Anomalous diffusion in QCD matter

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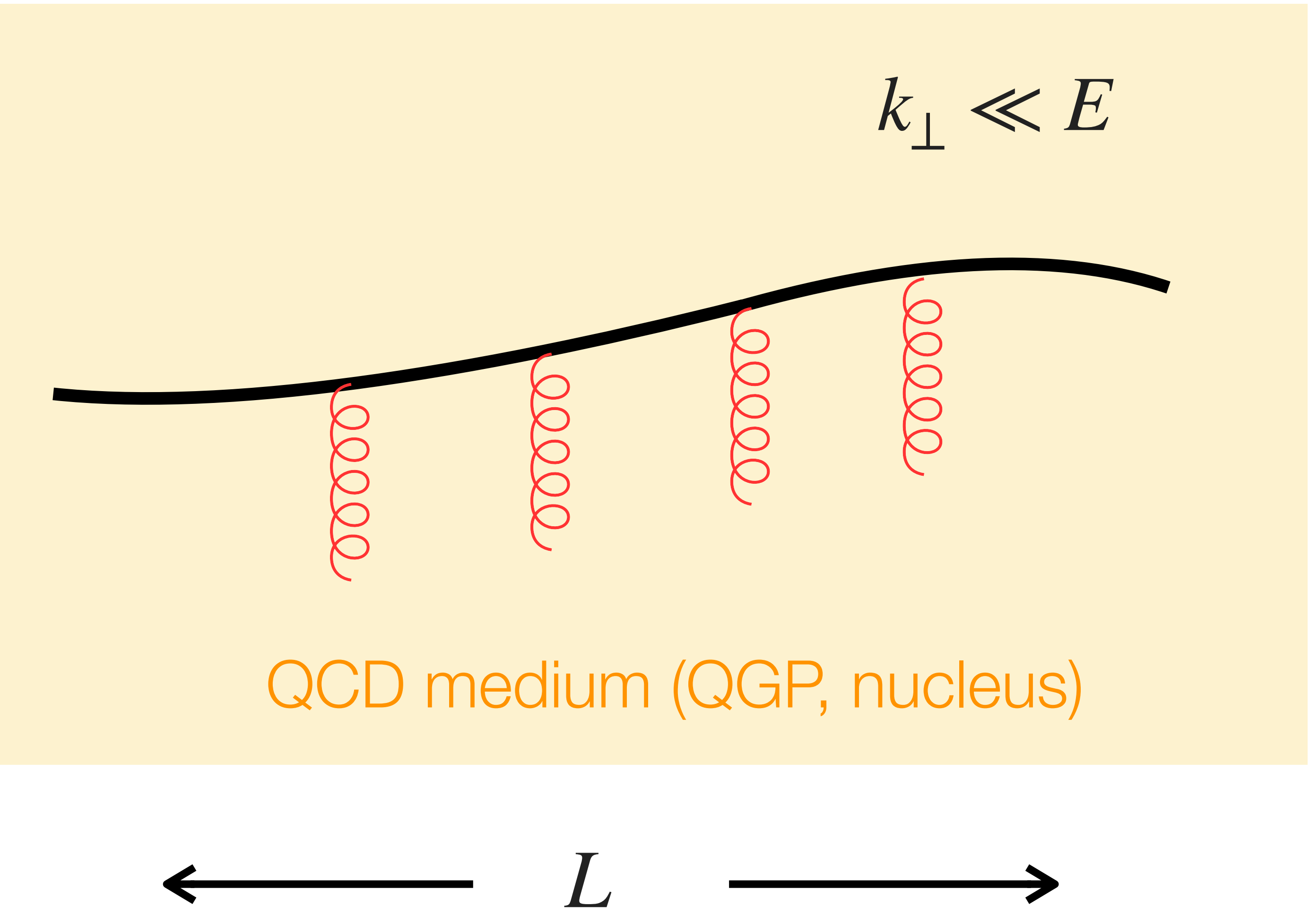

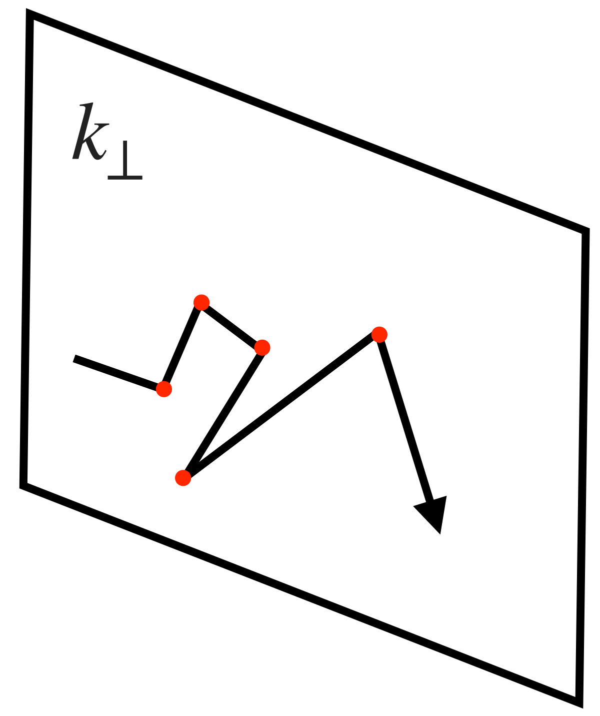

Transverse plane

Quark Matter 2022 @ Krakow, Poland
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Brookhaven National Laboratory

