MOAT REGIMES & THEIR SIGNATURES IN HEAVY-ION COLLISIONS

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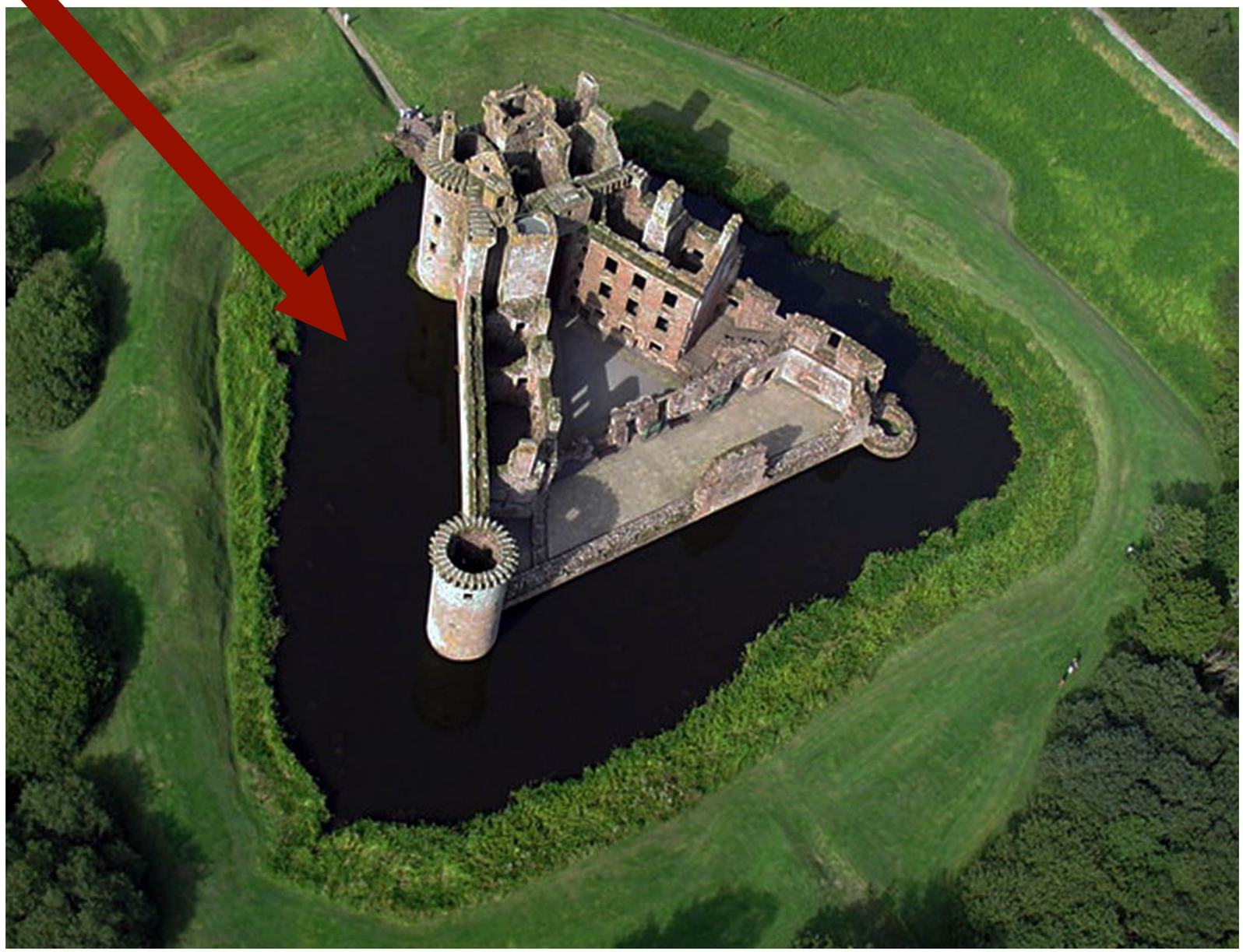
[Pisarski, FR, PRL 127 (2021)]

- QUARK MATTER 2022 -

KRAKOW - 06/04/2022

A MOAT

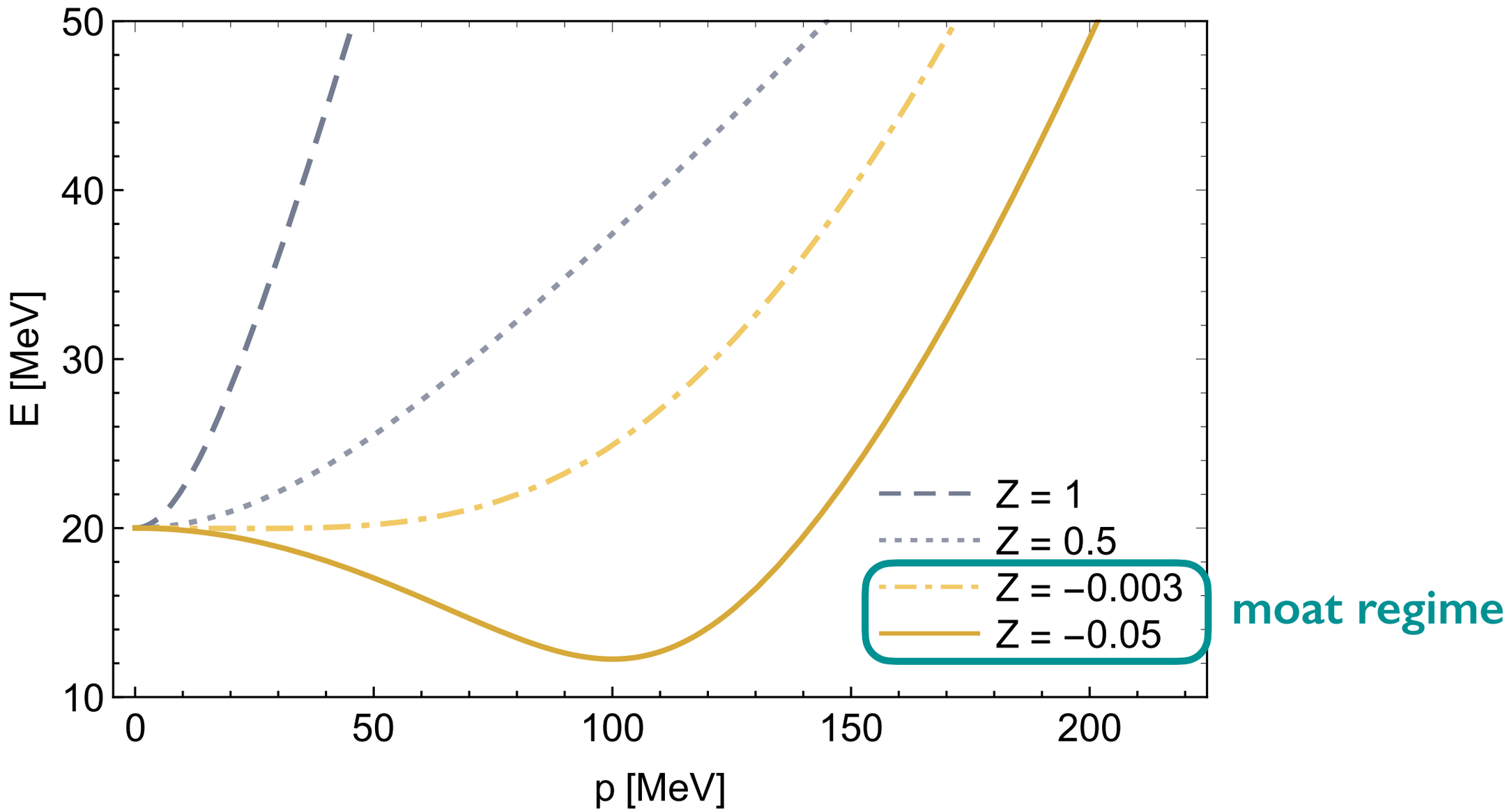


[Caerlaverock Castle, Scotland (source: Wikipedia)]

