MOAT REGIMES & THEIR SIGNATURES IN HEAVY-ION COLLISIONS

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[Pisarski, FR, PRL 127 (2021)]

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A MOAT

[Caerlaverock Castle, Scotland (source: Wikipedia)]
A MOAT

Energy dispersion of particle $\phi$:

$$E_\phi(p^2) = \sqrt{Zp^2 + W(p^2)^2 + m^2}$$

Particles are favored to have a nonzero momentum "gain energy by going faster"
WHERE DOES THE MOAT COME FROM?

heuristic picture:

spatial oscillation
\[ \cos(2\pi k_0 x) \]

momentum space peak
\[ \delta(p - k_0) \]

- particles subject to a spatial modulation are favored to have momentum \( k_0 \)

moat energy dispersion
(minimal energy at \( k_0 \))

\[ k_0^2 = -\frac{Z}{2W} \]

- typical for inhomogeneous/crystalline phases or a quantum pion liquid (Q\( \pi \)L)
WHERE CAN MOAT REGIMES APPEAR?

Expected at large $\mu$. Also QCD phase diagram?!

![Graph showing the phase diagram of QCD with regions marked for $Z < 0$, FRG crossover, and lattice data from Bazavov et al. '18 and Borsanyi et al. '20.]

- FRG: crossover
- Lattice: [Bazavov et al. '18]
- Lattice: [Borsanyi et al. '20]

indication for extended region with $Z < 0$ in QCD: **moat regime**
IMPLICATIONS OF THE MOAT

The energy gap might close at lower T and larger $\mu_B$:

$E > 0$ for all $p^2$

$E = 0$ at $p^2 > 0$:

instability towards formation of an inhomogeneous condensate
INHOMOGENEOUS PHASE
emerges if energy gap closes

- \( E_\phi(k_0^2) = 0 \): particles with momentum \( k_0 \) condense
- basic example: \( O(N) \) chiral spiral

\[
\begin{align*}
\phi &= \begin{pmatrix}
\sigma \\
\pi_{N-1} \\
\vdots \\
\pi_1
\end{pmatrix}, \\
\phi_0 &= \Delta 
\begin{pmatrix}
\cos(k_0 z) \\
\sin(k_0 z) \\
0 \\
\vdots \\
0
\end{pmatrix}
\end{align*}
\]

[Carignano, Buballa, Schaefer '14]
option 1: moat is a precursor for an inhomogeneous phase

possibilities: inhomogeneous chiral condensate or crystalline CSC
Inhomogeneous phases are mostly studied in mean-field. But associated spontaneous symmetry breaking gives rise to massless modes. Their fluctuations must be relevant!

Two types of symmetry breaking for inhomogeneous phases:

- continuous spatial symmetries (rotations, translations) broken down to discrete ones
- global flavor symmetries are broken (e.g. $O(N) \rightarrow O(N - 2)$ for chiral spiral)
SPATIAL SYMMETRY BREAKING

It has been argued that 1d modulations are favored against higher-dimensional ones

[Abuki, Ishibashi, Suzuki '12]
[Carignano, Buballa '12]

Goldstone bosons from spatial symmetry breaking (e.g. phonons) lead to Landau-Peierls instability of 1d inhomogeneous condensates (e.g. chiral density wave)

- Goldstone fluctuations lead to logarithmic IR divergences
  
  1d condensate is destroyed; the system is disordered

- algebraically instead of exponentially decaying correlations still possible
  
  quasi-long-range order (e.g. liquid crystals)

[Lee, Nakano, Tsue, Tatsumi, Friman '15]

Option 2: moat is a precursor for a liquid-crystal-like phase
FLAVOR SYMMETRY BREAKING

even "worse" for fluctuations of Goldstones from broken flavor symmetry

- basic example: fluctuations around $O(N)$ chiral spiral

$$\phi = \Delta \begin{pmatrix} \cos(k_0 z) \\ \sin(k_0 z) \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \begin{pmatrix} \delta \phi_\parallel \\ \delta \phi_\perp \end{pmatrix} \sim \frac{T}{W} k_0^{d-3} \int_{|\mathbf{p}| \sim k_0} \frac{d|\mathbf{p}|}{(|\mathbf{p}| - k_0)^2}$$

- transverse fluctuations lead to linear IR divergences at finite $T$ in any dimension

$\delta \phi_\perp$ disorders the system: no inhomogeneous phase for $N > 2$
not even quasi-long-range order (rigorous for $O(N)$ chiral spiral at $N \to \infty$)

instead, there is a quantum pion liquid

- disordered phase with a moat spectrum ($E > 0$ for all $\mathbf{p}^2$)
- spatial modulations: $\langle \phi(x)\phi(0) \rangle \sim e^{-m_r x} \cos(m_i x)$ for large $x$