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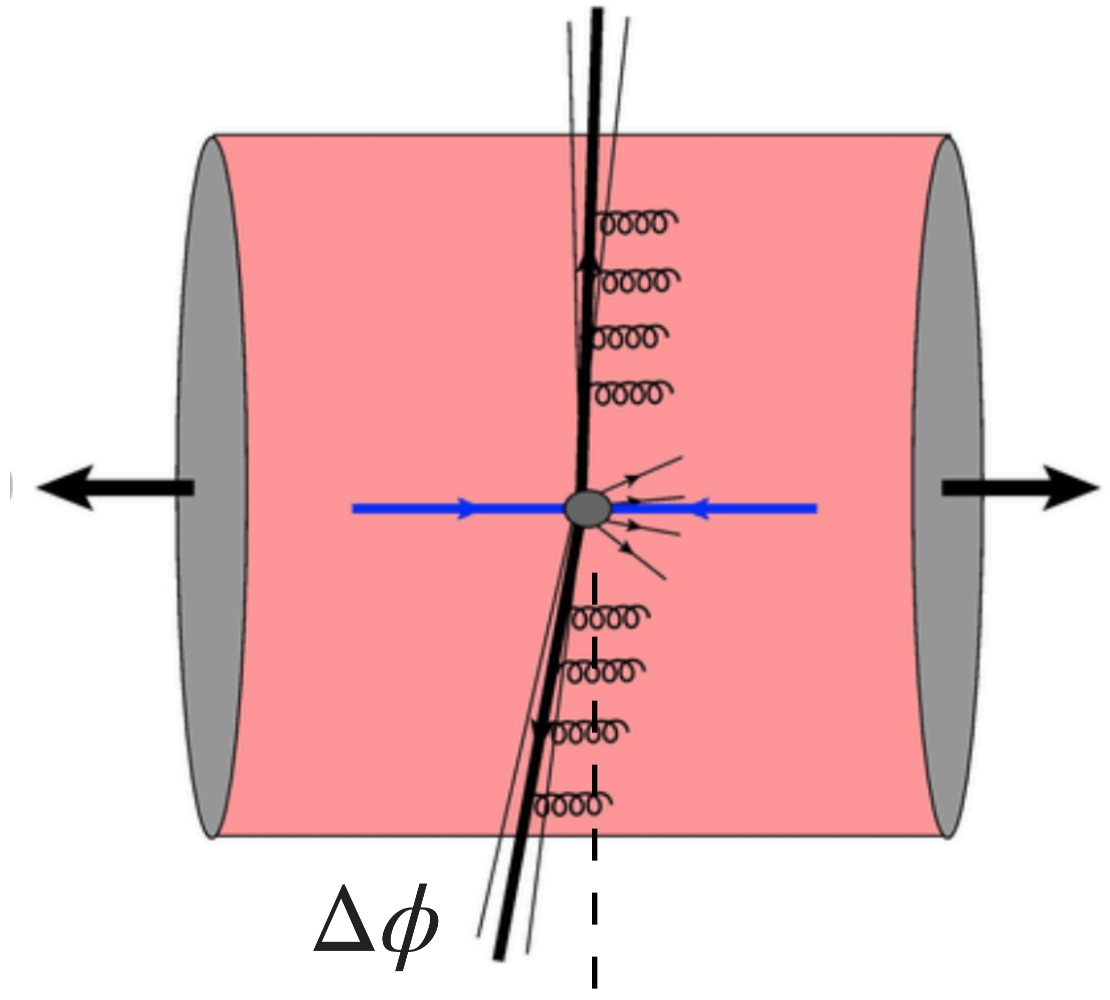
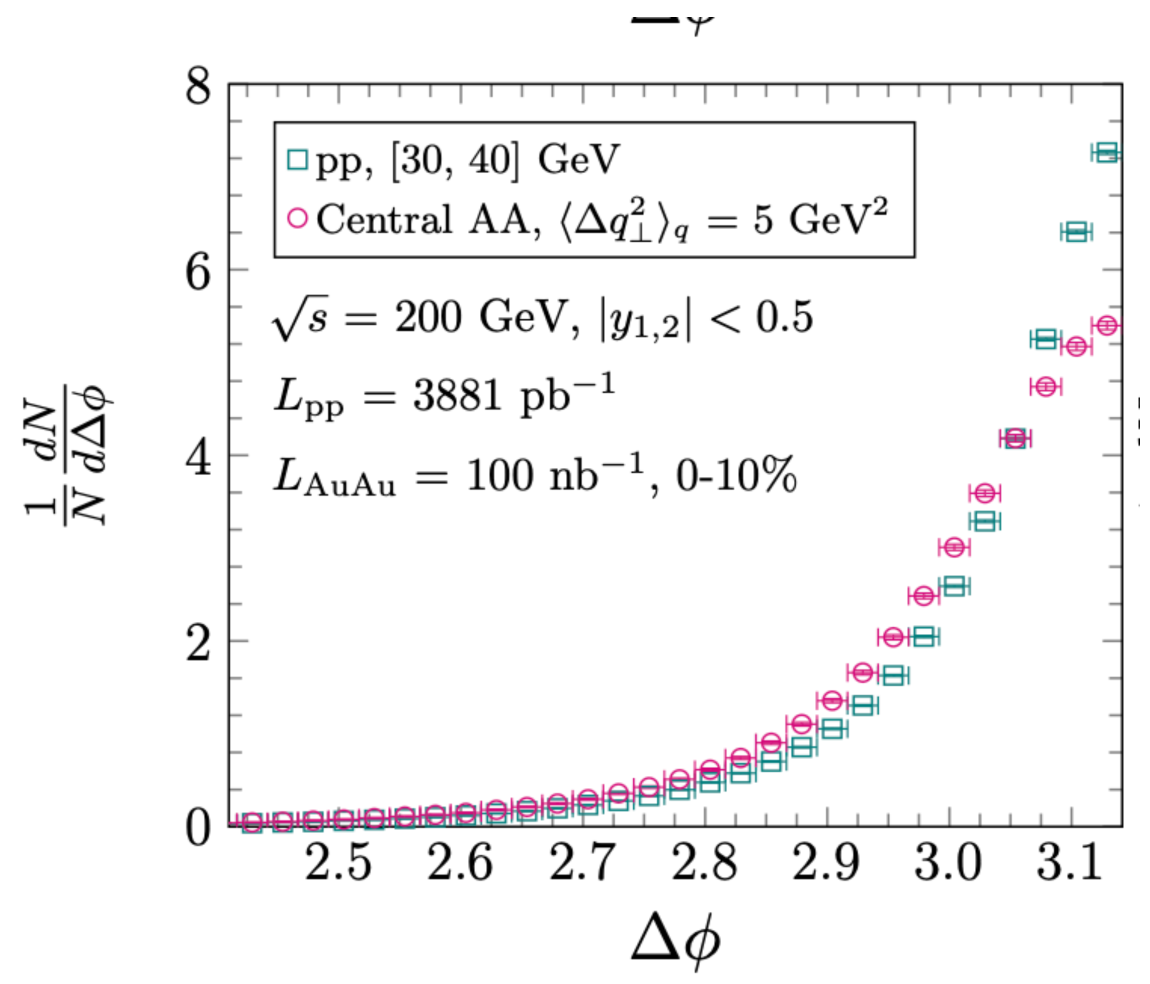
Transverse momentum broadening (TMB) in QCD matter: Normal diffusion at tree level

