Maximum entropy kinetic matching conditions for heavy-ion collisions

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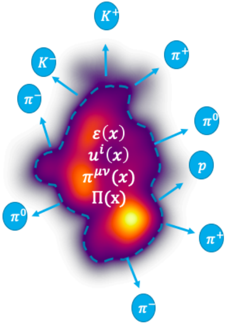
From hydrodynamic fields to particles

- Hydrodynamics ceases to be valid when $Kn \sim (\partial \cdot u)/T \gg 1$; requires a prescription for 'particlization':

$$E_p \frac{dN}{d^3p} = \int d\Sigma_\mu \, p^\mu \, f(x,p)$$

- Information available at freeze-out, $(\epsilon, u^\mu, \Pi, \pi^{\mu\nu})$, provide constraints on moments of $f(x,p)$.



- Standard approaches invoke multiple assumptions, $f \approx f_{eq}(1+\phi)$:

$$\phi_{Grad} = \frac{p^\mu p^\nu}{2(\epsilon+P)T^2}\left(\pi_{\mu\nu} + \frac{2}{5}\Pi\Delta_{\mu\nu}\right), \quad \text{(quadratic ansatz in momenta)}$$

$$\phi_{CE} = -\frac{\beta}{3\beta_\Pi}\left(3c_s^2(u\cdot p)^2 + p_{\langle\mu\rangle}p^{\langle\mu\rangle}\right)\frac{\Pi}{u\cdot p} + \frac{\beta}{2\beta_\pi}\frac{p_\mu p_\nu \pi^{\mu\nu}}{u\cdot p} \quad \text{(from simplified BE)}$$

- Viscous corrections present significant source of uncertainty in extraction of (η, ζ) due to uncontrolled physics approximations; also breaks down at large momenta: $f(x,p) < 0$.

The maximum-entropy principle [E. Jaynes, Phys. Rev. 106, 620 (1957)]

- The '**least-biased**' distribution that uses **all of, and only** information provided by hydro is one that maximizes non-equilibrium entropy,

$$s[f] = -\int dP\ (u\cdot p)\,\Phi[f],\ \ \Phi[f] \equiv f\ln(f) - \frac{1+\theta f}{\theta}\ln(1+\theta f),$$

($\theta = (-1, 0, 1)$ for FD, MB, BE statistics) subject to constraints,

$$\int dP\,(u\cdot p)^2\ f = \epsilon,\ \ -\frac{1}{3}\int dP\, p_{\langle\mu\rangle}p^{\langle\mu\rangle}\ f = P_{eq} + \Pi,\ \ \int dP\, p^{\langle\mu}p^{\nu\rangle}\ f = \pi^{\mu\nu}$$

- Using $(\delta s/\delta f) = 0$, the maximum-entropy distribution is

$$f_{\rm ME}(x,p) = \left[\exp\left(\Lambda(u\cdot p) - \frac{\lambda_\Pi}{u\cdot p}p_{\langle\alpha\rangle}p^{\langle\alpha\rangle} + \frac{\gamma_{\langle\alpha\beta\rangle}}{u\cdot p}p^{\langle\alpha}p^{\beta\rangle}\right) - \theta\right]^{-1}.$$

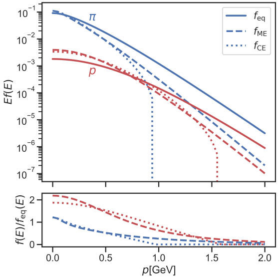
[D. Everett, C. Chattopadhyay, U. Heinz, Phys. Rev. C 103, 064902 (2021)]

- Key features of $f_{\rm ME}$:
  - Positive-definite for all momenta.
  - Non-linear dependence on $(\Pi, \pi^{\mu\nu})$; exact matching to $T^{\mu\nu}$ for entire range of viscous stresses allowed by kinetic theory.
  - Reduces to linearized Chapman-Enskog δf of RTA Boltzmann eq. for weak dissipative stresses.

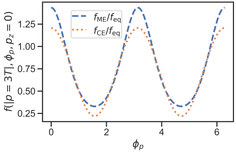
The maximum-entropy distribution

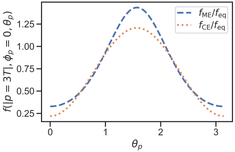
- 7 Lagrange parameters $(\Lambda, \lambda_\Pi, \gamma_{\langle\mu\nu\rangle}) \implies$ 7-d inversion problem; numerically expensive.
- However, matching condition implies shear matrix $\boldsymbol{\pi} \equiv \pi^{ij}_{LRF}$ is a power-series in $\boldsymbol{\gamma} \equiv \gamma^{ij}_{LRF}$; $[\boldsymbol{\pi}, \boldsymbol{\gamma}] = 0$.
- In eigenbasis of $\boldsymbol{\pi}$, reducible to 4-d inversion problem; tractable.

