Combinants and correlation functions in nuclear collisions

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Motivation

- Multiplicity distributions of charged particles produced in e^+e^-, proton-proton, and centrality selected Pb-Pb collisions exhibit, after closer inspection, peculiarly enhanced void probability and oscillatory behavior of the modified combinants.
- The set of combinants, C_j provides a similar measure of fluctuations as the set of cumulant factorial moments, K_q, which are very sensitive to the details of the multiplicity distribution and were frequently used in phenomenological analyses of data.
- While cumulants are best suited to the study of the densely populated region of phase space, combinants are better suited for the study of sparsely populated regions because calculation of C_j requires only a finite number of probabilities $P(N < j)$.

H. Ang *et al.*, Eur. Phys. J. A **56** 117
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H. Ang *et al.*, Mod. Phys. Lett. A **34** 1950324

Combinants, modified combinants, and correlation functions

The dynamics of multiparticle production process is hidden in the way in which the consecutive measured multiplicities N are connected. In the simplest case one assumes that the multiplicity N is directly influenced only by its neighboring multiplicities $(N \pm 1)$ in the way dictated by the simple recurrence relation:

$$(N+1)P(N+1) = g(N)P(N), \quad g(N) = \alpha + \beta N, \tag{1}$$

where $\beta > 0$ for negative binomial distribution (NBD), $\beta < 0$ for binomial distribution (BD) and $\beta = 0$ for Poisson distribution (PD).
We propose a more general form of the recurrence relation, used in counting statistics when dealing with multiplication effects in point processes. Contrary to Eq. (1), it now connects all multiplicities by means of some coefficients C_j, which define the corresponding $P(N)$ in the following way:

$$(N+1)P(N+1) = \langle N\rangle \sum_{j=0}^{N} C_j P(N-j). \tag{2}$$

The coefficients C_j contain the memory of particle $N+1$ about all the $N-j$ previously produced particles. They can be directly calculated from the experimentally measured $P(N)$ by reversing Eq. (2) and putting it in the form of the following recurrence formula for C_j:

$$\langle N\rangle C_j = (j+1)\left[\frac{P(j+1)}{P(0)}\right] - \langle N\rangle \sum_{i=0}^{j-1} C_i \left[\frac{P(j-i)}{P(0)}\right]. \tag{3}$$

The set of modified combinants, C_j, provides a similar measure of fluctuations as the set of cumulant factorial moments, K_q, which are very sensitive to the details of the multiplicity distribution and were frequently used in phenomenological analyses of data,

$$K_q = F_q - \sum_{i=1}^{q-1} \binom{q-1}{i-1} K_{q-i} F_i, \tag{4}$$

where

$$F_q = \sum_{N=q}^{\infty} N(N-1)(N-2)\dots(N-q+1)P(N), \tag{5}$$

are the factorial moments. The K_q can be expressed as an infinite series of the C_j,

$$K_q = \sum_{j=q}^{\infty} \frac{(j-1)!}{(j-q)!} \langle N\rangle C_{j-1}, \tag{6}$$

and, conversely, the C_j can be expressed in terms of the K_q,

$$C_j = \frac{1}{\langle N\rangle} \frac{1}{(j-1)!} \sum_{p=0}^{\infty} \frac{(-1)^p}{p!} K_{p+j}. \tag{7}$$

The modified combinants C_j defined by the recurrence relation (2) can be expressed by the generating function $G(z) = \sum_{N=0}^{\infty} P(N) z^N$ as:

$$\langle N\rangle C_j = \frac{1}{j!} \frac{d^{j+1} \ln G(z)}{dz^{j+1}}\bigg|_{z=0}. \tag{8}$$

The generating function can be shown to be a sum over the 'averaged' connected correlation function of all orders ($\xi_{(N)}$):

$$\ln G(z) = \sum_{N=1}^{\infty} \frac{(z-1)^N}{N!} \langle N\rangle^N \bar{\xi}_{(N)}, \tag{9}$$

where the correlation function

$$\bar{\xi}_{(N)} = \frac{1}{V^N} \int \cdots \int \xi_{(N)}(r_{ij})\, dV_1 \cdots dV_N. \tag{10}$$

