Proton-deuteron and deuteron-deuteron correlation functions and origin of light nuclei from relativistic heavy-ion collisions

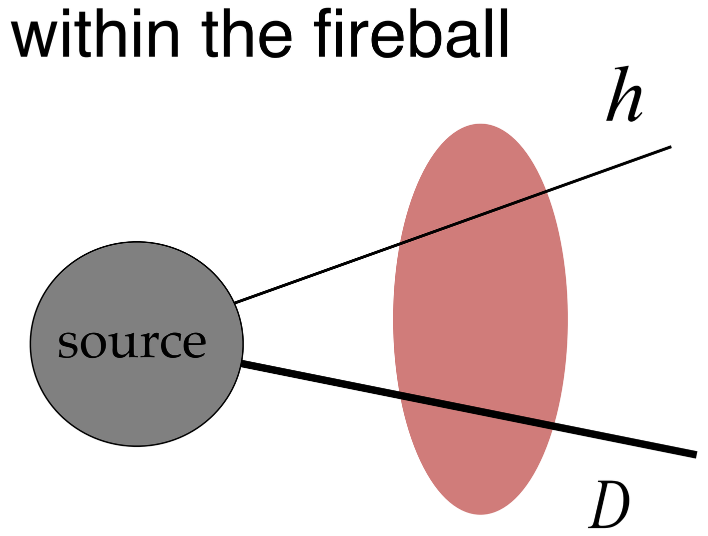
Patrycja Słoń, Stanisław Mrówczyński

Based on Phys. Rev. C 104, 024909 (2021) and Acta Physica Polonica B 51, 1739 (2020)

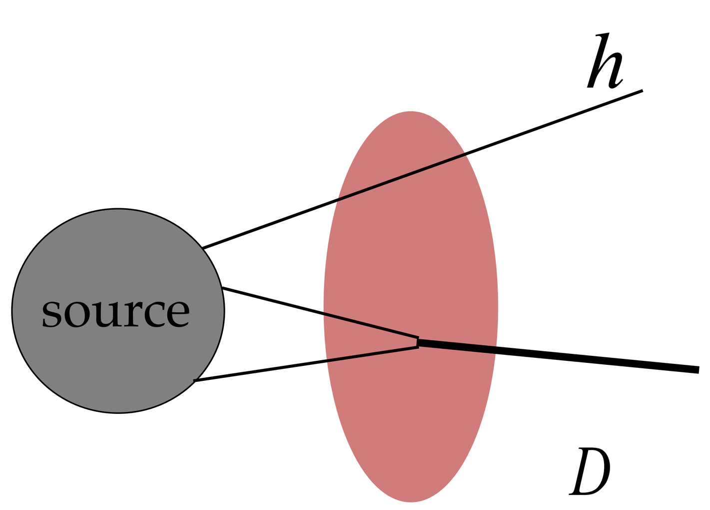
Production of light nuclei

Production of light nuclei is well described by two models:

- **thermodynamical** - light nuclei are formed within the fireball



- **coalescence** - light nuclei are formed due to final state interactions.



Notation

- $\mathbf{q}$ - relative momentum
- $\psi(\mathbf{r}_1, \mathbf{r}_2)$ - wave function
- $\mathscr{D}$ - source function
- $\mathbf{R}, \mathbf{P}$ - position and momentum of the center of mass
- $\mathscr{D}_r$ - 'relative' source
- indices 1,2 - protons
- indices 3,4 - neutrons
- R_s - parameter related to the average square radius of the source

Proton-deuteron correlation function

Hadron-deuteron and deuteron-deuteron correlations carry information about the source of deuterons. Measurement of the correlation function allows one to determine the process of production of light nuclei.

1. Deuterons treated as elementary (thermodynamical model)

- experimental definition of the correlation function

$$\frac{dP_{pD}}{d^3p_p\, d^3p_D} = \mathscr{R}(\mathbf{p}_p, \mathbf{p}_D)\frac{dP_p}{d^3p_p}\frac{dP_D}{d^3p_D}$$

- theoretical formula

$$\mathscr{R}(\mathbf{q}) = \int d^3r_p\, d^3r_D\, \mathscr{D}(\mathbf{r}_p)\mathscr{D}(\mathbf{r}_D)\, |\psi(\mathbf{r}_p, \mathbf{r}_D)|^2$$

For both cases we want to eliminate the center-of-mass motion of the $p-D$ pair. Therefore, we introduce the center-of-mass variables.

