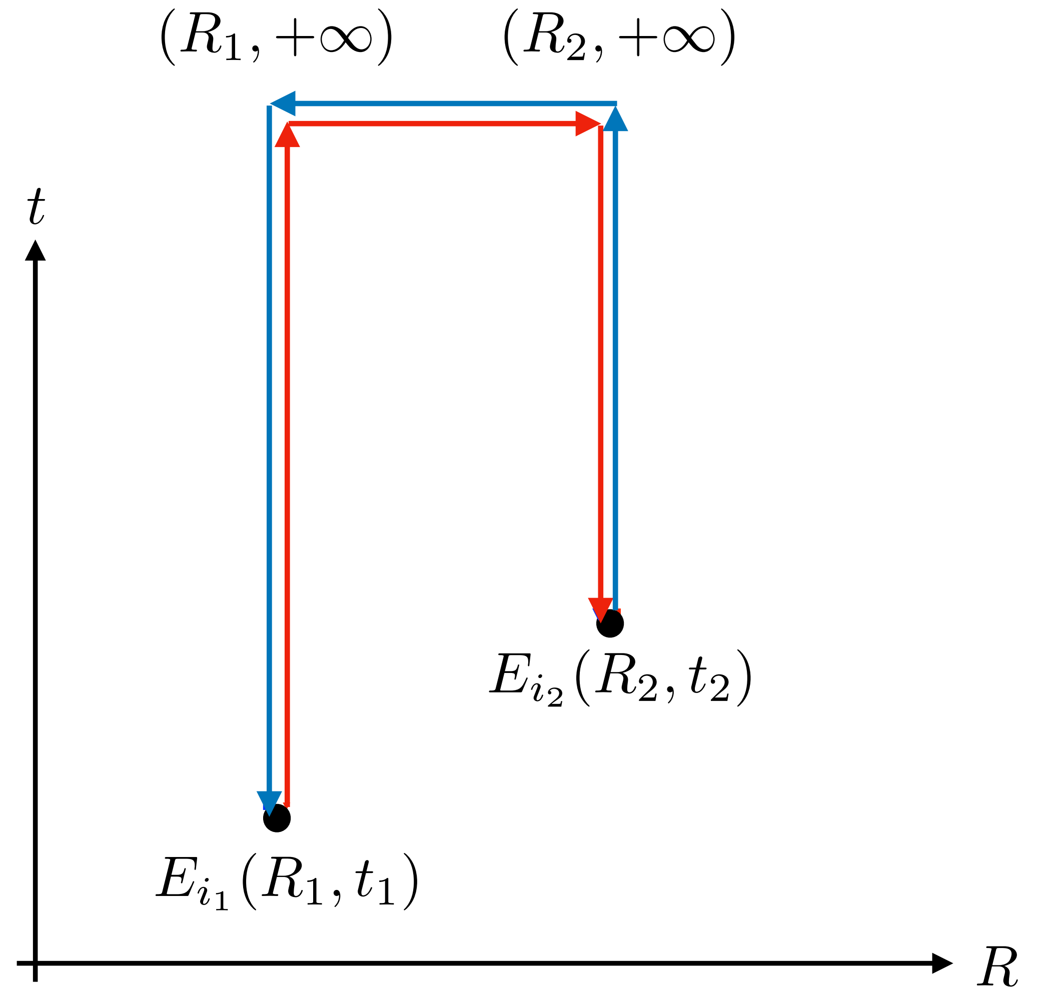
Chromoelectric correlators for quarkonia and heavy quarks: a comparison at weak and strong coupling

The following are the gauge-invariant correlators that determine (in the limit $R_1 \to R_2$):

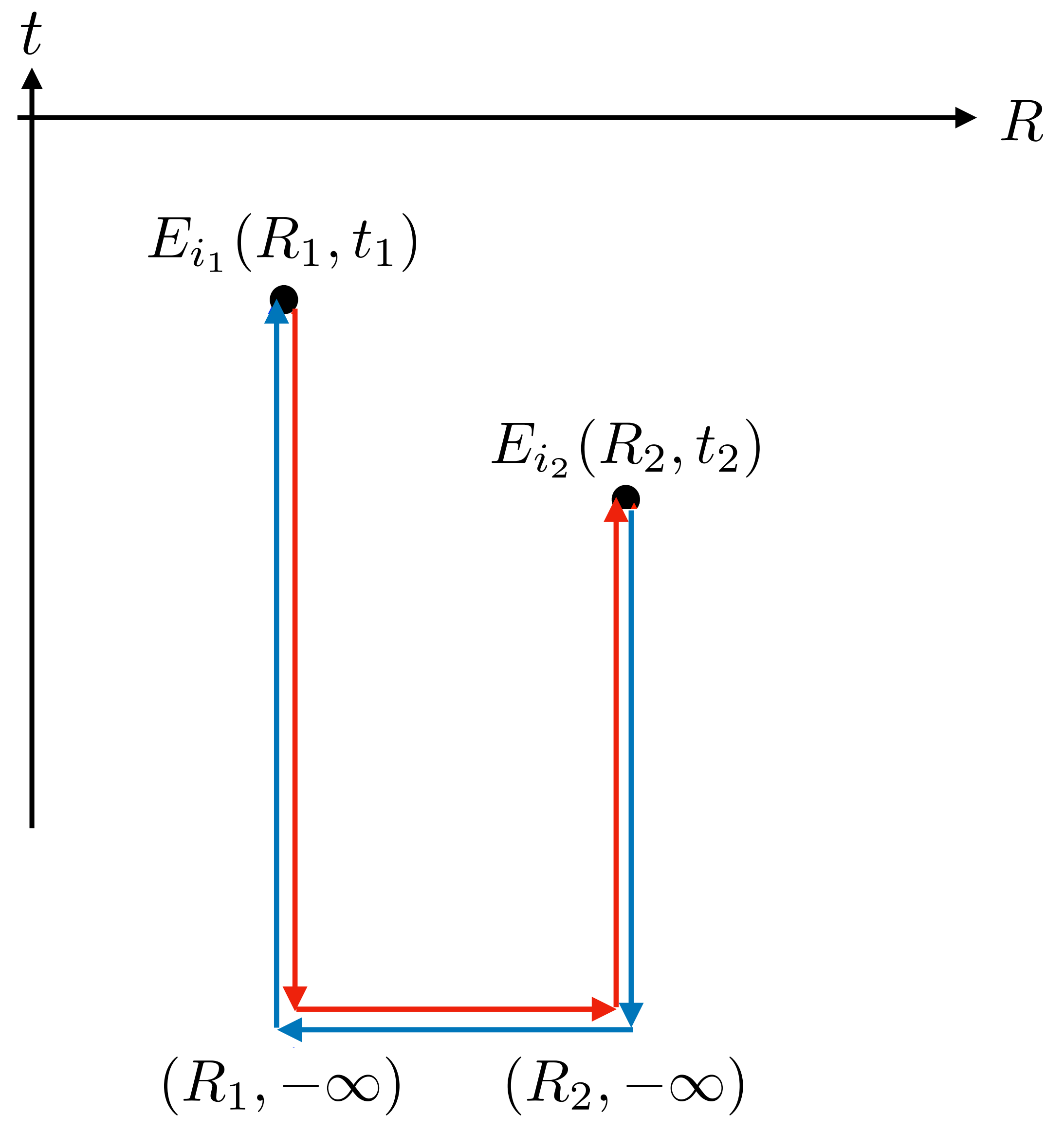
Quarkonia dissociation/recombination rates [1]:

