# Correlations from global baryon number conservation

Michał Barej

AGH University of Science and Technology, Kraków, Poland

In collaboration with Adam Bzdak

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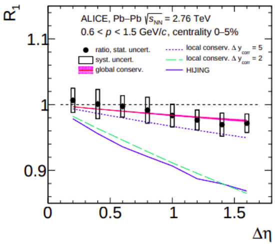
michal.barej@fis.agh.edu.pl

# Proton, antiproton factorial cumulants from baryon number conservation




"event-by-event baryon number conservation leads to subtle long-range correlations"

**S. Acharya *et al.* [ALICE], PLB 807, 135564 (2020)**

Notation:

- $\langle N_b \rangle$, $\langle \bar{N}_b \rangle$ - mean numbers of baryons and antibaryons *without* baryon number conservation,
- $\langle N_b \rangle_c$, $\langle \bar{N}_b \rangle_c$ - same but *with* baryon number conservation,
- $p$ $(\bar{p})$ - probability that the baryon (antibaryon) is in acceptance and observed as a proton (antiproton),
- $z = \sqrt{\langle N_b \rangle \langle \bar{N}_b \rangle}$, $z_c = \sqrt{\langle N_b \rangle_c \langle \bar{N}_b \rangle_c}$,
- $\langle N \rangle_c = \langle N_b \rangle_c + \langle \bar{N}_b \rangle_c$,
- $\Delta = z_c^2 - z^2$,
- $\gamma = z_c^2 + \Delta \langle N \rangle_c$,
- $\beta = \gamma(\langle N \rangle_c + 2) + 2\Delta^2$.
- $z_c$, $\langle N \rangle_c$, $\Delta$, $\gamma$, $\beta$ all depend on $B$ and $z$ only.

- MB, A. Bzdak, PRC **102**, no.6, 064908 (2020)
- Assumptions:
  - baryons and antibaryons numbers follow Poisson distribution (no correlations),
  - binomial acceptance,
  - global baryon number conservation as the only source of correlations.
- Proton, antiproton factorial cumulants:
  - $\hat{C}^{(1,0)} = p\langle N_b \rangle_c$,
  - $\hat{C}^{(2,0)} = -p^2 \left(\langle N_b \rangle_c + \Delta\right)$,
  - $\hat{C}^{(1,1)} = -p\bar{p}\Delta$,
  - $\hat{C}^{(3,0)} = p^3 \left[2! \left(\langle N_b \rangle_c + \Delta + \frac{1}{2}\gamma\right)\right]$,
  - $\hat{C}^{(2,1)} = p^2 \bar{p}\, \gamma$,
  - $\hat{C}^{(4,0)} = -p^4 \left[3! \left(\langle N_b \rangle_c + \Delta + \frac{1}{2}\gamma\right) + \beta\right]$,
  - $\hat{C}^{(3,1)} = -p^3 \bar{p} \beta$,
  - $\hat{C}^{(2,2)} = -p^2 \bar{p}^2 \left(\beta - \gamma\right)$,
  - higher-order factorial cumulants in the paper.
- Results are $\neq 0 \Rightarrow$ global baryon number conservation introduces correlations.
- Using our formulas it is possible and worth verifying if all experimentally measured factorial cumulants reflect the baryon number conservation.

# Baryon number conservation and short-range correlations

- V. Vovchenko, O. Savchuk, R.V. Poberezhnyuk, M.I. Gorenstein, V. Koch, PLB **811**, 135868 (2020)
- They assume:
  - global baryon number conservation,
  - subensemble acceptance,
  - grand-canonical cumulants,
  - subsystem is large enough to be close to the thermodynamic limit.
- They calculate the cumulants in the subsystem (*with* baryon number conservation) $\kappa_n^{(B)}$ in terms of the global cumulants *without* baryon number conservation $\kappa_m^{(G)}$ :

$\kappa_2^{(B)} \approx \bar{f} f \kappa_2^{(G)}$,

$\kappa_3^{(B)} \approx \bar{f} f (1\text{-}2f) \kappa_3^{(G)}$,

$\kappa_4^{(B)} \approx \bar{f} f \left[ \kappa_4^{(G)} - 3\bar{f} f \left( \kappa_4^{(G)} + \frac{(\kappa_3^{(G)})^2}{\kappa_2^{(G)}} \right) \right]$,

$f$ - fraction of particles in the first subsystem,

$\bar{f} = 1\text{-}f$.