- center-of-mass variables

$$\mathbf{R} = \frac{m_p\mathbf{r}_p + m_D\mathbf{r}_D}{m_p + m_D}, \qquad \mathbf{r} = \mathbf{r}_p - \mathbf{r}_D$$

- wave function

$$\psi(\mathbf{r}_p, \mathbf{r}_D) = e^{i\mathbf{RP}}\phi_{\mathbf{q}}(\mathbf{r})$$

- correlation function

$$\mathscr{R}(\mathbf{q}) = \int d^3r\, \mathscr{D}_r(\mathbf{r})\, |\phi_{\mathbf{q}}(\mathbf{r})|^2$$

2. Deuterons treated as a bound state (coalescence model)

- experimental definition of the correlation function

$$\frac{dP_{pD}}{d^3p_p\, d^3p_D} = \mathscr{R}(\mathbf{p}_p, \mathbf{p}_D)\, \mathscr{A}\frac{dP_n}{d^3p_n}\left(\frac{dP_p}{d^3p_p}\right)^2$$

- theoretical formula

$$\mathscr{R}(\mathbf{q})\, \mathscr{A} = \frac{3}{4}(2\pi)^3 \int d^3r_1\, d^3r_2\, d^3r_3\, \mathscr{D}(\mathbf{r}_1)\mathscr{D}(\mathbf{r}_2)\mathscr{D}(\mathbf{r}_3)\, |\psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)|^2$$

Deuteron formation rate $\mathscr{A}$

- experimental definition

$$\frac{dP_D}{d^3p} = \mathscr{A}\frac{dP_p}{d^3(p/2)}\frac{dP_n}{d^3(p/2)}$$

- theoretical formula

$$\mathscr{A} = \frac{3}{4}(2\pi)^3 \int d^3r_1\, d^3r_3 \mathscr{D}(\mathbf{r}_1)\mathscr{D}(\mathbf{r}_3)\, |\psi_D(\mathbf{r}_1, \mathbf{r}_3)|^2$$

- center-of-mass (Jacobi) variables

$$\mathbf{R} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + m_3\mathbf{r}_3}{m_1 + m_2 + m_3},$$

$$\mathbf{r}_{13} = \mathbf{r}_1 - \mathbf{r}_3, \qquad \mathbf{r} = \mathbf{r}_1 - \frac{m_2\mathbf{r}_2 + m_3\mathbf{r}_3}{m_2 + m_3}$$

- wave function

$$\psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = e^{i\mathbf{PR}}\, \phi_{\mathbf{q}}(\mathbf{r})\, \varphi_D(\mathbf{r}_{13})$$

- deuteron formation rate

$$\mathscr{A} = \frac{3}{4}(2\pi)^3 \int d^3r_{13}\, \mathscr{D}_r(\mathbf{r}_{13})\, |\varphi_D(\mathbf{r}_{13})|^2$$

- correlation function

$$\mathscr{R}(\mathbf{q}) = \int d^3r\, \mathscr{D}_{3r}(\mathbf{r})\, |\phi_{\mathbf{q}}(\mathbf{r})|^2$$

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Parameters used in the computation

1. We take into account the Coulomb effect by multiplying the correlation functions by the Gamow factor

$$G(q) = \frac{2\pi}{a_B q} \frac{1}{\exp\left(\frac{2\pi}{a_B q}\right) - 1}$$

where
- $a_B^{-1} = \mu\alpha$ - Bohr radius
- μ - reduced mass of the system
- α - fine-structure constant