A MOAT

energy dispersion of particle ϕ:

$$E_\phi(\mathbf{p}^2) = \sqrt{Z\,\mathbf{p}^2 + W\left(\mathbf{p}^2\right)^2 + m^2}$$



→ particles are favored to have a nonzero momentum

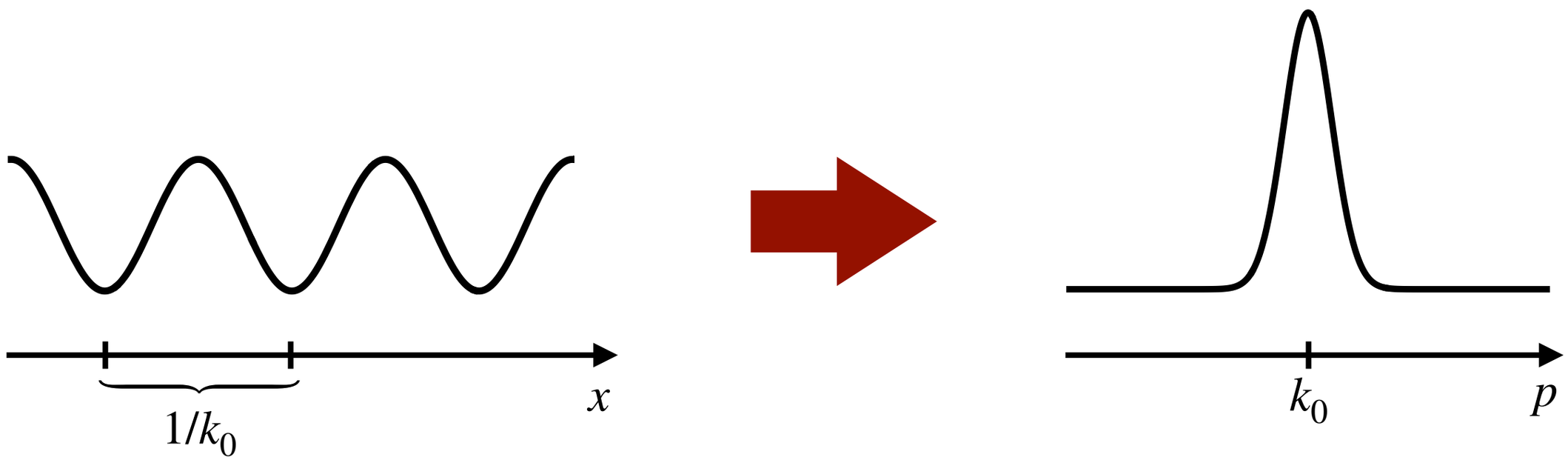
"gain energy by going faster"

WHERE DOES THE MOAT COME FROM?

heuristic picture:

spatial oscillation
$\cos(2\pi k_0 x)$

momentum space peak
$\delta(p - k_0)$



- particles subject to a spatial modulation are favored to have momentum k_0

→ moat energy dispersion
(minimal energy at k_0)

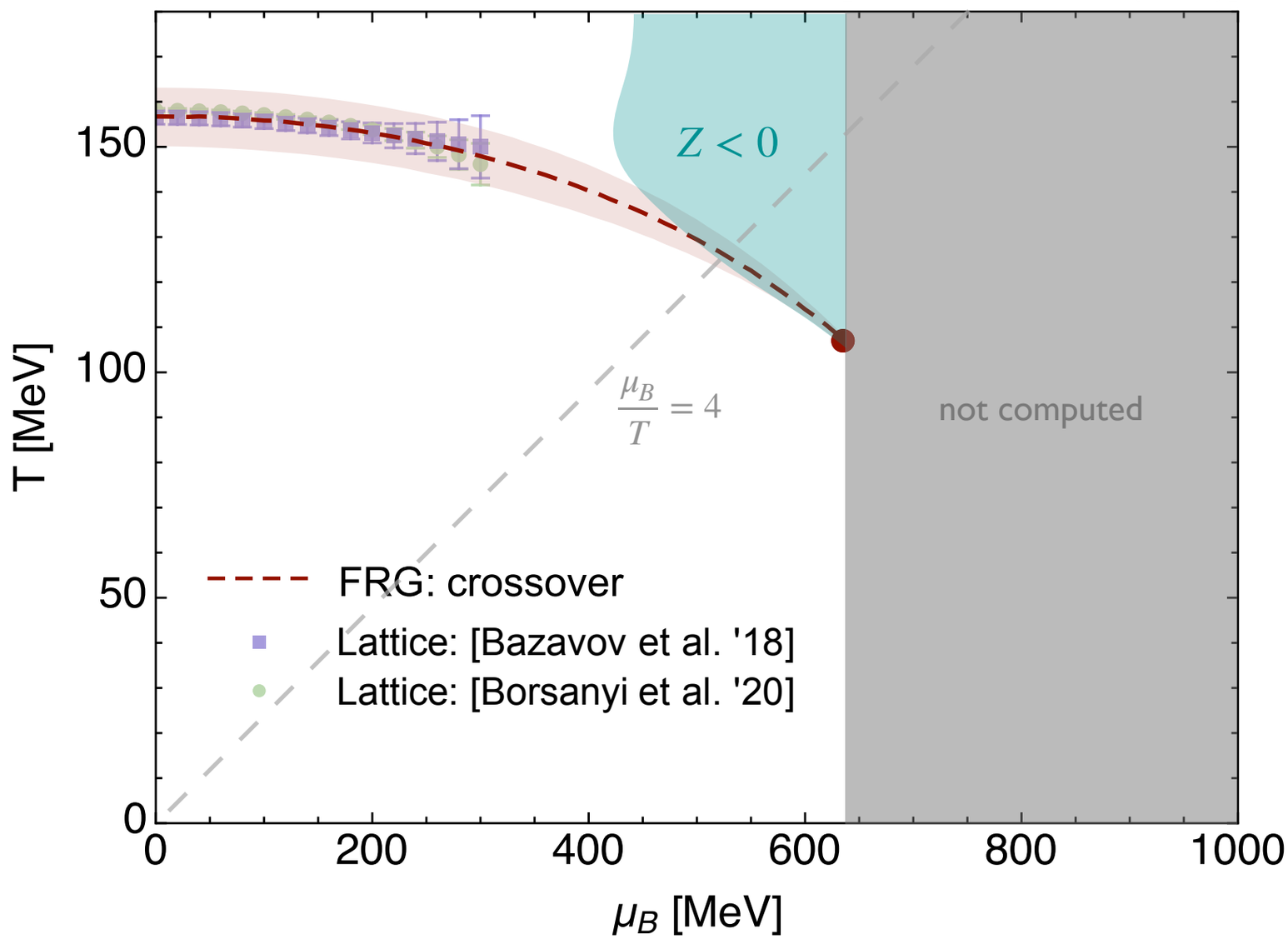
$k_0^2 = -Z/(2W)$

- typical for inhomogeneous/crystalline phases or a quantum pion liquid (QπL)

WHERE CAN MOAT REGIMES APPEAR?

