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Towards a causal and stable first-order theory of viscous chiral hydrodynamics

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in collaboration with

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Quark Matter 2022

Kraków, April 4-10, 2022

Chiral hydrodynamics from kinetic theory

e.g., Chen, Son, Stephanov, PRL 115, no.2, 021601 (2015); Yang, PRD 98, no.7, 076019 (2018)

- Consider ensemble of particles (and antiparticles) with chirality $\pm$
- Distribution function

$$f_{\mathrm{eq},\pm}(x,p) = \left[\exp(g_\pm) + 1\right]^{-1}$$

$$g_\pm(x,p) = -\beta\cdot p - \frac{\mu_\pm}{T} - \underbrace{\frac{1}{2}S^{\mu\nu}\varpi_{\mu\nu}}_{\text{Spin-vorticity coupling}}$$

  $S^{\mu\nu}$ - Rank-2 spin tensor,
  Thermal vorticity - $\varpi^{\mu\nu} = -\frac{1}{2}(\partial^\mu\beta^\nu - \partial^\nu\beta^\mu)$
- Hydrodynamic densities from distribution function: energy-momentum tensor $T^{\mu\nu}$, vector and axial vector currents J^μ_V, J^μ_A

$$\partial_\mu T^{\mu\nu} = 0 \qquad \partial_\mu J^\mu_V = 0 \qquad \partial_\mu J^\mu_A = 0$$

This is great. But can one solve it?

Ideal chiral hydrodynamics from kinetic theory

Consider the full nonlinear system: $\partial_\mu T^{\mu\nu} = 0,\ \partial_\mu J^\mu_V = 0,\ \partial_\mu J^\mu_A = 0$

ES, Bemfica, Disconzi, Noronha, 2104.02110 (2021)

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu + P\Delta^{\mu\nu} + \xi_T(\omega^\mu u^\nu + \omega^\nu u^\mu)$$
$$J^\mu_V = n_V u^\mu + \xi_V \omega^\mu$$
$$J^\mu_A = n_A u^\mu + \xi_A \omega^\mu$$

e.g., Chen, Son, Stephanov, PRL **115**, no.2, 021601 (2015); Yang, PRD **98**, no.7, 076019 (2018)

Vorticity - $\omega^\mu = (1/2)\epsilon^{\mu\nu\alpha\beta} u_\nu \partial_\alpha u_\beta$

- One can prove:

Characteristic determinant $= 0$

System is ill posed:
No unique solution exists for **arbitrary** initial data!

- **Conventional ideal case**: $\xi_T, \xi_V, \xi_A = 0 \Longrightarrow$ Well-posed, causal and stable

Ideal chiral hydrodynamics in Landau frame

ES, Bemfica, Disconzi, Noronha, 2104.02110 (2021)

- Shift of velocity

$$u^\mu = u_L^\mu - \frac{\xi_T \omega_L^\mu}{\varepsilon + P}$$

- Constitutive relations in Landau frame

$$\begin{aligned} T^{\mu\nu} &= \varepsilon u_L^\mu u_L^\nu + P\Delta^{\mu\nu} \\ J_V^\mu &= n_V u_L^\mu + \xi_V \omega_L^\mu \\ J_A^\mu &= n_A u_L^\mu + \xi_A \omega_L^\mu \end{aligned}$$

The theory is well-posed, causal and stable!

- Definition of hydrodynamic variables (hydrodynamic frames) matter even in the ideal case

First-order viscous chiral hydrodynamics

Bemfica, Disconzi, Noronha, ES (to appear)

- Constitutive relations in a general hydrodynamic frame

$$T^{\mu\nu} = (\varepsilon + \mathcal{A})\left(u^\mu u^\nu + \frac{\Delta^{\mu\nu}}{3}\right) + \mathcal{Q}^\mu u^\nu + \mathcal{Q}^\nu u^\mu + \pi^{\mu\nu}$$
$$J_V^\mu = (n_V + \mathcal{N}_V)u^\mu + \mathcal{J}_V^\mu$$
$$J_A^\mu = (n_A + \mathcal{N}_A)u^\mu + \mathcal{J}_A^\mu$$

- Consider the theory

$$\mathcal{A} = a_1 \mathcal{D}\varepsilon, \quad \mathcal{Q}^\mu = b_1 \Delta^{\mu\lambda}\mathcal{D}_\lambda \varepsilon, \quad \pi^{\mu\nu} = -2\eta\sigma^{\mu\nu}$$
$$\mathcal{N}_V = c_{V1}\mathcal{D}\varepsilon + c_{V2}\mathcal{D}n_V + c_{V3}\mathcal{D}n_A$$
$$\mathcal{J}_V^\mu = e_{V1}\Delta^{\mu\lambda}\mathcal{D}_\lambda\varepsilon + e_{V2}\Delta^{\mu\lambda}\mathcal{D}_\lambda n_V + e_{V3}\Delta^{\mu\lambda}\mathcal{D}_\lambda n_A + \xi_V\omega^\mu$$
$$\mathcal{N}_A = c_{A1}\mathcal{D}\varepsilon + c_{A2}\mathcal{D}n_V + c_{A3}\mathcal{D}n_A$$
$$\mathcal{J}_A^\mu = e_{A1}\Delta^{\mu\lambda}\mathcal{D}_\lambda\varepsilon + e_{A2}\Delta^{\mu\lambda}\mathcal{D}_\lambda n_V + e_{A3}\Delta^{\mu\lambda}\mathcal{D}_\lambda n_A + \xi_A\omega^\mu$$

$\mathcal{D}^\mu$ - Weyl derivative

One can prove: System is well-posed, causal and stable!

Conclusions

- Relativistic hydrodynamics from chiral kinetic theory (dissipationless) is ill-posed and acausal. Cannot be solved with numerical simulations!
- Ideal chiral hydrodynamics in Landau frame is well posed: hydrodynamic frames are important
- New first-order viscous chiral hydrodynamics is well posed, causal and stable (to appear)
- Outlook: First-order relativistic spin hydrodynamics

BACKUP

Initial-value problem

- **Nonlinear** system of partial differential equations

$$\mathcal{A}(\Psi, \partial)\Psi = \mathcal{B}$$

  - Ψ - vector of unknowns
  - $\mathcal{A}(\Psi, \partial)$ - Principal part contains higher-order derivatives
  - $\mathcal{B}$ - Coefficients depending on unknowns and lower-order derivatives

- Arbitrary initial data along hypersurface $\Sigma = \{\phi(x) = 0\}$

- **To find solution:** Need to express higher order derivatives in terms of lower-order ones

  $\implies \mathcal{A}$ must be invertible

$$\text{Characteristic determinant} = \det[\mathcal{A}(\Psi_0, \varphi)] \neq 0$$

  Ψ_0 initial data, $\varphi_\mu = \partial_\mu \phi$

Well-posedness

(see e.g. R. Wald, General Relativity)

Initial-value problem is locally well-posed in some function space (e.g., analytic functions) along a hypersurface Σ if

1) **Arbitrary** initial data on Σ $\Rightarrow$ Exists unique solution in a neighborhood of Σ
2) "Small changes" of initial data $\Rightarrow$ "Small changes" of solution
3) For relativistic theories causality must hold

- Examples of well-posed theories: Ideal hydrodynamics, Klein Gordon, GR

- Given **arbitrary** initial data, if

$$\det[\mathcal{A}(\Psi_0, \varphi)] = 0 \quad \Longrightarrow \quad \text{System is ill-posed and acausal}$$

$\Longrightarrow$ No unique solution for **arbitrary** initial data