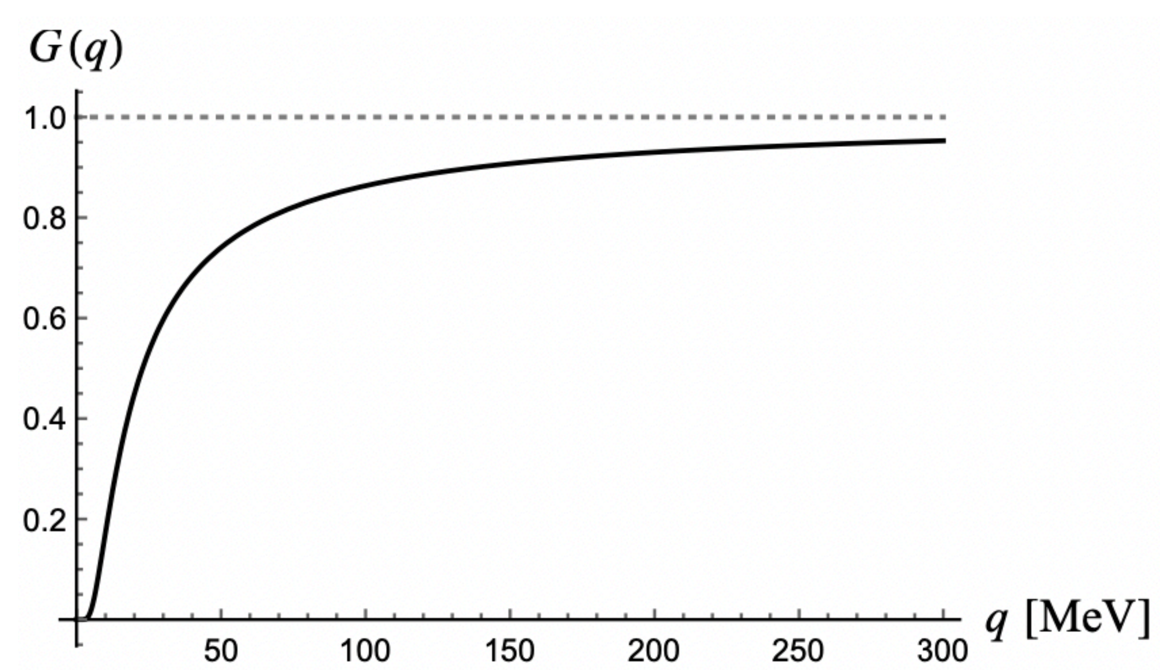

Fig. 1. The Gamow factor

2. Wave function in the asymptotic scattering form

$$\phi_{\mathbf{q}}(\mathbf{r}) = e^{iqz} + f(q)\frac{e^{iqr}}{r}$$

where
- $f(q) = \dfrac{-a}{1+iqa}$ - s-wave scattering aptitude
- a - scattering length

3. Isotropic Gaussian source function

$$\mathcal{D}(\mathbf{r}) = \left(\frac{1}{2\pi R_s^2}\right)^{3/2} e^{-\frac{\mathbf{r}^2}{2R_s^2}}$$

Proton-deuteron correlation function - results

'Relative' source and correlation function for both models

1. Deuterons treated as elementary (thermodynamical model)

$$\mathcal{D}_r(\mathbf{r}) = \left(\frac{1}{4\pi R_s^2}\right)^{3/2} e^{-\frac{\mathbf{r}^2}{4R_s^2}}, \qquad \mathcal{R}(\mathbf{q}) = \int d^3r\, \mathcal{D}_r(\mathbf{r})\,|\phi_{\mathbf{q}}(\mathbf{r})|^2$$

2. Deuterons treated as a bound state (coalescence model)

$$\mathcal{D}_{3r}(\mathbf{r}) = \left(\frac{1}{3\pi R_s^2}\right)^{3/2} e^{-\frac{\mathbf{r}^2}{3R_s^2}}, \qquad \mathcal{R}(\mathbf{q}) = \int d^3r\, \mathcal{D}_{3r}(\mathbf{r})\,|\phi_{\mathbf{q}}(\mathbf{r})|^2$$

For $p-D$ correlation function the source radius of deuterons formed due to final-state interactions is bigger by the factor of $\sqrt{4/3} \approx 1.15$ than that of directly emitted deuterons.