Glauber predicted that the maximal value of the N-body correlation function for thermal light is related to the order of the function by relationship $N!$. This $N!$ dependence is a consequence of Wick's theorem. Recently its validity is confirmed for massive particles. For NBD $\bar{\xi}_{(N)} = (N-1)!/k^{N-1}$, for BD $\bar{\xi}_{(N)} = (-1)^{N+1}(N-1)!/K^{N-1}$, and for PD $\bar{\xi}_{(1)} = 1$ and $\bar{\xi}_{(N>1)} = 0$.
$\bar{\xi}_{(N)}$ can be expressed in terms of C_j as:

$$\bar{\xi}_{(N)} = \frac{1}{\langle N\rangle^{N-1}} \sum_{j=N}^{\infty} \frac{(j-1)!}{(j-N)!} C_{j-1}. \tag{11}$$

S.D.M. White, MNRAS **186** 145
R.G. Dall *et al.*, Nature Physics **9** 341

Results

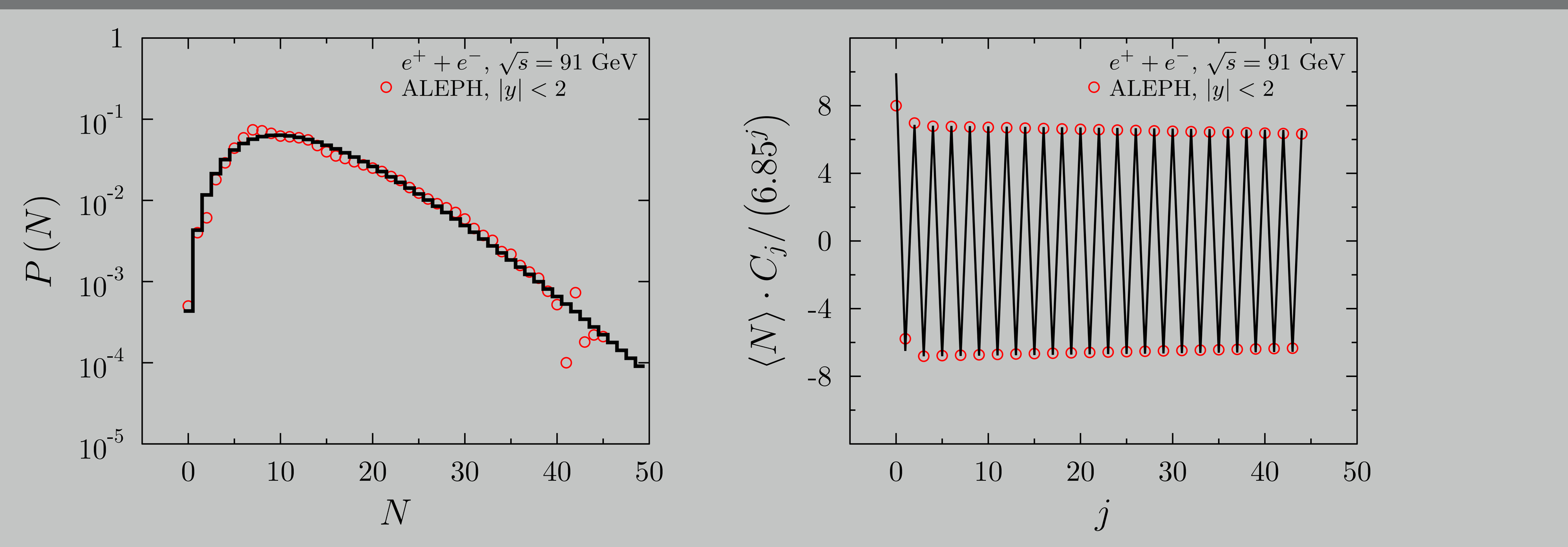

Fig. 1. Left panel: data on charged particles multiplicity distribution $P(N)$ measured in e^+e^- collisions by the ALEPH experiment at $\sqrt{s} = 91$ GeV. Right panel: the modified combinants C_j derived from these data (note the significant dependence of the amplitude on rank j. The oscillation amplitude of the plot has been scaled accordingly making it possible to plot the results. Otherwise the amplitudes would grow in a power-law fashion).