[1] X. Yao and T. Mehen, "Quarkonium Semiclassical Transport in Quark-Gluon Plasma: Factorization and Quantum Correction," JHEP 02 (2021) 062

$$\left\langle E_{i_2}^a \mathscr{W}^{ab} E_{i_1}^b \right\rangle_T$$



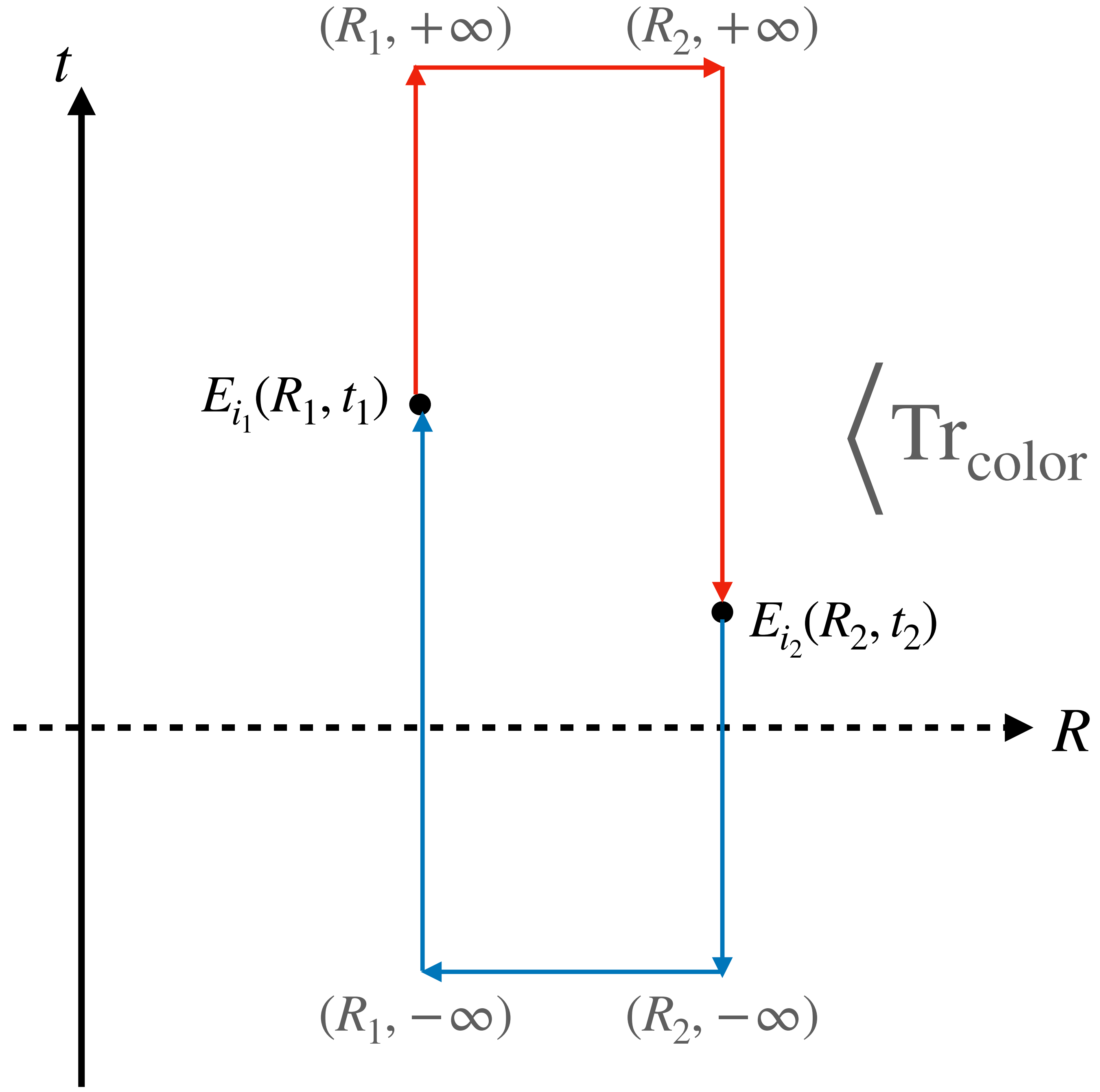
(dissociation)



(recombination)

Heavy quark momentum diffusion coefficient [2]:

[2] J. Casalderrey-Solana and D. Teaney, "Heavy quark diffusion in strongly coupled N=4 Yang-Mills," Phys. Rev. D 74 (2006) 085012



$$\left\langle \mathrm{Tr}_{\mathrm{color}} \left[U E_{i_2} U E_{i_1} U \right] \right\rangle_T$$

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Perturbative calculations of each correlator

Calculations for each correlator have been carried out at finite temperature up to 1-loop level in R_ξ covariant gauges. In terms of their respective (HTL-unresummed) spectral functions, they are:

- For Quarkonia [3]:

[3] T. Binder, K. Mukaida, B. Scheihing-Hitschfeld and X. Yao, "Non-Abelian Electric Field Correlator at NLO for Dark Matter Relic Abundance and Quakonium Transport," JHEP 01 (2022) 137

$$g^2 \varrho_E^{\mathrm{QA}}(p_0) = \frac{g^2(N_c^2-1)p_0^3}{(2\pi)^3}\left\{4\pi^2 + g^2\left[\left(\frac{11}{12}N_c - \frac{1}{3}N_f\right)\ln\left(\frac{\mu^2}{4p_0^2}\right) + \left(\frac{149}{36}+\frac{\pi^2}{3}\right)N_c - \frac{10}{9}N_f + F\left(\frac{p_0}{T}\right)\right]\right\}$$

- For Heavy Quarks [4]:

[4] Y. Burnier, M. Laine, J. Langelage and L. Mether, "Colour-electric spectral function at next-to-leading order," JHEP 08 (2010) 094

$$g^2 \rho_E^{\mathrm{HQ}}(p_0) = \frac{g^2(N_c^2-1)p_0^3}{(2\pi)^3}\left\{4\pi^2 + g^2\left[\left(\frac{11}{12}N_c - \frac{1}{3}N_f\right)\ln\left(\frac{\mu^2}{4p_0^2}\right) + \left(\frac{149}{36}-\frac{2\pi^2}{3}\right)N_c - \frac{10}{9}N_f + F\left(\frac{p_0}{T}\right)\right]\right\}$$

with the same temperature-dependent function $F(p_0/T)$.

At this order in perturbation theory, they differ by $g^4 N_c(N_c^2-1)p_0^3\pi^2/(2\pi)^3$. This verifies that

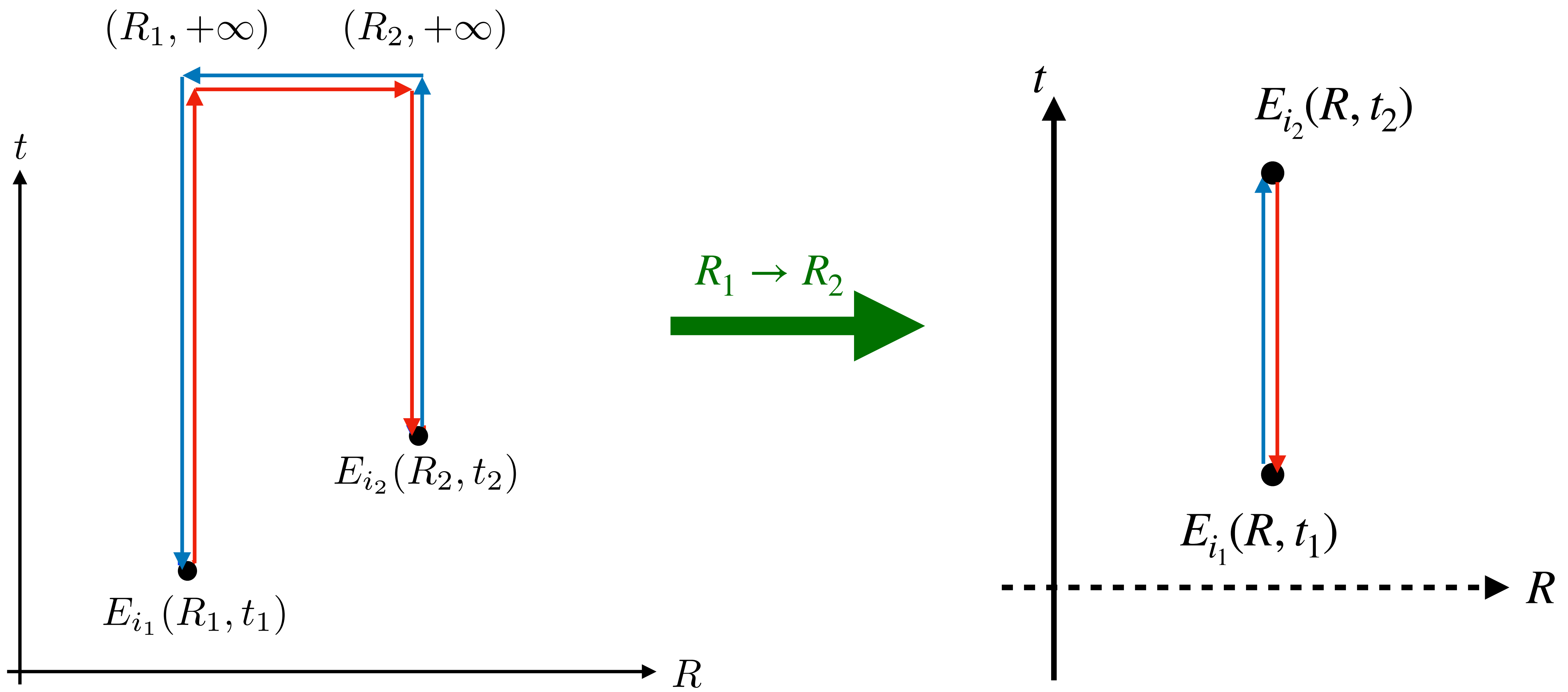
$$T_F\left\langle E_i^a(t)\mathscr{W}^{ab}(t,0)E_i^b(0)\right\rangle_T \neq \left\langle \mathrm{Tr}_{\mathrm{color}}\left[U(-\infty,t)E_i(t)U(t,0)E_i(0)U(0,-\infty)\right]\right\rangle_T.$$