The $p-D$ correlation function is averaged over the spin states.

$$\mathcal{R}(\mathbf{q}) = \frac{1}{3}\mathcal{R}_{1/2}(\mathbf{q}) + \frac{2}{3}\mathcal{R}_{3/2}(\mathbf{q})$$

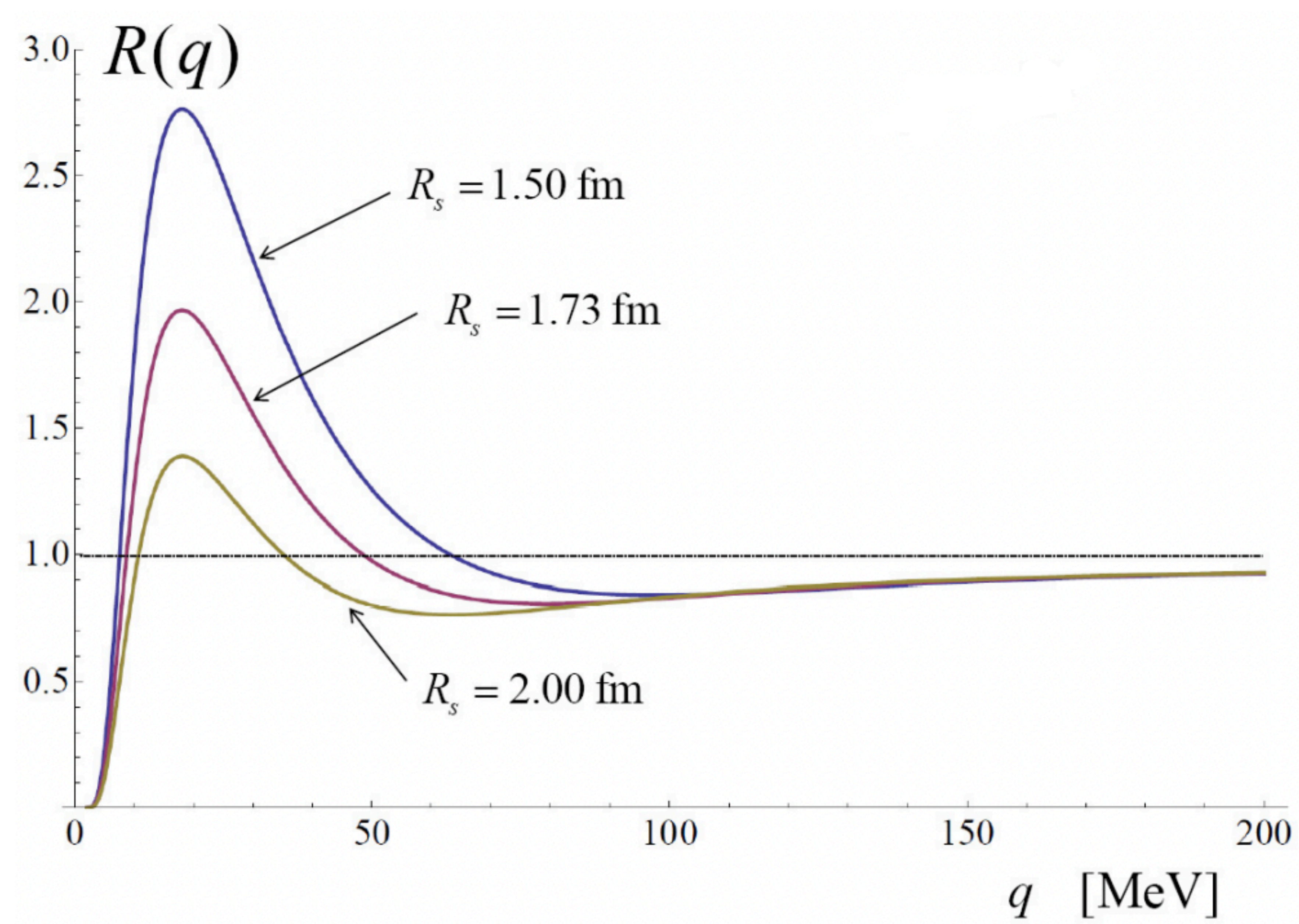

Fig. 2.The spin averaged $p-D$ correlation function

Numerical values

1. Scattering lengths
- $a_{1/2} = 4.0$ fm
- $a_{3/2} = 11.0$ fm

2. The source radius R_s is chosen in such a way that allows for direct comparison between the models:

$$2.00 = \sqrt{\frac{4}{3}} \cdot 1.73 = \frac{4}{3} \cdot 1.50$$

Conclusions

The correlation function strongly depends on R_s with the dependence becoming weaker as R_s grows. The analysis of higher p_T particles from noncentral events, when the sources are relatively small, is preferred. It should be possible to infer the R_s from experimentally measured $p-D$ function and compare it to R_s obtained from the $p-p$ correlation function. If deuterons are directly emitted from the fireball, the radii of proton and deuteron sources are the same. If deuterons are formed due to final state interactions, the radius of deuteron source is bigger by the factor of $\sqrt{4/3}$.