- TMB to probe the QGP in Heavy Ion Collisions: dijet azimuthal de-correlation, Rutherford scattering, jet quenching, ...
- High energy partons experience random kicks in hot or cold nuclear causing transverse momentum to increase over time.
- TMB distribution at leading order: Gaussian for $k_\perp < Q_s \sim \hat{q}\, L$ and exhibits the power law tail $k_\perp^{-4}$ for $k_\perp > Q_s$
- The typical event is decribed by brownian motion (normal diffusion) in TM space:

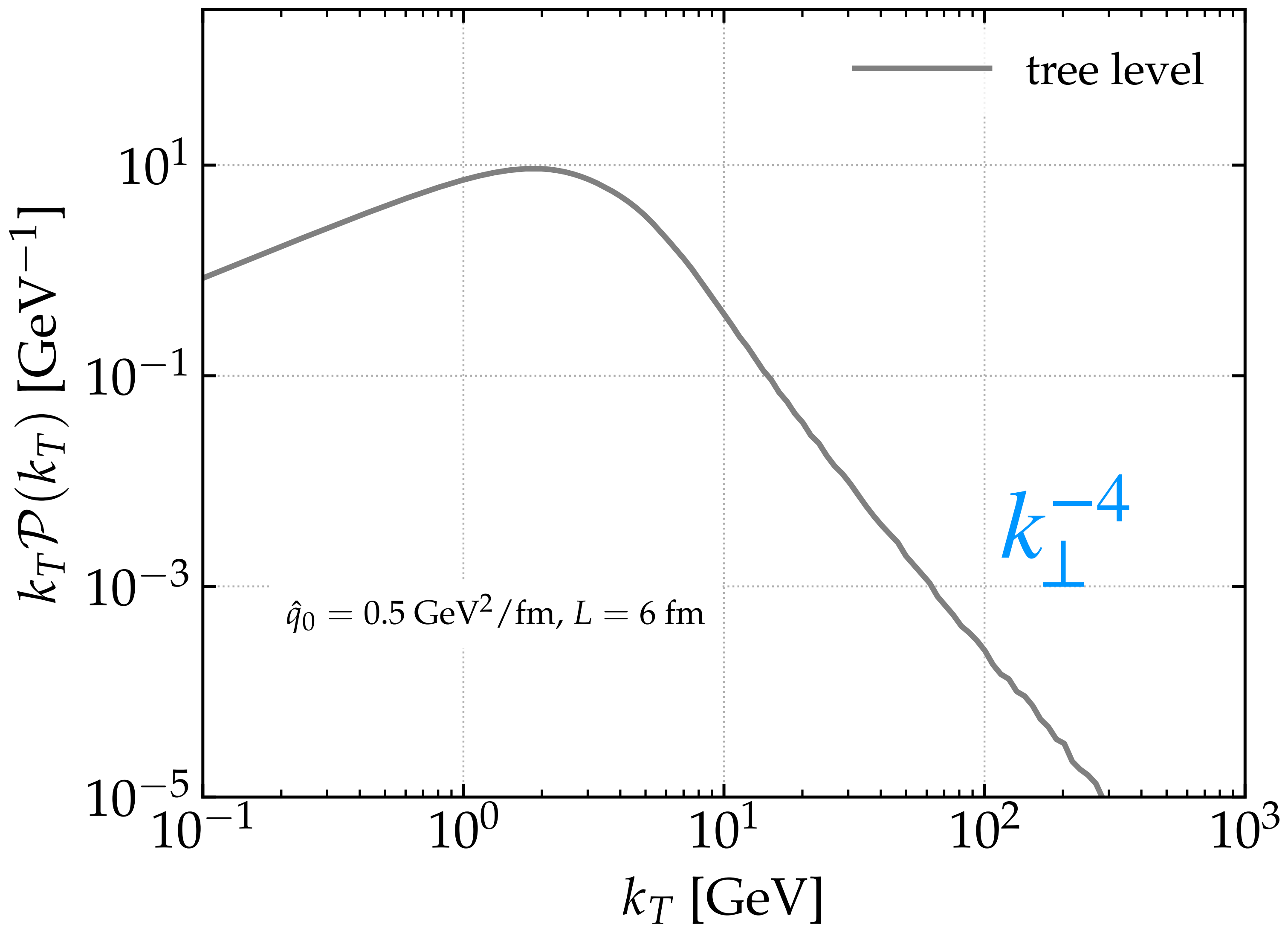
$$\langle k_\perp^2 \rangle_{\text{typ}} \propto \hat{q} L$$



Multiple scattering



Jia, Xiao, Yuan (2019)



