Loops in light cone perturbation theory

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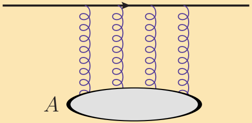
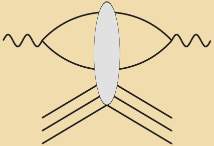
Quark Matter 2022, Kraków, Poland

Probing small-x color field, DIS dipole picture



- Amplitude for quark: **Wilson line**
  $\mathbb{P}\exp\left\{-ig\int^{x^+}\mathrm{d}y^+A^-(y^+,x^-,\mathbf{x})\right\} \underset{x^+\to\infty}{\approx} V(\mathbf{x}) \in \mathrm{SU}(N_\mathrm{c})$
- Amplitude for color dipole $\mathcal{N}(r=|\mathbf{x}-\mathbf{y}|) = 1 - \left\langle \frac{1}{N_\mathrm{c}} \mathrm{Tr}\, V^\dagger(\mathbf{x})V(\mathbf{y})\right\rangle$
- $r=0$: color transparency, $r \gg 1/Q_\mathrm{s}$: saturation

DIS: leading order dipole picture

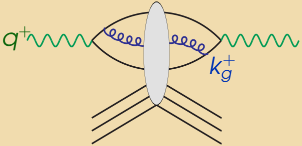


- $\gamma^* \to q\bar{q}$ in vacuum
- $q\bar{q}$ interacts eikonally with target
- σ^{tot} is $2\times$Im-part of amplitude

"Dipole model": Nikolaev, Zakharov 1991

Many fits to HERA data, starting with Golec-Biernat, Wüsthoff 1998

Leading Log and NLO: add gluon



- Soft gluon: large logarithm
  BK evolution Balitsky 1995, Kovchegov 1999

$$\int_{x_{Bj}} \frac{\mathrm{d}k_g^+}{k_g^+} \sim \ln\frac{1}{x_{Bj}}$$

- NLO: full gluon kinematics

Light cone wave function

- Know free particle Fock states: $|\gamma^*\rangle_0, \quad |q\bar{q}\rangle_0, \quad |q\bar{q}g\rangle_0$ etc.
- **Interacting** states are superpositions of these:

$$|\gamma^*\rangle = (1 + \dots)|\gamma^*\rangle_0 + \psi^{\gamma^* \to q\bar{q}} \otimes |q\bar{q}\rangle_0 + \psi^{\gamma^* \to q\bar{q}g} \otimes |q\bar{q}g\rangle_0 + \dots$$

- Calculate in QM perturbation theory, e.g. ground state $|0\rangle$ wavefunction:

$$\psi^{0 \to n} = \sum_n \frac{\langle n| \hat{V} |0\rangle}{E_n - E_0} + \dots$$

  - Here $1/\Delta E$ is $\sim$ the lifetime of the quantum fluctuation from 0 to n
- "Energy" E conjugate to "time", LC time is x^+ $\Longrightarrow$ LC energy k^- (energy "not conserved"!)

Connection to Feynman perturbation theory

- Matrix elements $\langle n| \hat{V} |m\rangle$ are vertices in Feynman rules
- LC energy denominators from propagators, integrating over k^- pole

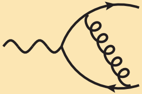
Natural for high energy scattering, x^+ is the relevant "time"

- Collinear factorization: LC quantize proton
- CGC/dipole picture of DIS: LC quantize probe photon

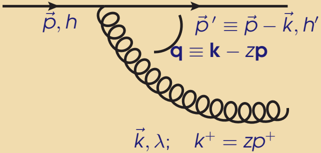
Massive quarks

Beuf, Paatelainen, T.L. 2112.03158 [hep-ph] 2103.14549 [hep-ph] + t.b.p.

- F_2^c important at HERA, EIC program, precision tool for saturation in DIS
- Calculate $\gamma^* \to q\bar{q}$ to one loop with **massive quarks**

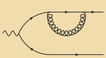


New theory challenge: mass renormalization in LCPT

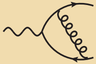


- 2 helicity nonflip-structures $\delta_{h,h'}$, $\propto q^i$ $\Longrightarrow$ more UV div.
- 1 helicity flip $\delta_{h,-h'}$, $\propto m_q$

$$\left[\bar{u}_{h'}(p')\not{\varepsilon}^*_\lambda(k)u_h(p)\right] \sim \overbrace{\bar{u}_{h'}\gamma^+u_h}^{\sim\delta_{h,h'}}\,\delta^{ij}q^i\varepsilon^{*j}_\lambda + \overbrace{\bar{u}_{h'}\gamma^+[\gamma^i,\gamma^j]u_h}^{\sim\delta_{h,h'}}\,q^i\varepsilon^{*j}_\lambda + \overbrace{\bar{u}_{h'}\gamma^+\gamma^ju_h}^{\sim\delta_{h,-h'}}\,m_q\varepsilon^{*j}_\lambda$$