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Deuteron-deuteron correlation function

1. Deuterons treated as elementary (thermodynamical model)

- experimental definition of the correlation function

$$\frac{dP_{DD}}{d^3p_1\, d^3p_2} = \mathscr{R}(\mathbf{p}_1, \mathbf{p}_2)\frac{dP_D}{d^3p_1}\frac{dP_D}{d^3p_2}$$

- theoretical formula

$$\mathscr{R}(\mathbf{q}) = \int d^3r_1\, d^3r_2\, \mathscr{D}(\mathbf{r}_1)\mathscr{D}(\mathbf{r}_2)\,|\,\psi(\mathbf{r}_1, \mathbf{r}_2)\,|^2$$

- center-of-mass variables

$$\mathbf{R} = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2), \qquad \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$$

- wave function

$$\psi(\mathbf{r}_1, \mathbf{r}_2) = e^{i\mathbf{RP}}\phi_{\mathbf{q}}(\mathbf{r})$$

- correlation function

$$\mathscr{R}(\mathbf{q}) = \int d^3r\, \mathscr{D}_r(\mathbf{r})\,|\,\phi_{\mathbf{q}}(\mathbf{r})\,|^2$$

2. Deuterons treated as a bound state (coalescence model)

- experimental definition of the correlation function

$$\frac{dP_{DD}}{d^3p_1\, d^3p_2} = 2\mathscr{R}(\mathbf{p}_1, \mathbf{p}_2)\,\mathscr{A}^2\left(\frac{dP_p}{d^3p_p}\right)^2\left(\frac{dP_n}{d^3p_n}\right)^2$$

- theoretical formula

$$2\mathscr{R}(\mathbf{q})\,\mathscr{A}^2 = \frac{3^2}{4^2}(2\pi)^6\sum_{i=1}^{2}\int d^3r_1\, d^3r_2\, d^3r_3\, d^3r_4\, \mathscr{D}(\mathbf{r}_1)\mathscr{D}(\mathbf{r}_2)\times$$
$$\times\,\mathscr{D}(\mathbf{r}_3)\mathscr{D}(\mathbf{r}_4)\,|\,\psi_i(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4)\,|^2$$

The factor of 2 takes into the consideration that the two deuterons can be built in two ways: $D(p_1, n_3)$ & $D(p_2, n_4)$ and $D(p_1, n_4)$ & $D(p_2, n_3)$, with corresponding wave functions ψ_1 and ψ_2, respectively.

Presented center-of-mass variables correspond to the configuration consistent with ψ_1. To compute the contribution from ψ_2 one should switch $\mathbf{r}_3 \leftrightarrow \mathbf{r}_4$.

- center-of-mass variables

$$\mathbf{R} = \frac{1}{4}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4),$$
$$\mathbf{r}_{13} = \mathbf{r}_1 - \mathbf{r}_3, \qquad \mathbf{r}_{24} = \mathbf{r}_2 - \mathbf{r}_4,$$
$$\mathbf{r} = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_3) - \frac{1}{2}(\mathbf{r}_2 + \mathbf{r}_4)$$

- wave function

$$\psi_1(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) = e^{i\mathbf{PR}}\phi_{\mathbf{q}}(\mathbf{r})\times$$
$$\times\,\varphi_D(\mathbf{r}_{13})\varphi_D(\mathbf{r}_{24})$$

- deuteron formation rate

$$\mathscr{A} = \frac{3}{4}(2\pi)^3\int d^3r_{13}\, \mathscr{D}_r(\mathbf{r}_{13})\,|\,\varphi_D(\mathbf{r}_{13})\,|^2$$

$$\mathscr{A} = \frac{3}{4}(2\pi)^3\int d^3r_{24}\, \mathscr{D}_r(\mathbf{r}_{24})\,|\,\varphi_D(\mathbf{r}_{24})\,|^2$$