Expected at large μ. Also QCD phase diagram?!

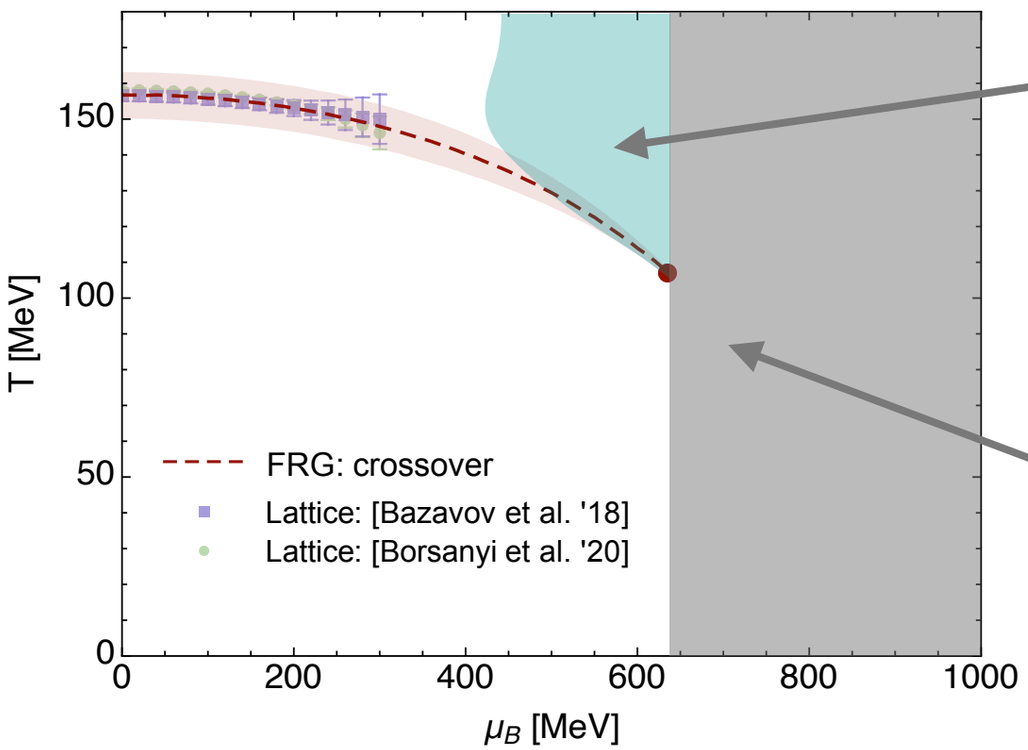
[Fu, Pawlowski, FR (2019)]

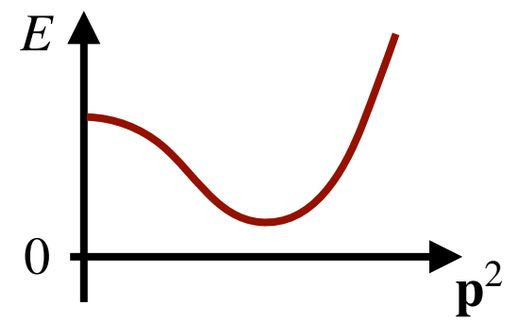


→ indication for extended region with $Z < 0$ in QCD: **moat regime**

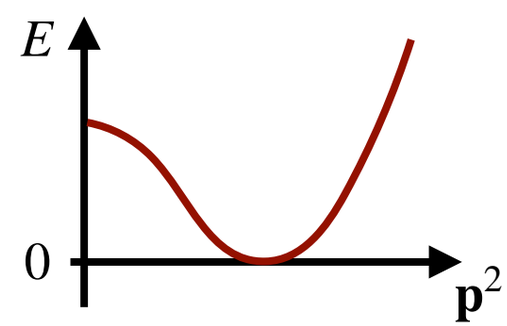
IMPLICATIONS OF THE MOAT

The energy gap might close at lower T and larger μ_B :





$E > 0$ for all $\mathbf{p}^2$



$E = 0$ at $\mathbf{p}^2 > 0$:

→ instability towards formation of an **inhomogeneous condensate**

INHOMOGENEOUS PHASE

emerges if energy gap closes

- $E_\phi(k_0^2) = 0$: particles with momentum k_0 condense
- basic example: $O(N)$ chiral spiral