The holographic counterparts in strongly coupled $\mathcal{N} = 4$ SYM

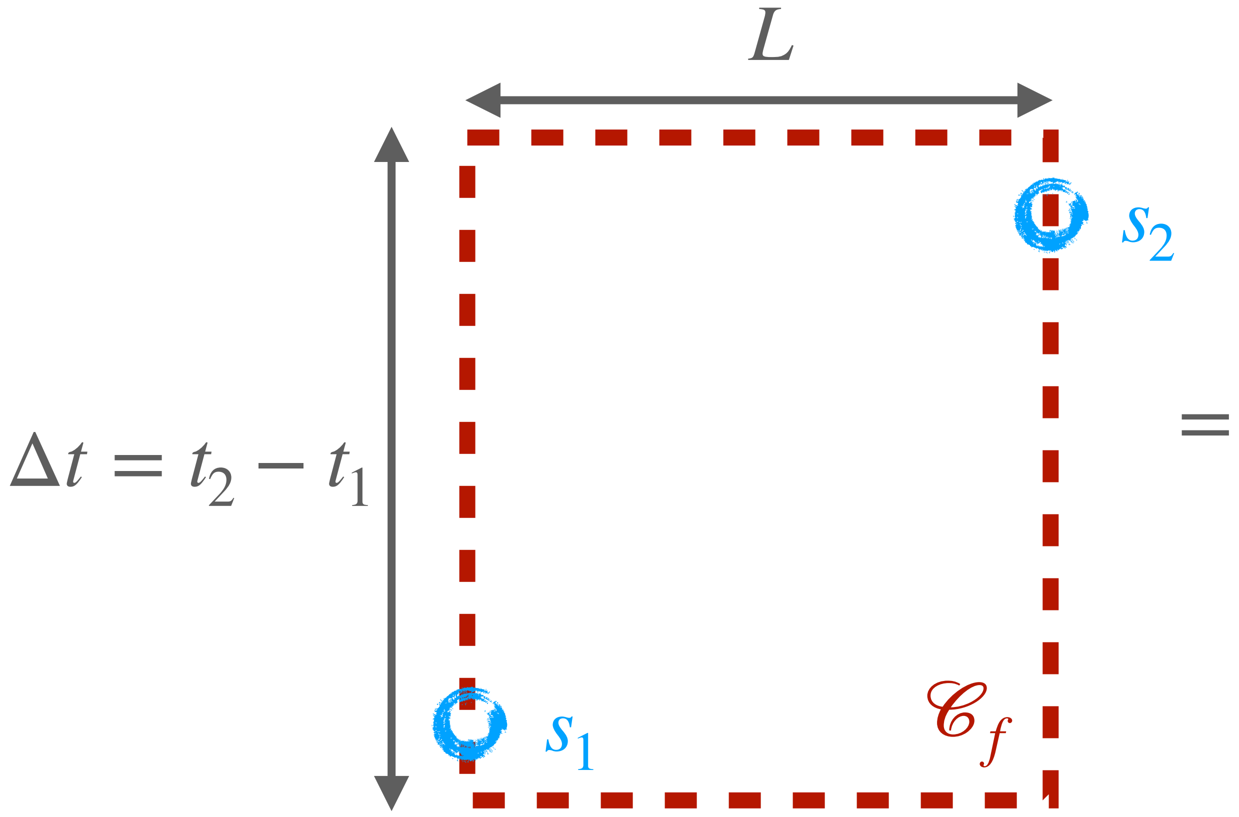
Both correlators may be formulated in the AdS/CFT language: $\langle W[\mathscr{C} = \partial\Sigma] \rangle_T = e^{iS_{\rm NG}[\Sigma]}$

Quarkonia

(G.N., B.S.H., X.Y., current work)



This can be evaluated by taking path variations in the following Wilson loop $\mathscr{C}_f$, where $f^i(s)$ is a small deformation at position s from the rectangular contour $\mathscr{C}_{f=0}$.