Option 3: moat signals a quantum pion liquid
IMPLICATIONS OF THE MOAT

the moat regime could be an indication that dense QCD has:

- option 1: inhomogeneous phase
  - only if there are no Goldstone bosons

- option 2: liquid-crystal-like
  - only if there are only Goldstones from spatial symmetry breaking

- option 3: quantum pion liquid
  - only if there are Goldstones flavor symmetry breaking

this will occur in the regions where inhomogeneous phases are expected
SEARCH FOR MOAT REGIMES [Pisarski, FR '21]

Characteristic feature: minimal energy at nonzero momentum

⇒ enhanced particle production at nonzero momentum

→ look for signatures in the momentum dependence of particle numbers and correlations

• consider heavy-ion collision
• particles at freeze-out "mapped" onto detector
• freeze out at certain temperature $T_f$

defines 3d hypersurface: 
freeze-out surface $\Sigma$

How does the moat regime affect particles on $\Sigma$?
GENERALIZED COOPER-FRYE FORMULA

compute particle numbers on the freeze-out surface

• probability distribution of finding a particle $\phi$ with momentum $p$ in thermal equilibrium: Wigner function

$$F_\phi(p) = 2\pi \rho_\phi(p_0, p) f(p_0)$$

• particles on $\Sigma$ boosted with fluid velocity $u^\mu(x)$:

- energy: $\tilde{p}_0 = u^\mu p_\mu$
- spatial momentum: $\tilde{p}^2 = (u^\mu u^\nu - g^\mu\nu) p_\mu p_\nu$

• particle spectrum from integrating particle number current over freeze-out surface:

$$\frac{d^3N_\phi}{dp^3} = \frac{2}{(2\pi)^3} \int d\Sigma \int d\tilde{p}_0 \frac{dp_0}{2\pi} p^\mu \Theta(\tilde{p}_0) F_\phi(\tilde{p})$$

$\sim$ particle number current density

• reduces to Cooper-Frye formula for free vacuum spectral function: $\rho_\phi(p) = \text{sign}(p_0) \delta[p_0^2 - (p^2 + m^2)]$
PARTICLE SPECTRUM IN A MOAT REGIME

transverse momentum spectrum

- use simple models for illustration (quasi-particle in moat regime, boost-inv. and transverse isotropic freeze-out at fixed proper time, blast wave fluid velocity)

- compare normal phase (gray, $W = 0$) to moat phase (yellow, $W = 2.5$ GeV$^{-2}$)

\[ \frac{d^3N}{p_T dp_T dy d\phi_p} \]

enhanced particle production at nonzero momentum!
maximum related to the wavenumber of the spatial modulation
PARTICLE NUMBER CORRELATIONS

- correlations sensitive to in-medium modifications
- moat regime is disordered: single particle correlations can capture relevant features

\[ \text{correlations on } \Sigma \text{ from (generalized) Cooper-Frye formula} \]

\[
\left< \prod_{i=1}^{n} \frac{d^3 N_i}{d \mathbf{p}_i^3} \right> = \left[ \prod_{i=1}^{n} \frac{2}{(2\pi)^3} \int d\Sigma_i^\mu \int \frac{dp_i^0}{2\pi} (p_i)_\mu \Theta(p_i^0) \right] \left< \prod_{i=1}^{n} F_{\phi}(\tilde{p}_i) \right>
\]

- fluctuations, e.g., of thermodynamic quantities lead to fluctuations of \( F_\phi \)
- consider small fluctuations in \( T, \mu_B, u \)
TRANSVERSE MOMENTUM CORRELATIONS

- normalized two-particle correlation

\[ \Delta n_{12} = \frac{\langle \frac{d^3N_\phi}{dp_1^3} \frac{d^3N_\phi}{dp_2^3} \rangle_c}{\langle \frac{d^3N_\phi}{dp_1^3} \rangle \langle \frac{d^3N_\phi}{dp_1^3} \rangle} \]

( relativley) flat two-particle \( p_T \) correlation in the normal phase
TRANSVERSE MOMENTUM CORRELATIONS

- normalized two-particle correlation

\[ \Delta n_{12} = \frac{\left\langle \frac{d^3N_\phi}{dp_1^3} \frac{d^3N_\phi}{dp_2^3} \right\rangle_c}{\left\langle \frac{d^3N_\phi}{dp_1^3} \right\rangle \left\langle \frac{d^3N_\phi}{dp_1^3} \right\rangle} \]

moat regime

pronounced peak and ridges at nonzero \( p_T \) related to wavenumber of spatial modulation!

huge enhancement:

\[ \frac{\Delta n_{12}(p_{peak})}{\Delta n_{12}(p_{peak})} \bigg|_{\text{moat}} \approx 10^2 \]
SUMMARY

