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Analytic and Semi-Analytic Calculations for Color Glass in the Weak Field Limit

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Gluon Fields for Colliding Nuclei

- Solutions to the Yang-Mills equations for nuclei colliding on the light cone are known in the form of a series expansion in time. [1]
- A partial re-summation in terms of Bessel functions (in momentum space) is possible in the weak field limit.

$$A_{(n)} = \frac{1}{n(n+2)} \sum_{k+l+m=n-2} \left[D^i_{(k)}, \left[D^i_{(l)}, A_{(m)} \right] \right] ,$$

$$A^i_{\perp(n)} = \frac{1}{n^2} \left(\sum_{k+l=n-2} \left[D^j_{(k)}, F^{ji}_{(l)} \right] + ig \sum_{k+l+m=n-4} \left[A_{(k)}, [D^i_{(l)}, A_{(m)}] \right] \right) .$$

$$A^{\mathrm{LO}}(\tau, \vec{k}_\perp) = \frac{2A^{\mathrm{LO}}_{(0)}(\mathrm{k}_\perp)}{k_\perp \tau} J_1\left(k_\perp \tau\right) ,$$

$$A^i_\perp(\tau, \vec{k}) = \epsilon^{ij} \frac{(i\vec{k})_j}{k^2} B^3_{(0)}(\vec{k}) \left[J_0(k\tau) - 1 \right] + A^i_{\perp(0)}(\vec{k})$$

- The goal of the calculation is to compute observables without a series expansion in time in the aforementioned limit.

[1] G, CHEN, R. FRIES, J. KAPUSTA, AND Y. LI, PHYS. REV. C 92, 064912 (2015)

Model Comparison

- The chain of derivation goes as follows: <ρρ> → <AA> → <FF>
- <ρρ> ∝ $\mu(\vec{R}) f(\vec{r})$
- The average charge area density $\mu(\vec{R})$ is determined by the IP Glasma model [2]. The transverse distribution function $f(\vec{r})$ must be chosen from a different model.

- MV Model [3]: $f(\vec{r}) = \delta(\vec{r})$
  - Pros:
    - Used in existing literature.
    - Functions involved in the calculation are well known special and hypergeometric functions.
  - Cons:
    - Singular solutions in need of UV regularization for observables.

- Gaussian Model: $f(\vec{r}) = \frac{1}{\sigma^2 \pi} e^{-r^2/\sigma^2}$
  - Pros:
    - Solutions remain UV regular.
  - Cons:
    - Non-existent in literature.
    - Current solutions are numeric.

[2] B. SCHENKE, P. TRIBEDY, AND R. VENUGOPALAN, PHYS. REV. C 86, 034908 (2012)
[3] L. MCLERRAN AND R. VENUGOPALAN, PHYS. REV. D 49, 2233 (1994)

Analytic Solution with the MV Model

- The MV model allows for analytic solutions in terms of the Meijer G function for the stress energy tensor. A simple example for the longitudinal energy density is quoted below. The solutions keep up to one gradient in the charge density.

$$\mu_1 \mu_2 g^6 \frac{N_c}{N_c^2 - 1} \frac{1}{128 \pi^2 m^3 \tau} \left(4\, G_{2,4}^{3,1}\left(4 m^2 \tau^2 \,\middle|\, \begin{matrix} 1, 1 \\ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \end{matrix}\right) - 4\, G_{2,4}^{3,1}\left(4 m^2 \tau^2 \,\middle|\, \begin{matrix} 1, 1 \\ \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2} \end{matrix}\right) + G_{2,4}^{3,1}\left(4 m^2 \tau^2 \,\middle|\, \begin{matrix} 1, 2 \\ \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{1}{2} \end{matrix}\right)\right) +$$

$$\nabla\mu_1 . \nabla\mu_2\, g^6 \frac{N_c}{N_c^2 - 1} \frac{1}{128 \pi^2 m^3 \tau}$$