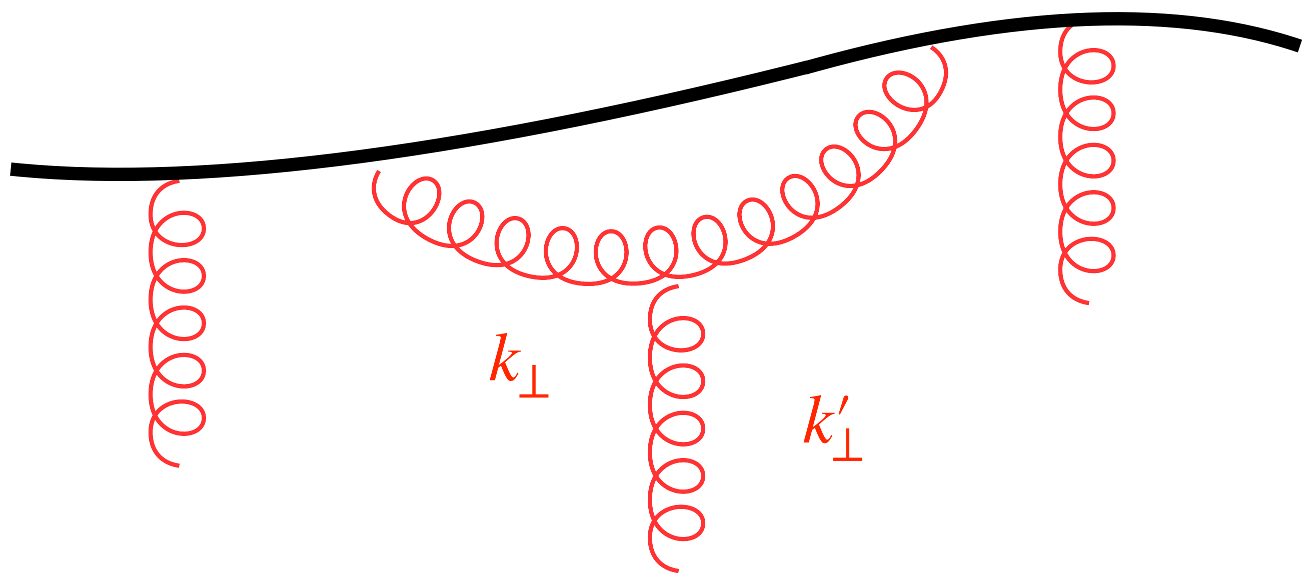
Rare Rutherford scattering

D'Eramo, Lekaveckas, Liu, Rajagopal (2012)

Q: What are the effects of quantum corrections on transverse momentum broadening?

- Potentially large double logs (DL) in transverse momentum broadening at NLO [Liou, Mueller, Wu (2013)]

$$\tau_0 \ll \tau \ll L$$

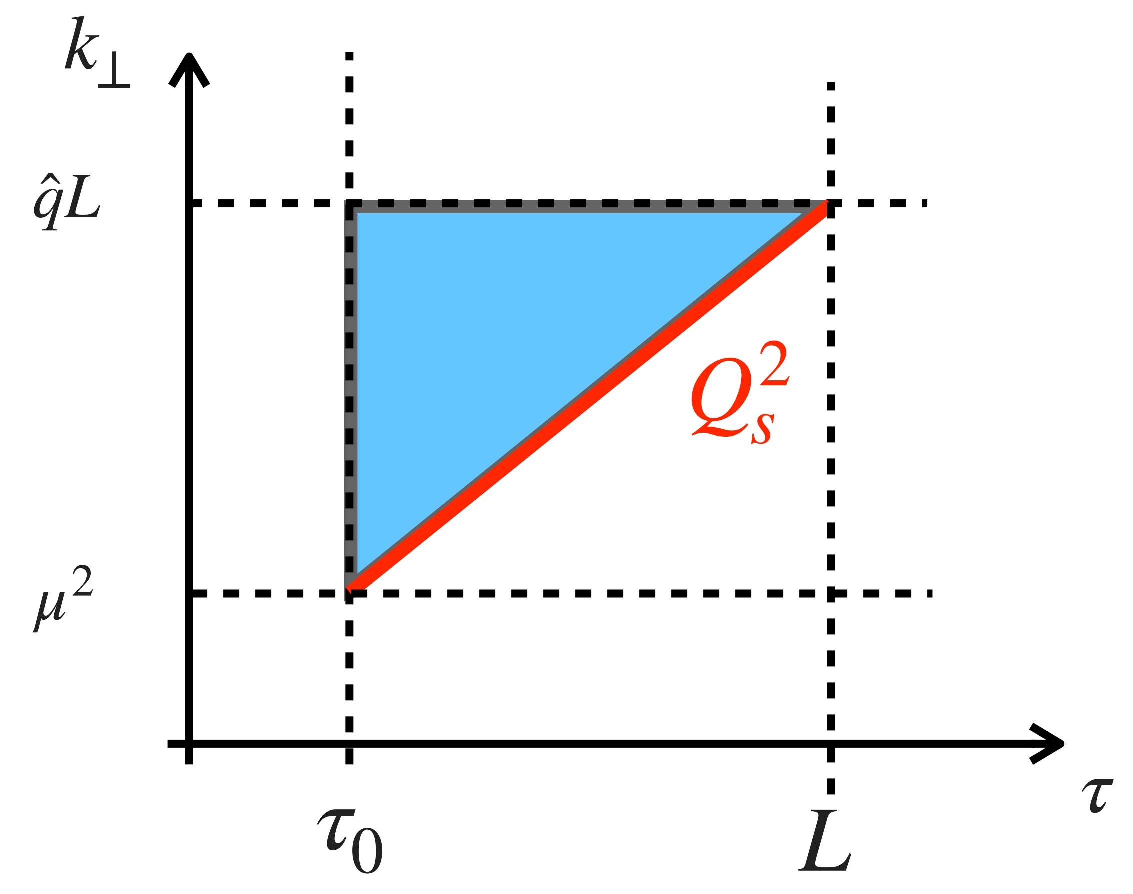


$$\tau_0 \sim \ell_{\rm mfp} \ll \tau = \omega/k_\perp^2 \ll L$$

$$\text{NLO} \sim \bar{\alpha} \int^{L} \frac{d\tau}{\tau} \int_{Q_s^2(\tau)}^{Q_s^2(L)} \frac{dk_\perp^2}{k_\perp^2}$$

$$\langle k_\perp^2 \rangle = \hat{q}_0 L \left(1 + \frac{\bar{\alpha}}{2} \log^2 \frac{L}{\tau_0} \right)$$

