# How to interpret results of Bayesian analyses

## Illustrated from the JETSCAPE analyses of properties of the quark-gluon-plasma

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# BAYESIAN BASICS:

- Parameter estimation: Given a model, what parameter values are compatible with experimental data, and with what precision can we determine them?
- Can answer with Bayesian inference — ideal for detailed and systematic treatment of uncertainty
- Experimental data ($D$) and parameters ($p$) are each associated with probability distributions
- Bayes' theorem relates conditional probabilities. E.g., $\Pr(D|p)$ is the probability of $D$, given $p$.
- The probability that both $D$ and $p$ are true is

$$\Pr(p\,\&D) = \Pr(p) \times \Pr(D|p) = \Pr(D) \times \Pr(p|D)$$
$$\text{prior} \times \text{likelihood} = \text{evidence} \times \text{posterior}$$

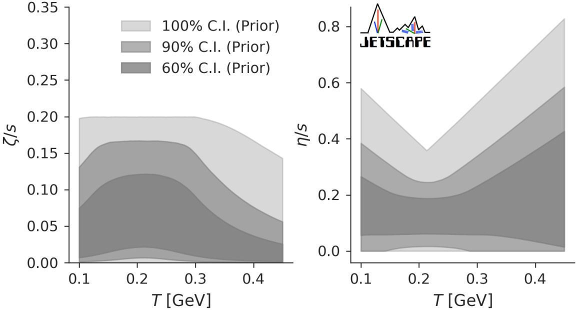
- We typically want to know $\Pr(p|D) \propto \Pr(p)\,\Pr(D|p)$
- $\implies$ need to choose a prior $\Pr(p)$ and compute the likelihood $\Pr(D|p)$ from comparison with data

$$\Pr(D|p) \propto e^{-\chi^2/2}$$
$$\text{with } \chi^2 = (D - \text{Model}(p))^T\,\Sigma^{-1}\,(D - \text{Model}(p))$$
$$\text{and } \Sigma = \text{uncertainty covariance (exp. and theor.)}$$

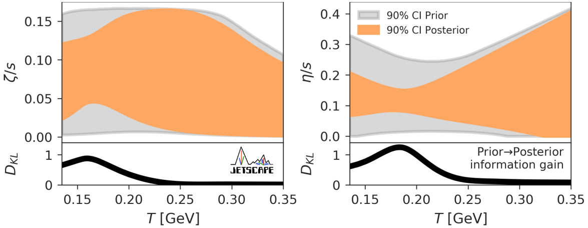
- Not a magic black box — Here are some tips for using and interpreting results:

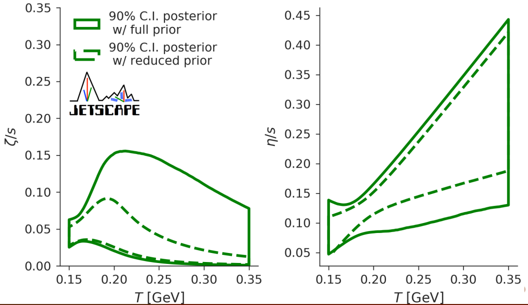
# Prior Pr($p$) and information gain $D_{KL}$

- The prior represents knowledge or belief about parameters before measurement
- There doesn't exist a neutral or uninformed choice
- The choice of prior can significantly affect the posterior
- Should compare prior and posterior. Can quantify the information gain



