac{\sqrt{\pi}}{4}\mu_D T\sigma r^3 G^{2,2}_{2,4}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{2} \\ \frac{1}{2},\frac{1}{2},-\frac{3}{2},-1\end{matrix}\,\middle|\, \frac{\mu_D^2 r^2}{4}\right) - \alpha\, T\,\phi(\mu_D r)\,.$$

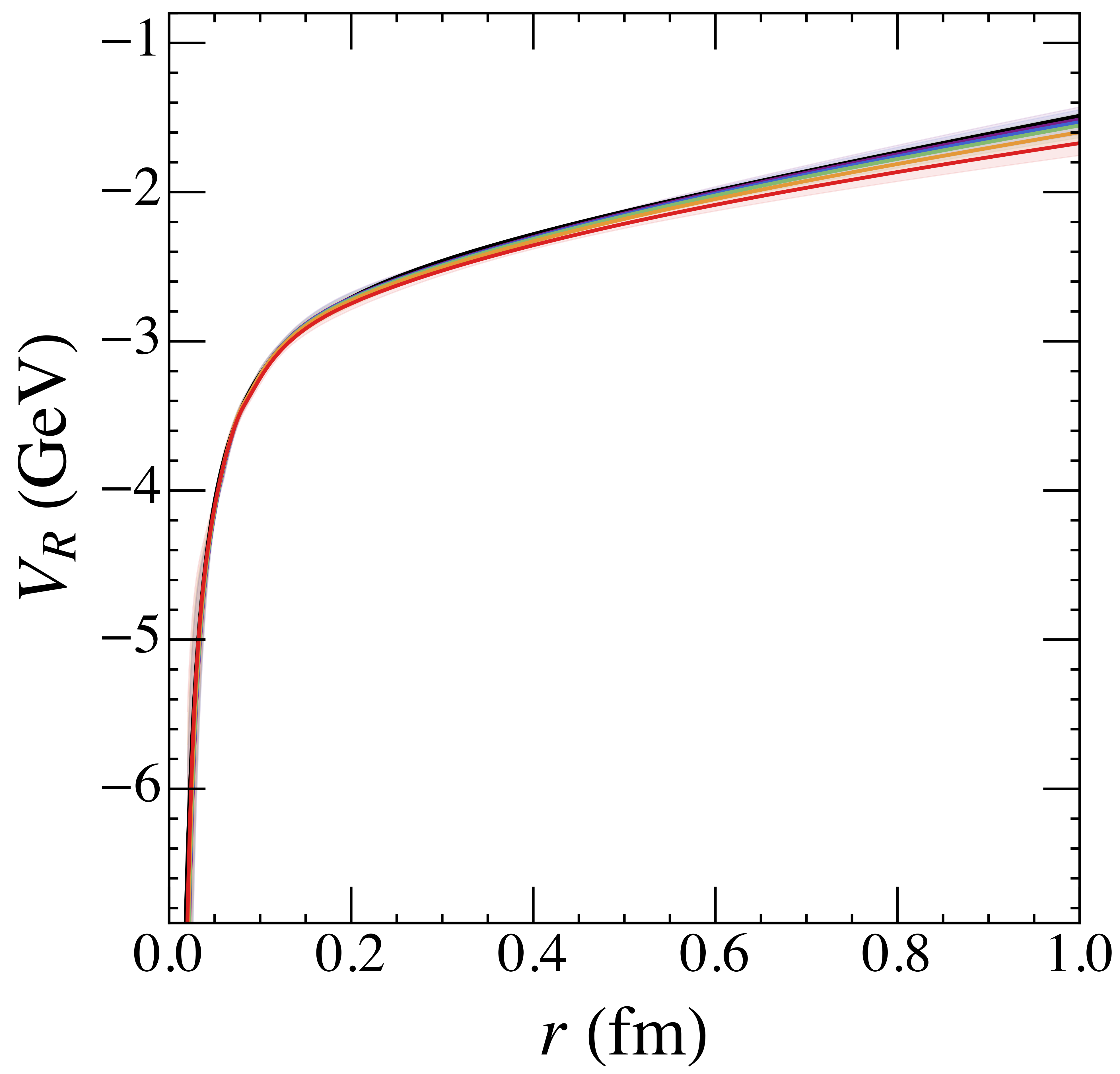
- Compute $\{m_n, \Gamma_n\}$ at five different T-points