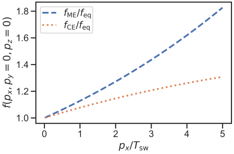
L.R.F. spectra using $\Pi = -P_{eq}/3$ at $T = 150$ MeV (no shear)



$(\pi^{xx} = \pi^{zz} = P_{eq}/5 \neq \pi^{yy})$ $(\pi^{xx} = \pi^{yy} = P_{eq}/5 \neq \pi^{zz})$, $T_{sw} = 150$ MeV.







Massless Boltzmann gas ($\Pi = 0$): Differences between $f_{\rm ME}$ and $f_{\rm CE}$ increase with $|\boldsymbol{p}|$; for fixed $|\boldsymbol{p}|$, differences maximum along eigen-directions of $\boldsymbol{\pi}$.

Hydrodynamics using maximum-entropy distribution

- Consider, for simplicity, a Bjorken system whose evolution is governed by RTA Boltzmann eq., $\frac{\partial f}{\partial \tau} = -\frac{1}{\tau_R}\,(f - f_{eq})$.
- The evolution of components of $T^{\mu\nu} = \mathrm{diag}(\epsilon, P_T, P_T, P_L)$ are,

$$\frac{d\epsilon}{d\tau} = -\frac{\epsilon + P_L}{\tau}, \quad \frac{dP_L}{d\tau} = -\frac{P_L - P_{eq}}{\tau_R} + \frac{\bar{\zeta}_z^L}{\tau}, \quad \frac{dP_T}{d\tau} = -\frac{P_T - P_{eq}}{\tau_R} + \frac{\bar{\zeta}_z^\perp}{\tau}.$$

- The system of equations are **not closed** as the couplings,

$$\bar{\zeta}_z^L = -3P_L + \int dP\, E_p^{-2}\, p_z^4\, f, \quad \bar{\zeta}_z^\perp = -P_T + \frac{1}{2}\int dP\, E_p^{-2}\, p_z^2\, p_T^2\, f,$$

  require knowledge of exact f; amounts to solving full kinetic theory.

- To obtain hydro, we must truncate using a distribution that knows only about $T^{\mu\nu}$; the least-biased of all such distributions is

$$f_{\mathrm{ME}} = \exp\Big(-\Lambda E_p - \frac{\lambda_\Pi}{E_p}\boldsymbol{p}^2 - \frac{\gamma}{E_p}\,(p_T^2/2 - p_z^2)\Big)\ [\text{for Boltzmann statistics}],$$

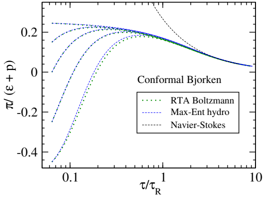
  with Lagrange parameters **matched** to (ϵ, P_L, P_T).

- 'Max-Ent hydro' can be generalized for arbitrary flow profiles.

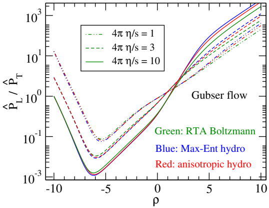
Far-from-equilibrium dynamics [Preliminary results]

Conformal fluids:

(I) Bjorken

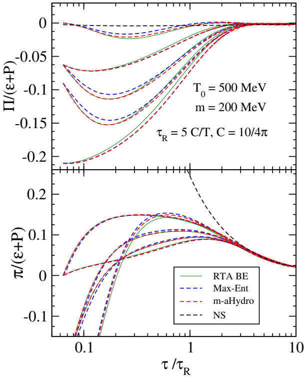


(II) Gubser flow (non-trivial transverse flow)



ρ is de-Sitter 'time'

Non-conformal Bjorken flow



Modified aHydro ansatz:

$$\tilde{f} = e^{-\beta_{RS}\sqrt{p_T^2+(1+\xi)p_z^2}+\alpha}$$

- Max-Ent hydro provides excellent description of out-of-equilibrium dynamics even near regimes of longitudinal and transverse free-streaming.