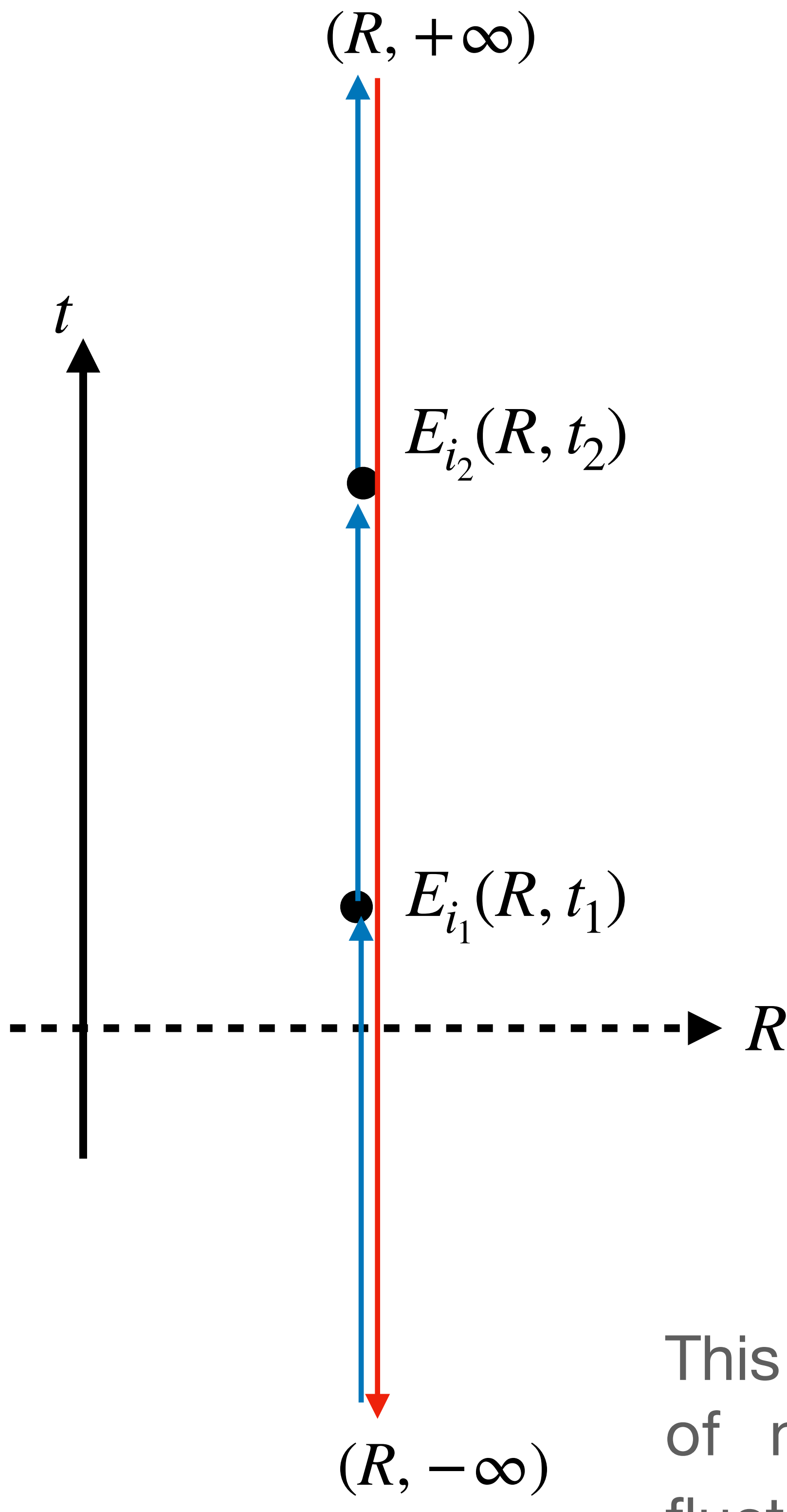


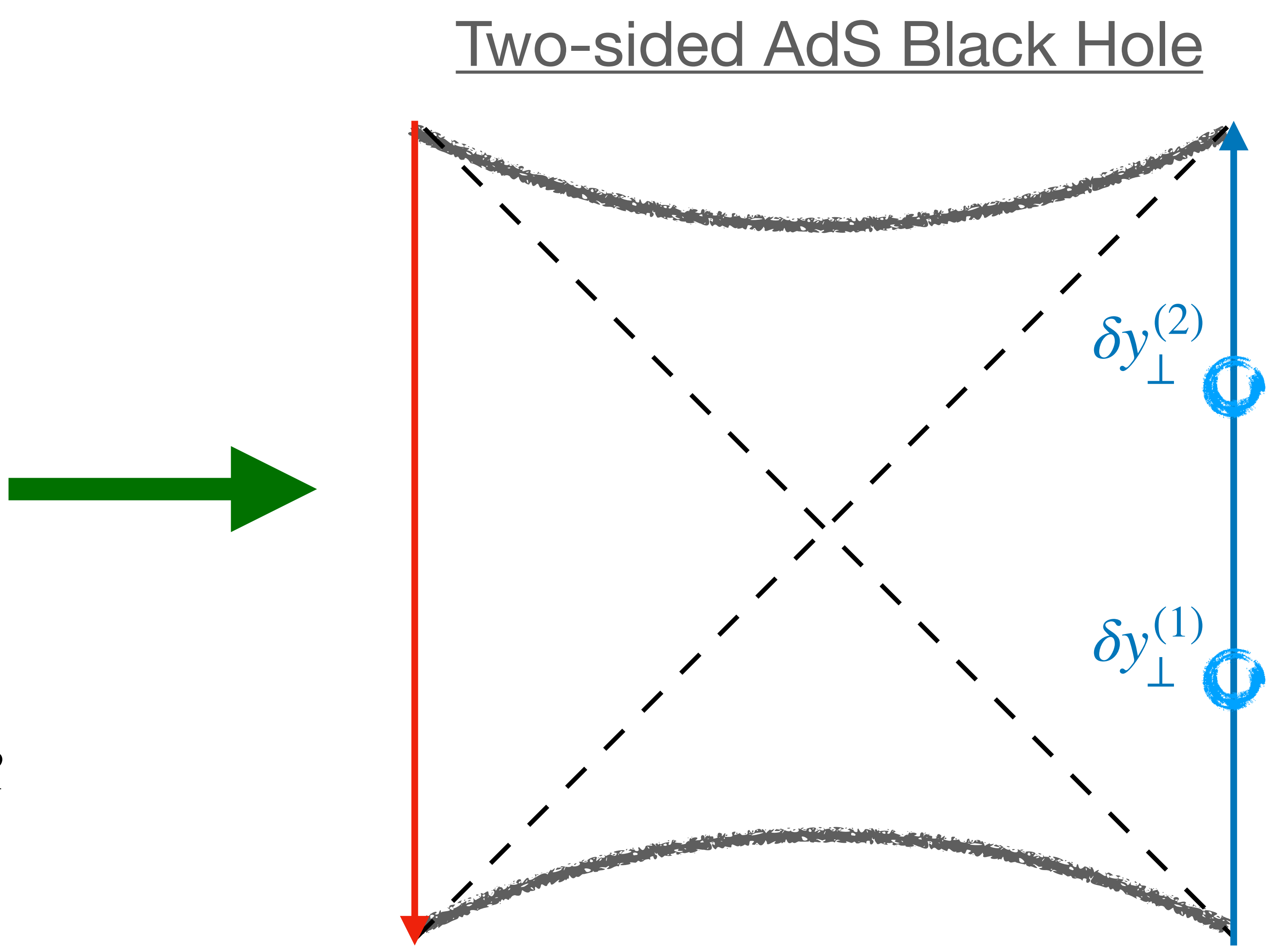
$$= \frac{1}{\left\langle W[\mathscr{C}_f] \right\rangle_T} \frac{\delta}{\delta f^i(s_2)} \frac{\delta}{\delta f^i(s_1)} \left\langle W[\mathscr{C}_f] \right\rangle_T \Big|_{f=0}$$

The correlator of interest is recovered by taking $L \to 0$.