Moats arise in regimes with spatial modulations

- expected to occur at large $\mu_B$
- are precursors for inhomogeneous, liquid-crystal-like or quantum pion liquid phases
- quantum fluctuations are crucial

Enhanced production of moat particles at nonzero momentum

- characteristic peaks (and ridges) in particle spectra and correlations at nonzero $p_T$
- requires good resolution at low momentum

Opportunity to discover novel phases with low-energy heavy-ion collisions

So far: basic description of qualitative effects
To do: quantitative description of moat regimes
PARTICLE SPECTRUM IN A MOAT PHASE

use simple models to show general structure

Particle in a moat regime:

- low-energy model of free bosons in a moat regime \((Z < 0, W > 0)\):
  \[
  \mathcal{L}_0 = \frac{1}{2} (\partial_0 \phi)^2 + \frac{Z}{2} (\partial_i \phi)^2 + \frac{W}{2} (\partial^2 \phi)^2 + \frac{m_{\text{eff}}^2}{2} \phi^2
  \]

- gives simple in-medium spectral function
  \[
  \rho_\phi(p_0, p^2) = \text{sign}(p_0) \delta[p_0^2 - E^2_\phi(p^2)] \quad \text{with} \quad E_\phi(p^2) = \sqrt{Z p^2 + W(p^2)^2 + m_{\text{eff}}^2}
  \]

- boost symmetry broken! (but spatial rotation symmetry still intact)

Fluid velocity and freeze-out surface from hydro evolution

- boost invariant freeze-out at fixed temperature \(T_f\) and fixed proper time \(\tau_f\) \(= \sqrt{t^2 - z^2}\)

- blast wave approximation for the fluid velocity:
  \[
  u^r = \bar{u} \frac{r}{\bar{R}} \theta(\bar{R} - r)
  \]

[Schnedermann, Sollfrank, Heinz (1993)]
[Teaney (2003)]
PARTICLE SPECTRUM IN A MOAT PHASE
use simple models to show general structure

model parameters:

• pick a beam energy of $\sqrt{s} = 5$ GeV and read off thermodynamic and blast wave parameters:

$$T_f = 115 \text{ MeV} \quad \bar{u} = 0.3$$
$$\mu_{B,f} = 536 \text{ MeV} \quad \bar{R} = 8 \text{ fm}$$
$$\tau_f = 5 \text{ fm}/c$$

[Andronic, Braun-Munzinger, Redlich, Stachel (2018)]

[Andronic, Braun-Munzinger, Redlich, Stachel (2018)]

thermodynamics (used later) from a hadron resonance gas

[Braun-Munzinger, Redlich, Stachel (2003)]

moat parameters: purely illustrative

if $Z < 0$: $W = 2.5 \text{ GeV}^{-2}$
THERMODYNAMIC CORRELATIONS

- correlations \( \langle \ldots \rangle \) from thermodynamic average
- weight configurations with the change in entropy due to fluctuations, \( \Delta s^\mu \) \[\text{[Landau, Lifshitz (vol. 5)\]}\]

Generating functional of (connected) thermodynamic correlations:

\[
W[J] = \ln \int \mathcal{D}\kappa(x) \exp \left( d\Sigma_\mu \left[ \Delta s^\mu(x) + J(x)_{i\nu} \hat{\nu}^\mu \kappa_i^\nu(x) \right] \right)
\]

normal to \( \Sigma \)

- connected n-particle correlations \( \langle \kappa^n \rangle_c \) from

\[
\frac{\delta^n W[J]}{\delta J^n} \bigg|_{J=0}
\]

- change of entropy in an ideal fluid \( (T^\mu_\nu = \epsilon u^\mu u_\nu + p\Delta^\mu_\nu) \) with Gaussian fluctuations:

\[
\hat{\nu}_\mu \Delta s^\mu = -\frac{1}{2} \kappa_{i\mu}(x) \mathcal{F}_{ij}^\mu \kappa_{j\nu}(x)
\]

Local fluctuations!

\[
\mathcal{F}_{ij}^\mu = \frac{1}{T} \begin{pmatrix}
\hat{u} \frac{\partial s}{\partial T} & \hat{u} \frac{\partial s}{\partial \mu_B} & s \hat{\nu}^\nu \\
\hat{u} \frac{\partial s}{\partial \mu_B} & \hat{u} \frac{\partial n_B}{\partial \mu_B} & n_B \hat{\nu}^\nu \\
\hat{s} \hat{\nu}^\mu & n_B \hat{\nu}^\nu & -\hat{u} (Ts + \mu_B n_B) g^{\mu\nu}_{ij}
\end{pmatrix}
\]

Fluctuation matrix \( (\hat{u} = \hat{\nu}_\mu u_\mu) \)