- correlation function

$$\mathscr{R}(\mathbf{q}) = \int d^3r\, \mathscr{D}_{4r}(\mathbf{r})\,|\,\phi_{\mathbf{q}}(\mathbf{r})\,|^2$$

Wave function of identical deuterons

For identical particles the spatial wave function $\phi_{\mathbf{q}}$ should be symmetrized or antisymmetrized. It must be symmetric for $S = 0, 2$, and antisymmetric for $S = 1$. Therefore, the wave function should be replaced by

$$\phi_{\mathbf{q}}(\mathbf{r}) \to \frac{1}{\sqrt{2}}\big(\phi_{\mathbf{q}}(\mathbf{r}) + (-1)^S\phi_{\mathbf{q}}(-\mathbf{r})\big)$$

where $S = 0, 1, 2$ is the total spin of a $D{-}D$ pair.

Correlation function of identical deuterons

The new form of the spatial wave function $\phi_{\mathbf{q}}$ produces different results for the correlation functions with $S = 0, 2$ and $S = 1$. In the case of $S = 1$ the correlation function coincides with that of noninteracting identical fermions.

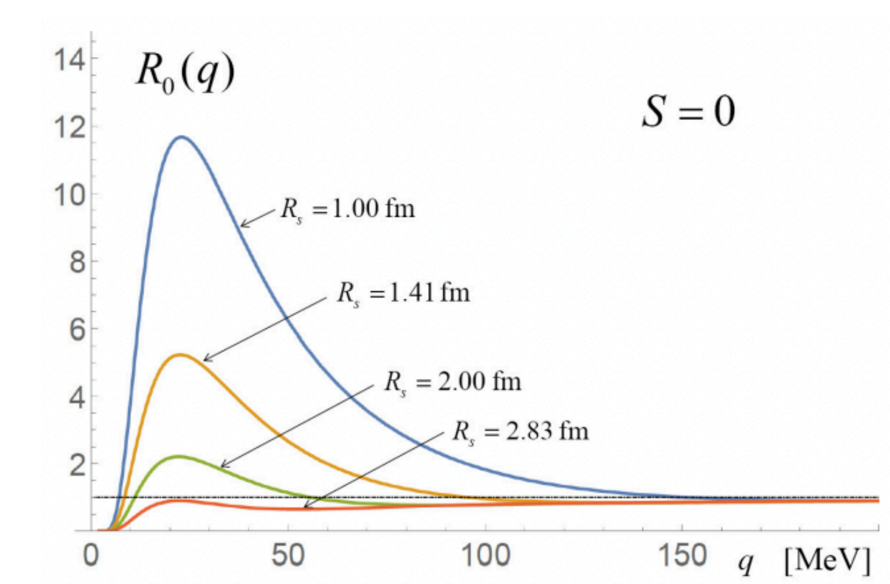

Fig. 3. The $D{-}D$ correlation function of $S = 0$

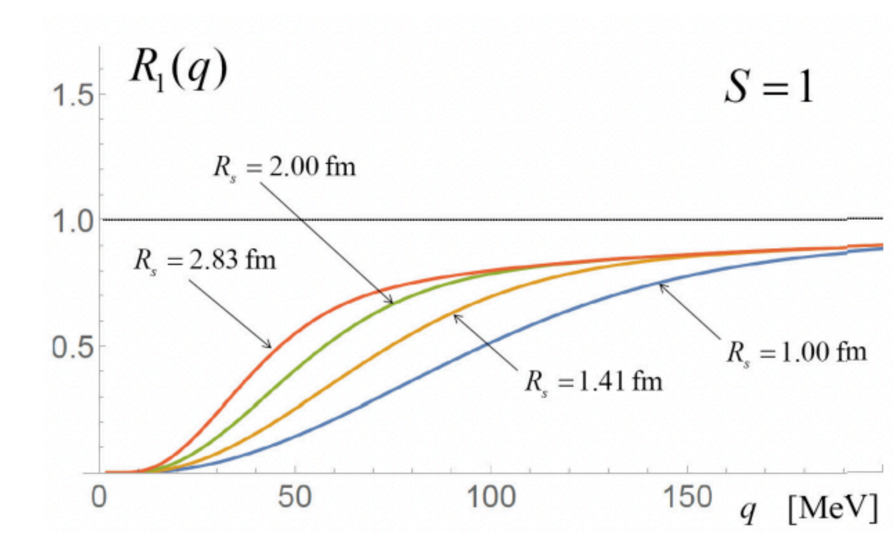

Fig. 4. The $D{-}D$ correlation function of $S = 1$

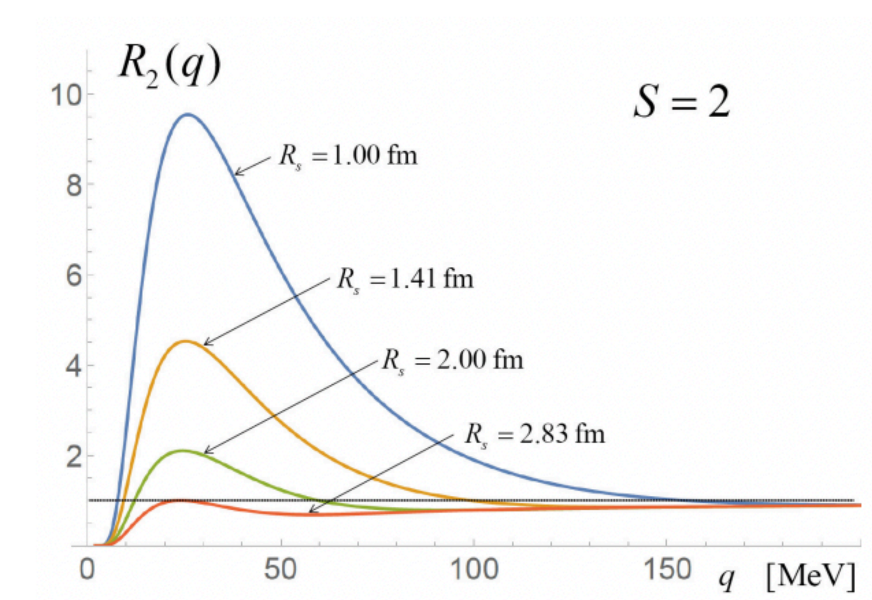