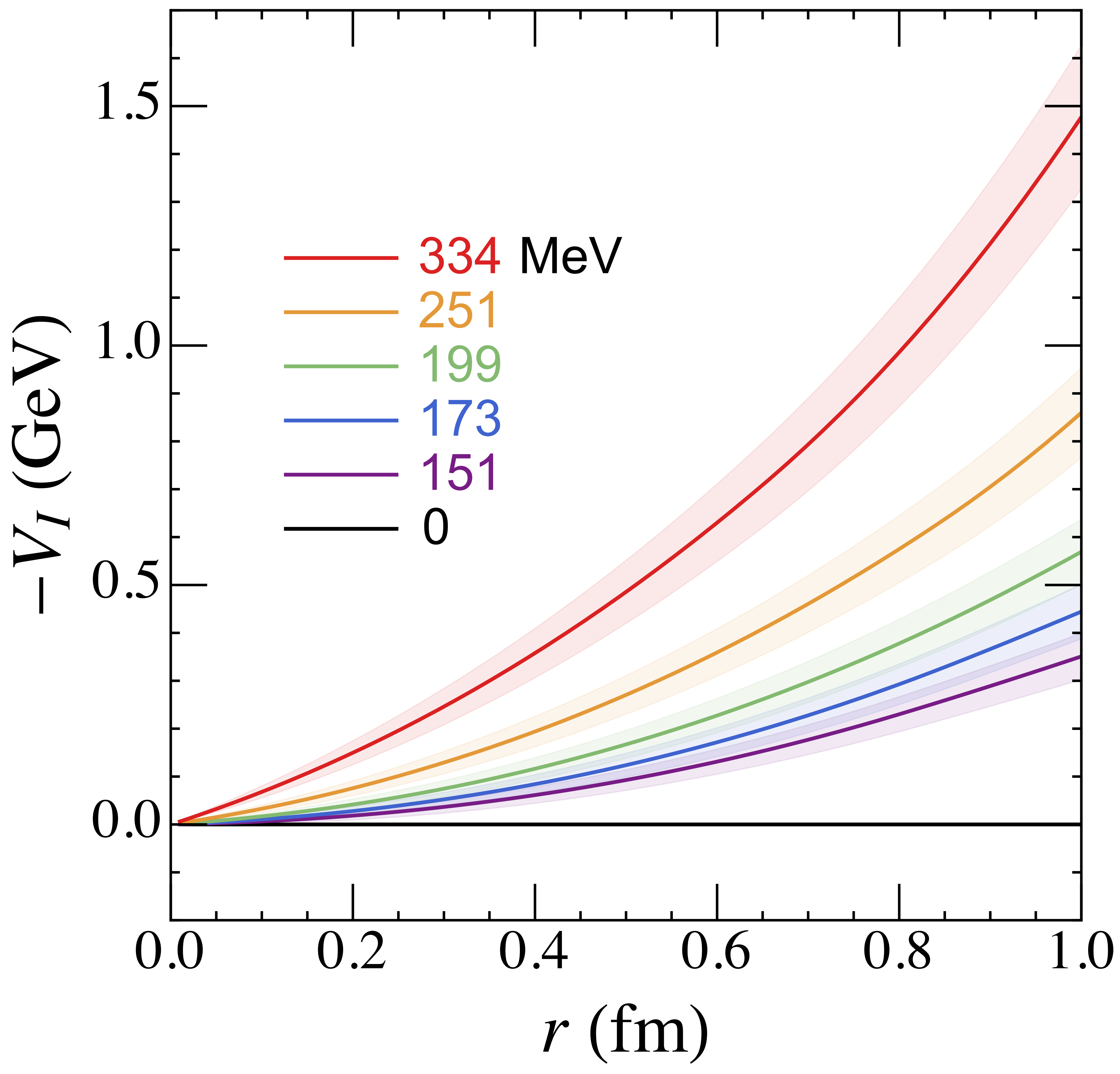
- Learn the potential using DNN (dash + band)



Gray region: extrapolations

What physics we have learned from $V_{\mathrm{DNN}}(T, r)$?





☑ color screening

⦿ weaker than existing LQCD result

☑ increase w/ T and r

⦿ stronger than HTL