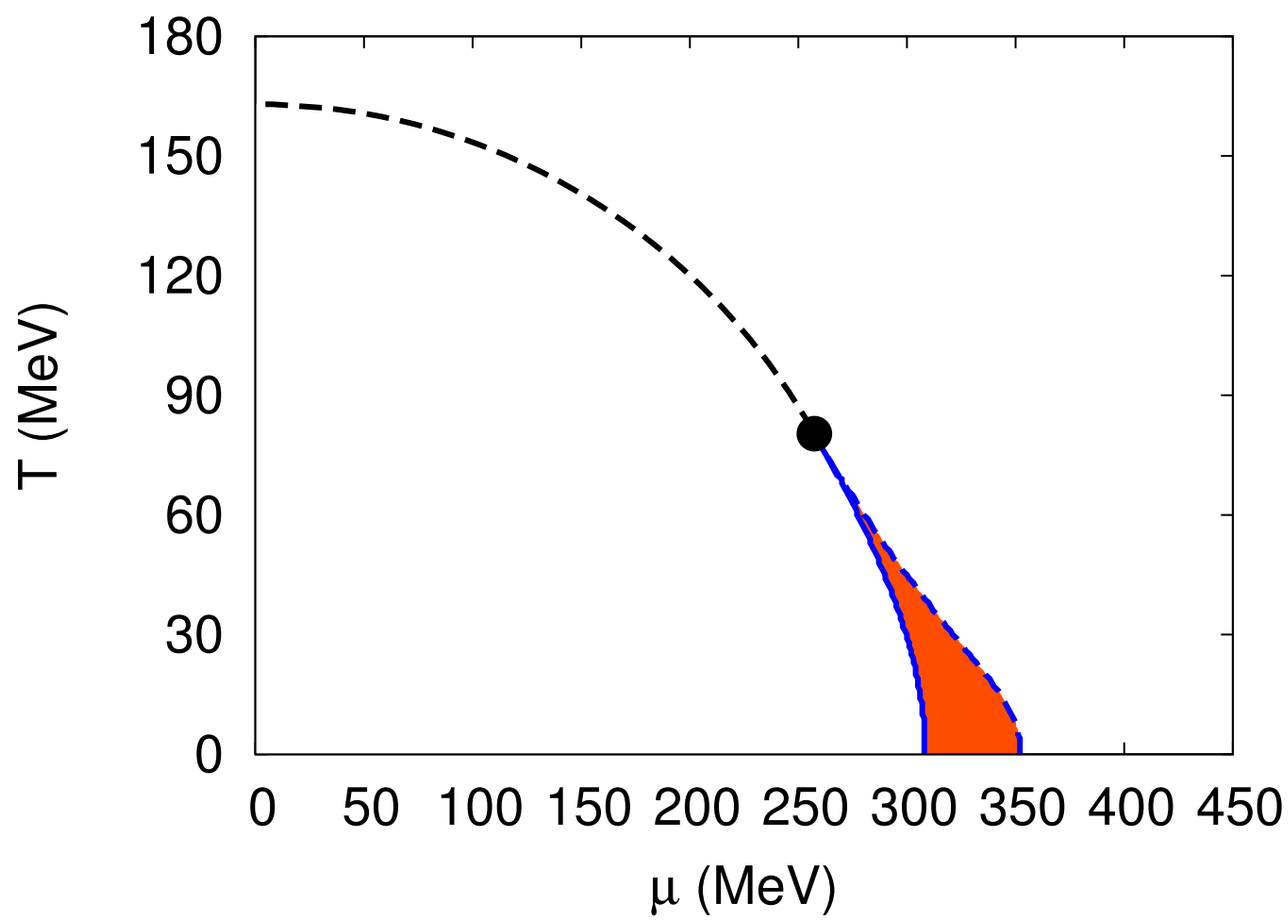
field

$$\phi = \begin{pmatrix} \sigma \\ \pi_{N-1} \\ \pi_{N-2} \\ \vdots \\ \pi_1 \end{pmatrix}$$

condensate/VEV

$$\phi_0 = \Delta \begin{pmatrix} \cos(k_0\, z) \\ \sin(k_0\, z) \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

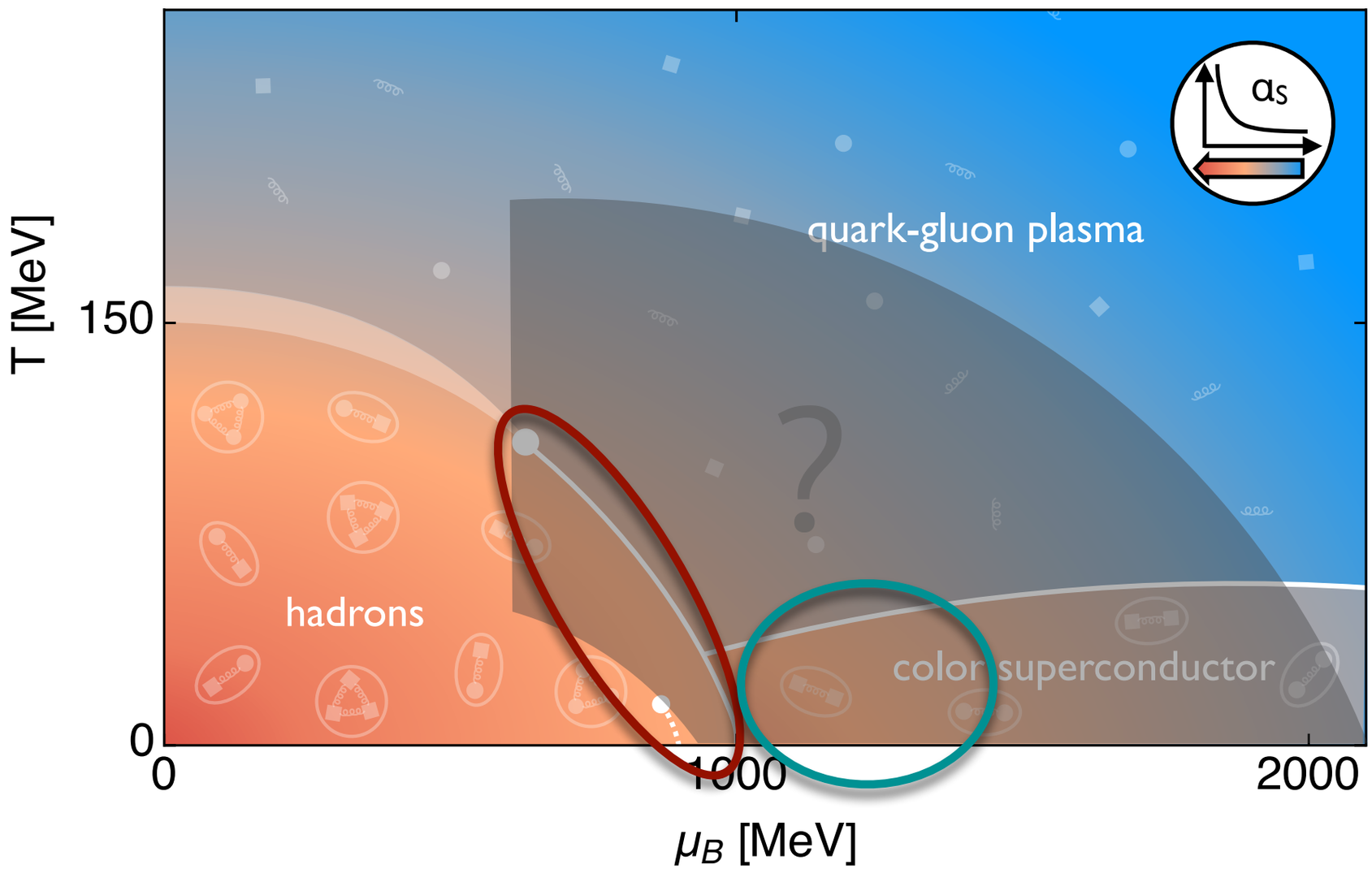
mean-field phase diagram



[Carignano, Buballa, Schaefer '14]

IMPLICATIONS OF THE MOAT

option 1: moat is a precursor for an inhomogeneous phase



possibilities: inhomogeneous chiral condensate or crystalline CSC

INHOM. PHASES & FLUCTUATIONS

Inhomogeneous phases are mostly studied in mean-field.

But associated spontaneous symmetry breaking gives rise to massless modes.

Their fluctuations must be relevant!

Two types of symmetry breaking for inhomogeneous phases:

- continuous spatial symmetries (rotations, translations) broken down to discrete ones
- global flavor symmetries are broken (e.g. $O(N) \to O(N-2)$ for chiral spiral)

SPATIAL SYMMETRY BREAKING

It has been argued that 1d modulations are favored against higher-dimensional ones

[Abuki, Ishibashi, Suzuki '12]
[Carignano, Buballa '12]

Goldstone bosons from spatial symmetry breaking (e.g. phonons) lead to Landau-Peierls instability of 1d inhomogeneous condensates (e.g. chiral density wave)

- Goldstone fluctuations lead to **logarithmic IR divergences**

  → 1d condensate is destroyed; the system is disordered

- algebraically instead of exponentially decaying correlations still possible

  → quasi-long-range order (e.g. liquid crystals)

[Landau, Lifshitz, Stat. Phys. I, §137]
[Lee, Nakano, Tsue, Tatsumi, Friman '15]

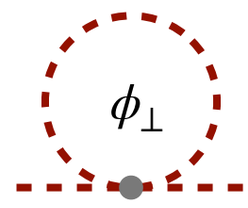
Option 2: moat is a precursor for a liquid-crystal-like phase