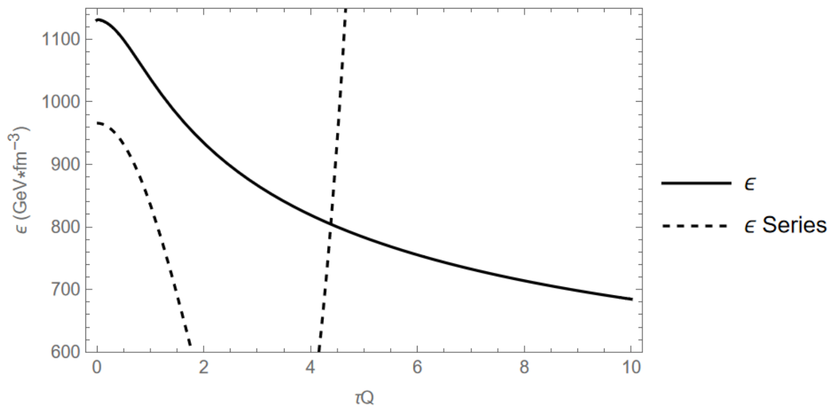
$$\left(2 m^3 \tau^5 G_{2,4}^{3,1}\left(4 m^2 \tau^2 \,\middle|\, \begin{matrix} -\frac{1}{2}, \frac{1}{2} \\ -1, 0, 1, -2 \end{matrix}\right) + 2 m \tau^3 G_{2,4}^{3,1}\left(4 m^2 \tau^2 \,\middle|\, \begin{matrix} \frac{1}{2}, \frac{1}{2} \\ 0, 0, 0, -1 \end{matrix}\right) - \tau^2 G_{2,4}^{3,1}\left(4 m^2 \tau^2 \,\middle|\, \begin{matrix} 1, 1 \\ \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{1}{2} \end{matrix}\right) + \frac{1}{2 m^2} G_{2,4}^{3,1}\left(4 m^2 \tau^2 \,\middle|\, \begin{matrix} 1, 2 \\ \frac{3}{2}, \frac{3}{2}, \frac{3}{2}, \frac{1}{2} \end{matrix}\right)\right)$$

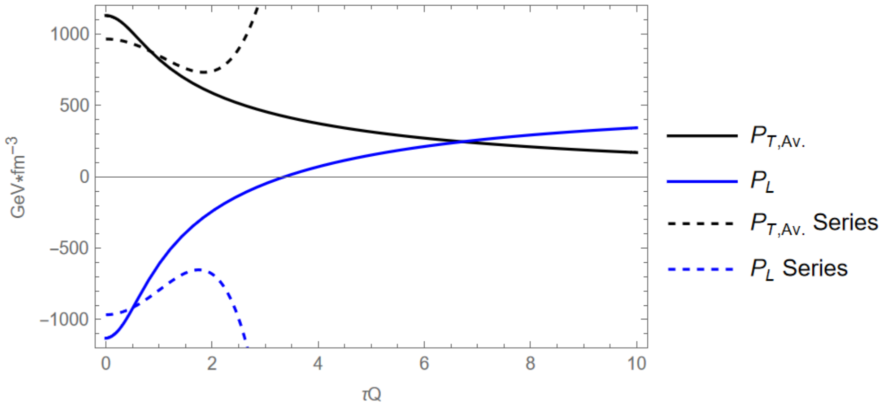
- This solution is singular at $\tau = 0$. The Gaussian model can be taken to be a regularization scheme by identifying $1/\sigma$ with the UV cutoff Q. The two models agree up to ~10% difference across all observables.

- One integral must be taken numerically when using the Gaussian model. Those results follow.