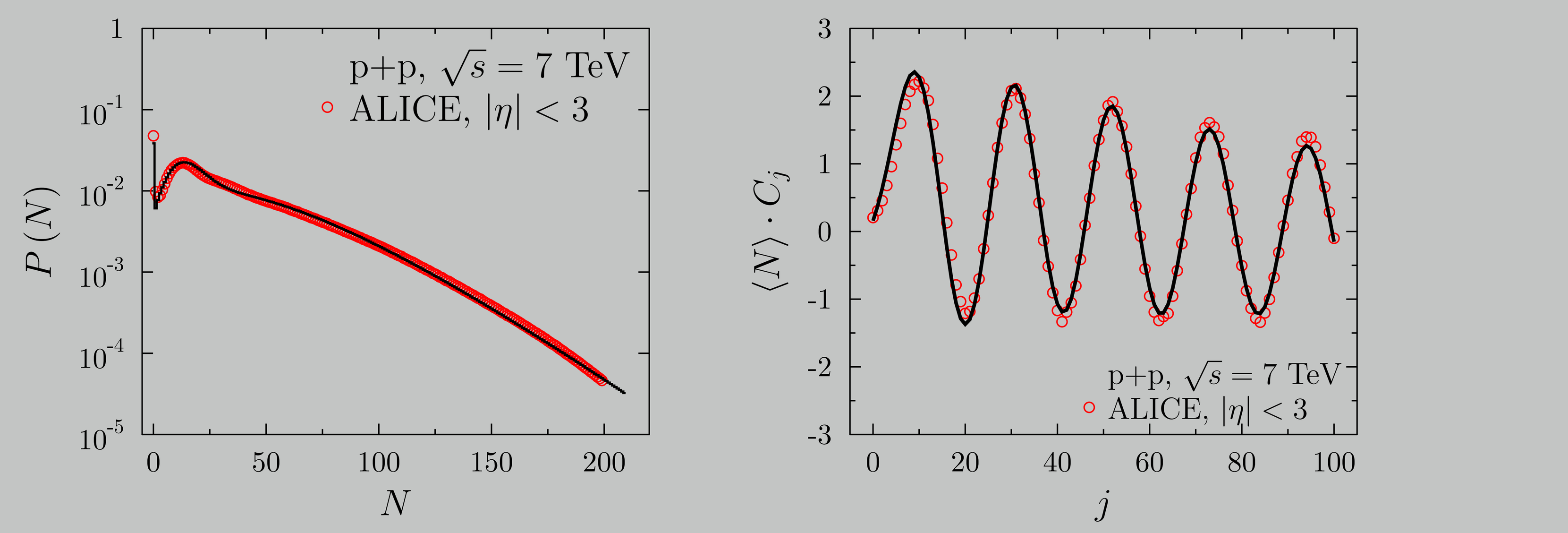

Fig. 2. Left panel: Charged particle multiplicity distribution $P(N)$ measured in proton-proton collisions by ALICE at $\sqrt{s} = 7$ TeV. Right panel: The corresponding modified combinants C_j emerging from it fitted using a two-compound distribution.

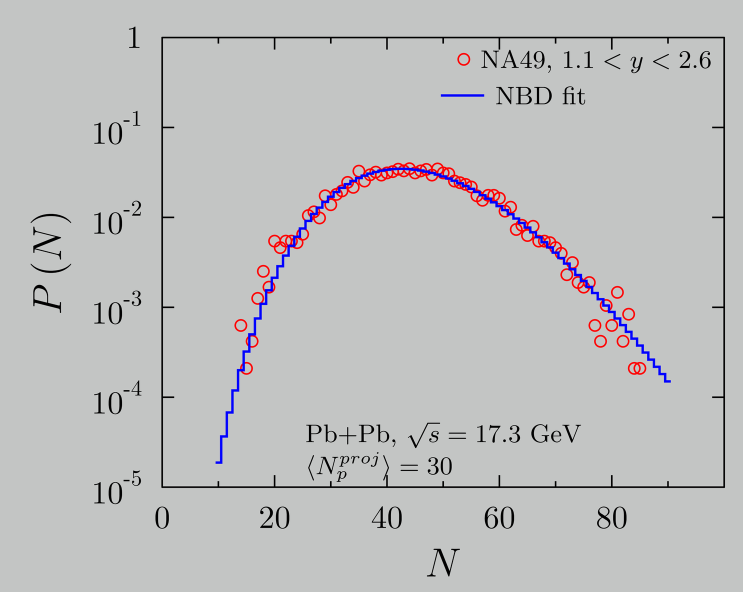

Fig. 3. Charged particle multiplicity distribution $P(N)$ measured in semi-peripheral Pb+Pb collisions by NA49 experiment at $\sqrt{s_{NN}} = 17.3$ GeV.