FLAVOR SYMMETRY BREAKING

even "worse" for fluctuations of Goldstones from broken flavor symmetry

- basic example: fluctuations around $O(N)$ chiral spiral

$$\phi = \Delta \begin{pmatrix} \cos(k_0\, z) \\ \sin(k_0\, z) \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \begin{pmatrix} \delta\phi_{\parallel} \\ \delta\phi_{\perp} \end{pmatrix} \longrightarrow$$



$$\sim \frac{T}{W}\, k_0^{d-3} \int_{|\mathbf{p}|\sim k_0} \frac{d\,|\mathbf{p}|}{(\,|\mathbf{p}| - k_0)^2}$$

- transverse fluctuations lead to linear IR divergences at finite T in any dimension

$\longrightarrow$ $\delta\phi_\perp$ disorders the system: **no inhomogeneous phase for $N > 2$**
not even quasi-long-range order (rigorous for $O(N)$ chiral spiral at $N \to \infty$)

instead, there is a **quantum pion liquid**

- disordered phase with a moat spectrum ($E > 0$ for all $\mathbf{p}^2$)
- spatial modulations: $\langle \phi(x)\phi(0)\rangle \sim e^{-m_r\, x} \cos(m_i\, x)$ for large x

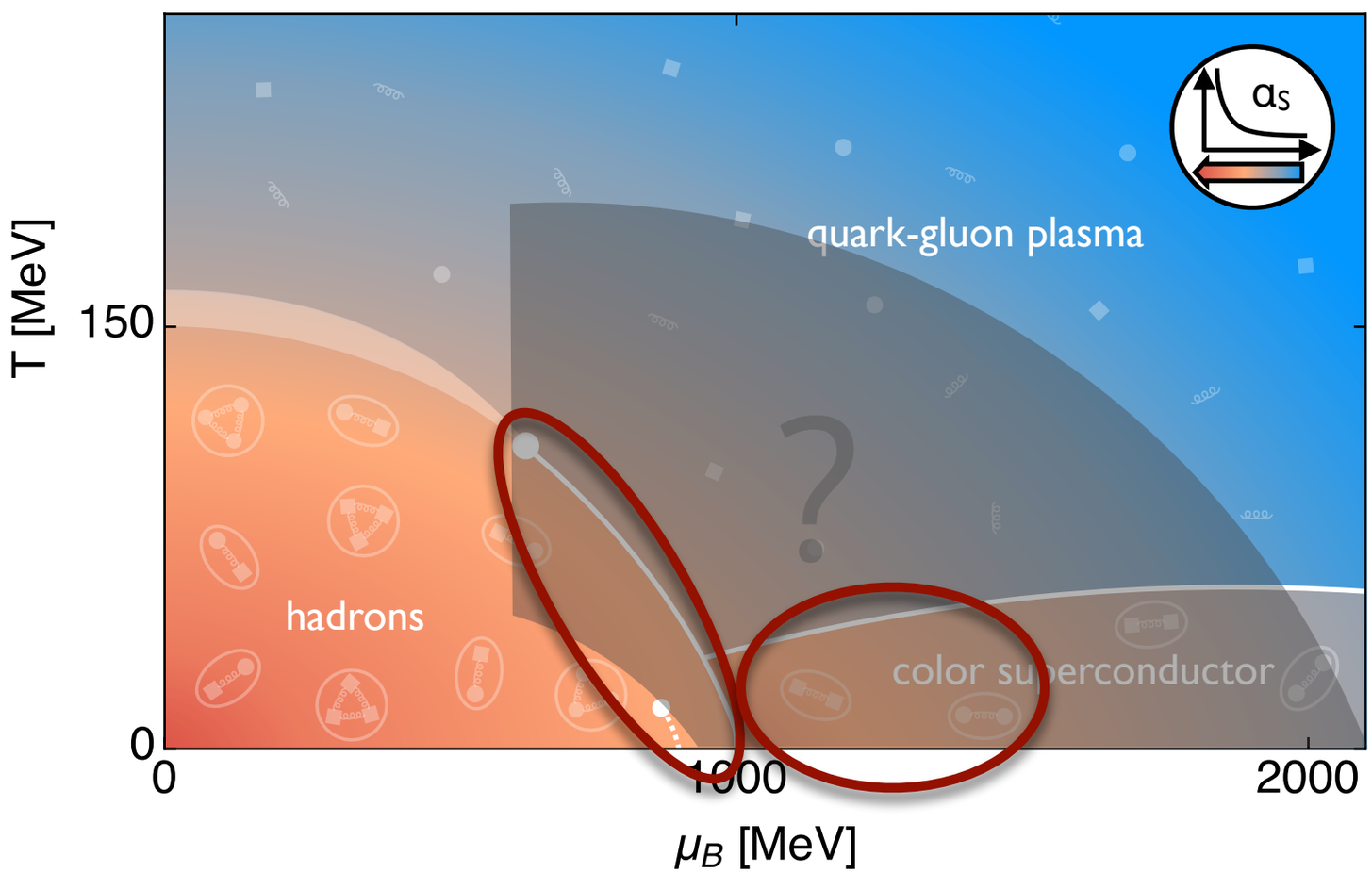
[Pisarski, Tsvelik, Valgushev '20]
[Pisarski '21]

Option 3: moat signals a quantum pion liquid

IMPLICATIONS OF THE MOAT

the moat regime could be an indication that dense QCD has:

option 1:	option 2:	option 3:
inhomogeneous phase	**liquid-crystal-like**	**quantum pion liquid**
• only if there are no Goldstone bosons	• only if there are only Goldstones from spatial symmetry breaking	• only if there are Goldstones flavor symmetry breaking



this will occur in the regions where inhomogeneous phases are expected

SEARCH FOR MOAT REGIMES [Pisarski, FR '21]

Characteristic feature: minimal energy at nonzero momentum

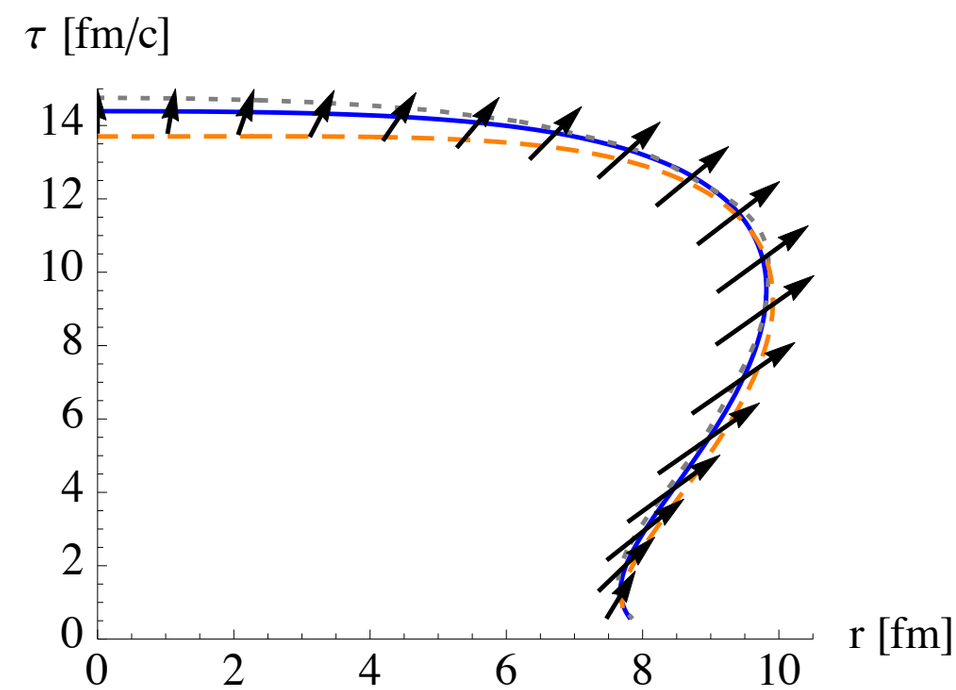
$\Rightarrow$ **enhanced particle production at nonzero momentum**

$\rightarrow$ look for signatures in the **momentum dependence** of particle numbers and correlations

- consider heavy-ion collision
- particles at freeze-out "mapped" onto detector
- freeze out at certain temperature T_f

$\rightarrow$ defines 3d hypersurface:
freeze-out surface Σ

How does the moat regime affect particles on Σ?