Heavy Quarks

[2] J. Casalderrey-Solana and D. Teaney, "Heavy quark diffusion in strongly coupled N=4 Yang-Mills," Phys. Rev. D 74 (2006) 085012





This was evaluated by solving the classical equations of motion from the Nambu-Goto action for the fluctuations $\delta y_\perp$ sourced by $\delta y_\perp^{(1)}$ and evaluating the response at $\delta y_\perp^{(2)}$ with in-falling boundary conditions at the right black hole horizon.

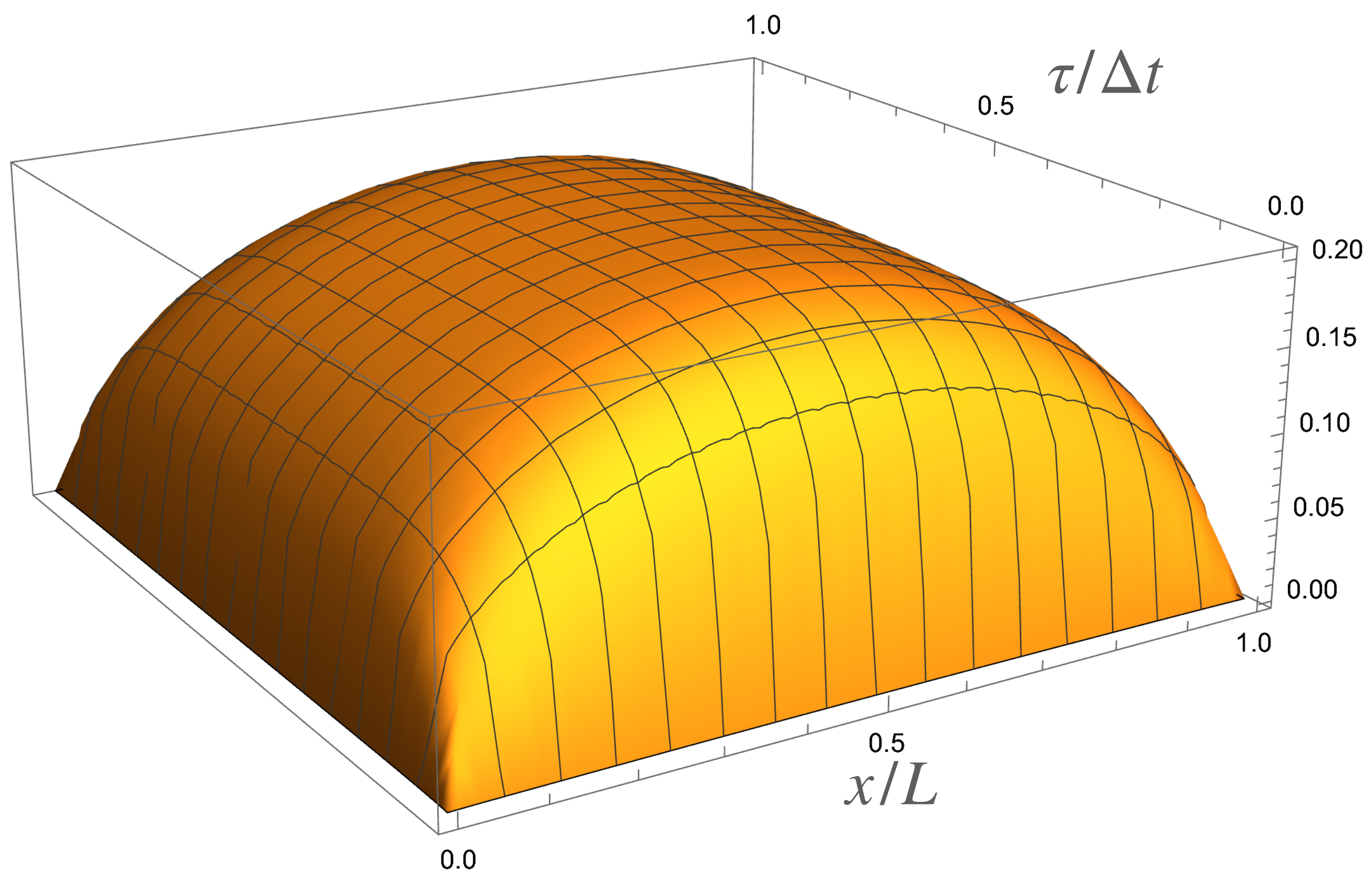
The well-known heavy quark diffusion coefficient follows from here by integrating over $t_2 - t_1$, obtaining: $\kappa = \pi\sqrt{\lambda}T^3$