DL phase-space



- DL's in the single scattering regime with multiple-scattering boundary $Q_s^2 \equiv \hat{q}(Q_s, \tau)\,\tau$ (screening of mass singularity)
- All orders DL resummed and absorbed in a redefinition of $\hat{q}$ [Blaizot, MT (2014), Iancu (2014)]

$$\frac{\partial}{\partial \ln \tau} \hat{q}(k_\perp, \tau) = \bar{\alpha} \int_{Q_s^2(\tau)}^{k_\perp^2} \frac{dk_\perp'^2}{k_\perp'^2} \hat{q}(k'_\perp, \tau)$$

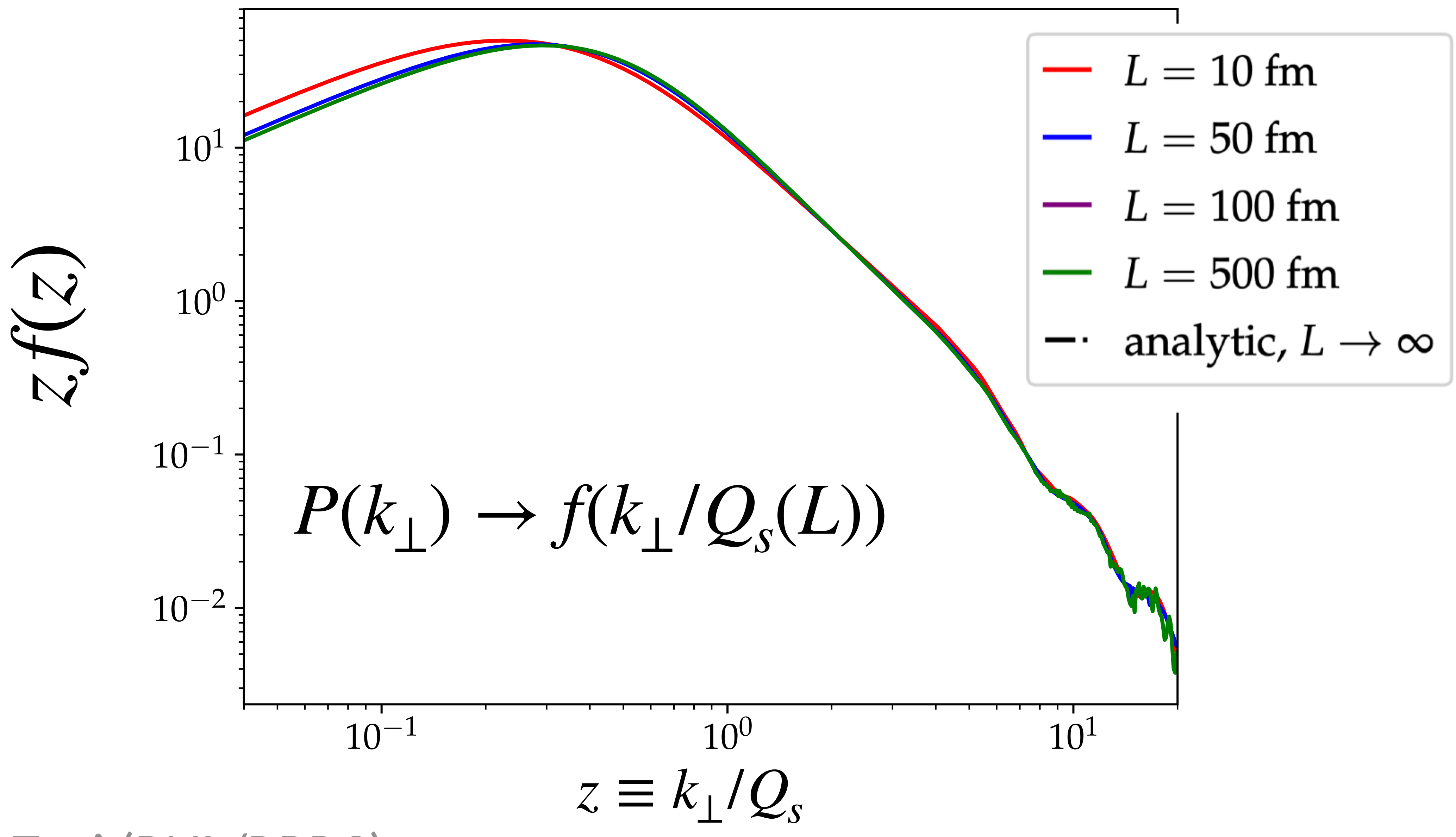
Universality, geometric scaling and super-diffusion