Fig. 5. The $D{-}D$ correlation function of $S = 2$

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Deuteron-deuteron correlation function - results

'Relative' source and correlation function for both models

1. **Deuterons treated as elementary (thermodynamical model)**

$$\mathscr{D}_r(\mathbf{r}) = \left(\frac{1}{4\pi R_s^2}\right)^{3/2} e^{-\frac{\mathbf{r}^2}{4R_s^2}}, \qquad \mathscr{R}(\mathbf{q}) = \int d^3r\, \mathscr{D}_r(\mathbf{r})\, |\phi_{\mathbf{q}}(\mathbf{r})|^2$$

2. **Deuterons treated as a bound state (coalescence model)**

$$\mathscr{D}_{4r}(\mathbf{r}) = \left(\frac{1}{2\pi R_s^2}\right)^{3/2} e^{-\frac{\mathbf{r}^2}{2R_s^2}}, \qquad \mathscr{R}(\mathbf{q}) = \int d^3r\, \mathscr{D}_{4r}(\mathbf{r})\, |\phi_{\mathbf{q}}(\mathbf{r})|^2$$

For $D-D$ correlation function the source radius of deuterons formed due to final-state interactions is bigger by the factor of $\sqrt{2} \approx 1.41$ than that of directly emitted deuterons.

The $D-D$ correlation function is averaged over the spin states.

$$\mathscr{R}(\mathbf{q}) = \frac{1}{9}\mathscr{R}_0(\mathbf{q}) + \frac{3}{9}\mathscr{R}_1(\mathbf{q}) + \frac{5}{9}\mathscr{R}_2(\mathbf{q})$$