- MB, A. Bzdak, in preparation
- Assumptions:
  - one kind of particles, e.g. protons,
  - global baryon number conservation,
  - subensemble acceptance,
  - short-range correlations: $\hat{C}_k = \alpha_k \langle N \rangle$, $\alpha_k$ - k-particle correlation strength
  - $\langle N \rangle = B$
- Our calculation is fully analytical.
- We derived the factorial cumulant generating function:

$$G_B(z) = \ln \left[ \frac{A}{B!} \frac{d^B}{dx^B} \exp\left( \sum_{k=1}^{\infty} \frac{[(xz-1)^k f + (x-1)^k \bar{f}]\alpha_k B}{k!} \right) \Bigg|_{x=0} \right]$$

$A$ - normalization const.

- From this we calculate factorial cumulants: $\hat{C}_k^{(B)} = \frac{d^k}{dz^k} G_B(z) \Big|_{z=1}$ and cumulants via the appropriate relations.
- We showed analitycally that the cumulants calculated by V. Vovchenko et al. come from $G_B(z)$ at the large $B$ limit.

# Correction to the cumulants in subsystem

- MB, A. Bzdak, in preparation
- Using $G_B(z)$ we obtain

$$\kappa_n^{(B)} = \underbrace{B\kappa_n^{(B,LO)}}_{\text{Vovchenko et al.}} + \underbrace{\kappa_n^{(B,NLO)}}_{\text{our correction}} + \left[\frac{1}{B}\kappa_n^{(B,NNLO)} + \dots\right]$$

- Results:
  - $\kappa_2^{(B,NLO)} = \frac{1}{2}\bar{f}f\frac{(\bar{\kappa}_3^{(G)})^2+2\bar{\kappa}_3^{(G)}-\bar{\kappa}_2^{(G)}-\bar{\kappa}_4^{(G)}(1+\bar{\kappa}_2^{(G)})}{(1+\bar{\kappa}_2^{(G)})^2}$,
  - $\kappa_3^{(B,NLO)} = \frac{1}{2}f\bar{f}(1-2f)\frac{\bar{\kappa}_3^{(G)}-\bar{\kappa}_2^{(G)}+\bar{\kappa}_4^{(G)}+\bar{\kappa}_3^{(G)}\bar{\kappa}_4^{(G)}-\bar{\kappa}_5^{(G)}(1+\bar{\kappa}_2^{(G)})}{(1+\bar{\kappa}_2^{(G)})^2}$,
  - $$\begin{aligned}\kappa_4^{(B,NLO)} = -\frac{1}{2}f\bar{f}\Bigg\{ & \frac{\bar{\kappa}_2^{(G)}-\bar{\kappa}_3^{(G)}}{(\bar{\kappa}_2^{(G)}+1)^2} - 3f\bar{f}\Bigg[\Bigg[\left((\bar{\kappa}_2^{(G)})^2 - 8\bar{\kappa}_2^{(G)} - 1\right)\bar{\kappa}_3^{(G)} \\ & + \bar{\kappa}_2^{(G)}\left((\bar{\kappa}_2^{(G)})^2 + 5\bar{\kappa}_2^{(G)} + 2\right) + 2(\bar{\kappa}_3^{(G)})^4 + 8(\bar{\kappa}_3^{(G)})^3 \\ & -(5\bar{\kappa}_2^{(G)} - 7)(\bar{\kappa}_3^{(G)})^2\Bigg] / (\bar{\kappa}_2^{(G)}+1)^4 \\ & + \frac{(2\bar{\kappa}_2^{(G)}-3)\bar{\kappa}_4^{(G)}-5(\bar{\kappa}_3^{(G)})^2\bar{\kappa}_4^{(G)}-10\bar{\kappa}_3^{(G)}\bar{\kappa}_4^{(G)}}{(\bar{\kappa}_2^{(G)}+1)^3} + \frac{(\bar{\kappa}_4^{(G)})^2}{(\bar{\kappa}_2^{(G)}+1)^2}\Bigg] \\ & - \frac{(1+3f\bar{f})(\bar{\kappa}_3^{(G)}+1)\bar{\kappa}_5^{(G)}}{(\bar{\kappa}_2^{(G)}+1)^2} + \frac{(1-3f\bar{f})\bar{\kappa}_6^{(G)}}{\bar{\kappa}_2^{(G)}+1}\Bigg\},\end{aligned}$$
  - where $\bar{\kappa}_n^{(G)} = \frac{\kappa_n^{(G)}}{B} - 1$ (equals 0 for Poisson distribution).
- We have checked by analytical calculations with numerical values (Mathematica) that we obtain more accurate results when using the correction.