- For large system size TMB tends to a universal distribution that obeys geometric scaling (self-similarity) due to non-linear dynamics

$$\hat{q}(x_\perp, L)\, L \equiv Q_s^2(L)\; g\left(z = \frac{1}{x_\perp^2 Q_s^2(L)} \right)$$

$$g(z) = z^\beta \left(1 + \beta \ln z\right) \qquad z > 1$$

$$g(z) = z^{2\beta} \qquad z < 1$$

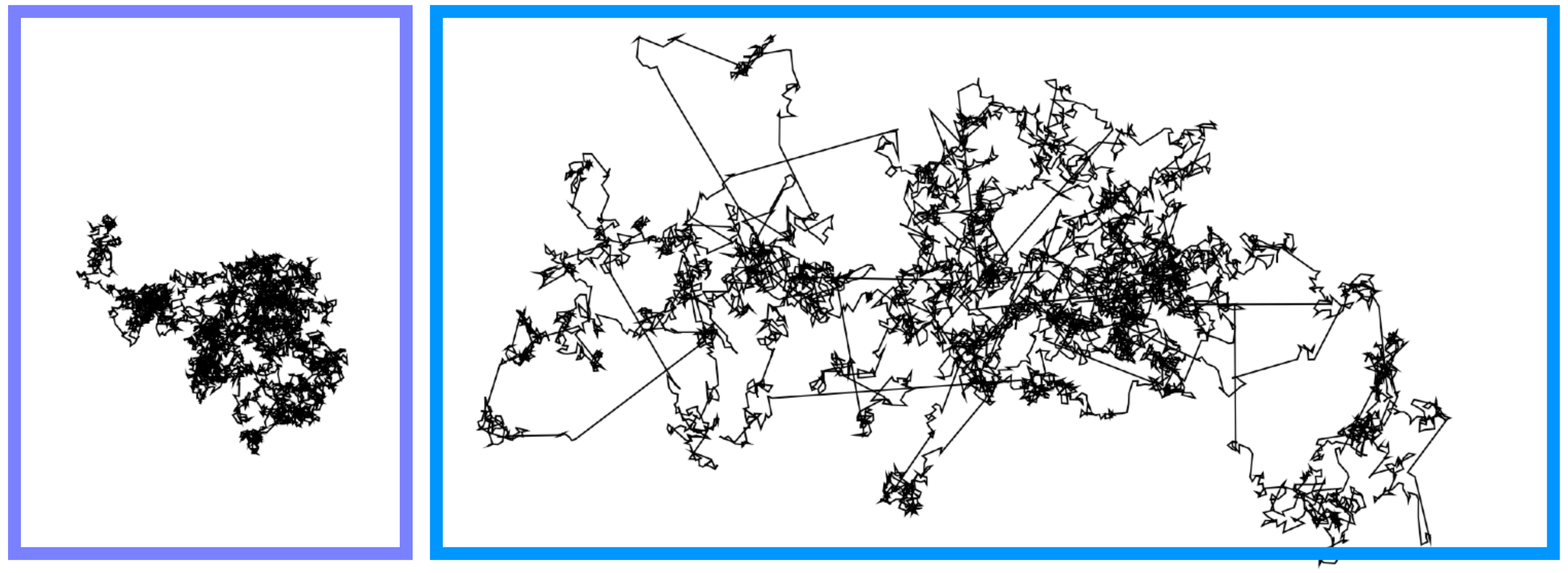