- 2 flips: UV-divergence $\propto m_q^2$
- Renormalize m_q in LO energy denominator



- 1 flip: UV-divergence $\propto m_q$
- Renormalize m_q in LO vertex

$\gamma^* \to q\bar{q}$ with massive quarks

New result Beuf, Paatelainen, T.L. 2112.03158, 2204.02486 full LC gauge 1-loop structure of $\gamma^* \to Q\bar{Q}$

$$\widetilde{\psi}^{\gamma_T^* \to q\bar{q}}_{\text{NLO}} = -\frac{ee_f}{2\pi}\left(\frac{\alpha_s C_F}{2\pi}\right)\left\{\left[\left(\frac{k_0^+ - k_1^+}{q^+}\right)\delta^{ij}\bar{u}(0)\gamma^+ v(1) + \frac{1}{2}\bar{u}(0)\gamma^+[\gamma^i,\gamma^j]v(1)\right]\mathcal{F}\left[P_T{}^i \mathcal{V}^T\right] + \bar{u}(0)\gamma^+ v(1)\mathcal{F}\left[P_T{}^j \mathcal{N}^T\right]\right.$$

$$\left. + m\bar{u}(0)\gamma^+\gamma^i v(1)\mathcal{F}\left[\left(\frac{P_T{}^i P_T{}^j}{P_T{}^2} - \frac{\delta^{ij}}{2}\right)\mathcal{S}^T\right] - m\bar{u}(0)\gamma^+\gamma^j v(1)\mathcal{F}\left[\mathcal{V}^T + \mathcal{M}^T - \frac{\mathcal{S}^T}{2}\right]\right\}\varepsilon^j_\lambda .$$

$$\mathcal{F}\left[\mathbf{P}^i \mathcal{V}^T\right] = \frac{i\mathbf{x}_{01}^i}{|\mathbf{x}_{01}|}\left(\frac{\kappa_z}{2\pi|\mathbf{x}_{01}|}\right)^{\frac{D}{2}-2}\left\{\left[\frac{3}{2} + \log\left(\frac{\alpha}{z}\right) + \log\left(\frac{\alpha}{1-z}\right)\right]\left\{\frac{(4\pi)^{2-\frac{D}{2}}}{(2-\frac{D}{2})}\Gamma\left(3-\frac{D}{2}\right) + \log\left(\frac{|\mathbf{x}_{01}|^2\mu^2}{4}\right)\right.\right.$$

$$\left.\left. + 2\gamma_E\right\} + \frac{1}{2}\frac{(D_s-4)}{(D-4)}\right\}\kappa_z K_{\frac{D}{2}-1}(|\mathbf{x}_{01}|\kappa_z) + \frac{i\mathbf{x}_{01}^i}{|\mathbf{x}_{01}|}\left\{\left[\frac{5}{2} - \frac{\pi^2}{3} + \log^2\left(\frac{z}{1-z}\right) - \Omega^T_{\mathcal{V}} + L\right]\kappa_z K_1(|\mathbf{x}_{01}|\kappa_z) + I^T_{\mathcal{V}}\right\}$$

$$\mathcal{F}\left[\mathbf{P}^j \mathcal{N}^T\right] = \frac{i\mathbf{x}_{01}^j}{|\mathbf{x}_{01}|}\left\{\Omega^T_{\mathcal{N}}\,\kappa_z K_1(|\mathbf{x}_{01}|\kappa_z) + I^T_{\mathcal{N}}\right\}$$

$$\mathcal{F}\left[\left(\frac{\mathbf{P}^i\mathbf{P}^j}{\mathbf{P}^2} - \frac{\delta^{ij}}{2}\right)\mathcal{S}^T\right] = \frac{(1-z)}{2}\left[\frac{\mathbf{x}_{01}^i\mathbf{x}_{01}^j}{|\mathbf{x}_{01}|^2} - \frac{\delta^{ij}}{2}\right]\int_0^z \frac{\mathrm{d}\chi}{(1-\chi)}\int_0^\infty \frac{\mathrm{d}u}{(u+1)^2}|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}$$

$$\times K_1\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}\right) + [z \leftrightarrow 1-z].$$