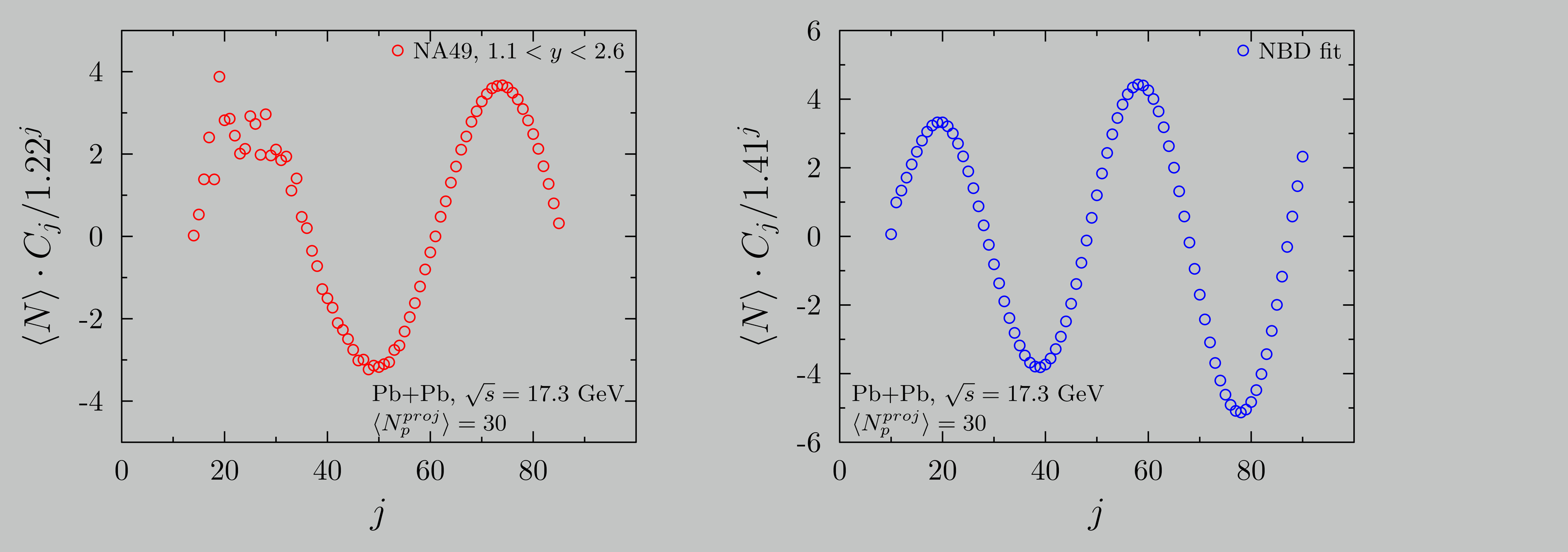

Fig. 4. Left panel: the modified combinants C_j derived from the NA49 data. Right panel: the modified combinants C_j derived from the NBD fit to the NA49 data.

Data from:
ALEPH Coll., Z. Phys. C **69** 15
ALICE Coll., Eur. Phys. J. C **77** 852
NA49 Coll., Phys. Rev. C **75** 064904

Conclusions

- Modified combinants, C_j, deduced from the measured multiplicity distributions of charged particles, $P(N)$, together with the already measured void probabilities, could provide additional information on the dynamics of the particle production.
- A detailed analysis of the modified combinants derived from the experimental $P(N)$'s reveals differences between the various processes. In e^+e^- annihilation, the C_j's oscillate with a period of 2 with amplitudes increasing as a power-law. On the other hand, pp and $p\bar{p}$ collisions produce C_j's oscillating with approximately 10 times the period of their e^+e^- counterparts, with decaying amplitudes.
- C_j derived from $P(N)$'s obtained in centrality selected nucleus-nucleus collisions are still under investigation.

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