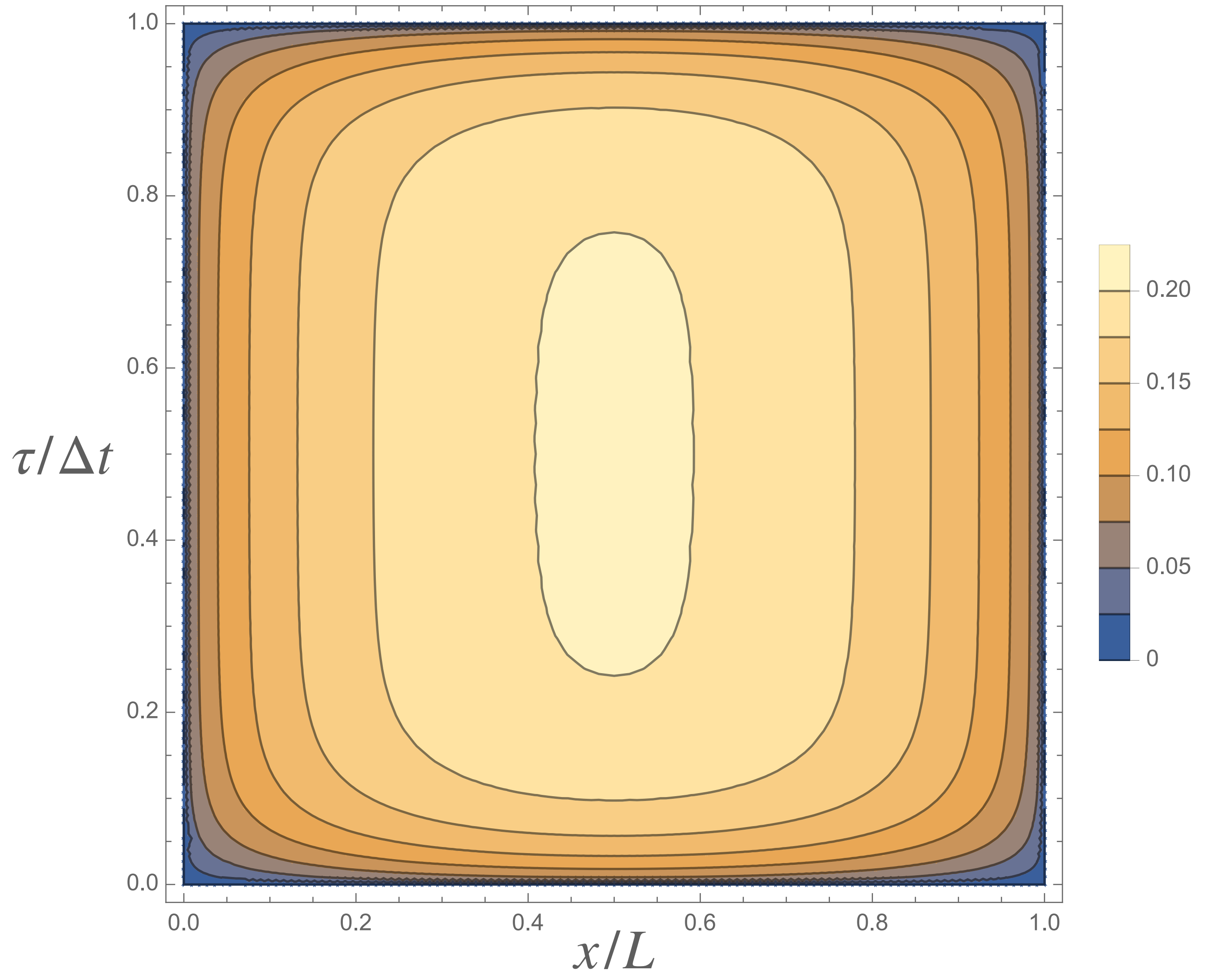
Evaluation of the quarkonia correlator using AdS/CFT

Our evaluation of the quarkonia correlator using AdS/CFT is in progress. We are currently working through the following steps (in Euclidean signature), using the Chebyshev spectral method [5]:

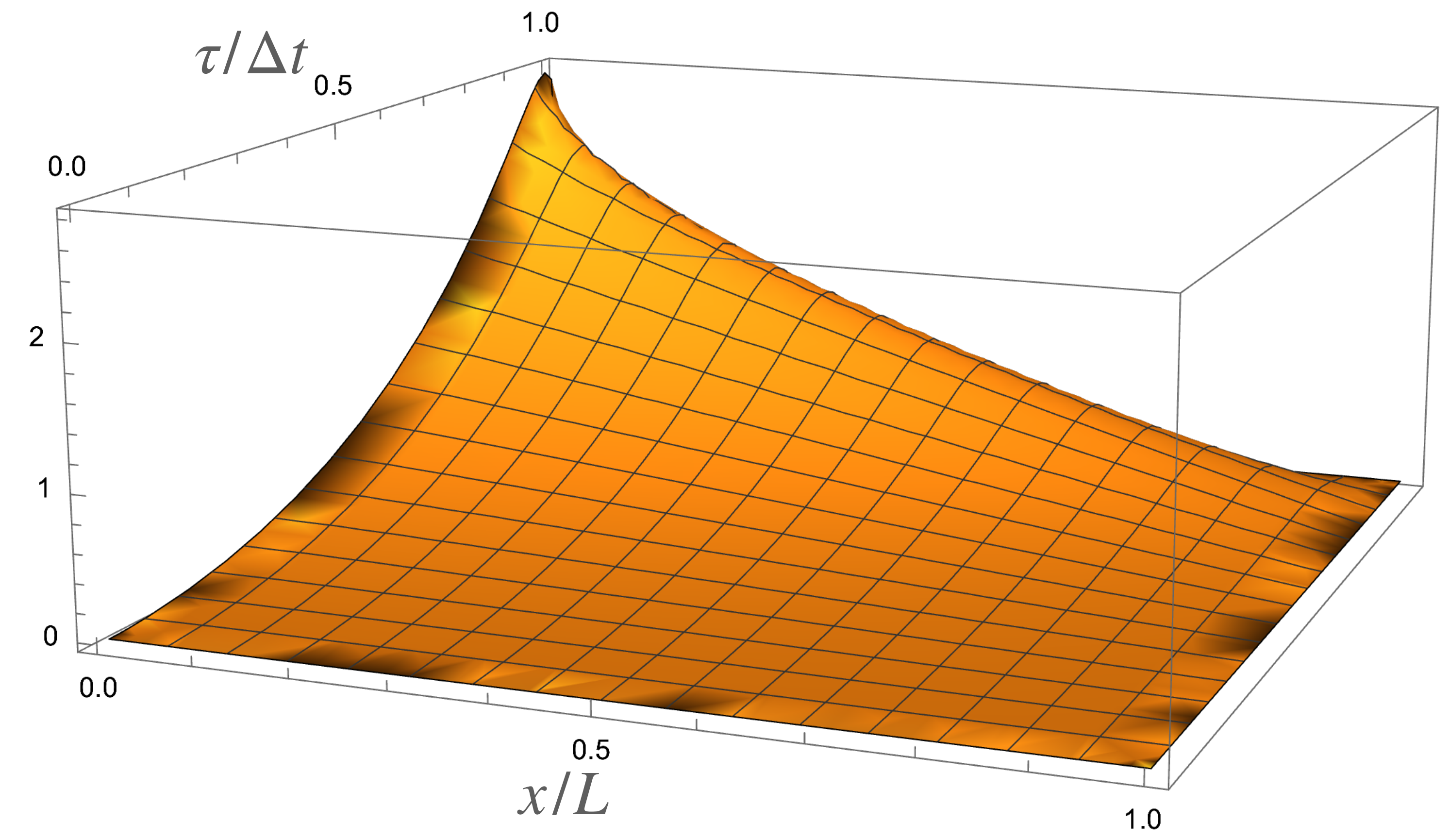
[5] J.P. Boyd, “Chebyshev and Fourier Spectral Methods,” Dover books on Mathematics (2001)