$$\mathcal{F}\left[\mathcal{V}^T + \mathcal{M}^T - \frac{\mathcal{S}^T}{2}\right] = \left(\frac{\kappa_z}{2\pi|\mathbf{x}_{01}|}\right)^{\frac{D}{2}-2}\left\{\left[\frac{3}{2} + \log\left(\frac{\alpha}{z}\right) + \log\left(\frac{\alpha}{1-z}\right)\right]\left\{\frac{(4\pi)^{2-\frac{D}{2}}}{(2-\frac{D}{2})}\Gamma\left(3-\frac{D}{2}\right) + \log\left(\frac{|\mathbf{x}_{01}|^2\mu^2}{4}\right)\right.\right.$$

$$\left.\left. + 2\gamma_E\right\} + \frac{1}{2}\frac{(D_s-4)}{(D-4)}\right\}K_{\frac{D}{2}-2}(|\mathbf{x}_{01}|\kappa_z) + \left\{3 - \frac{\pi^2}{3} + \log^2\left(\frac{z}{1-z}\right) - \Omega^T_{\mathcal{V}} + L\right\}K_0(|\mathbf{x}_{01}|\kappa_z) + I^T_{\mathcal{VMS}},$$

$$\Omega^T_{\mathcal{V}} = -\left(1+\frac{1}{2z}\right)\left[\log(1-z) + \gamma\log\left(\frac{1+\gamma}{1+\gamma-2z}\right)\right] + \frac{1}{2z}\left[\left(z+\frac{1}{2}\right)(1-\gamma) + \frac{m^2}{Q^2}\right]\log\left(\frac{\kappa_z^2}{m^2}\right) + [z \leftrightarrow 1-z]$$

$$I^T_{\mathcal{V}} = \int_0^1 \frac{\mathrm{d}\xi}{\xi}\left(\frac{2\log(\xi)}{(1-\xi)} - \frac{(1+\xi)}{2}\right)\left\{\sqrt{\kappa_z^2 + \frac{\xi(1-z)}{(1-\xi)}m^2}K_1\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + \frac{\xi(1-z)}{(1-\xi)}m^2}\right) - [\xi \to 0]\right\}$$

$$- \int_0^1 \mathrm{d}\xi\left(\frac{\log(\xi)}{(1-\xi)^2} + \frac{z}{(1-x)} + \frac{z}{2}\right)\frac{(1-z)m^2}{\sqrt{\kappa_z^2 + \frac{\xi(1-z)}{(1-\xi)}m^2}}K_1\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + \frac{\xi(1-z)}{(1-\xi)}m^2}\right)$$

$$- \int_0^z \frac{\mathrm{d}\chi}{(1-\chi)}\int_0^\infty \frac{\mathrm{d}u}{u(u+1)}\frac{m^2}{\kappa_\chi^2}\left[2\chi + \left(\frac{u}{1+u}\right)^2\frac{1}{z}(z-\chi)(1-2\chi)\right]$$

$$\times\left\{\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\xi)}\kappa_\chi^2}K_1\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\xi)}\kappa_\chi^2}\right) - [u \to 0]\right\}$$

$$- \int_0^z \frac{\mathrm{d}\chi}{(1-\chi)^2}\int_0^\infty \frac{\mathrm{d}u}{(u+1)}(z-\chi)\left[1 - \frac{2u}{1+u}(z-\chi) + \left(\frac{u}{1+u}\right)^2\frac{1}{z}(z-\chi)^2\right]$$

$$\times\frac{m^2}{\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}}K_1\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}\right) + [z \leftrightarrow 1-z].$$

$$\Omega^T_{\mathcal{N}} = \frac{z+1-2z^2}{z}\left[\log(1-z) + \gamma\log\left(\frac{1+\gamma}{1+\gamma-2z}\right)\right] - \frac{(1-z)}{z}\left[\frac{2z+1}{2}(1-\gamma) + \frac{m^2}{Q^2}\right]\log\left(\frac{\kappa_z^2}{m^2}\right) - [z \leftrightarrow 1-z] \qquad (9)$$

$$I^T_{\mathcal{N}} = \frac{2(1-z)}{z}\int_0^z \mathrm{d}\chi\int_0^\infty \frac{\mathrm{d}u}{(u+1)^3}\left\{\left[(2+u)uz + u^2\chi\right]\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}\,K_1\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}\right)\right. \qquad (10)$$