Asymptotically non-Gaussian: Lévy distribution in Fourier space

$$S(x_\perp, L) \to \mathrm{e}^{-(x_\perp^2 Q_s^2(L))^{1-2\beta}}$$

Anomalous scaling: super diffusive process

$$Q_s^2(L) \propto L^{1+2\beta} \qquad \beta \simeq \sqrt{\bar{\alpha}}$$

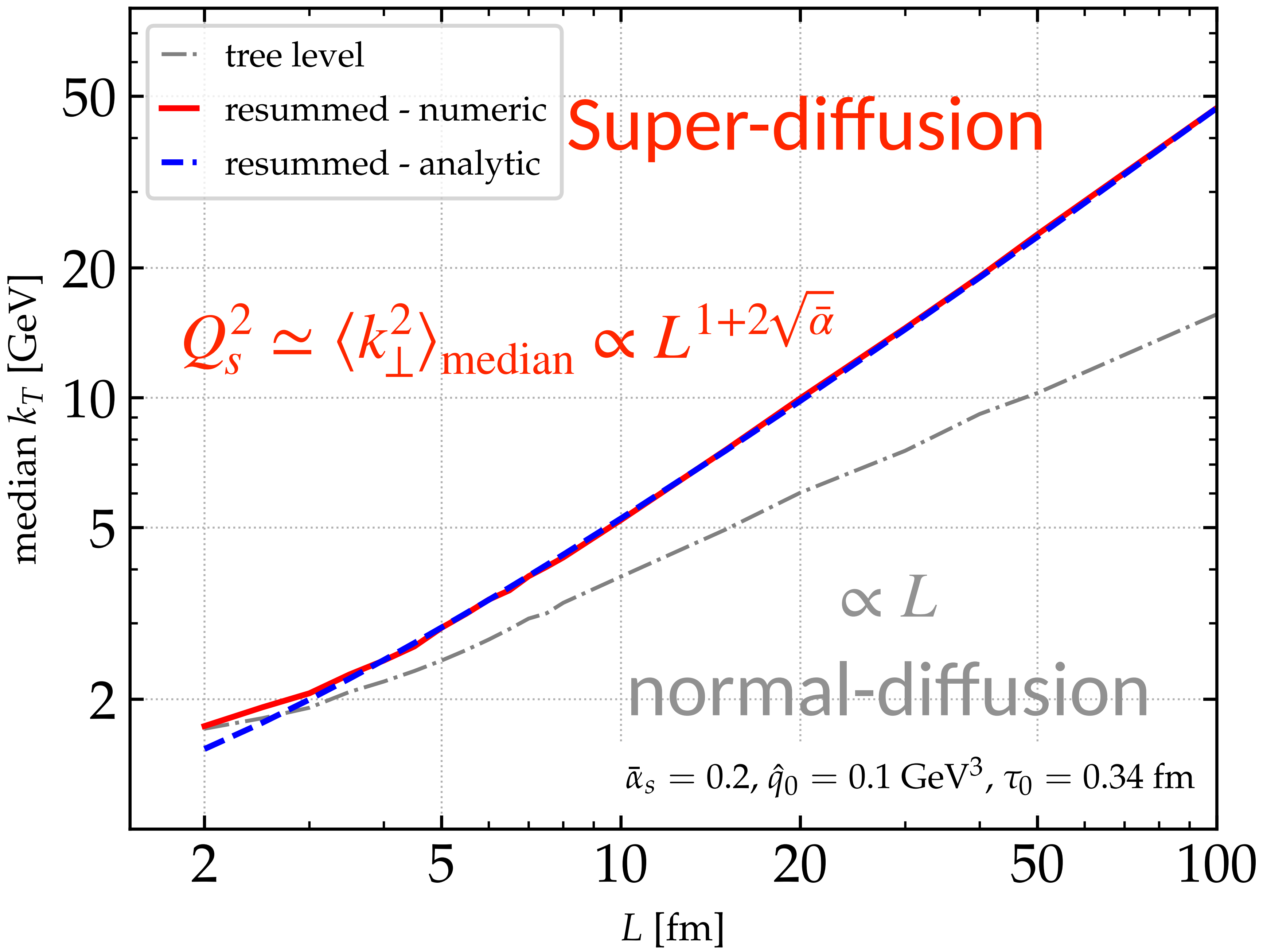


Brownian motion

Lévy random walk

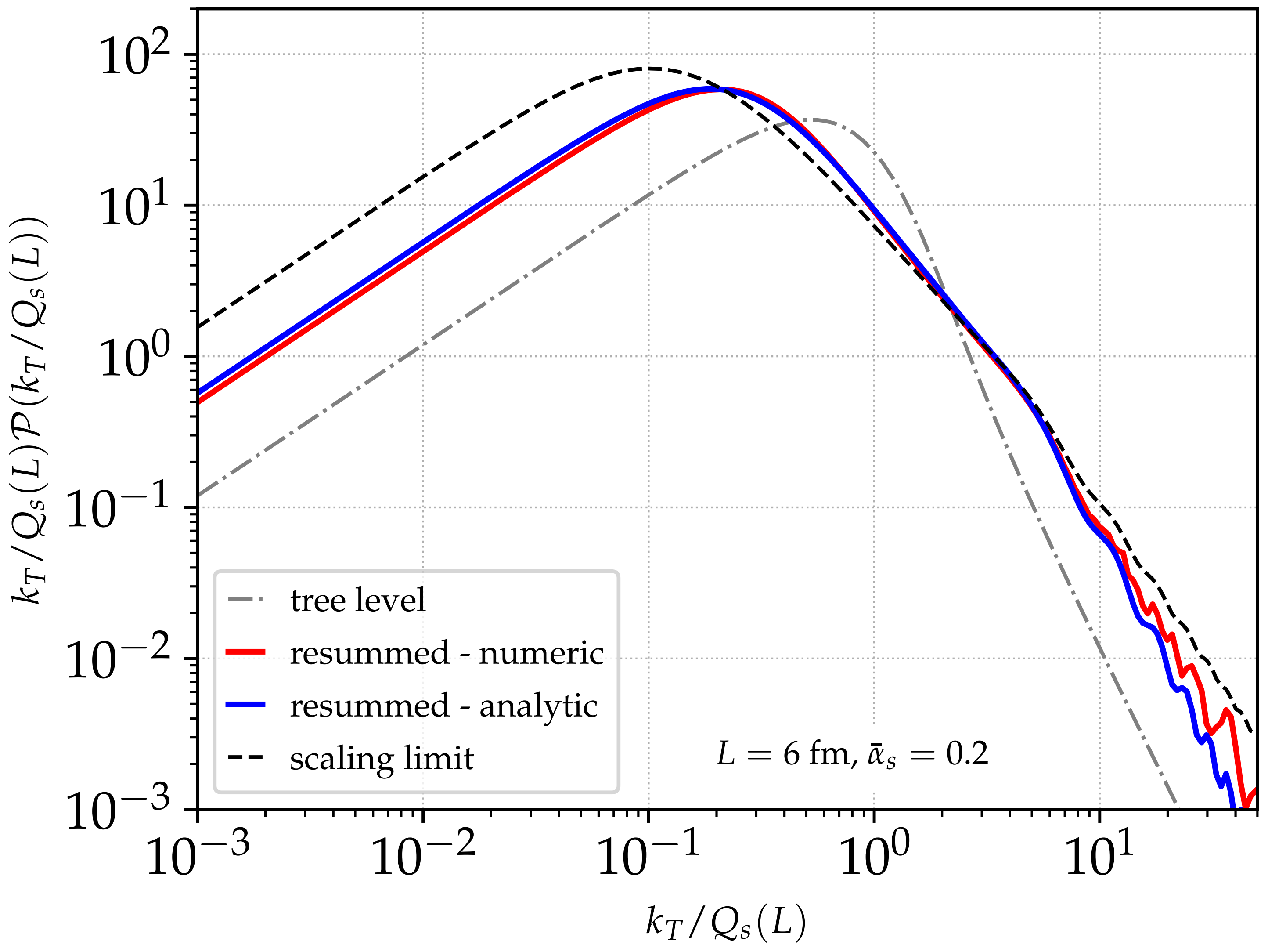
Analytic solutions (asymptotic analyzis)