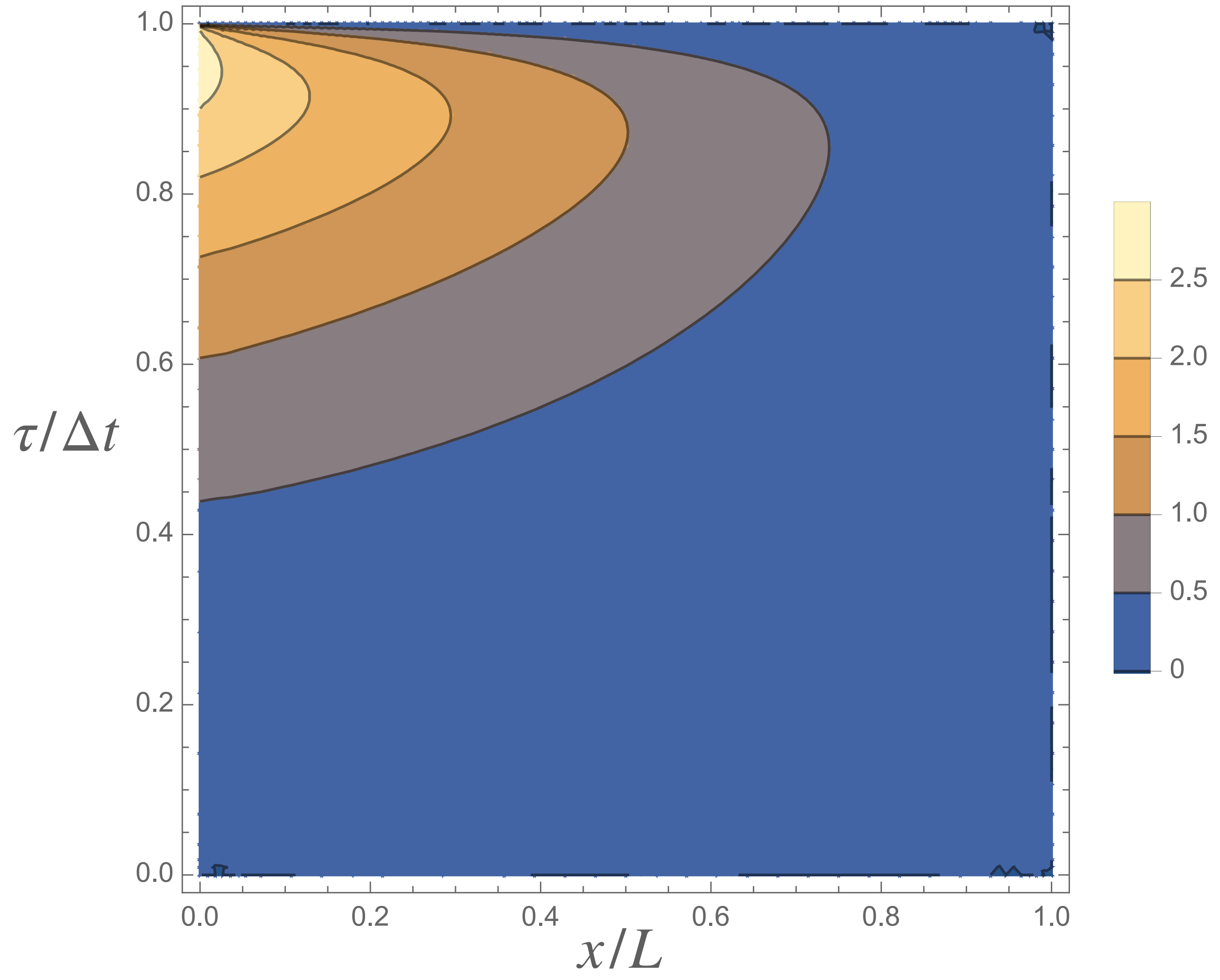
1) Solve for the background worldsheet solution:





2) Solve for the fluctuations with a source as a boundary condition:





3) Extrapolate in the limit $L \to 0$:

In progress. Stay tuned!

Summary and conclusions

- The electric field correlators that are relevant for quarkonia recombination/dissociation are different to those that are relevant for single heavy quarks.
- They have been demonstrated to be different in perturbation theory, with each calculation being gauge-invariant.
- The heavy quark correlator in a thermal medium has been evaluated in strongly coupled $\mathcal{N} = 4$ SYM. However, the quarkonia correlator was only recently formulated in its current form [1], and an AdS/CFT calculation of it is currently underway.
- Goal of this work: determine the degree to which the two correlators differ at strong coupling at finite temperature.