Bound states and resonances in thermal models

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Outline and goals

1. Gas of particles at nonzero T: how to describe in a QFT framework the effect of the interaction on the pressure and other thermodynamic quantities?

2. If the particles attract each other strongly enough to form a bound state in vacuum, how to include this bound state at non-zero T? Is it enough to add the corresponding thermal integral for the bound state? Does it count as ‘one’?

3. We address these questions in two QFT: φ^3 and Sφ^2 using the so-called phase-shift (or S-matrix) formalism. The phase-shifts are vacuum’s quantities!

Results based on:
S. Samanta and F. Giacosa,
QFT treatment of a bound state in a thermal gas,

S. Samanta and F. Giacosa,
Thermal role of bound states and resonances in scalar QFT,
[arXiv:2110.14752 [hep-ph]].
\[ L = \frac{1}{2} (\partial_\mu \varphi)^2 - \frac{1}{2} m^2 \varphi^2 - g \frac{\varphi^3}{3!}, \]

\[ A(s, t, u) = -\frac{g^2}{s - m^2 + i\epsilon} - \frac{g^2}{t - m^2 + i\epsilon} - \frac{g^2}{u - m^2 + i\epsilon}. \]

**Partial wave amplitudes:**

\[ A_l(s) = \frac{1}{2} \int_{-1}^{+1} d\xi A(s, \theta) P_l(\xi) \]

**Unitarization:**

\[ A_k^U(s) = [A_k^{-1}(s) - \Sigma(s)]^{-1} \]

\[ \Sigma \text{ is the } \varphi\varphi \text{ loop} \]

**Pressure of the free gas:**

\[ P_{\varphi, \text{free}} = -T \int_k \ln \left[ 1 - e^{-\beta \sqrt{k^2 + m^2}} \right] \]

**Interaction’s pressure (with phase-shift formula):**

\[ P_{\varphi\varphi, \text{int}} = -T \int_2^\infty dx \frac{1}{\pi} \frac{d\delta_0^U(s = x^2)}{dx} \int_k \ln \left[ 1 - e^{-\beta \sqrt{k^2 + x^2}} \right] \]
Bound state

The bound state equation (s-channel and s-wave, for s below the threshold) reads:

\[ A_0^{-1}(s) - \Sigma(s) = 0. \]

Bound state exists for \( g > g_c \).

(Eventual) bound state contribution

\[
P_B = -\theta(g_c - g)T \int_k \ln \left[ 1 - e^{-\beta \sqrt{k^2 + M_B^2}} \right]
\]

Total pressure (including the bound state)

\[
P_{tot}^U = P_B + P_{\varphi,\text{free}} + P_{\varphi\varphi-\text{int}}^U
\]

Note: the bound state appears abruptly for \( g > g_c \).

Is the pressure continuous as function of \( g \)?
Results: pressure as function of g and of T

The s-wave discontinuity exactly cancels the bound-state one!
The pressure is continuous as function of g!

Pressure vs T: the (attractive) interaction generates an increase of the pressure.

Note: this is different from the classic van-der-Waals gas, but it is indeed expected: attraction means an increase of the density of states.
Results: role of the interaction.

\[ P_{tot}^U = \eta P_{\varphi,\text{free}} \]

\[ \eta = \frac{P_{tot}^U}{P_{\varphi,\text{free}}} = 1 + \frac{P_{\varphi-\text{int}}^U}{P_{\varphi,\text{free}}} \]

No bound state. Interaction may cause even a 25% increase w.r.t. the free gas!

\[ P_{tot}^U = P_{\varphi,\text{free}} + \zeta P_B \]

\[ \zeta = \frac{P_{\varphi-\text{int}}^U + P_B}{P_B} \]

With bound state: simply adding the bound state implies an overestimation; a partial cancellation takes place.

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We repeat the steps for a theory with a resonance $S$.

\[ \mathcal{L} = \frac{1}{2} (\partial_\mu \varphi)^2 - \frac{1}{2} m^2 \varphi^2 + \frac{1}{2} (\partial_\mu S)^2 - \frac{1}{2} M^2 S^2 + \frac{g}{2!} S \varphi^2, \]

\[ A(s, t, u) = \frac{-g^2}{s - M^2 + i\epsilon} + \frac{-g^2}{t - M^2 + i\epsilon} + \frac{-g^2}{u - M^2 + i\epsilon}. \]

Then:

**Left**: No bound state; **Right**: with bound state $B$

S is an unstable resonance of $M > 2m$. Then:

\[ \Gamma_S = \frac{g^2}{2} \sqrt{\frac{M^2}{4} - m^2} \]

We thus have the resonance $S$ and eventually the bound state $B$.

Left: No bound state; Right: with bound state $B$. 

\[ P_{\text{tot}}^U = \eta_S (P_{\varphi,\text{free}} + P_{S,\text{free}}) \quad \quad P_{\text{tot}}^U = P_{\varphi,\text{free}} + \zeta_S (P_{S,\text{free}} + P_B) \]

\[ \eta_S = \frac{P_{\text{tot}}^U}{P_{\varphi,\text{free}} + P_{S,\text{free}}} \quad \quad \zeta_S = \frac{P_{\varphi,\text{int}}^U + P_B}{P_{S,\text{free}} + P_B} \]
Conclusions: the bound state (if it forms) counts as one (independently on the binding energy), but the net result is diminished by the residual interaction. (The degree of cancellation depends on various details.)

Final comment: the results suggest that the multiplicity of a bound state is just that of thermal integral! This is in agreement with the thermal model treatment of nuclei.

Thank You!
Technical detail: the loop function

Unitarization

\[ I(s) = \text{Im} \Sigma(s) = \frac{1}{2} \frac{\sqrt{\frac{s}{4} - m^2}}{8 \pi \sqrt{s}} \text{ for } \sqrt{s} > 2m. \]

The loop function \( \Sigma(s) \) for a complex \( s \) is chosen by considering two subtractions:

\[ \Sigma(s) = -\frac{(s - m^2)(s - 3m^2)}{\pi} \int_{4m^2}^{\infty} \frac{\frac{1}{2} \frac{\sqrt{s' - m^2}}{8 \pi \sqrt{s'}}}{(s - s' + i \varepsilon)(s' - m^2)(s' - 3m^2)} ds'. \]

- The subtractions guarantee that \( \Sigma(s = m^2) = 0 \) and \( \Sigma(s = 3m^2) = 0 \).
- In this way, the choice of \( \Sigma(s) \)
  (i) preserves the pole corresponding to \( s = m^2 \)
  (ii) assures that the unitarized amplitude diverges at the branch point \( s = 3m^2 \) generated by the single-particle pole for \( m^2 \) along the \( t \) and \( u \) channels
  (log 0 at the 2nd term of \( A_0 \))