Causal instabilities of Chern-Simons magnetohydrodynamics

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Add $F^{\mu\nu\star}F_{\mu\nu} = \partial_\mu\left(\epsilon^{\mu\nu\rho\sigma}A_\nu F_{\rho\sigma}\right) \equiv \partial_\mu J^\mu_{\text{CS}}$ with a non-constant factor $\Theta(x)$ to the Maxwell theory (e.g. [Kharzeev (2010)])

$$\mathcal{L}_{\text{MCS}} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + A_\mu J^\mu + \frac{C_A}{4} \overset{\text{integration by part}}{P_\mu J^\mu_{\text{CS}}} \qquad P_\mu = \partial_\mu\Theta = \left(\dot{\Theta}, \vec{\mathbf{P}}\right)$$

Maxwell equation $\rightarrow$ MCS equation

$$\partial_\mu {}^\star F^{\mu\nu} = 0 \qquad \partial_\mu F^{\mu\nu} = J^\mu + C_A {}^\star F^{\mu\nu}\partial_\nu\Theta$$

Energy-momentum conservation [Sadooghi and Shokri (2018)]

$$\partial_\mu T^{\mu\nu}_{\text{Fluid}} = F^{\nu\lambda}J_\lambda - \frac{C_A}{4}\left(F^{\mu\nu\star}F_{\mu\nu}\right)P^\nu$$

$$\boldsymbol{\nabla}\cdot\vec{\mathbf{E}} = n_e + C_A\vec{\mathbf{P}}\cdot\vec{\mathbf{B}}$$

$$\boldsymbol{\nabla}\times\vec{\mathbf{B}} - \frac{\partial\vec{\mathbf{E}}}{\partial t} = \vec{\mathbf{J}} + \overset{\text{Chiral Magnetic}}{\underbrace{C_A\dot{\boldsymbol{\Theta}}}_{\mu_5}\vec{\mathbf{B}}} - \overset{\text{Anomalous Hall}}{C_A\vec{\mathbf{P}}\times\vec{\mathbf{E}}}$$

MCS Thermodynamics, and linear stability

Second law of thermodynamics demands new thermodynamic relations [Ozonder (2010)]:

$$\mathrm{d}\epsilon = T\,\mathrm{d}s + \mu_e\,\mathrm{d}n_e + \mathcal{R}_\Theta\,\mathrm{d}\Theta \qquad \mathcal{R}_\Theta = -\frac{C_A}{4} F^{\mu\nu\star} F_{\mu\nu}$$

$$\mathrm{d}p = s\,\mathrm{d}T + n_e\,\mathrm{d}\mu_e - \mathcal{R}_\Theta\,\mathrm{d}\Theta \quad \text{but still} \quad s = \frac{1}{T}(\epsilon + p - n_e\mu_e)$$

Assume a hydrostatic configuration for an **uncharged** fluid

$$\epsilon = \epsilon_0 \quad u^\mu = (1, \mathbf{0}) \quad B^\mu = (0, \boldsymbol{B}_0) \quad E^\mu = 0 \quad \Theta \overset{\mu_5=0}{=} \Theta_0(\boldsymbol{x}) \quad n_{e,0} = \boldsymbol{P}_0 \cdot \boldsymbol{B}_0 = 0$$

Linear stability analysis: Perturb quantities using $\delta X(\omega, \boldsymbol{k}) \exp\left(-i\omega t + i\vec{\mathbf{k}}\cdot\vec{\mathbf{x}}\right)$ and linearize the EOM to find $M\delta X = 0 \rightarrow$. Finally find the dispersion relations $\omega = \omega(\vec{\mathbf{k}})$ from $\det(M) = 0$

Non-hydro modes: $\omega(\mathbf{0}) \neq 0$ Stable: $\mathrm{Im}(\omega) < 0$ Causal: $\frac{1}{|\boldsymbol{k}|} \lim_{|\boldsymbol{k}|\to\infty} \mathrm{Re}(\omega) \leq 1$

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The nonchiral channel

Resistive non-dissipative MHD is linearly stable and causal: 2 slow and 2 fast magnetosonic modes [Gedalin (1993)], plus a genuine nonhydro mode.

$$\omega_{\mathrm{slow.},\pm} = \pm v_s k \sqrt{\mathcal{A}-\mathcal{B}} - \frac{\mathrm{i} v_s^2 (1-v_a^2)\left(1-v_s^2(\mathcal{A}-\mathcal{B})\right)(1-\mathcal{A}+\mathcal{B})}{4\sigma_e \mathcal{B}} k^2 + \mathcal{O}\left(k^3\right)$$

The chiral channel

CSMHD is unstable but causal: 2 Alfven modes [Gedalin (1993)], plus a genuine nonhydro mode ($0 \leq \Delta < 2\pi$ is the angle between $\mathbf{P}_0$ and $\hat{k}_\perp = \sin\theta$).

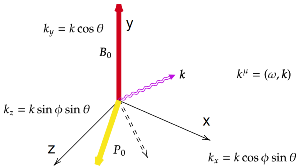
$$\omega_{\mathrm{Alfven},\pm} = \pm v_a k \cos\theta - \frac{\mathrm{i}(1-v_a^2)(1-v_a^2\cos^2(\theta))}{2\sigma_{\mathrm{eff.}}} k^2 + \mathcal{O}\left(k^3\right) \quad \sigma_{\mathrm{eff.}} \equiv \sigma_e - C_A P \sin\Delta \tan\theta$$

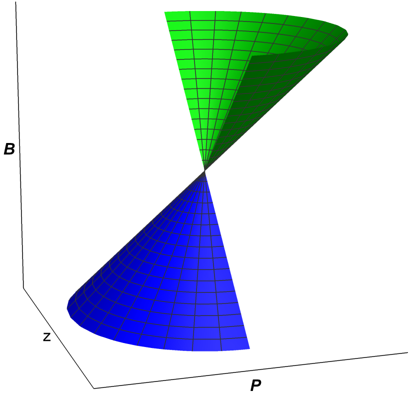
Parity breakdown: The Alfven modes are damped in one direction, and amplified in the opposite one.

The only possible constraint for stability is $\boldsymbol{\nabla}\Theta|_{\mathrm{equilibrium}} = 0$

The instability illustrated

The cone is depicted for $\theta = \theta_c = \text{atan}\left(\frac{\sigma_e}{C_A P \sin\Delta}\right)$ (At θ_c, $\sigma_{\text{eff.}} = 0$): unstable inside, stable outside





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