[Floerchinger, Wiedemann '13]

GENERALIZED COOPER-FRYE FORMULA

compute particle numbers on the freeze-out surface

- probability distribution of finding a particle ϕ with momentum p in thermal equilibrium: Wigner function

$$F_\phi(p) = 2\pi\, \rho_\phi(p_0, \mathbf{p}) f(p_0)$$

spectral function

- particles on Σ boosted with fluid velocity $u^\mu(x)$:

$$\text{energy: } \breve{p}_0 = u^\mu p_\mu$$

$$\text{spatial momentum: } \breve{\mathbf{p}}^2 = \left(u^\mu u^\nu - g^{\mu\nu}\right) p_\mu p_\nu$$

- particle spectrum from integrating particle number current over freeze-out surface:

$$\frac{d^3 N_\phi}{d\mathbf{p}^3} = \frac{2}{(2\pi)^3} \int_\Sigma d\Sigma_\mu \underbrace{\int \frac{dp_0}{2\pi}\, p^\mu\, \Theta(\breve{p}_0)\, F_\phi(\breve{p})}_{\sim \text{ particle number current density}}$$

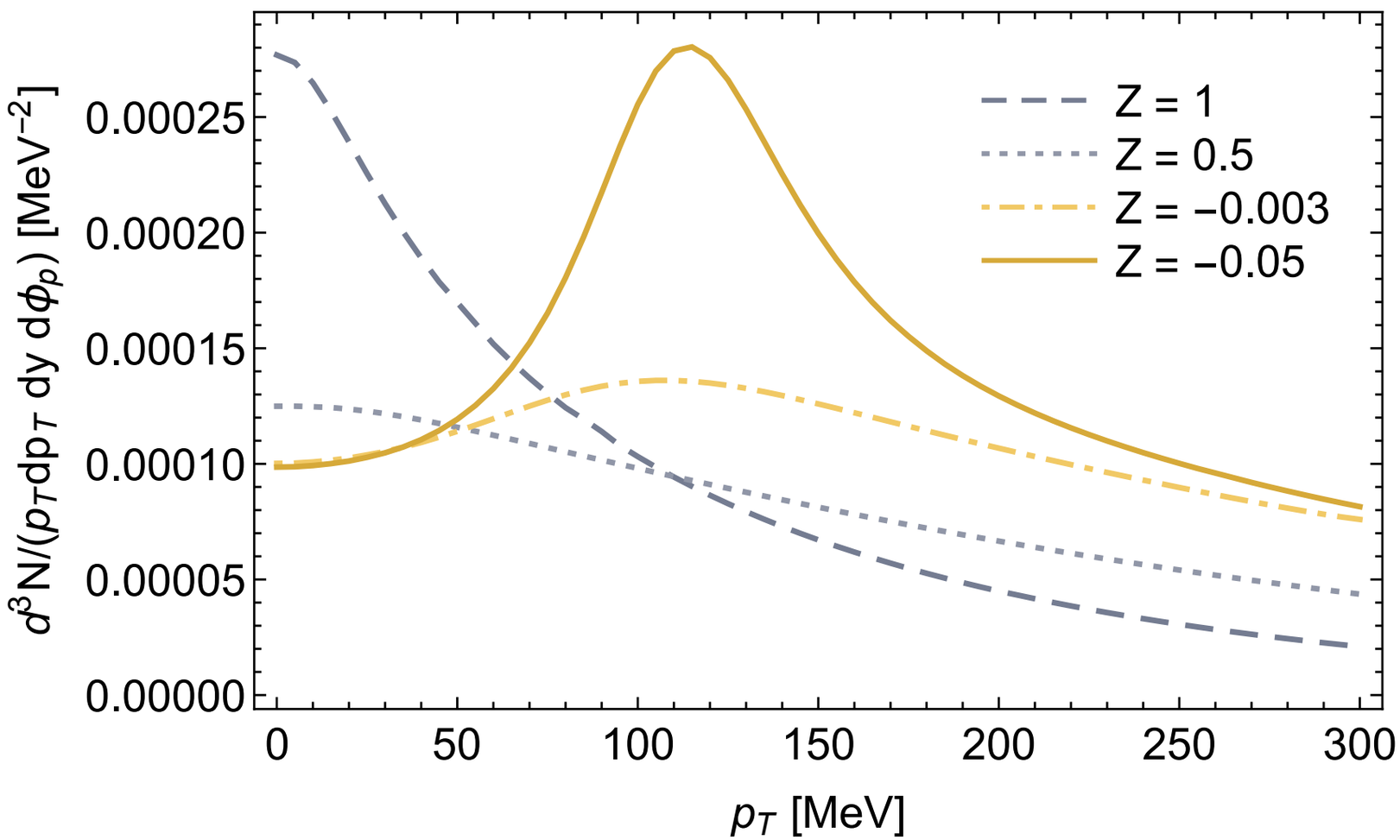
- reduces to Cooper-Frye formula for free vacuum spectral function: $\rho_\phi(p) = \mathrm{sign}(\mathrm{p}_0)\, \delta\left[\mathrm{p}_0^2 - \left(\mathbf{p}^2 + \mathrm{m}^2\right)\right]$

PARTICLE SPECTRUM IN A MOAT REGIME

transverse momentum spectrum

- use simple models for illustration (quasi-particle in moat regime, boost-inv. and transverse isotropic freeze-out at fixed proper time, blast wave fluid velocity)
- compare normal phase (gray, $W = 0$) to moat phase (yellow, $W = 2.5\,\mathrm{GeV}^{-2}$)

$$\frac{d^3N}{p_T\,dp_T\,dy\,d\phi_p}$$



enhanced particle production at nonzero momentum!

maximum related to the wavenumber of the spatial modulation

PARTICLE NUMBER CORRELATIONS

- correlations sensitive to in-medium modifications
- moat regime is disordered: single particle correlations can capture relevant features

→ **correlations on Σ from (generalized) Cooper-Frye formula** [Pisarski, FR (2021)] [Floerchinger (unpublished)]

n-particle correlation:

$$\left\langle \prod_{i=1}^{n} \frac{d^3 N_\phi}{d\mathbf{p}_i^3} \right\rangle = \left[\prod_{i=1}^{n} \frac{2}{(2\pi)^3} \int d\Sigma_i^\mu \int \frac{dp_i^0}{2\pi} (p_i)_\mu \, \Theta(\breve{p}_i^0) \right] \left\langle \prod_{i=1}^{n} F_\phi(\breve{p}_i) \right\rangle$$