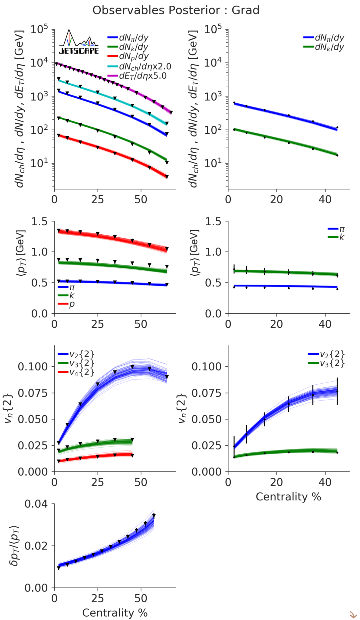

$$D_{KL} \equiv \sum_p \Pr(p) \log \left[ \frac{\Pr(p)}{\Pr(p|D)} \right]$$

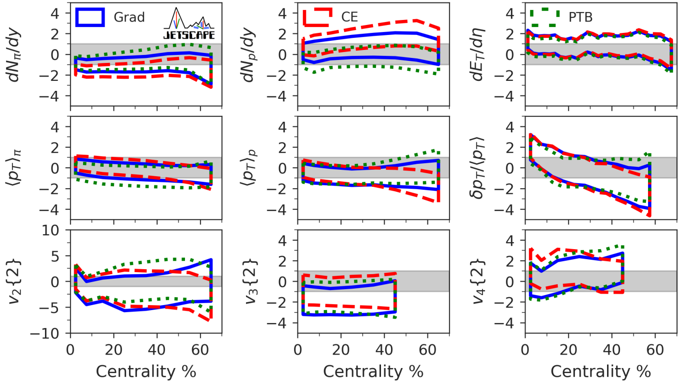


# Evaluating model success

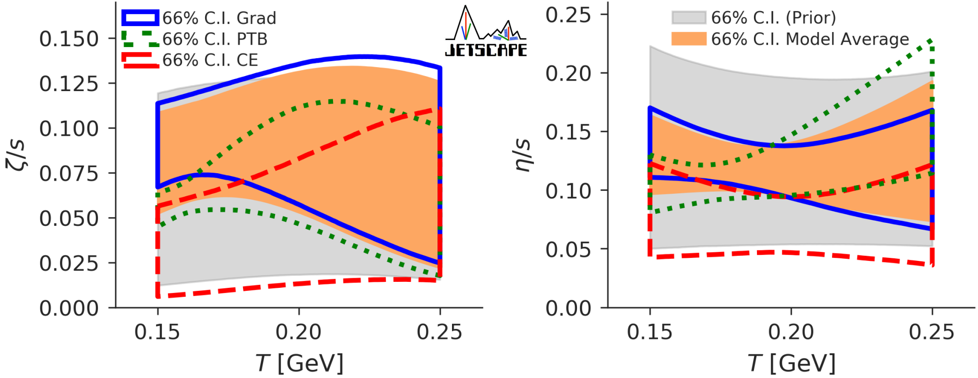
- Posterior does not tell the overall quality of model/fit (only *relative* quality at different parameter points)
- Must evaluate success of model separately
- E.g., direct observable comparison of posterior predictive distributions (right), or discrepancy relative to experimental uncertainty (below):



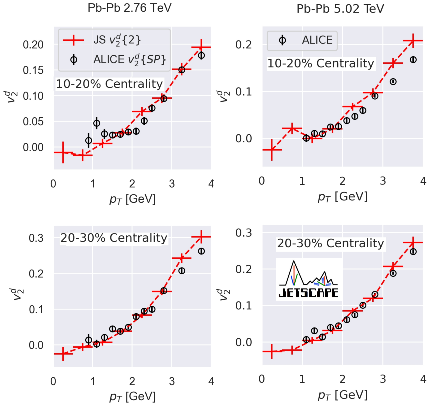
# Comparing/selecting models and observables and Bayesian averaging

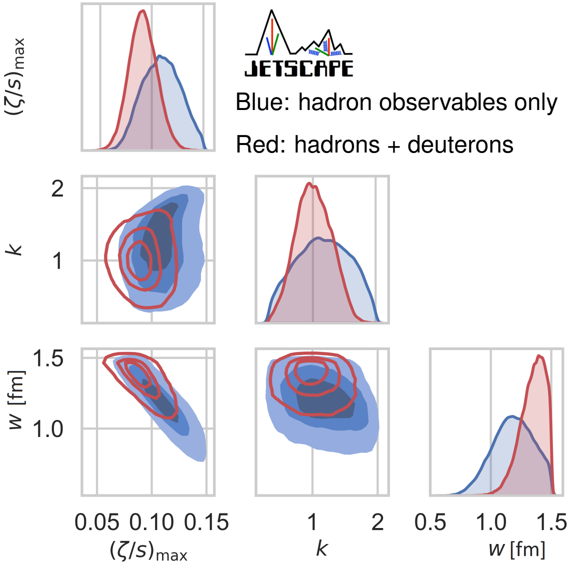
- Results are always interpreted in the context of a particular model — if something is missing from a model, this error does not appear in the results
- Can compare multiple models with Bayesian evidence $\mathrm{Pr}^{(i)}(D) = \int dp\, \mathrm{Pr}^{(i)}(D|p)\, \mathrm{Pr}(p)$
- E.g., models for the hadron distribution at hydro→kinetic theory transition
- Grad:PTB:CE $\simeq$ 5000:2000:1 $\Longrightarrow$ CE disfavored by data
- Probability-weighted Bayesian model average: $\mathrm{Pr}_{\mathrm{BMA}}(p, D) \propto \sum_i \mathrm{Pr}^{(i)}(D)\, \mathrm{Pr}^{(i)}(p|D)$



# Directed study example: deuterons (arXiv:2203.08286)

- Bayesian methods can be used for smaller, directed studies
- Heavier particles such as deuterons have a larger sensitivity to bulk viscosity
- $\implies$ Deuteron measurements can be used to better constrain $\zeta/s$




Blue: hadron observables only

Red: hadrons + deuterons

# JETSCAPE Contributions at Quark Matter 2022

- Talks
  - Yasuki Tachibana
    T03, Tues 16:30
    *Comprehensive Study of Multi-scale Jet-medium Interaction*
    arXiv:2204.01163
  - Raymond Ehlers
    T04, Wed 16:20
    *Bayesian analysis of QGP jet transport using multi-scale modeling applied to inclusive hadron and reconstructed jet data*
- Posters
  - Christine Nattrass
    Session 1 T04_2, Wed 17:30–18:30
    *Multi-scale probe of the jet-medium interaction via internal jet structure modification*
  - Chathuranga Sirimanna
    Session 2 T13, Wed 18:30–19:30
    *Photon-Jet correlations in central heavy-ion collisions with JETSCAPE*
  - Abhijit Majumder
    Session 2 T03, Wed 18:30–19:30
    *Comprehensive study of multi-scale jet-medium interaction*
  - Arjun Sengupta
    Session 2 T14_1, Wed 18:30–19:30
    *A Systematic Study of In-Medium Hadronization of Jet Showers with JETSCAPE and Hybrid Hadronization*
  - Wenkai Fan
    Session 3 T11_3, Fri 14:00–15:00
    *Heavy flavor production in heavy ion collisions with JETSCAPE*