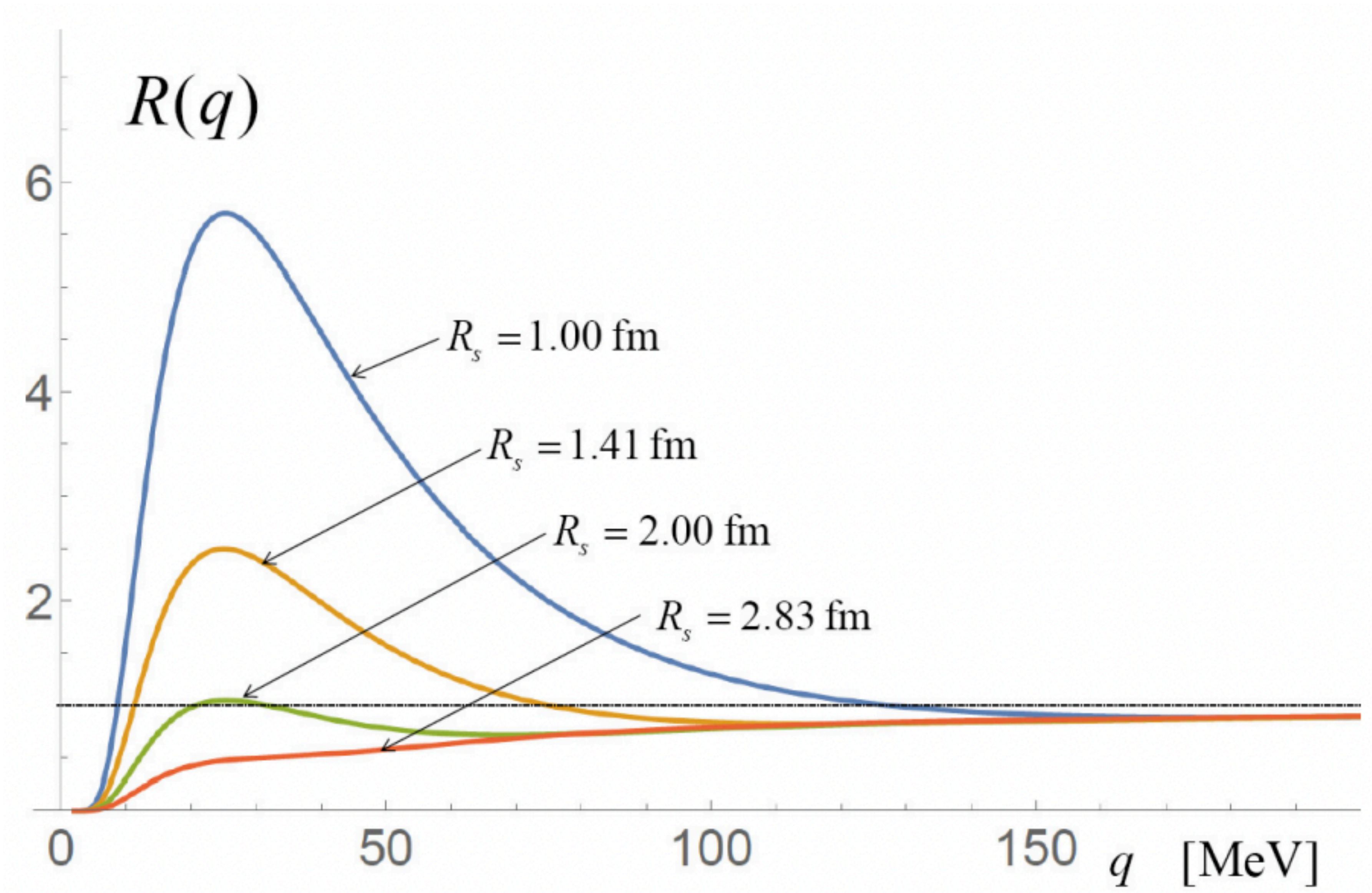

Fig. 6. The spin averaged $D-D$ correlation function

Numerical values

1. Scattering lengths
   - $a_0 = (10.2 + 0.2i)$ fm
   - $a_2 = 7.5$ fm

2. The source radius R_s is chosen in such a way that allows for direct comparison between the models:

$$2.83 = \sqrt{2}\cdot 2.00 = 2\cdot 1.41 = 2^2\cdot 1.00$$

Conclusions

The correlation function strongly depends on R_s with the dependence becoming weaker as R_s grows. The analysis of higher p_T particles from noncentral events, when the sources are relatively small, is preferred. It should be possible to infer the R_s from experimentally measured $D-D$ function and compare it to R_s obtained from the $p-p$ correlation function. If deuterons are directly emitted from the fireball, the radii of proton and deuteron sources are the same. If deuterons are formed due to final state interactions, the radius of deuteron source is bigger by the factor of $\sqrt{2}$.