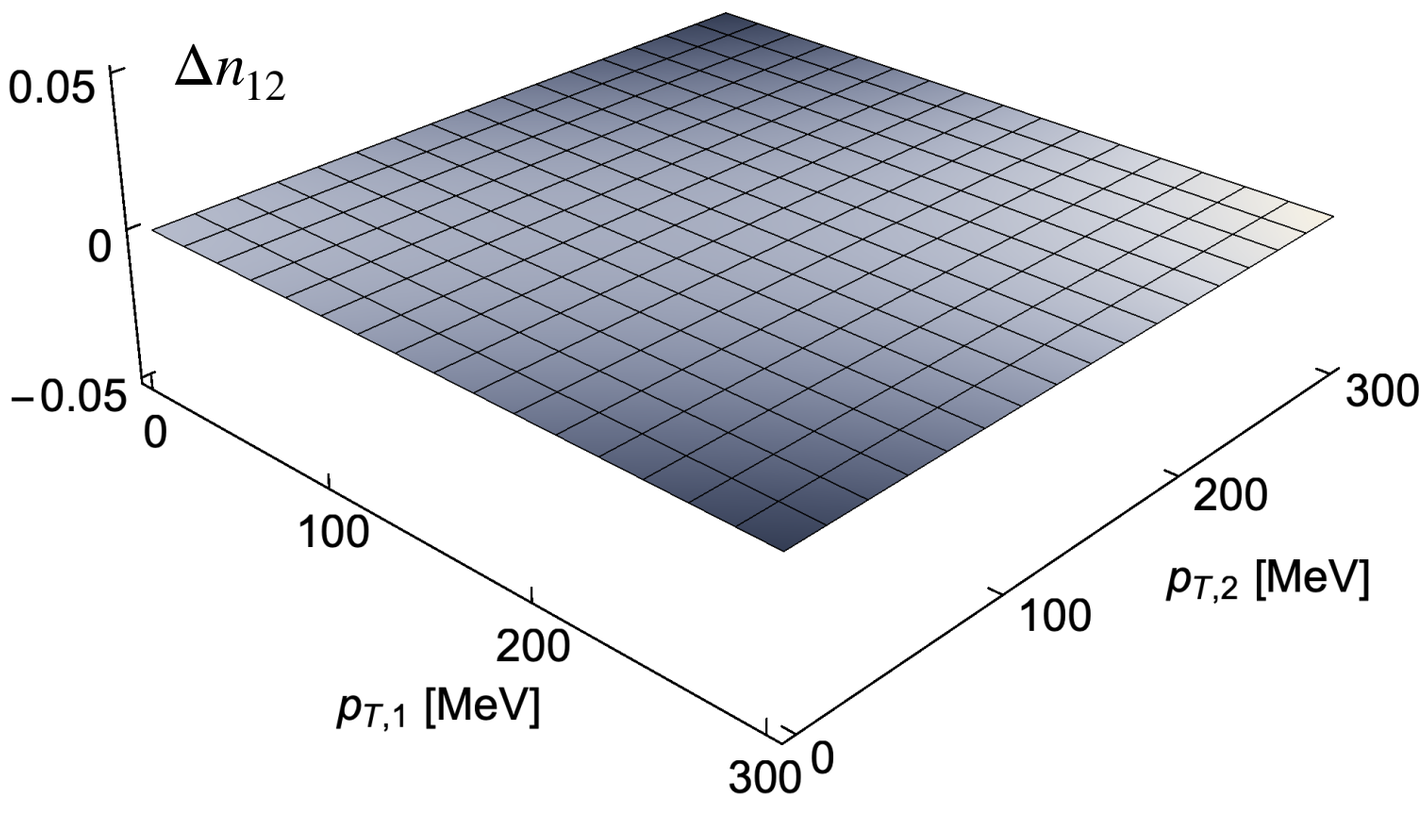
thermodynamic average

- fluctuations, e.g., of thermodynamic quantities lead to fluctuations of F_ϕ
- consider small fluctuations in T, μ_B, u

TRANSVERSE MOMENTUM CORRELATIONS

- normalized two-particle correlation $$\Delta n_{12} = \frac{\left\langle \frac{d^3N_\phi}{d\mathbf{p}_1^3} \frac{d^3N_\phi}{d\mathbf{p}_2^3} \right\rangle_c}{\left\langle \frac{d^3N_\phi}{d\mathbf{p}_1^3} \right\rangle \left\langle \frac{d^3N_\phi}{d\mathbf{p}_1^3} \right\rangle}$$