- The typical transverse momentum squared expansion at late times
- Nonlocal quantum corrections: anomalous system size dependence (super diffusion)



$$\rho_s(Y) = \log Q_s^2(Y) = cY + b \log Y + \text{const}\,.$$

- Universal pre-asymptotic solution provides a good description of numerical simulations
- wider distribution due to heavy Lévy tail



$$Y \equiv \log \frac{L}{\tau_0}$$

$$x \equiv \log \frac{k_\perp^2}{Q_s^2}$$

$$b = -\frac{2}{3(1-\beta)}$$

$$c \simeq 1 + 2\sqrt{\bar\alpha}$$

$$\frac{\hat q(Y,k_\perp)\,L}{Q_s^2} = \exp\left(\beta x - \frac{\beta x^2}{4cY}\right)\left[1 + \beta x + \frac{bx}{c^2Y}\left(1 + \frac{\beta(c+4)x}{6}\right) + \mathcal{O}(Y^{-2})\right]$$

Blue: asymptotic limit. Orange: pre-asymptotic $\mathcal{O}(1/Y)$

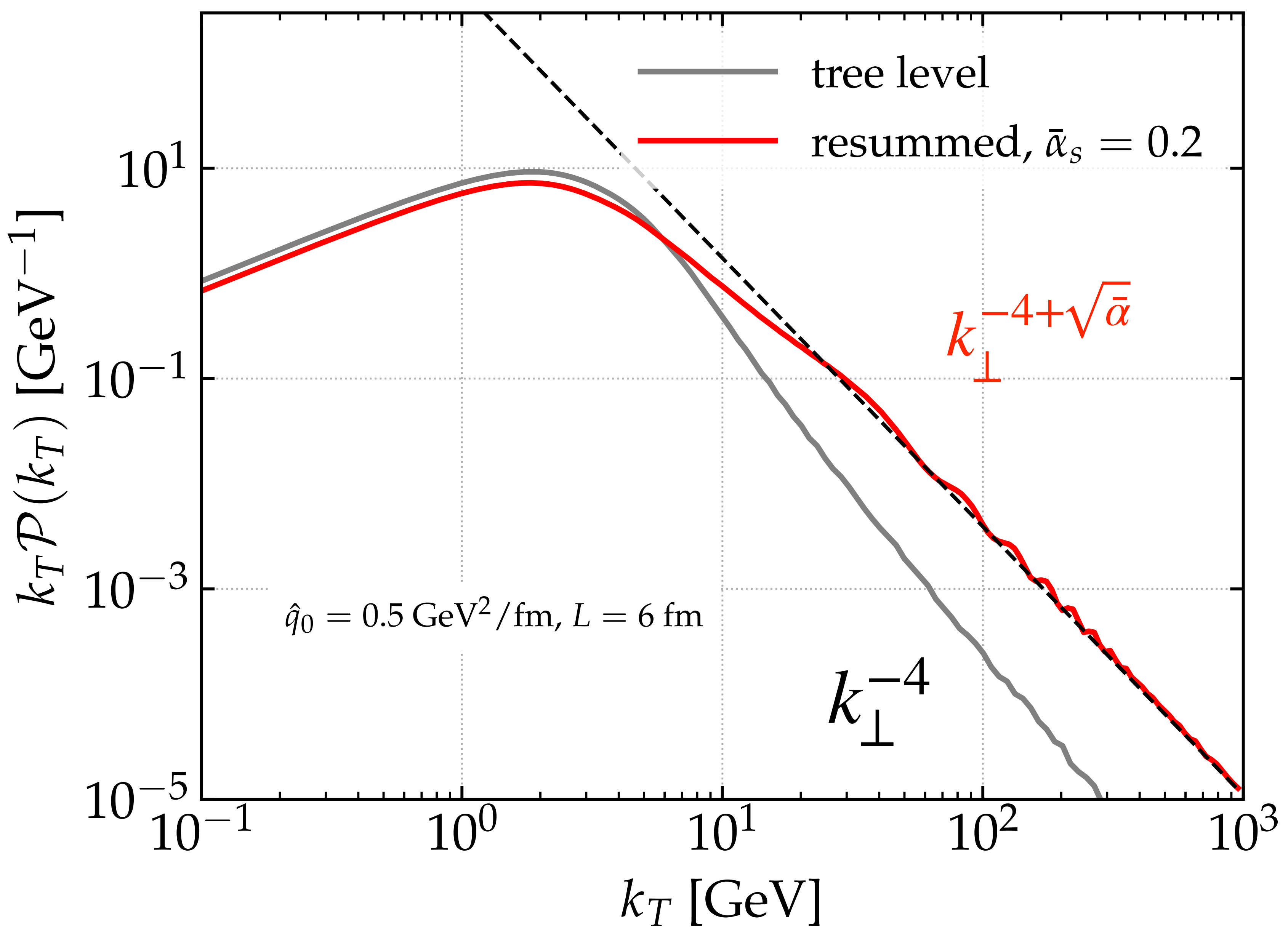
Departure from Rutherford: Heavy Lévy tail

- Extended geometric scaling window: $Q_s(L) < k_\perp \ll Q_s^2/\mu$ (in small-x: Mueller, Triantafyllopoulos (2002) Iancu, Itakura, McLerran (2002)
- Modified Rutherford scattering: Asymptotic behavior after resummation (heavy tail)

- Modified scaling variable for running coupling

$$x \equiv \log \frac{k_\perp^2}{Q_s^2} \;\rightarrow\; x \equiv \frac{\log \frac{k_\perp^2}{Q_s^2}}{\sqrt{Y}} \sim \sqrt{\bar{\alpha}(Y)} \log \frac{k_\perp^2}{Q_s^2}$$

$$\rho_s(Y) = \log Q_s^2(Y) = Y + 4\sqrt{bY} + 3\xi_1(4bY)^{1/6} + O(\ln Y)$$



- TMB is a super-diffusive process due to logarithmically enhanced quantum corrections
- TMB exhibits geometric scaling and heavy tails akin to Lévy random walks
- Exploiting an analogy with saturation physics we find exact universal asymptotic and pre-asymptotic solutions for the transverse momentum distribution
- Outlook: investigate anomalous diffusion in QCD matter in experiment (HIC), compute NLL …