$$\left. + \frac{m^2}{\kappa_\chi^2}\left(\frac{z}{1-z} + \frac{\chi}{1-\chi}[u - 2z - 2u\chi]\right)\left[\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}\,K_1\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}\right) - [u \to 0]\right]\right\} - [z \leftrightarrow 1-z].$$

$$I^T_{\mathcal{VMS}} = \int_0^1 \frac{\mathrm{d}\xi}{\xi}\left(\frac{2\log(\xi)}{(1-\xi)} - \frac{(1+\xi)}{2}\right)\left\{K_0\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + \frac{\xi(1-z)}{(1-\xi)}m^2}\right) - [\xi \to 0]\right\}$$

$$+ \int_0^1 \mathrm{d}\xi\left(-\frac{3(1-z)}{2(1-\xi)} + \frac{(1-z)}{2}\right)K_0\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + \frac{\xi(1-z)}{(1-\xi)}m^2}\right)$$

$$+ \int_0^z \frac{\mathrm{d}\chi}{(1-\chi)}\int_0^\infty \frac{\mathrm{d}u}{(u+1)^2}\left\{-z - \frac{u}{(1+u)}\frac{(z+u\chi)}{z}(\chi - (1-z))\right\}K_0\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}\right)$$

$$+ \int_0^z \mathrm{d}\chi\int_0^\infty \frac{\mathrm{d}u}{(u+1)^3}\left\{\frac{\kappa_z^2}{\kappa_\chi^2}\left[1 + u\frac{\chi(1-\chi)}{z(1-z)}\right] - \frac{m^2}{\kappa_\chi^2}\frac{\chi}{(1-\chi)}\left[2\frac{(1+u)^2}{u} + \frac{u}{z(1-z)}(z-\chi)^2\right]\right\}$$

$$\times\left\{K_0\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}\right) - [u \to 0]\right\} + [z \leftrightarrow 1-z].$$

Cross section, real and virtual correction

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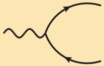
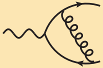
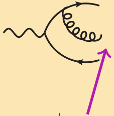
Evaluate cross section as $\sigma^{\mathrm{NLO}}_{L,T} = \sigma^{\mathrm{LO}}_{L,T} + \sigma^{\mathrm{dip}}_{L,T} + \sigma^{qg}_{L,T,\mathrm{sub.}}$



$$\Rightarrow \quad \sigma^{\mathrm{LO}} \sim \int_0^1 \mathrm{d}z_1 \int_{\mathbf{x}_0,\mathbf{x}_1} |\psi^{\mathrm{LO}}_{\gamma^*\to q\bar{q}}(z_1,\mathbf{x}_0,\mathbf{x}_1)|^2 \mathcal{N}_{01}(x_{Bj})$$



$$- * \Rightarrow \quad \sigma^{\mathrm{dip}} \sim \alpha_s C_\mathrm{F} \int_{\mathbf{x}_0,\mathbf{x}_1,z_1} \left|\psi^{\mathrm{LO}}_{\gamma^*\to q\bar{q}}\right|^2 \left[\frac{1}{2}\ln^2\left(\frac{z_1}{1-z_1}\right) - \frac{\pi^2}{6} + \frac{5}{2}\right] \mathcal{N}_{01}(x_{Bj})$$



$$+ * \Rightarrow \quad \sigma^{qg}_{\mathrm{sub.}} \sim \alpha_s C_\mathrm{F} \int_{z_1,z_2,\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \mathrm{d}z_2 \Big[|\psi_{\gamma^*\to q\bar{q}g}(z_1,z_2,\{\mathbf{x}_i\})|^2 \mathcal{N}_{012}(X(z_2)) - |\psi_{\gamma^*\to q\bar{q}g}(z_1,0,\{\mathbf{x}_i\})|^2 \mathcal{N}_{012}(X(z_2)) \Big].$$

* UV-divergence cancellation — LL: subtract leading log, already in BK-evolved $\mathcal{N}$

- Parametrically $X(z_2) \sim x_{Bj}$, but $X(z_2) \sim 1/z_2$ essential!
  ("k_T-factorization" with fixed rapidity scale is unstable @ NLO. Analogous probem in $p + A \to h + X$)
- Factorization already demonstrated for massless quarks see 2007.01645 [hep-ph] poster Hänninen

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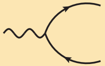

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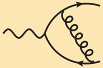

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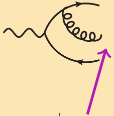

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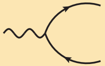

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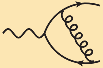

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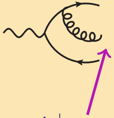

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- LL

$k_g^+ \sim z_2$

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