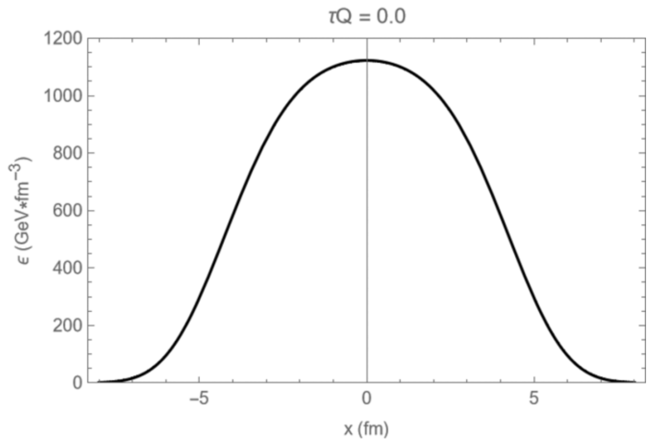
Energy Density and Pressure

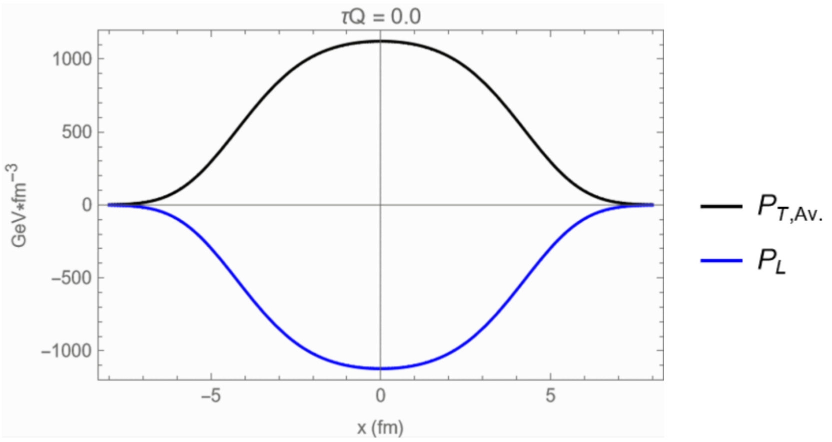
Const. μ





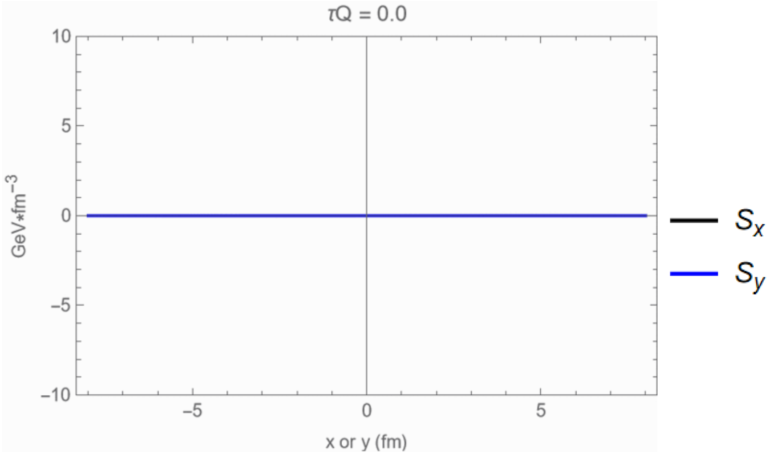
Realistic μ
Animations



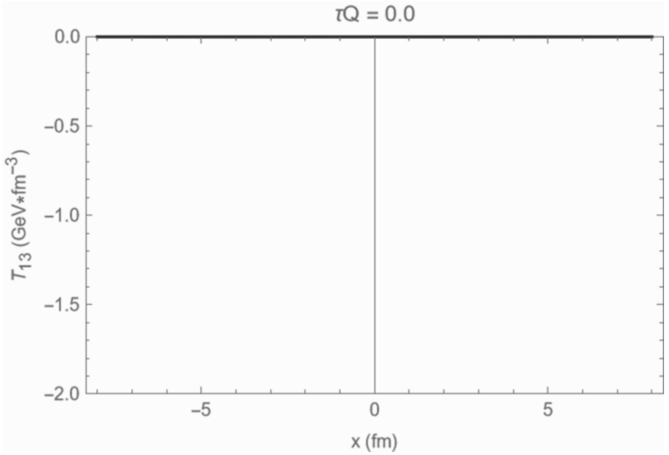


AU+AU COLLISION AT RHIC ENERGY WITH AN IMPACT PARAMETER OF 3FM.
REALISTIC MU TAKEN AT A CROSS SECTION ALONG THE X AXIS.
SERIES EXPANSIONS RESULTS OF [1]

Poynting Vector, Sheer Stress, and Angular Momentum



- The difference in magnitude between the two components leads to elliptic flow.



- The sheer stress in the x direction. Also how the angular momentum would be carried if present.

- To the degree of accuracy which this calculation was taken, the angular momentum in the system and subsequent flow is consistent with zero.

- The momentum broadening coefficient calculation is still on-going. See [4] for a comparable calculation on the lattice.

[4] A. IPP, D. MULLER, AND D. SCHUH, PHYS. REV. D 102, 074001 (2020)