normal phase

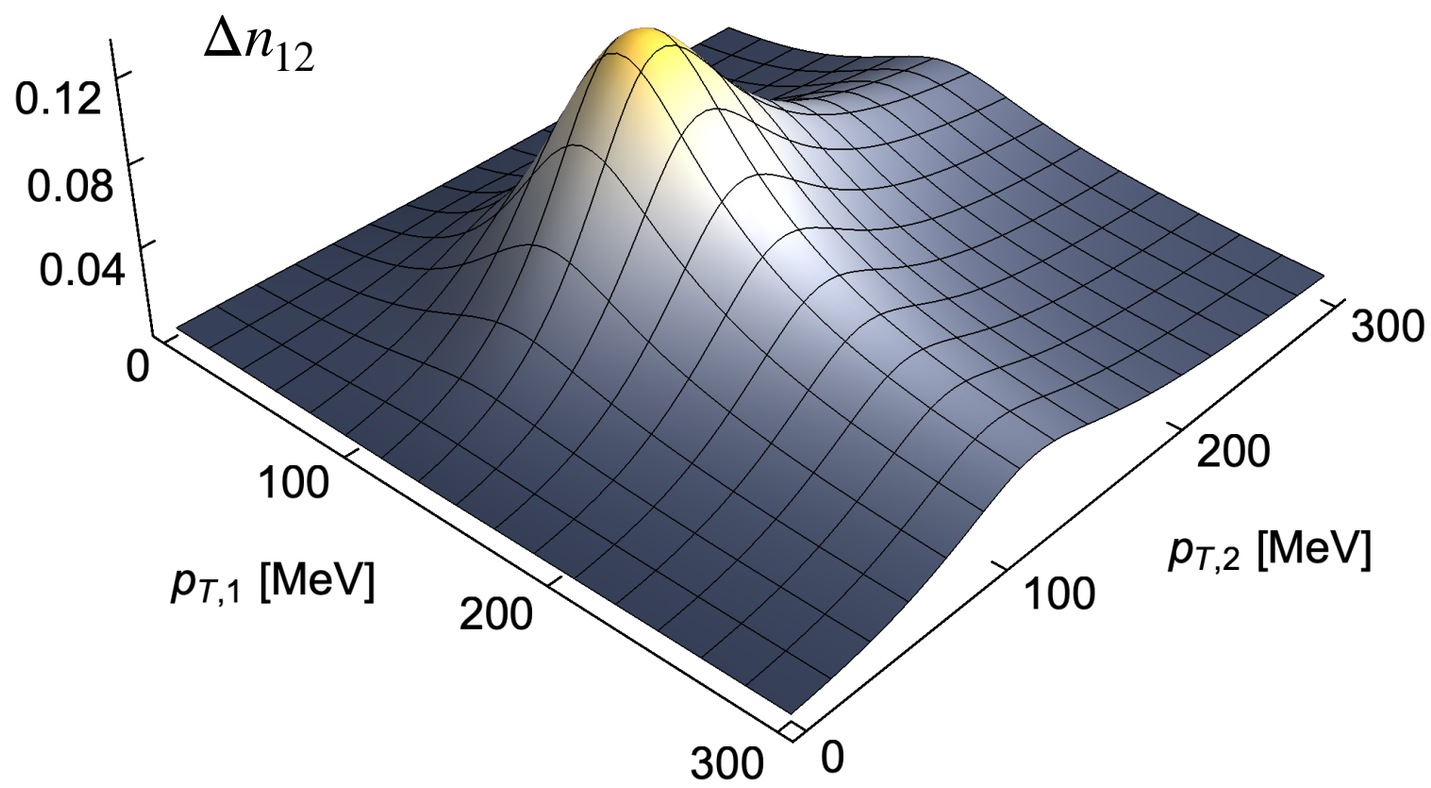


(relatively) flat two-particle p_T correlation in the normal phase

TRANSVERSE MOMENTUM CORRELATIONS

- normalized two-particle correlation $$\Delta n_{12} = \frac{\left\langle \frac{d^3N_\phi}{d\mathbf{p}_1^3} \frac{d^3N_\phi}{d\mathbf{p}_2^3} \right\rangle_c}{\left\langle \frac{d^3N_\phi}{d\mathbf{p}_1^3} \right\rangle \left\langle \frac{d^3N_\phi}{d\mathbf{p}_1^3} \right\rangle}$$

moat regime



pronounced peak and ridges at nonzero p_T related to wavenumber of spatial modulation!

huge enhancement:

$$\frac{\Delta n_{12}(p_{\rm peak})\Big|_{\rm moat}}{\Delta n_{12}(p_{\rm peak})\Big|_{\rm normal}} \approx 10^2$$

SUMMARY

Moats arise in regimes with spatial modulations

- expected to occur at large μ_B
- are precursors for inhomogeneous, liquid-crystal-like or quantum pion liquid phases
- quantum fluctuations are crucial

Enhanced production of moat particles at nonzero momentum

- characteristic peaks (and ridges) in particle spectra and correlations at nonzero p_T
- requires good resolution at low momentum

Opportunity to discover novel phases with low-energy heavy-ion collisions

So far: basic description of qualitative effects
To do: quantitative description of moat regimes

BACKUP

PARTICLE SPECTRUM IN A MOAT PHASE

use simple models to show general structure

Particle in a moat regime:

- **low-energy model** of free bosons in a moat regime $(Z < 0, W > 0)$:

$$\mathscr{L}_0 = \frac{1}{2}\left(\partial_0\phi\right)^2 + \frac{Z}{2}\left(\partial_i\phi\right)^2 + \frac{W}{2}\left(\partial_i^2\phi\right)^2 + \frac{m_{\text{eff}}^2}{2}\phi^2$$

- gives simple in-medium spectral function

$$\rho_\phi(p_0, \mathbf{p}^2) = \text{sign}(\text{p}_0)\,\delta\left[\text{p}_0^2 - \text{E}_\phi^2(\mathbf{p}^2)\right] \quad \text{with} \quad E_\phi(\mathbf{p}^2) = \sqrt{Z\,\mathbf{p}^2 + W\left(\mathbf{p}^2\right)^2 + m_{\text{eff}}^2}$$

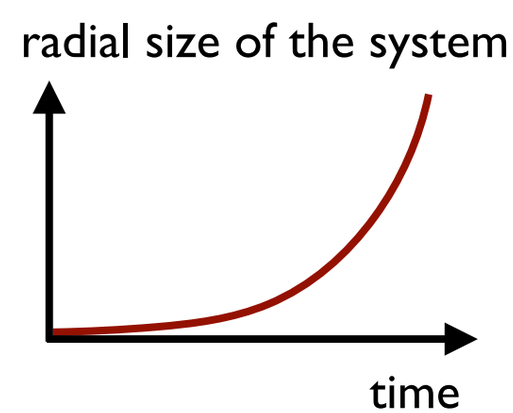
- boost symmetry broken! (but spatial rotation symmetry still intact)

Fluid velocity and freeze-out surface from hydro evolution

- boost invariant freeze-out at fixed temperature T_f and fixed proper time τ_f $(= \sqrt{t^2 - z^2})$

- blast wave approximation for the fluid velocity:

$$u^r = \bar{u}\,\frac{r}{\bar{R}}\,\theta(\bar{R} - r)$$



[Schnedermann, Sollfrank, Heinz (1993)]
[Teaney (2003)]

PARTICLE SPECTRUM IN A MOAT PHASE

use simple models to show general structure

model parameters:

- pick a beam energy of $\sqrt{s} = 5\,\mathrm{GeV}$ and read off thermodynamic and blast wave parameters:

$$T_f = 115\,\mathrm{MeV}$$
$$\mu_{B,f} = 536\,\mathrm{MeV}$$

[Andronic, Braun-Munzinger, Redlich, Stachel (2018)]

$$\bar{u} = 0.3$$
$$\bar{R} = 8\,\mathrm{fm}$$
$$\tau_f = 5\,\mathrm{fm/c}$$

[Zhang, Ma, Chen, Zhong (2016)]

- thermodynamics (used later) from a hadron resonance gas [Braun-Munzinger, Redlich, Stachel (2003)]

- moat parameters: purely illustrative

$$\text{if } Z < 0: \quad W = 2.5\,\mathrm{GeV}^{-2}$$

THERMODYNAMIC CORRELATIONS

- correlations $\langle \ldots \rangle$ from thermodynamic average
- weight configurations with the change in entropy due to fluctuations, Δs^μ [Landau, Lifshitz (vol. 5)]

$\longrightarrow$ generating functional of (connected) thermodynamic correlations

$$W[J] = \ln \int \mathscr{D}\kappa(x) \, \exp \int d\Sigma_\mu \left[\Delta s^\mu(x) + \; J(x)_{i\nu} \, \hat{v}^\mu \, \kappa_i^\nu(x) \right]$$

normal to Σ

- connected n-particle correlations $\langle \kappa^n \rangle_c$ from $\left. \dfrac{\delta^n W[J]}{\delta J^n} \right|_{J=0}$
- change of entropy in an ideal fluid $(T^{\mu\nu} = \epsilon \, u^\mu u^\nu + p \Delta^{\mu\nu})$ with Gaussian fluctuations:

$$\hat{v}_\mu \Delta s^\mu = -\frac{1}{2} \kappa_{i\mu}(x) \, \mathscr{F}_{ij}^{\mu\nu}(x) \, \kappa_{j\nu}(x)$$

local fluctuations!

$$\mathscr{F}_{ij}^{\mu\nu} = \frac{1}{T} \begin{pmatrix} \hat{u} \dfrac{\partial s}{\partial T} & \hat{u} \dfrac{\partial s}{\partial \mu_B} & s \hat{v}^\nu \\ \hat{u} \dfrac{\partial s}{\partial \mu_B} & \hat{u} \dfrac{\partial n_B}{\partial \mu_B} & n_B \hat{v}^\nu \\ s \hat{v}^\mu & n_B \hat{v}^\mu & -\hat{u} \, (Ts + \mu_B \, n_B) g^{\mu\nu} \end{pmatrix}_{ij}$$

fluctuation matrix $(\hat{u} = \hat{v}^\mu \, u_\mu)$